SIXTH EUROPEAN ROTORCRAFT AND POWERED LIFT AIRCRAFT FORUM

Paper No. 19

APPLICATION OF STRAIN GAUGE PATTERN ANALYSIS

D. R. Gaukroger
D. B. Payen
A. R. Walker

Royal Aircraft Establishment,
Farnborough, Hampshire, England

SUMMARY

Strain gauge pattern analysis, developed at the Royal Aircraft Establishment for measuring vibration modes of a rotating blade, is described. The analysis enables displacements in flap, lag and torsion to be derived from patterns of strain gauge responses, which are related to displacements through a set of calibration modes. The latter can be obtained with the blade non-rotating, and conventional strain gauge calibration is avoided.

A theoretical study of the effects of measurement errors has shown that the technique is insensitive to random errors of a magnitude that might be expected in practice. Results obtained from a model rotor are given, and a blade-shrouding technique has been used to obtain modes with and without aerodynamic forces.

September 16-19, 1980
Bristol, England

THE UNIVERSITY, BRISTOL, BS8 1HR, ENGLAND

1 INTRODUCTION

The method of strain gauge pattern analysis, or SPA for short, was originally developed at RAE[1,2] to provide experimental data to check the accuracy of helicopter blade mode calculations. The requirements at that time were to measure the flap, lag and twist components of the modes of a rotating blade at a sufficiently large number of spanwise stations to enable a detailed comparison to be made with calculated modes. Several possible approaches were studied, only to be discarded on grounds of accuracy or complexity or cost. For example, optical methods have the attraction that they require minimal blade instrumentation, but they cannot achieve the accuracy required without great complexity of optical equipment, and are generally unsuitable for determining phase relationships. Accelerometers which would, perhaps, be the natural form of transducer for modal measurements in any other branch of dynamics, are unsuitable for a rotating blade because of the large centrifugal acceleration components of the accelerometer signals which arise from changes of blade slope. Strain gauges, which can be installed on or in a blade rather more easily than can accelerometers, are somewhat unattractive if analysis of their signals is directed first to obtaining strain and thence to deriving displacement from strain by theoretical methods based on a knowledge of the blade stiffness distributions. However, the attractiveness of the strain gauge for rotor model work (where its small mass is a distinct advantage over any other type of transducer) its low cost, its lack of undesirable phase characteristics, its standard signal conditioning requirements, led to a re-examination of the ways in which a set of strain gauge responses could be analysed. The result was the SPA method of analysis, which retains all the advantageous aspects of the strain gauge and avoids many of the disadvantages of its use. The principles of the method have already been published[3] and some early examples given of its application. The aim of this paper is to show later developments, and to indicate possible applications which appear to merit consideration.

Although SPA was developed primarily for use on helicopter blades, there is probably a wide field of application - and not necessarily confined to rotors or rotating systems. The cost of a strain gauge relative to the cost of a miniature accelerometer, and the sensitivity of the accelerometer to external acceleration fields suggest that for applications such as the measurement of missile or wind tunnel model distortions under manoeuvre or buffet conditions, SPA might be an economic alternative to conventional techniques of measurement.

2 STRAIN GAUGE PATTERN ANALYSIS THEORY

For the benefit of those who may not be familiar with SPA, the technique and the simple theoretical background necessary for its application will be restated. The principle of the technique depends on two conditions being satisfied. The first is that the set of displacements of a structure which are to be determined can be expressed as a linear sum of several known sets of displacements. The second condition is that there shall be a unique relationship between any set of displacements and the corresponding set of strain gauge responses. If both these conditions are satisfied, an unknown set of displacements can be determined by measuring the corresponding pattern of strain gauge responses.

Let us suppose that p sets of displacements and corresponding strain gauge responses have been measured. Each set may consist of displacements at n points and strain gauge responses at r points. These sets will be called 'calibration modes'; the displacements for the jth set will be denoted by δ_j and the strain gauge responses by s_j . Under some test condition for which we wish to determine a set of n displacements $\bar{\delta}$, we measure a set of r strain gauge responses $\bar{s}$. If the first of the two basic conditions is satisfied there will be a set of constants $\alpha_1, \alpha_2 \dots \alpha_p$ such that

$$\alpha_1\delta_1 + \alpha_2\delta_2 \dots \alpha_p\delta_p = \bar{\delta} \qquad (1)$$

and if the second assumption is satisfied there will be a similar equation relating strain gauge responses:

$$\alpha_1 s_1 + \alpha_2 s_2 \dots \alpha_p s_p = \bar{s} \ . \qquad (2)$$

The α terms can be determined from equation (2) and then used in equation (1) to find $\bar{\delta}$.

Obviously the solution to equation (2) depends on the values of p and r - the numbers of calibration modes and strain gauges. The number of gauges must be not less than the number of calibration modes - and, in practice, will usually be much greater. If $r > p$, it will be necessary to solve equation (2) for a best fit of the calibration mode strain gauge responses to the responses $\bar{s}$. If G is the $(r \times p)$ matrix of the sets $s_1, s_2 \dots s_p$ and $\bar{\alpha}$ the column matrix of the p α-terms, it is easy to show that

$$\bar{\alpha} = (G^T G)^{-1} G^T \bar{s} \qquad (3)$$

determines $\bar{\alpha}$ for a least-squares-error best fit. Substituting for $\bar{\alpha}$ in equation (1) gives the set of required displacements

$$\bar{\delta} = D\bar{\alpha} \qquad (4)$$

where D is the $(n \times p)$ matrix of the sets $\delta_1, \delta_2 \dots \delta_p$. It will be noted that although the number of strain gauges must not be less than the number of calibration modes, there is no constraint on the number of points at which displacements are measured. However, the requirement to satisfy both the conditions mentioned earlier is important in deciding on the values of n, r and p . Normally n will be determined by the detail in which the unknown displacements are required. It is then necessary to choose r , the number of strain gauges, and their distribution such that a unique relationship exists between the strain gauge responses and the associated displacements. If r is much smaller than n , it is easy to see that a local displacement might not produce a strain gauge response; the unique relationship does not then exist. Similarly, if all the strain gauges on a helicopter blade were attached to the inboard half, they would not respond to distortions of the outboard sections alone. To some extent, of course, the number and distribution of strain gauges is governed by the complexity of the displacements which are to be measured; if lower order modes of a helicopter blade are to be measured, fewer gauges will be required than for higher order modes. A sound working rule is that the number of gauges r should be roughly the same as the number of points n at which displacements are required, and the gauges should be distributed more or less evenly across the structure.

It is worth noting some of the advantages and disadvantages of the SPA technique. Since we are only concerned with patterns of strain gauge signals, and not with actual values of strain, it is unnecessary, when fixing the gauges, to ensure a high degree of accuracy in positioning, orientation, or gauge matching. It will not matter, for example, if a row of gauges which are to measure flapping motion of a helicopter blade are not fixed exactly where they should have been, or if their orientations are several degrees askew, or if they all have different sensitivities and cross-sensitivities to lag and twist strain. Nor will it matter if the signal conditioning equipment for each gauge has different characteristics. All these factors are eliminated in the analysis process. What is important - essential, in fact - is that the gauges and their associated equipment retain the same characteristics throughout the experiment. That is to say, the characteristics must be the same for the test conditions as for the measurement of calibration modes. Despite the welcome relaxations in strain gauge instrumentation that the SPA technique allows, it is probably unwise to depart too far from accepted standards. Some of the advantage of a weighting process (which will be referred to later) can be lost if the strain gauge responses do not even approximate to a credible strain distribution.

Some care is needed if the motion to be measured contains rigid body components. These components will produce no strain gauge response and it is then necessary to install other forms of transducer to measure the relevant components. The way in which these transducer signals are used depends on the nature of the rigid-body motion. For example, flap oscillations of an articulated rotor blade will have a rigid-body component due to rotation about the hinge, and a hinge transducer is necessary. If calibration modes for the blade are measured with the hinge locked then the hinge transducer will have to be calibrated separately and its readings in the test conditions converted directly into flap angles; on the other hand, if calibration modes are measured with the hinge free, the hinge transducer output can be treated as an element in the pattern of strain gauge responses and no separate calibration is necessary. In fact, from the modal point of view, hinge-free calibration modes would be preferred in this case, so there is then an opportunity to check the derived rigid-body motion by direct calibration of the hinge transducer.

We have assumed, so far, that the displacements to be measured can always be expressed as a linear sum of the calibration mode displacements. In measuring the modes of a rotating helicopter blade using calibration modes obtained with the blade non-rotating, the condition requires that each rotating-blade mode shape can be expressed as a sum of non-rotating-blade mode shapes. Bearing in mind that the number of calibration modes is limited by practical considerations, it might appear that the requirement is sometimes difficult to meet. This is not so because it is always possible to measure new calibration modes which have been tailored to overcome the difficulty, and can be added to or replace some of the existing set. A simple illustration of this can be seen in the measurement of the fundamental flap mode of a rotating blade. Let us suppose that a good fit cannot be obtained between the strain gauge pattern for this mode and a linear sum of the gauge patterns for the calibration modes, which consist of the first three or four flap modes of the non-rotating blade. A better fit will certainly be obtained by increasing the number of calibration modes - which means measuring several higher order modes. A simpler procedure is to make one or two alterations to the loading on the non-rotating blade and to measure an additional fundamental flap mode with each alteration. For example, an added mass attached to the blade at mid-span will give a mode with increased strain over the inboard sections; an added mass at the tip with a spring to 'earth' at mid-span can give a mode with increased strain over the outboard sections. These are both valid and valuable additions to the original set of calibration modes which will improve the curve-fitting problem in the fundamental mode. Moreover, since the quality of the fit can be examined after each addition, the process can be terminated as soon as a satisfactory fit has been obtained. Also, the process is self-checking in that there is no point in proceeding to a calculation of displacements unless there is a good fit of strain gauge responses.

We turn now to a weighting process mentioned earlier which has been found to enhance the accuracy of results for rotor blades, and is generally appropriate for any cantilevered structure. It is obvious that for cantilever modes such as are obtained for a non-articulated blade, strain is large at the root and zero at the tip whereas displacement is zero at the root and large at the tip. In applying SPA, the relative proportions of the calibration modes are found in terms of the strain patterns, and the proportions will be found such that a best fit is obtained near the root where these strains are large, and there will be a less accurate fit near the tip where the strains are small. When these strains are transformed into displacements, any tip errors (which may appear small in the strain regime) become relatively large. This difficulty can be overcome by weighting the strain measurements before carrying out the SPA analysis. It is obvious that the required weighting should transform a pattern of strain values into one that resembles the corresponding displacements. This can be realised by weighting in such a way that each set of strains is, in effect, integrated along the span from the root - a single integration for torsion strain, and double integration for flap or lag bending strain. There is no need for accurate integration, and the weighting matrices that have been developed were arrived at by assuming that all the gauges of each pattern were equally spaced spanwise, and that the strain values varied linearly between adjacent points. Where no strain values are available for the blade root, values of zero are assumed. Although from a physical point of view this is an incorrect assumption, it does not invalidate the weighting process, and its advantage is that it avoids having to use finite values that will almost certainly be in error, however they are derived. The reason for this is that if finite root strain values have to be assumed for all the calibration modes, it is essential that their values should be consistent with the outputs of a set of imaginary strain gauges at the root.

The result of weighting these strains in this way will not, of course, give an accurate picture of blade displacements, nor is it intended to do so. What it ensures is that the weighted pattern of strains retains a unique relationship to the true displacements and that the maxima and minima of the weighted strains and displacements approximately coincide.

3 MEASUREMENT ERRORS

In applying SPA to the measurement of vibration modes of a helicopter blade, the blade must be instrumented with strain gauges that are responsive to motions in the three main directions of flap, lag and torsion. It is not necessary that each gauge should be positioned to be sensitive to each motion, but that there should be sufficient gauges sensitive to each motion to give adequate definition of that motion along the length of the blade. It is clear that, in practice, measurement errors may be present not only in the responses of the strain gauges, but also in the calibration mode measurements represented by the matrices G and D. In the case of the D matrix, it is possible to reduce errors by smoothing the measurements in each mode, and using smoothed values as the elements of D. Similar treatment of the G matrix is unacceptable, because the position, orientation and sensitivity of each gauge may be such that smooth variation of strain gauge response along the blade would not occur even in the absence of experimental error. It was decided that, in exploring the effects of measurement error in the calibration modes, both the G and D matrix elements should be treated in the same way. This ignores the advantages of smoothing the D matrix, and therefore represents a greater degree of measurement degradation than need be tolerated in practice. The effects of measurement errors were investigated theoretically taking a typical semi-rigid helicopter rotor as a model. The calibration modes were the displacements in the flap, lag and torsion for the first 10 blade normal modes in the non-rotating condition. Sets of strain data as well as displacements were calculated for the blade at its normal operating speed. The strain data were then used as measured strain gauge patterns and the displacements were used to compare with those derived from the strain data using the SPA technique. Neither aerodynamic forces nor structural damping were included in the calculations so that the mode shapes are the undamped normal modes which in the rotating case are modified by centrifugal forces only. One effect of this simplification of modal data is that all the blade responses in any mode are in phase or anti-phase and the problems associated with phase differences in the strain readings do not arise. As a datum for the investigation, the error-free calculated strain data (representing what will subsequently be called strain gauge responses) in each of the 10 modes at the normal operating speed of the rotor were used to derive the corresponding mode shapes. For the first eight modes, the fit achieved between the weighted strain gauge responses and the summation of weighted calibration strains is excellent, and justified proceeding to the derivation of the mode shapes, which are also in excellent agreement with the calculated values except for small errors in the eighth mode. In the ninth mode, shown in Figure 1, despite acceptable agreement in the strain regime, the errors in mode shape are noticeable. In the tenth mode, which is the first overtone torsion mode, there is poor agreement between weighted strain gauge responses and the weighted calibration strains, as can be seen in Figure 2. In these circumstances, no credible mode shape is to be expected and it can be seen that even the major torsional component is incorrectly derived in general shape. The reason for this discrepancy lies in the inadequacy of the torsion components in the calibration modes, in that they do not enable a linear combination to represent the mid-span sign change. If it had been necessary to determine this mode shape accurately, calibration modes with suitable torsional components would have to be included. For the present investigation, the tenth mode was omitted from the subsequent analyses.

Random errors were now introduced into the strain gauge responses for the modes of the blade at normal operating speed, and the analysis processes were repeated. The maximum errors allowed were 5% of the value of each element. For the lower order modes, small errors in the derived mode shapes were evident, but generally, the derivation was accurate, particularly so in the smaller components. Figure 3 shows the fifth mode - the third flap mode - where there are small errors in the flap components at outboard sections.

Since random errors were introduced into the strain gauge responses, it would be expected that processing an average of several sets of responses would result in better accuracy in the derived modes. This is so, although it would be necessary to average what might be an impractically large number of sets to achieve a high order of accuracy, comparable to the no-error results.

Random errors were then introduced into the calibration mode strain gauge and displacement data. The strain gauge reponses for the modes at normal operating speed of the rotor were error-free, and these responses were processed as before. Generally, the derived modes were comparable in accuracy to those of the earlier analysis; though accuracy was less for higher order modes, as may be seen in Figure 4, which shows the ninth mode.

The main point that emerges is that even with errors in measurement of 5% of the true value, the derived modes from the SPA analysis are found with an accuracy which is entirely acceptable for the lower order modes, and is not unacceptable for higher orders, if the objective is to compare measured and calculated mode shapes. This is an encouraging result since it should always be possible to keep experimental errors in the calibration modes to less than the 5% introduced here, and with smoothing of the elements of the displacement matrix D , errors in the calibration data should be minimal. In laboratory measurements on models, errors may also be small with careful experimentation. For wind tunnel or flight measurements however, errors may well be larger, and the possibility of averaging sets of readings to improve accuracy should certainly be explored.

4 APPLICATION TO ROTOR MODELS

The first application of SPA to an actual rotor blade was designed to demonstrate that correct answers could be obtained under realistic test conditions. Such a demonstration could not be made on a rotating blade since there was no other method of mode measurement which could be used as a datum for comparison. What was done was to modify the blade, after measuring the calibration modes, and to determine, using SPA, the (non-rotating) modes of the modified blade - which could, of course, be accurately measured. The blade modifications consisted of adding masses at several positions and applying a spanwise steady force at the tip so as to give modes having the same general characteristics as those of the rotating blade. The results of these tests have been given in an earlier report[4], and it is sufficient to note here that they fully confirmed the viability of the SPA technique under practical test conditions.

We turn now to measurements of the modes of a rotating model rotor blade. A blade consisting of a carbon fibre spar with a segmented balsa wood fairing was attached to a semi-rigid hub. The model was the subject of a theoretical investigation to assess various mode calculation programmes, and the aim of the tests was to provide accurate measurement of the lower order modes for comparison with the calculations.

Thirty sets of strain gauges were installed at equal spacing along the spar, 10 in each of the flap, lag and torsion directions. A small electromagnetic exciter was fitted at the hub, and aligned so that it would provide force inputs in all three directions and thus would excite all the lower order modes. The arrangement is shown in Figure 5. With the blade non-rotating, the first 10 modes were excited and for each mode the displacements and associated strain gauge responses were measured. These constituted the calibration modes. The blade was then rotated, and the modes were again excited and measurements made of the strain gauge patterns. The analysis was then applied to derive displacements in the rotating blade modes. A typical mode is shown in Figure 6. It will be seen that both in-phase and quadrature components of the motion are shown - the phase datum being the response of a reference gauge sensitive to motion in the direction of the meain component of the mode. It is, of course, inevitable that with excitation applied at only one point, the mode excited will not be a pure mode, but will exhibit phase differences along the blade. There are several ways of handling complex responses. The most satisfactory procedure is to process separately the real and imaginary parts of the responses, and this is valid provided the calibration modes are not complex. If they are, it is necessary to perform the analysis in complex arithmetic. In fact, with a structure such as a helicopter blade there is little difficulty in exciting pure calibration modes, though multipoint excitation may be required for some higher order modes.

Because one of the prime purposes of the SPA development was to measure blade modal properties for comparison with blade mode theory, some thought has been given to ways of testing a rotating model blade without aerodynamic forces playing a significant part in the dynamic behaviour. The obvious solution is to make the tests in a vacuum chamber, but considerations of availability, cost and instrumentation problems make it desirable to seek an alternative solution. A scheme that has been tried is to shroud the blade by enclosing it in a tubular container; this was unsuccessful because of the level of unsteady aerodynamic forcing on the container. A modification of this scheme is to enclose the blade within a segment of a disc, and Figures 7 and 8 show the disc (constructed in balsa wood with a stretched plastic sheet cover) which is increased in depth at the centre so as to enclose all the rotating parts. Figure 9 shows the segment containing the blade with its cover removed. Shrouding the blade in this way allows blades to be tested in what might be termed 'still-air' conditions and achieves its object in removing the bulk of oscillatory aerodynamic forces. A confirmation of the effectiveness of the disc is that flap-mode damping does not rise with rotational speed when the blade is shrouded. An advantageous by-product of shrouding is that the blade responses are much cleaner than those of the unshrouded blade thus improving the accuracy of measurement and analysis.

An example of results obtained with the shrouding disc can be seen in Figure 10 where the shapes of a flap mode with and without shrouding are compared at the same rotational speed. The differences are due to aerodynamic effects. Results of this type might enable some experimental check to be made on the accuracy of blade aerodynamic theories.

5 CONCLUDING REMARKS

In this paper we have outlined the strain pattern analysis method of determining displacements of a structure from a measurement of strain gauge responses. Although the applications considered have been primarily directed towards the measurement of modes of rotor blades, the technique clearly has applications in a much wider field. Even in the helicopter field, there seems to be no reason why SPA should not be used to determine blade distortions in flight so as to obtain a complete picture of behaviour at successive increments of azimuth during a cycle of the rotor. Outside the rotor field, the technique could be applied in many situations where conventional motion transducers are difficult and expensive to install or where accelerometers, though preferred, would be adversely affected by external force fields. An obvious application in the first category would be to flutter or buffet models in wind tunnels, and in the second category to weapons and missiles subjected to high steady accelerations. Whatever wider applications may be found, there is now plenty of evidence that SPA can provide the helicopter dynamicist with experimental data which has hitherto been lacking.

REFERENCES

1 D.R. Gaukroger, C.J.W. Hassal. Note on a proposed method of measuring the vibration modes of a rotating model blade. Unpublished MOD Paper (1975)

2 C.J.W. Hassal. Development and initial application of a technique to measure vibration mode shapes of a rotating blade. RAE Technical Report 77064 (1977)

3 D.R. Gaukroger, C.J.W. Hassal. Measurement of vibratory displacements of a rotating blade. Vertica, Vol 2, pp 111-120 (1978)

4 C.J.W. Hassal, D.R. Gaukroger. A theoretical investigation of the effect of measurement errors in deriving vibration modes from strain gauge responses. RAE Technical Report 79055 (1979)

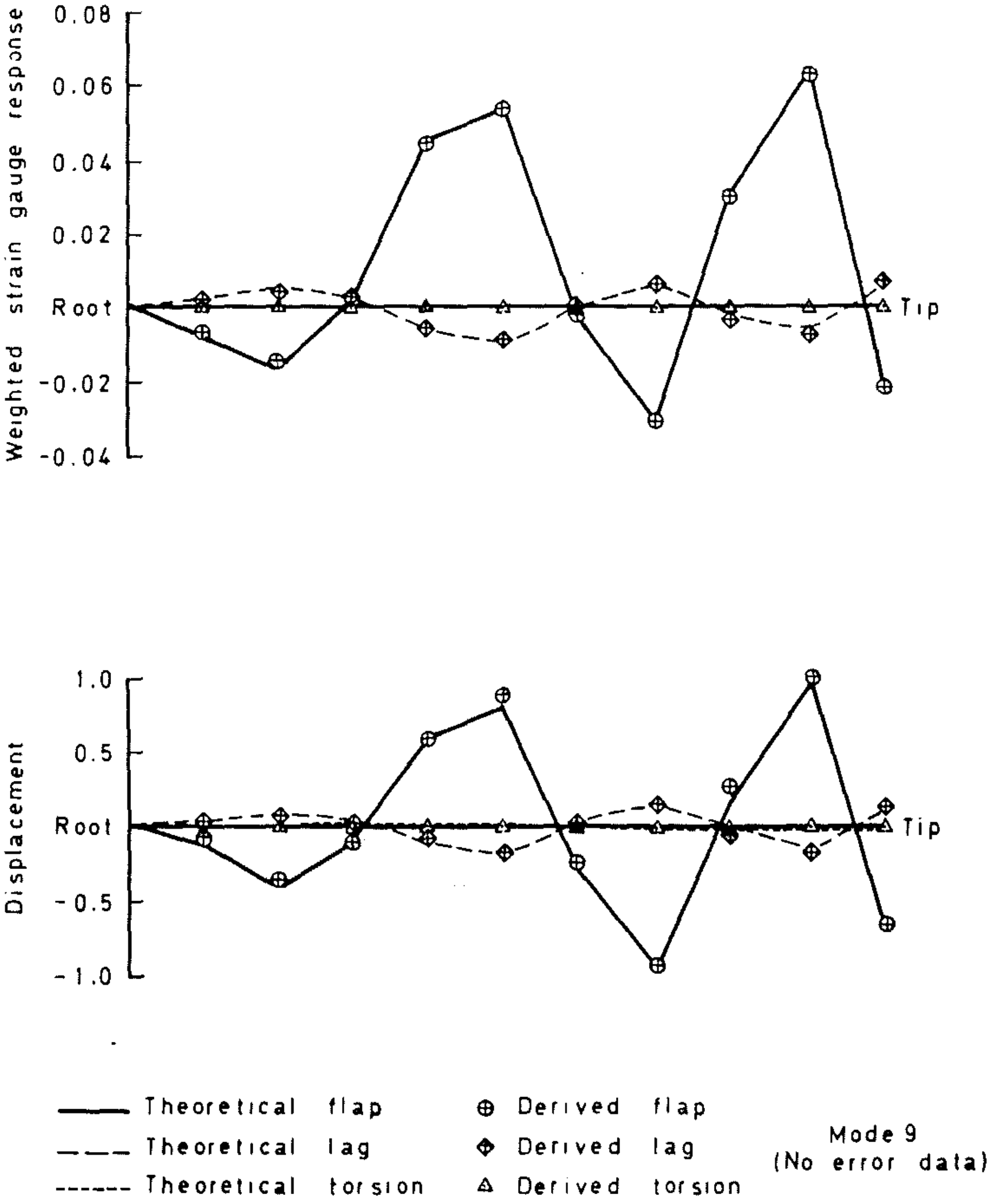

Fig 1 Comparison of calculated and derived results – 5th flap mode

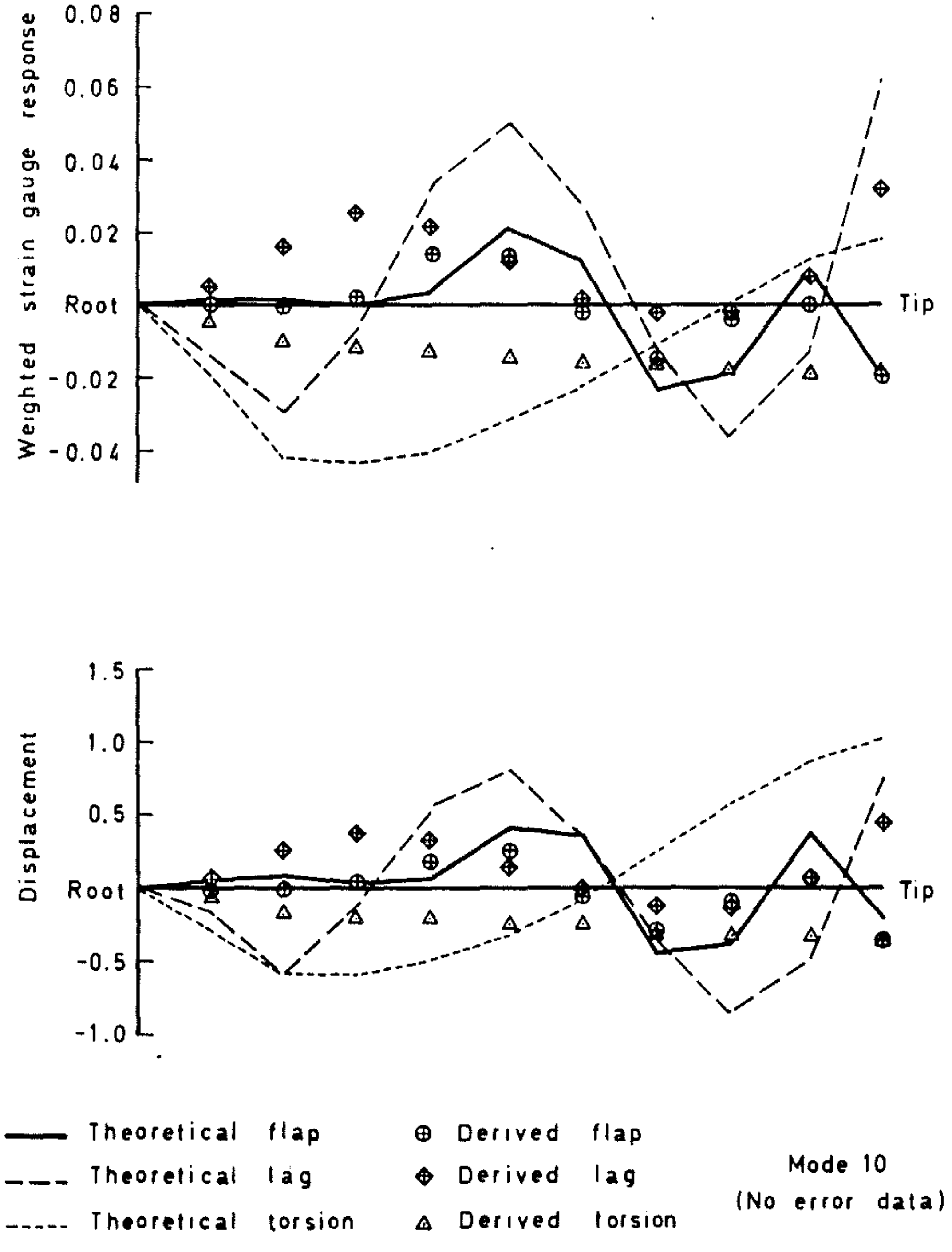

Fig 2 Comparison of calculated and derived results — 2nd torsion mode

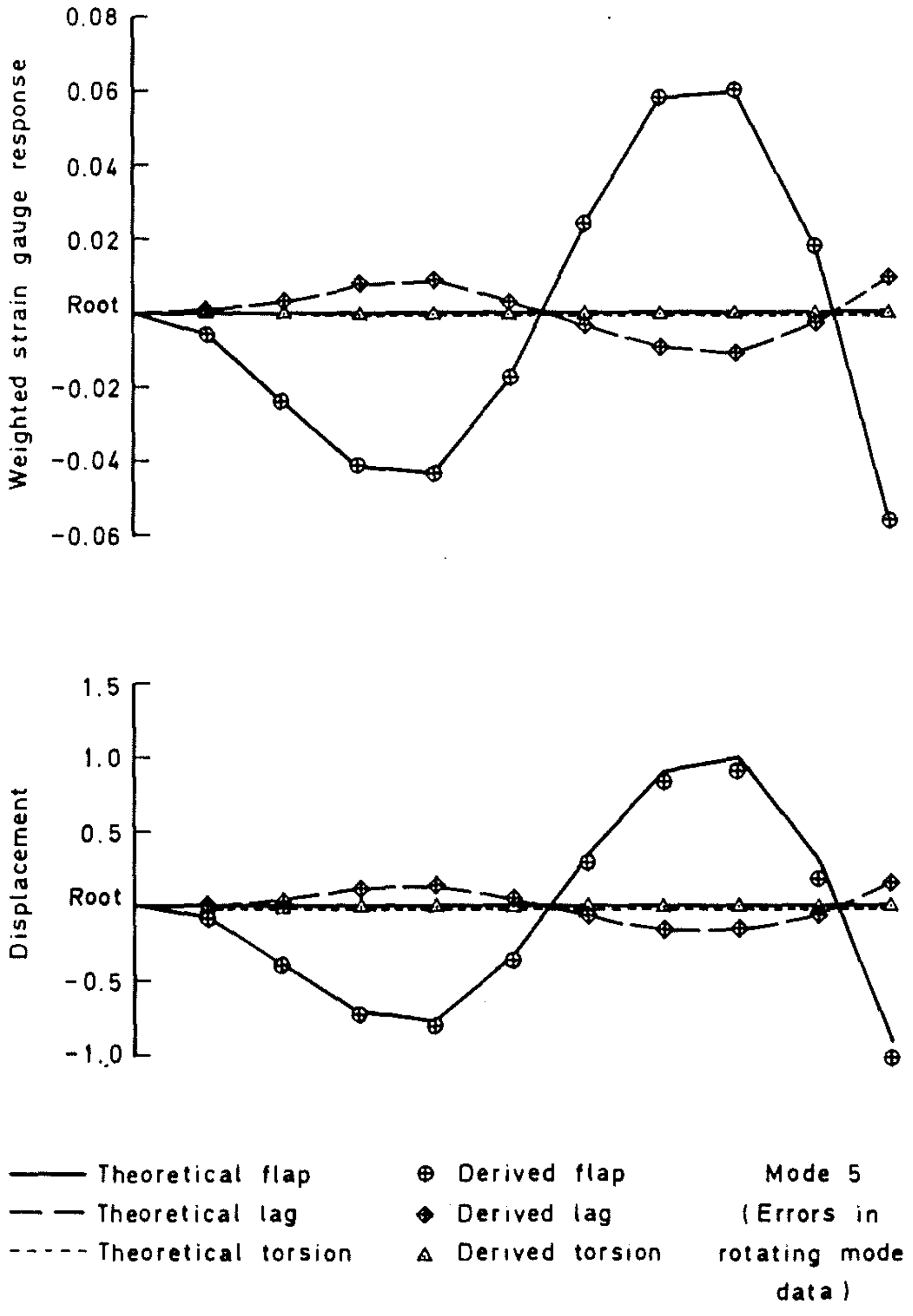

Fig 3 Comparison of calculated & derived results – 3rd flap mode

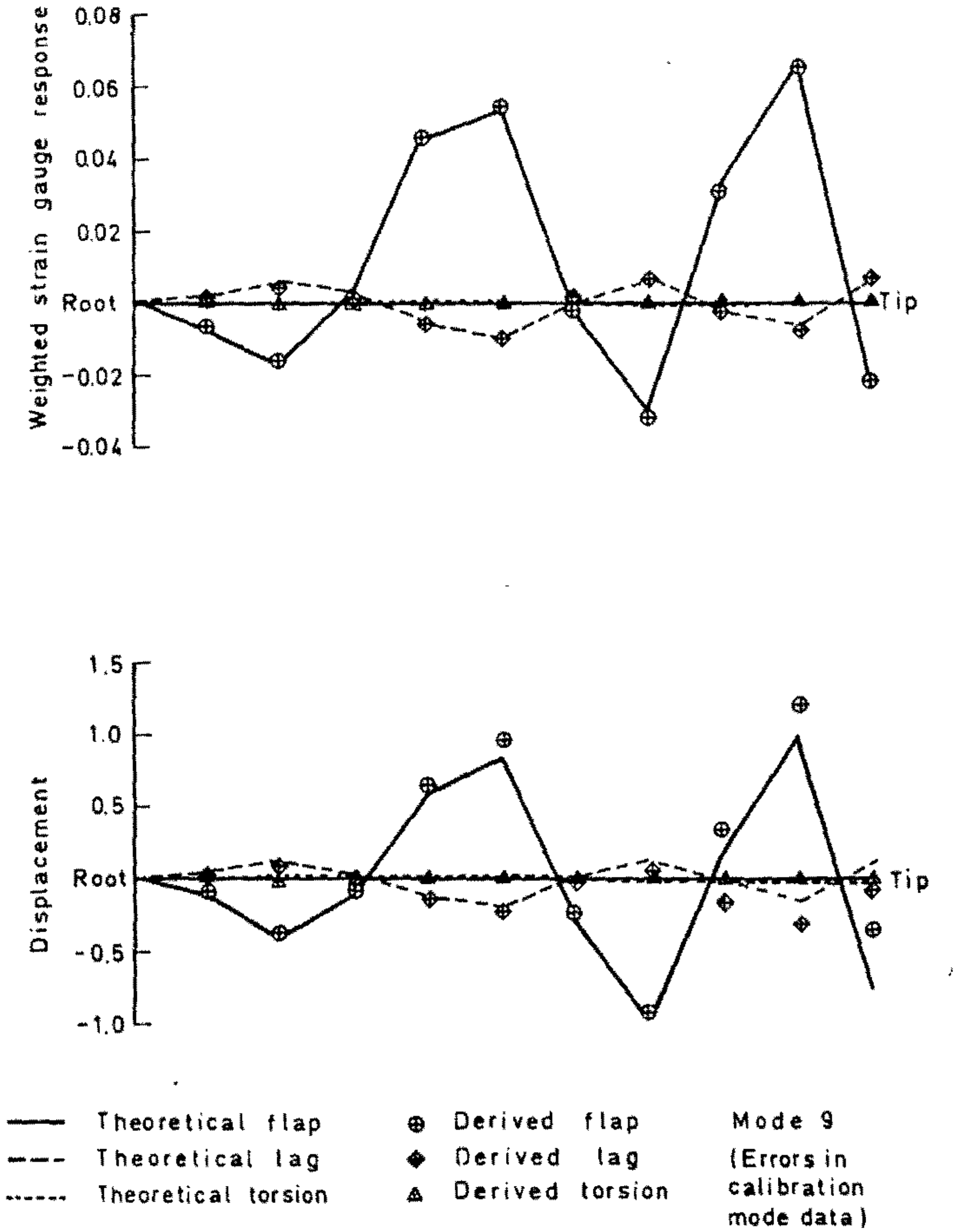

Fig 4 Comparison of calculated and derived results — 5th flap mode

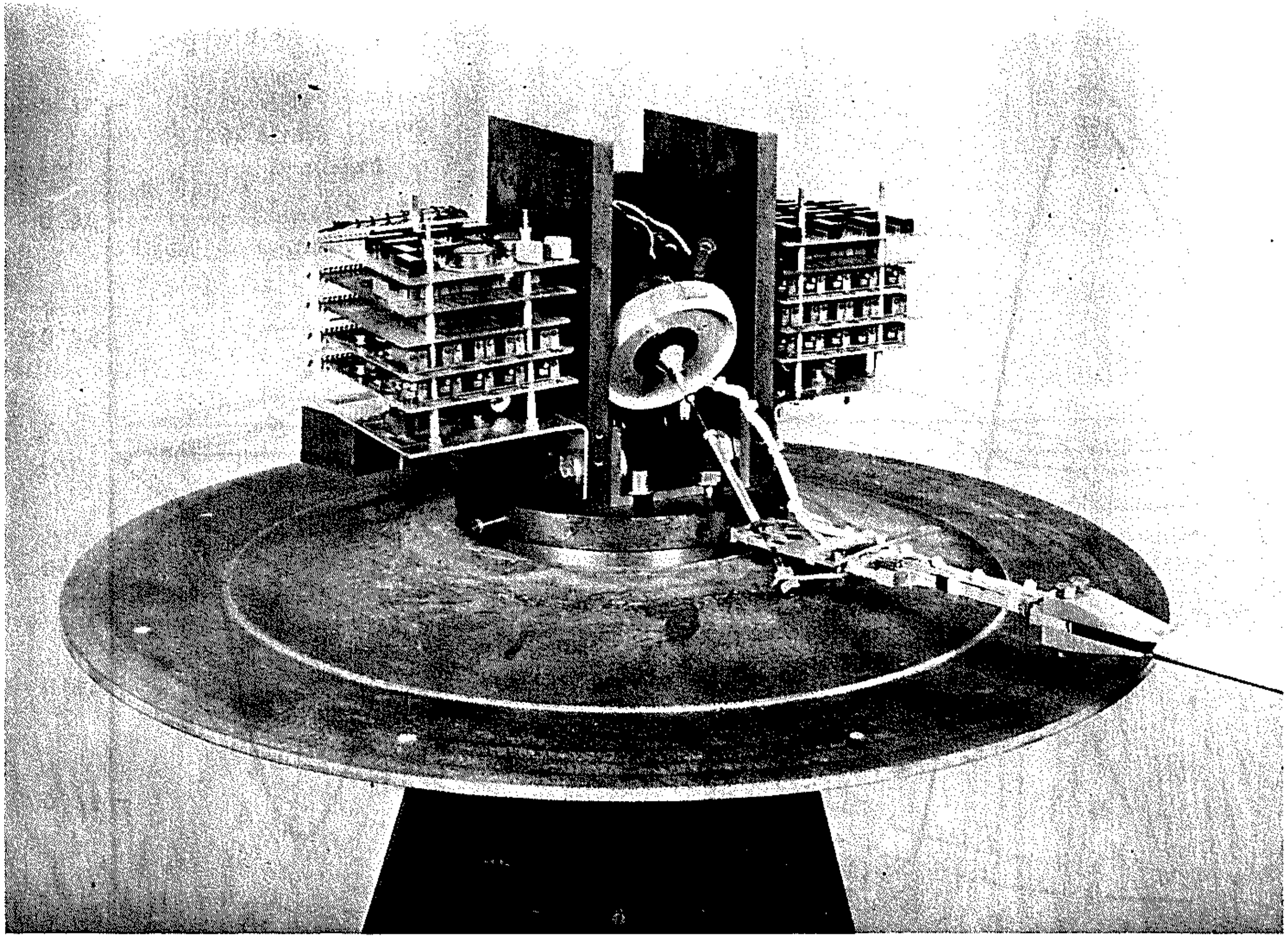

Fig 5 General arrangement of test rig

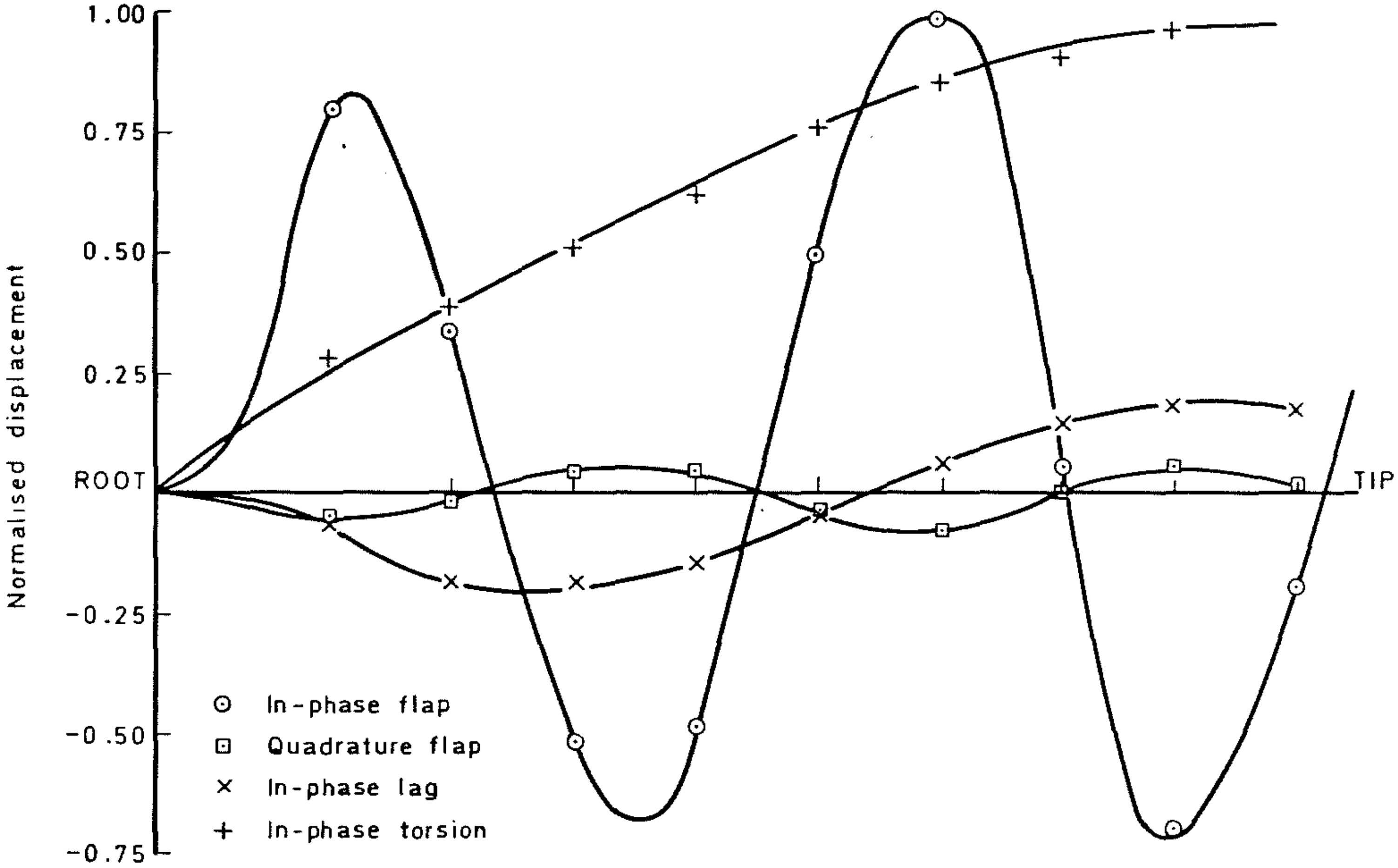

Fig 6 Components of blade fundamental torsion mode

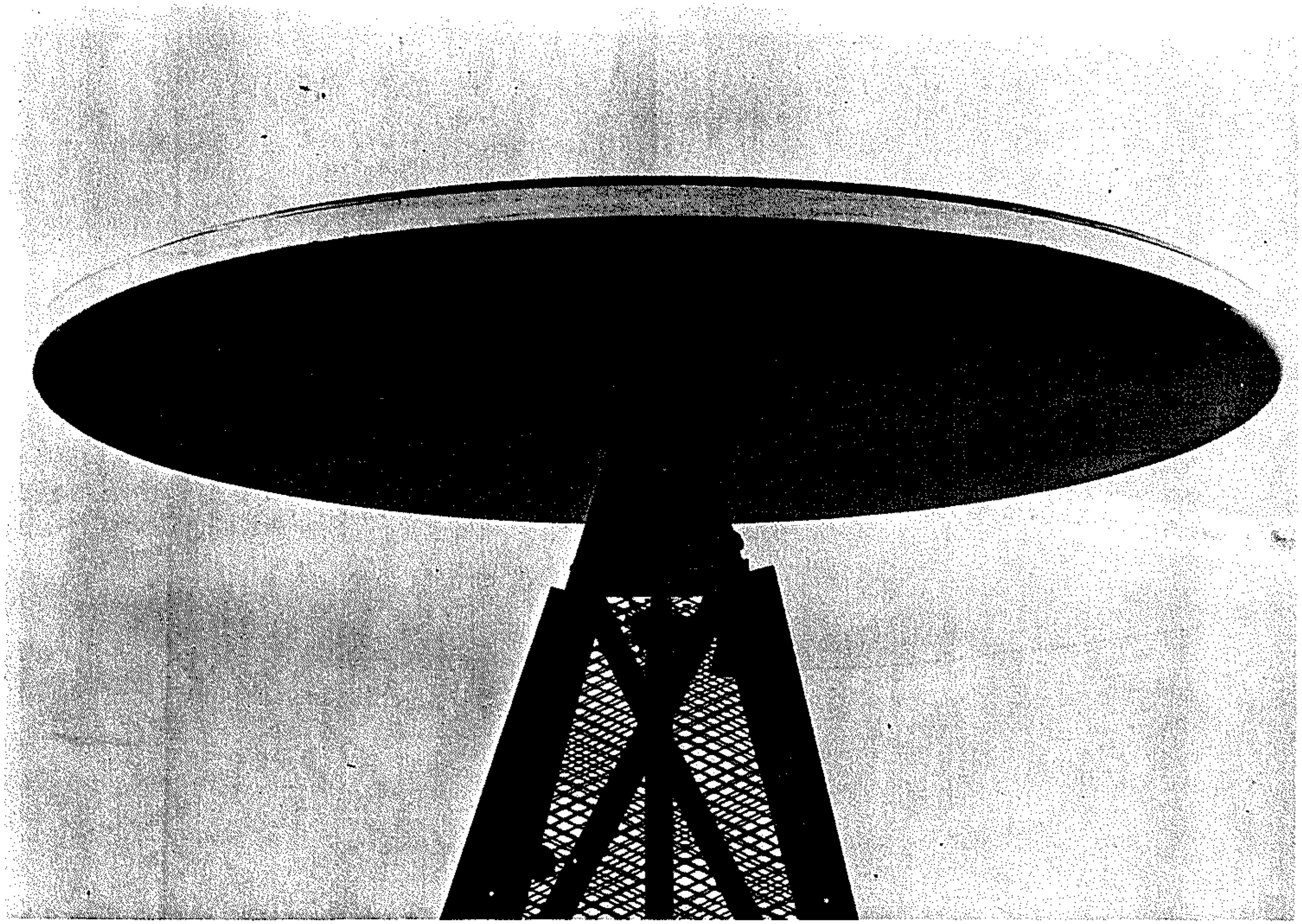

Fig 7 Shrouding disc

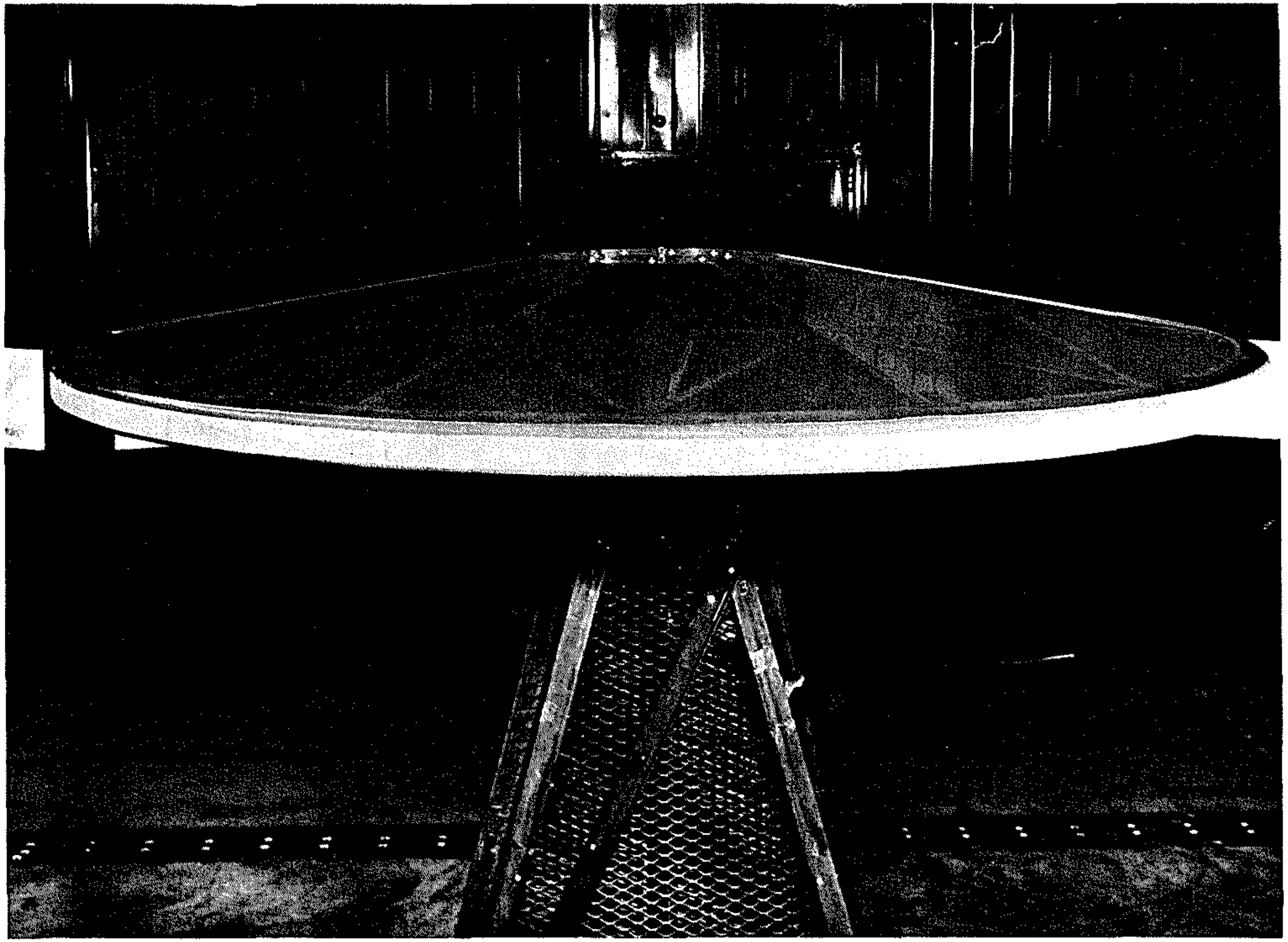

Fig 8   Shrouding disc

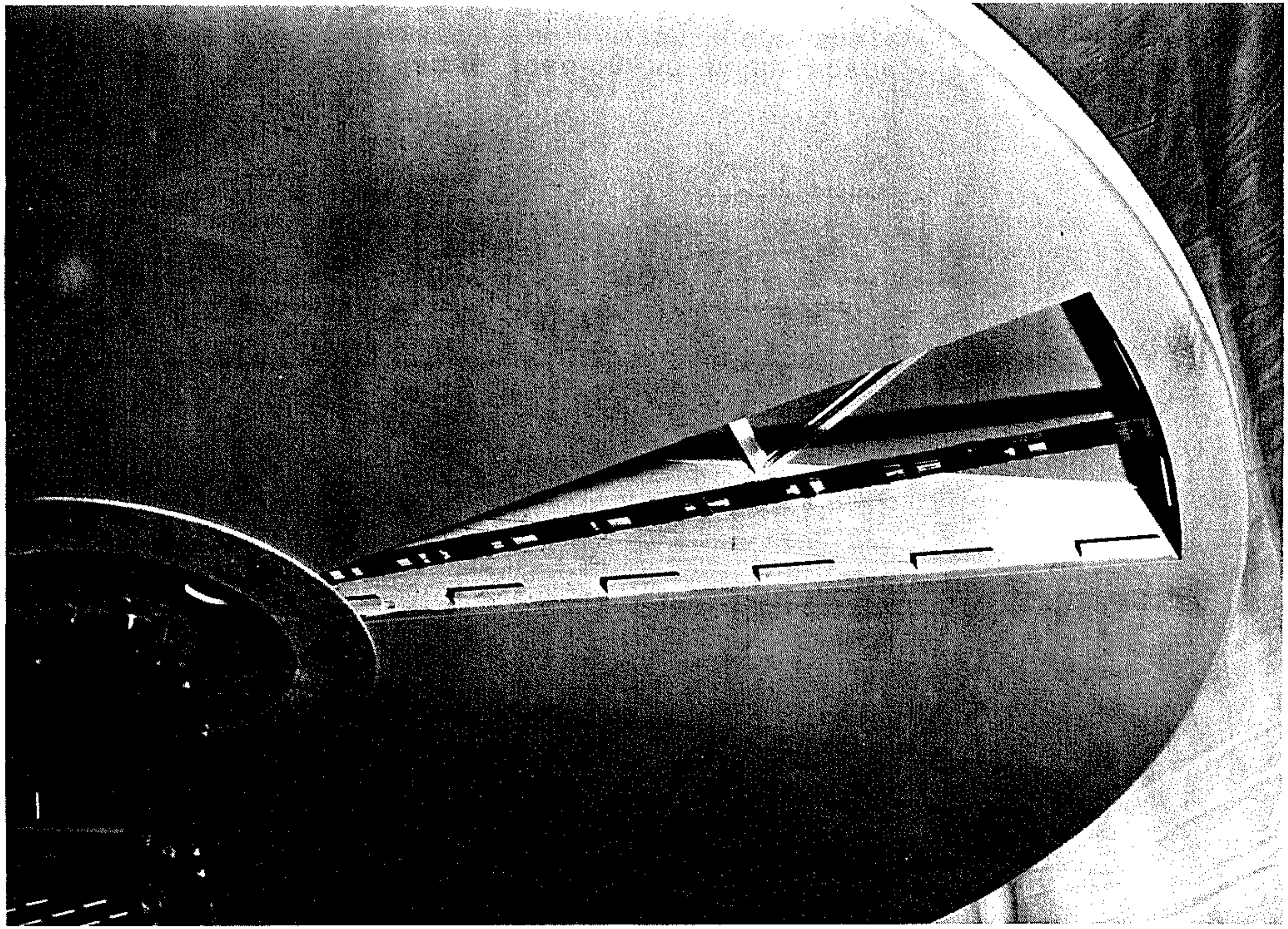

Fig 9   Blade installation in disc

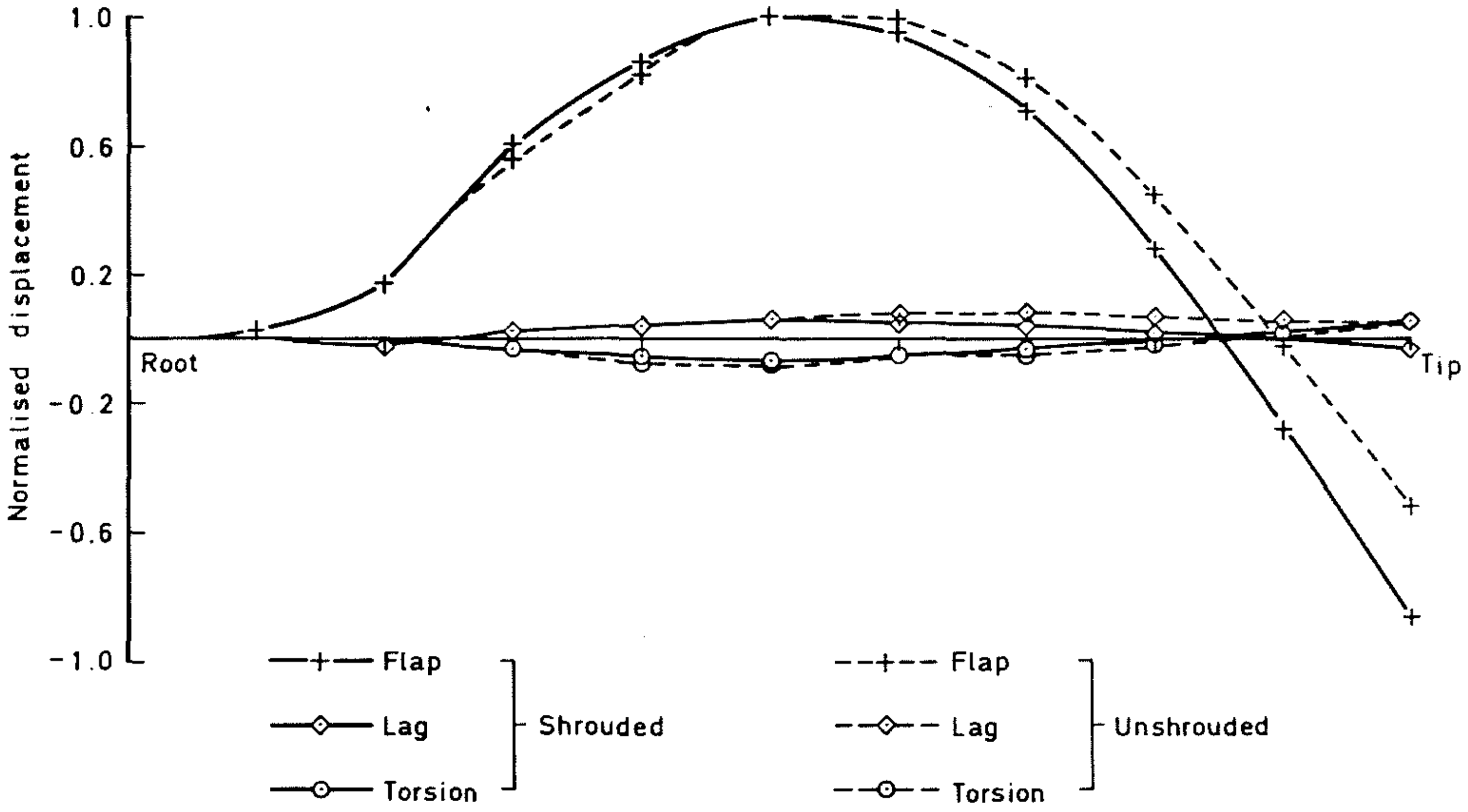

Fig 10 Effect of aerodynamic loading on mode shape – Second flap mode