TWENTY FIRST EUROPEAN ROTORCRAFT FORUM

Paper No VII. 10

ROBUST CONTROL LAW DESIGN FOR THE BELL -205 HELICOPTER

BY

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August 30 - September 1, 1995

SAINT - PETERSBURG, RUSSIA

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Abstract

The open loop behaviour of the Bell-205 is highly nonlinear, unstable and cross-axis coupled over the full operating envelope. In this paper a two degrees-of-freedom (2DOF) H_∞ approach to the control law synthesis for the Bell-205 model is presented. The controller provides robust stability against coprime factor uncertainty and forces the system to follow a pre-specified reference model [11]. The 2DOF controller can be written in an observer-based form [12] which is useful for implementation and scheduling across different operating point designs.

1 Introduction

The design of a helicopter flight control system is of great importance in maintaining system stability and performance and thus reducing a pilot's workload. The Control Systems Research Group at Leicester University has worked for several years on the design of advanced control laws for Lynx-like helicopters using the simulation facilities of the Defence Research Agency at Bedford, eg. [1]-[3]. We have demonstrated through ground-based piloted simulations that multivariable control techniques, especially H^∞ optimal control, can play a significant role in meeting the high performance requirements demanded in future vehicles. The group has now turned its attention to the design of advanced control laws for an experimental fly-by-wire helicopter, the Canadian Bell-205.

The open loop behaviour of the Bell 205 (a typical single rotor helicopter) is highly non-linear and cross-axis coupled. Many control laws have been tested with the majority of them designed using classical techniques. However, modern control techniques seem to offer a better solution to this complex control problem, by providing a multivariable framework for designs of control systems with high performance requirements. Issues such as performance and robustness are assessed simultaneously in the design cycle and the trade-offs between them can be established relatively easy. A practical implementation of a control law requires a controller which stabilizes the plant against parametric uncertainty and decouples the Attitute-Command-Attitude-Hold (ACAH) type of response. Additionally it must be simple, easily scheduled and implementable. The currently implemented control laws on the Bell-205 are based on classical techniques and require extensive use of gain scheduling in order to cover the full flight envelope.

In this work a 2DOF approach to the H_∞ loop-shaping design procedure, as introduced by Hoyle et.al. in [4], is applied to the Bell 205. The main objective is to design a full-authority control system that: a) robustly stabilizes the helicopter with respect to model uncertainty, b)provides high level of decoupling between the selected outputs and c) satisfies the ADS-33C level 1 criteria. In Walker et.al. [2]-[3] it was demonstrated on a high-bandwidth Lynx-type helicopter, that the 2DOF approach provides an elegant framework for designing control laws to meet strict performance requirements. Additionally, the advantage of these controllers was that they possessed a particular structure that could be used for practical implementation and scheduling across different operating point designs.

This paper is organised as follows: Section 3 contains the necessary background to the 2DOF approach, the controller structure as well as the method of inequalities used for tuning the responses. Section 4 presents the mathematical model of the Bell 205 helicopter used in this work. Section 5 discusses the design procedure, and the results are presented in Section 6. Finally, some implementation issues and conclusions are discussion in Section 7.

2 Theoretical Background

A transfer function in the state space form can be represented by

$$G(s) := C(sI - A)^{-1}B + D := \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \qquad (1)$$

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RH_∞ denotes the space of all rational functions analytic and bounded in the right half complex plane.

2.1 Normalized Coprime Factors and Uncertain Models

The pair $(M,N) \in RH_\infty$ constitutes a normalized left coprime factorization of a plant model G if

$$G = M^{-1}N$$

In this case, it is known that there exists $U, V \in RH_\infty$ such that

$$MV + NU = I$$

A state space construction for a normalized left coprime factorization can be obtained from the well known formula

$$[M \quad N] = \left[\begin{array}{c|cc} A+HC & B & H \\ \hline C & 0 & I \end{array}\right]$$

where $H=-YC^T$ is the non negative stabilising solution to the algebraic Riccati equation

$$AY + YA^T - YC^TCY + BB^T = 0$$

If Δ_M, Δ_N are stable, norm-bounded transfer functions representing the uncertainty in the nominal plant model

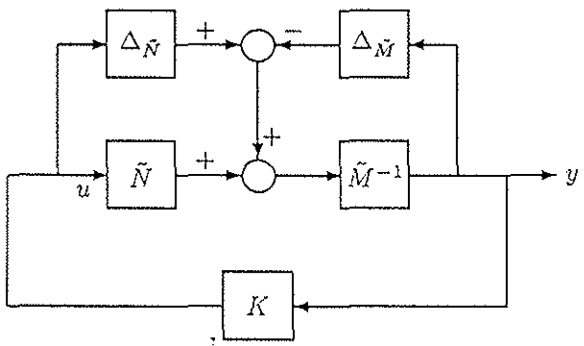

Figure 1: 1 DOF Scheme.

then the perturbed plant transfer function can be written as

$$G_\Delta = (M + \Delta_M)^{-1}(N + \Delta_N)$$

The robust stabilisation objective is to stabilise the family of perturbed plants defined by

$$G_\varepsilon := \{G_\Delta : \| \Delta_M, \Delta_N \|_\infty < \varepsilon\}$$

using a feedback controller (fig. 1). To maximise the robust stability of the closed loop system one must minimise

$$\gamma = \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \right\|_\infty \qquad (2)$$

where the lowest value for γ is given by

$$\gamma = (1 + \lambda_{\max}(XZ))^{1/2} \qquad (3)$$

In (3) X and Z are the stabilising solutions of the generalised control and filtering algebraic Riccati equations respectively. Detailed results on normalised coprime factorization and robust stabilisation can be found in [10]. The two degrees-of-freedom approach, as introduced in [4] (Figure 2) includes a model matching problem in addition to the robust stability minimisation problem described above.

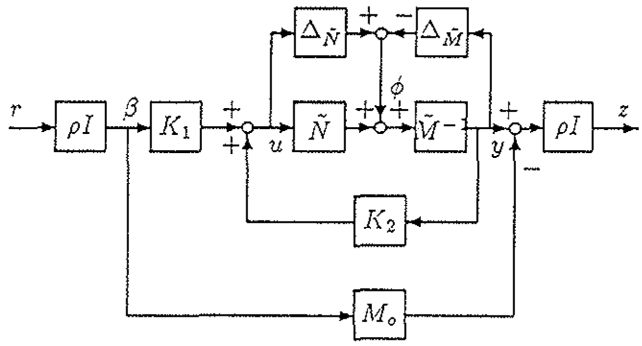

Figure 2: 2 DOF Scheme.

The closed loop response follows that of a specified model (M_0) and the controller K is partitioned as $K=[K_1 \; K_2]$ where K_1 is the prefilter and K_2 is the feedback controller. From figure 2 and the state space equations of the plant and the ideal model M_0 the problem can be formulated in the standard control configuration (SCC) form:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_0 \\ \hline u \\ y \\ z \\ \hline \beta \\ y \end{bmatrix} = \left[\begin{array}{cc|cc|c} A & 0 & 0 & -H & B \\ 0 & A_0 & B_0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I \\ C & 0 & 0 & I & 0 \\ \rho F_s & -\rho^2 C_0 & 0 & \rho F_s & 0 \\ \hline 0 & 0 & \rho I & 0 & 0 \\ C & 0 & 0 & I & 0 \end{array}\right] \cdot \begin{bmatrix} x \\ x_0 \\ r \\ \phi \\ u \end{bmatrix}$$

In the SCC above, F_s ensured that only the conrolled outputs would be used in the model matching problem, whilst ρ was a diagonal matrix in order to provide an additional parameter for model matching and tuning. Standard algorithms performing the γ-iteration were utilised to carry out the minimisation of the H_∞ performance criterion. The controller was written in an observer form as in [5] where the solution to the control Riccati equation X_∞ was partioned as $X_\infty = [X_{\infty 11} \; X_{\infty 12}]$:

$$\begin{bmatrix} \dot{\hat{x}} \\ x_0 \end{bmatrix} = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right] \cdot \begin{bmatrix} \hat{x} \\ x_0 \end{bmatrix} + \left[\begin{array}{c|c} 0 & -H \\ \hline \rho B_0 & 0 \end{array}\right] \cdot \begin{bmatrix} r \\ y \end{bmatrix}$$

where
$A_c = A + HC - BB^T X_{\infty 11}$;
$B_c = -BB^T X_{\infty 12}$; $C_c = 0$; $D_c = A_0$;
and

$$u = -B^T [X_{\infty 11} X_{\infty 12}] \cdot \begin{bmatrix} \hat{x} \\ x_0 \end{bmatrix}$$

2.2 The Method of Inequalities

The method of inequalities (MOI) is a computer-aided multi-objective design approach, where desired performance is represented by a set of algebraic inequalities. The design problem is expressed as

$$\varphi_i(p) \leq \epsilon_i \tag{4}$$

where ε_i are real numbers and p a vector chosen from a set of real functions. For control system design the functions $\varphi_i(p)$ are functionals of the system responses, for example coupling between the channels, maximum sensitivity values or bandwidths. The solution of the set of inequalities is obtained by means of numerical search algorithms such as the moving of boundary process (MBP). In the 2DOF design procedure described above, the MOI was used for fine tuning of the time responses, as well as for the minimisation of the maximum value of the sensitivity function. After a good initial condition using the loop shaping design procedure [11] was found, the MBP optimised the final values that are described in the design section of this paper.

3 The Helicopter Model

The model used for this work is the basic 6-degrees-of-freedom model provided in [8] linearised at 10 knots. A comparison of a low-speed model provided in [8] with the actual flight data can be found in [11]. The model was represented in the state-space form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{5}$$

where the states and their units are described below:
- u - forward velocity - m/s
- w - vertical velocity - m/s
- q - pitch rate - deg/s
- v - lateral velocity - m/s
- p - roll rate - deg/s
- r - yaw rate - deg/s
- ϑ - pitch attitude - deg
- ϕ - roll attitude - deg

The actuator dynamics were modelled as first order pade-approximations and cascaded with the plant. This resulted in a 12 state design model. The outputs chosen for control were [w,ϑ,ϕ,r] with the pitch and roll rates fed back to the controller.

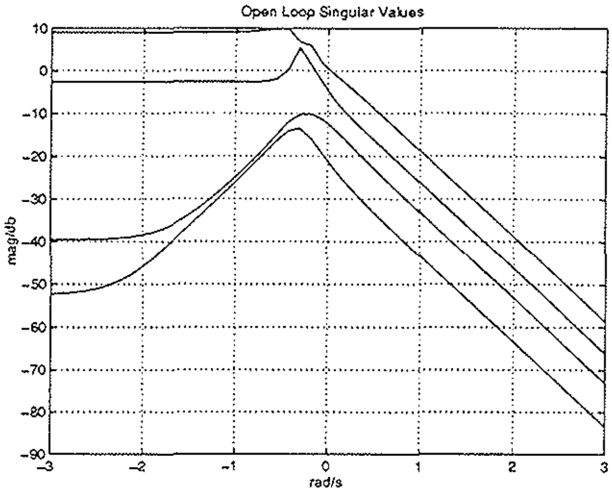

Figure 3: Uncompensated plant

4 Design Procedure

The performance limitations of the BELL 205 mainly depend on the control power of the teetering rotor system and a lightly damped structural transmission mode of the fuegelage at about 14 rad/s. These factors put very strict limits on the achievable bandwidth of the helicopter. Also, the model used in this work includes the dynamics of the stabilizer bar, causing the bandwidth to have slightly lower values. With these considerations in mind the weights were chosen as simple as possible in order to produce a satisfactory response. The tasks followed for the design of the compensator are described below:

The singular values of the open loop plant are shown in fig. 3.

From this figure it can be seen that the plant was almost singular at low frequencies, the tracking in the two channels was poor, whilst the condidtion number was 1154. Therefore, integral action was necessary to boost the low frequency gain in order to provide good tracking properties and disturbance rejection. The roll-off at the cross over frequencies was reduced by introducing zeros in the W_1 weighting. The final structure of the W_1 weighting was:

$$W_1 = \begin{bmatrix} \frac{s+5.8}{s+10^{-10}} & 0 & 0 & 0 \\ 0 & \frac{s+0.1}{s+10^{-10}} & 0 & 0 \\ 0 & 0 & \frac{s+0.9}{s+10^{-10}} & 0 \\ 0 & 0 & 0 & \frac{s+5.8}{s+10^{-10}} \end{bmatrix} \tag{6}$$

The values for the weighting W_2 were fixed at $diag\{1, 1, 1, 1, 0.08, 0.1\}$ so that it de-sensitised the two additional outputs.

The shaped plant $G_s = W_2 G W_1$ was aligned at 2.4 rad/s. An additional gain matrix K_g in the forward loop was used to control the actuator usage. After some trial and error, it was selected as $diag\{2.9, 0.1, 1.12, 1.55, 1, 1\}$.

A step response model was chosen so as to reflect specifications of the handling qualities for an Attitude-Command-Attitude-Hold (ACAH) design type. The model incorporated a second order transfer function for all the controlled channels.

$$M_0 := \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

A damping factor of $[0.9, 0.96, 0.9, 0.9]$ was assumed and the parameter ω_n was chosen as to give an appropriate rise time for the chosen outputs. The rise time here is defined as the time to reach the first peak. This selection gave a rise time of 2.4 sec for the vertical velocity, 5.4 sec for the pitch, 3.48 sec for the roll and 1.8 sec for the yaw channel.

The γ-iteration gave an optimal cost γ=2.71 and a slightly suboptimal controller was chosen with γ=2.84. This is known to prevent a fast pole appearing in the closed loop system and resulting in improved responses of the plant. The ρ parameter was decided to be $[1.3, 1.3, 1.3, 1.3]$. From the robustness point of view the smallest ρ would provide the better robustness results, but the controller would be unable to follow the step response model M_0.

The controller obtained by the γ-iteration was cascaded with the weights W_2 and W_1, and partitioned as $K=[K_1\ K_2]$. The prefilter K_1 was scaled with a gain matrix $S = K_1^{-1}(0) \cdot K_2(0)$ so that the closed loop transfer function $(I - GK_2)^{-1}GK_1$ matched the unit matrix at the steady-state.

5 Design Results

Figures (4-7) show the singular value plots of the shaped and unshaped plant, sensitivity function, complementary sensitivity function and the final loop shape.

From the plot of the $\sigma(I + GK)^{-1}$ it can be deduced that for each channel; a gain margin of $[1.43, 0.76]$ and a phase margin of ± 17.4 are guaranteed. These margins appear to be poor, as the unstructured singular value tends to give conservative results. Where perturbation information is available, it is known that the structured singular value is a less conservative measure of robustness. From the final loop shape it can be seen that the controller boosted the low frequency gain which rolls off before 10 rad/sec. This provides a margin of 4rad/sec from the lightly damped fugelage transmission mode and it does not excite any unmodelled dynamics known to exist beyond this frequency.

5.1 Time Simulation

Two types of time simulations were performed in order to demonstrate the achievements and the potential hazards of this design. The first was a linear simulation of the

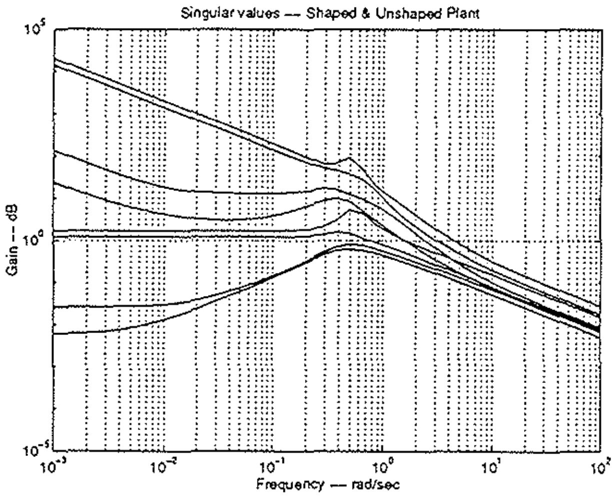

Figure 4: Singular values of the shaped and Unshaped plants

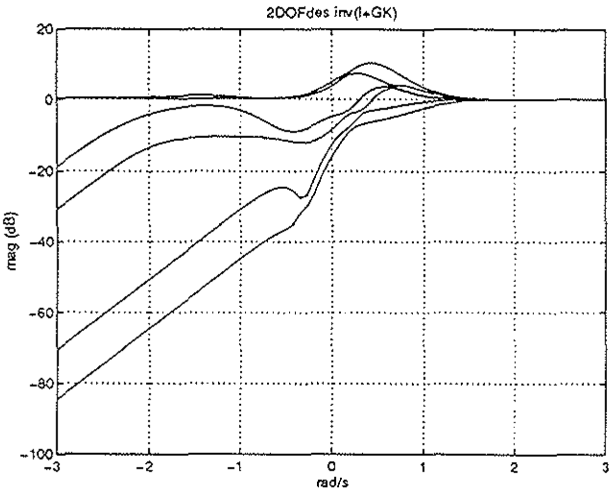

Figure 5: Sensitivity function

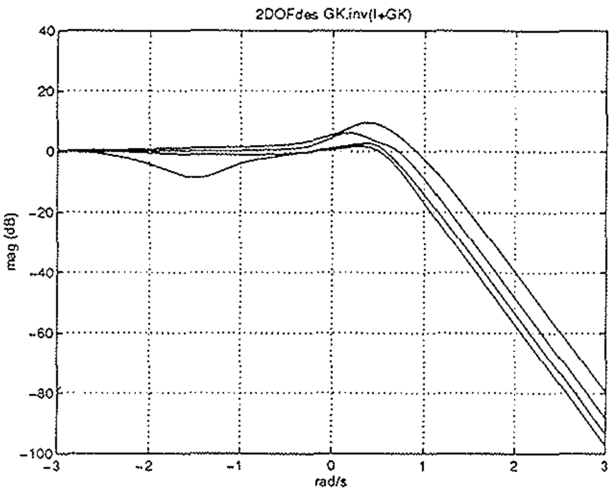

Figure 6: Complementary sensitivity

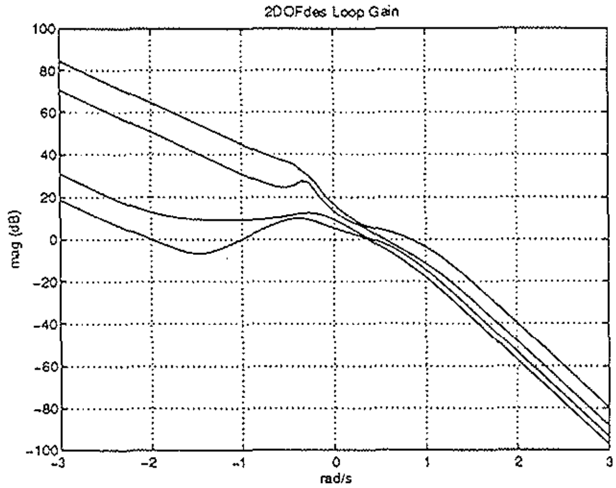

Figure 7: Loop Gain frequency response

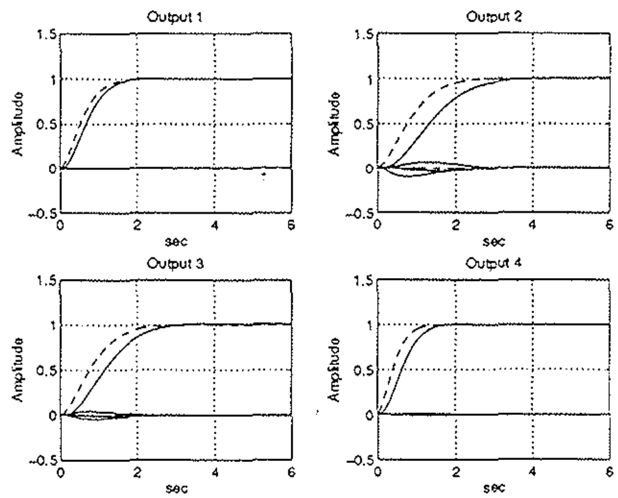

Figure 8: Linear Responses

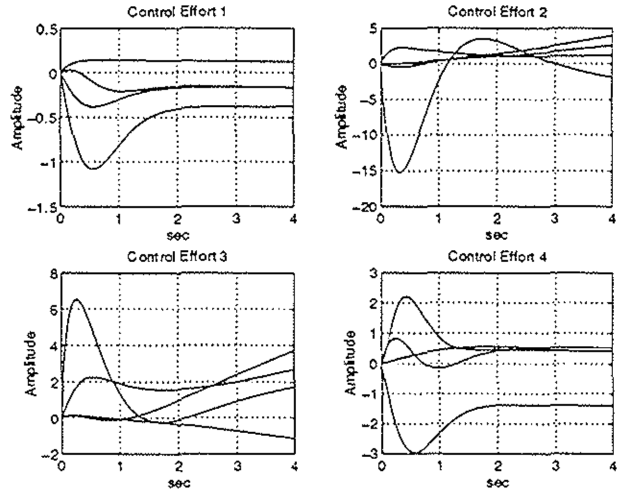

Figure 9: Controller Action

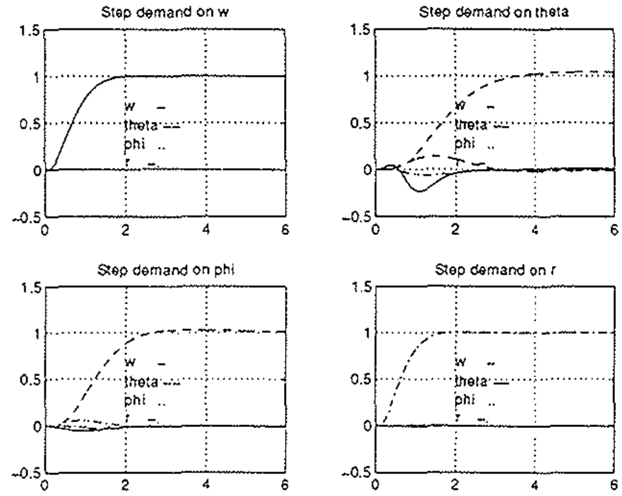

Figure 10: Nonlinear Step Responses

closed loop system with the responses of the reference model superimposed see fig. 8.

From this figure it can be seen that a satisfactory level of decoupling between the controlled outputs was achieved. The control effert for each of the responses is shown in fig. 9.

The velocity, roll and yaw responses stay within the actuator limits. The pitch response though slightly hit the rate actuator limit for about 1.5 sec. As it can be seen from the nonlinear simulations in fig. 10 the saturation of the actuator did not cause any serious instability problems, but increased the coupling of the pitch with the roll and velocity responses. It is clearly demonstrated that the loop gain direction affects the other two channels.

5.2 Handling Qualities Evaluation

In this section pitch and roll - the two important channels for an ACAH response type - are presented against the

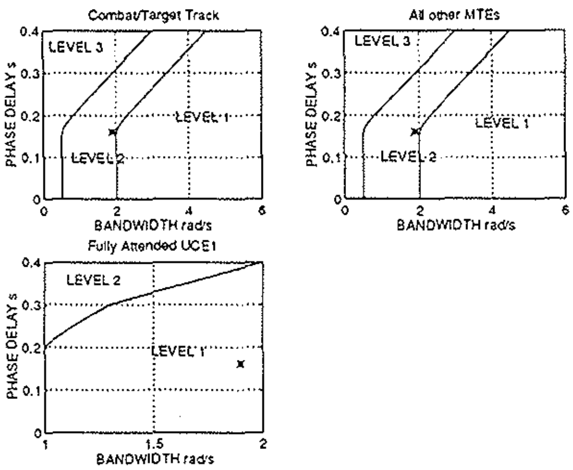

Figure 11: Short term pitch

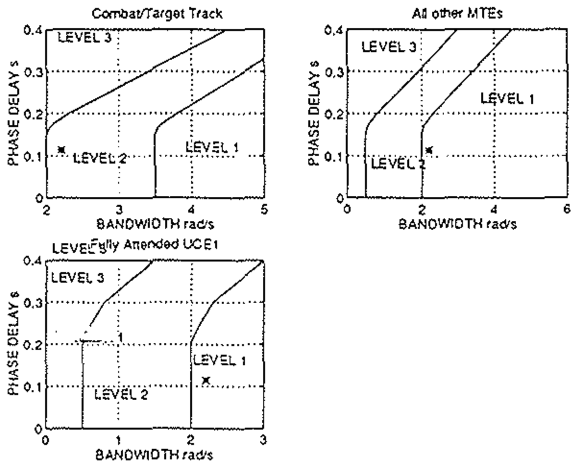

Figure 12: Short term roll

latest requirements specification for helicopters (ADS-33C). The performance of a helicopter is assessed according to three levels, where level-1 is the best and level-3 the worst. Figure 11 depicts the pitch bandwidth and phase delay against the ADS-33C requirements. Similarly the roll characteristics against the ADS-33C are shown in figure 12. The bandwidths and phase delays calculated were $\omega = 1.9 rad/s$, $\tau = 0.1s$ for pitch and $\omega = 2.2 rad/sec$, $\tau = 0.1s$ for roll respectively.

6 Discussion

The analysis presented in this paper demonstrated that there is potential for considering antiwindup schemes in the control law since the control signals were close to the actuator limits. The controller consisted of a plant observer and a reference model. This structure could be used to yield a significant saving in the real time computations. The state equation of a conventional unstructured 28-state controller would require 28*28+28*6=952 multiplications and 28*7+28*5=896 additions. The state equation of the observer would require 20*20+20*6=520 multiplications and 20*18+20*5=480 additions. The reference model would require 8*8+8*6=112 multiplications and 8*7+8*5=96 additions to update its state equation. Therefore, a total of 632 multiplications and 576 additions would be necessary to perform in real time. This indicates a reduction of 33 and 36 per cent in multiplications and additions, respectively. Finally the locality of the control law designed for the BELL 205 helicopter, assumed a Linear-Time-Invariant plant model with modelling uncertainties and deficiencies. The Linear-Parameter-Varying nature of the plant dictates the use of gain scheduling in order to handle the global performance and robustness requirements. The benefits from a scheduled control law with adequate robustness properties, would limit the need for extensive linear designs and improve the full envelope capability of future generation helicopters.

7 Acknowledgements

We are grateful to Dr. Kevin Goheen of Carleton University, Ottawa and Mr. Stewart Baillie of the Canadian Flight Research Laboratory for many helpful discussions regarding the Bell-205 helicopter model.

References

[1] A. Yue and I. Postlethwaite, "Improvement of Helicopter Handling Qualities Using H^∞ Optimisation",IEE Proc., Part D, 137, 115-129, 1990

[2] D.J.Walker and I. Postlethwaite, "Full Authority Active Control System Design for a High Performance Helicopter", 16th European Rotorcraft Forum, Glasgow, September, 1990.

[3] D.J.Walker and I. Postlethwaite, "A Full-Flight Envelope High-Bandwidth Rotorcraft Flight Control System",Proc. 30th IEEE Conf. on Decision and Control, Brighton, December, 1991.

[4] Hoyle D.J., Hyde R. A., and Limebeer D.J.N., "An H^∞ approach to two degree of freedom design," Proc. 30th CDC, pp. 1581-1585, England 1991.

[5] Walker D. "On the structute of a 2-degrees-of-freedom controller",International Journal of Control, to appear.

[6] D.J.Walker,
I.Postlethwaite, J.Howitt and N.P.Foster, "Rotorcraft Flying Qualities Improvement Using Advanced Control", American Helicopter Society/NASA Conference, San Francisco, January, 1993.

[7] I. Postlethwaite and D.J.Walker, "Advanced Control of High Performance Rotorcraft", IMA Conf. on Aerospace Vehicle Dynamics and Control, Cranfield Institute of Technology.

[8] Heffley R.K., Jewell W.F., Lehman M. J., Winkle R.A., " A Compilation and Analysis of Helicopter Handling Qualities Data", NASA Report 3144, 1979.

[9] I. Postlethwaite and D.J.Walker, "The Design of Helicopter Flight Control Systems Using Advanced H^∞ Control, Proc. American Control Conference, Baltimore, June 1994.

[10] McFarlane D.C., Glover K. "Robust Controller Design Using Normalized Coprime Factor Descriptions, Lecture Notes in Control and Information Sciences, 1989.

[11] Baillie S.W., Morgan M.J. and Goheen K.R. "Practical Experiences in control Systems Design Using the NRC Airborne Simulator", Flight Mechanics Panel Symposium, Turin Italy, 9-13 May 1994.

[12] D.J.Walker and I. Postlethwaite, "Discrete time H^∞ Control Laws for a High Performance Helicopter", Proc. 30th IEEE Conf. on Decision and Control, Brighton, December, 1991.

[13] I. Postlethwaite and D.J.Walker, "The Design of Helicopter Flight Control Systems Using Advanced H^∞ Control, Proc. American Control Conference, Baltimore, June 1994.