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STRUCTURAL DYNAMIC PHYSICAL MODELS OF HELICOPTERS BY IDENTIFICATION TECHNIQUE

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ABSTRACT

The techniques and procedures for the design of structural dynamic physical models have been discussed in this paper. The design procedures for these scale models are based on the actual quantitative structural dynamic information available on a given full scale structure such as the helicopter or a major component of the helicopter. The available structural dynamic information can be in the form of a finite element model or the experimental data on the full scale structure. In the past, the structural dynamic physical scale models have been designed and fabricated either by near replication of the full scale structure or by the use of the experience factor and lumped mass arrangements.

The design objective in this paper is to simulate selected natural frequencies and mode shapes of the full scale structure. The procedure also includes some possible constraints such as the preservation of aerodynamic surfaces and minimum thicknesses. Whenever possible the design can be restricted to utilize commercially available hardware.

The design procedure has been developed by using structural dynamic identification techniques and optimization procedures. The developed procedure has been used to illustrate the design of a half-scale model of the tail boom of a selected helicopter. The constraint on the aerodynamic surface has been removed in the example design.

1. INTRODUCTION

The design process of a helicopter is very complex. In this process, the vibration problems and their solutions fall into the category of comparatively difficult tasks that need many new solution procedures. For example, a problem of interest can be explained as follows.

At present, a reliable procedure for obtaining the vibration and structural dynamic response of a given helicopter uses flight test information. Such a procedure is expensive and is not available for design analysis. An alternate approach under development for the past few years is the formulation of analytical models (1), (2) that can accurately predict the structural dynamic response of the helicopter. The state-of-the-art of the development of such mathematical models employs finite element techniques. In many cases, NASTRAN program has been used to develop the models. Most of the developed models have been for airframe-only conditions (1), (2). In spite of the simplicity of consideration of only the airframe, the developed mathematical model yields results that do not always agree with structural dynamic test results that have been obtained for an airframe-only conditions (3), (4). Many procedures have been used to improve the mathematical models to achieve an agreement with the test result. Some of these modifications that have been based on identification techniques have been further complicated by the presence of complex modes in the test results and approximations in the formulation of the type of finite elements and their stiffnesses.

Therefore, in this paper, another alternate procedure has been presented for obtaining the structural dynamic response of helicopters at the design analysis stage. The procedure depends on the development of a physical scale model. In particular the discussions in the paper are restricted only to the design of such physical scale models.

2. BACKGROUND

Scale models have been used, in the past, to obtain natural frequencies and mode shapes of several complicated structures. Some of the recent applications are in the fields of space shuttle design (5), Saturn rockets (6), ship board machinery (7), fuselage design of a fixed wing aircraft (8), nuclear power stations (9), reactor vessels (10), automobiles (11) and helicopter airframes (12). The specific purpose of the model has differed in each use. For example, in the case of the space shuttle a quarter-scale model has been designed, fabricated and tested. The design has been achieved by a near scale replication of the full scale structure. For some elements, only the stiffness has been scaled. The purpose of the quarter-scale model has been to measure the dynamic characteristics of the four major components of the shuttle individually and in their mated configuration. The measurements have been transformed to correspond to the full scale structure and have been compared with analytical predictions to gain confidence in the design.

Most of the physical models, with the exception of the helicopter airframe model, have been scale models. The scale models do not always reproduce the exact natural frequencies and mode shapes. The purpose of the paper is to discuss a structural dynamic physical model that is not a near replication of the full scale structure. The objective is to develop scale models that reproduce a selected number of frequencies and mode shapes exactly or to a reasonable degree of tolerance. The objective is also to obtain a procedure that will provide a choice of simple structural dynamically similar models with a geometry that will result in easy fabrication.

It is logical to ask the following question. Why do we need such models? Of course, the primary reasons for all model studies are the economy, simplicity in the design of test fixtures and instrumentation procedures that are managable. It is also safe to state that each model design is generally restricted to a class of special studies. In this paper, the restriction is to the study in the field of structural dynamics. By using a model that can reproduce selected number of natural frequencies and mode shapes, the dynamic response of the helicopter structure can be studied for different inputs. Similarly, force

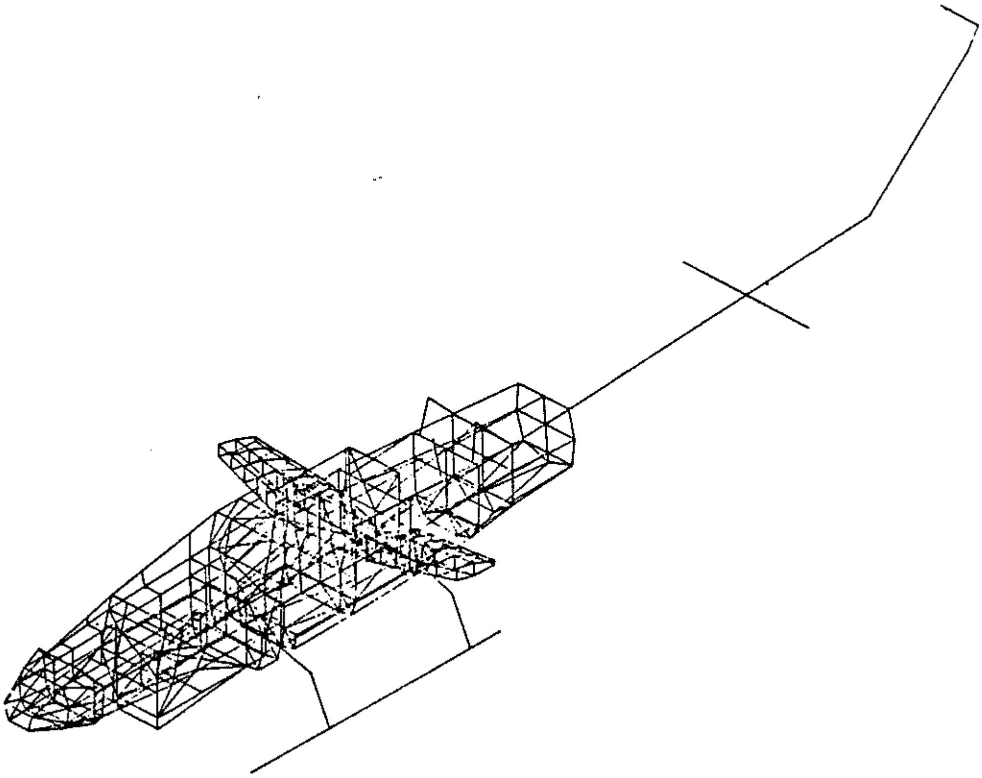


Figure 1. 489 node finite element representation of a helicopter

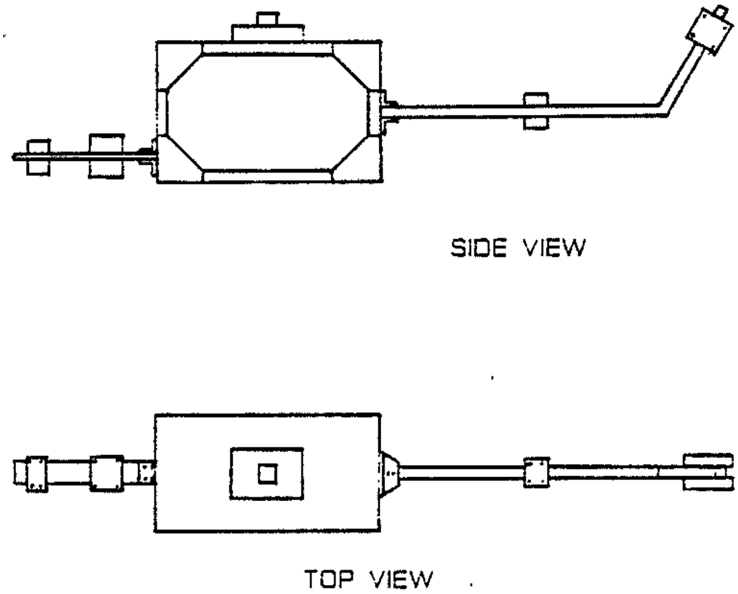


Figure 2. Intended model geometry

determination studies can be conducted by using measured accelerations and dynamic calibration data. The model can also be used for validating structural dynamic system identification techniques and experimental techniques that can be later used for full scale aircraft. A very useful potential application is in the study of rotor-airframe interaction studies in a wind tunnel. This application, however, requires the constraint of maintaining the aerodynamic surface.

Another question that might be asked is as follows. Why is it not preferable to use an analytical finite element model instead of a physical model? One answer to the question is obvious. As in the case of shuttle studies (9), experimental measurements always improve the design confidence. In fields such as rotor-airframe interaction or damping studies, physical model studies definitely have an edge and can provide the needed inputs to the analytical models.

3. MODELLING TECHNIQUE

It is assumed that an a priori finite model of the full scale structure is either available or can be formulated to a reasonable degree of accuracy. For purposes of discussion the number of degrees of freedom is denoted by n. The finite element representation of a helicopter with 489 nodes is illustrated in Figure 1. The details of the actual structure is more complicated. The objective requires that the model should have a simple geometry with fewer stations at which the cross sectional properties change. As an illustration, the model representation can be in the form that is illustrated in Figure 2. The finite element representation of the model in Figure 2 is illustrated in Figure 3.

At this stage, there are two different ways, in which the model design can be achieved. In the first procedure, suitable member dimensions and appropriate nonstructural masses are assumed as initial estimates. With these initial estimates, a suitable structural dynamic system identification procedure can now be used to obtain the appropriate dimensions for each member that simulate the selected number of natural frequencies ω_i and mode shapes x_i. An alternate procedure is to condense the matrices from their sizes nxn to mxm that correspond to the reduced number of stations s. By decomposing the condensed stiffness matrix to element stiffness matrices, the appropriate geometrical properties of the elements are designed by using an optimization procedure. In order to maintain a reasonable degree of accuracy in the simulation of the natural frequencies and mode shapes, it is necessary to retain several internal nodes in the condensed element during the optimization or design procedures. Thus, a full scale model has been designed with a simple configuration and fewer stations at which the geometrical properties change. Appropriate structural dynamic scaling principles are now used to obtain a half-scale or a quarter-scale model as desired.

A finite element model with appropriate internal nodes is now formulated for the scale-model. The mode shapes and natural frequencies are determined. Structural dynamic system identification procedures are again used to improve the model design parameters. This identification procedure is needed for two reasons. The first is to correct any loss of accuracy in the scaling procedure. The second is to obtain reasonable dimensions for the model design that can be easily incorporated into a fabrication procedure.

4. DESIGN OF A SPECIFIC MODEL

Instead of modelling the complete helicopter with 489 nodes, a major component of the same helicopter has been considered. The primary reasons for selecting a component instead of the complete helicopter is the simplicity in

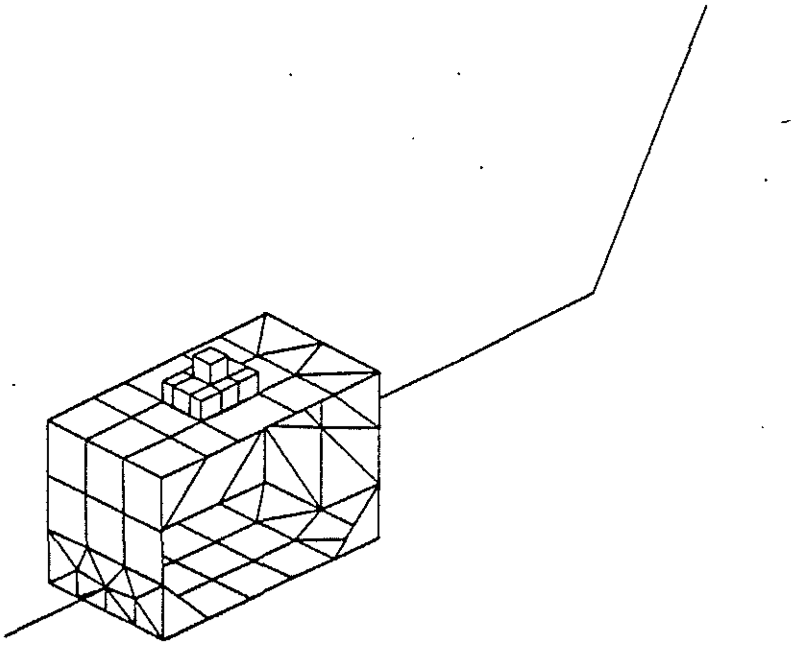


Figure 3. Finite element representation of the model

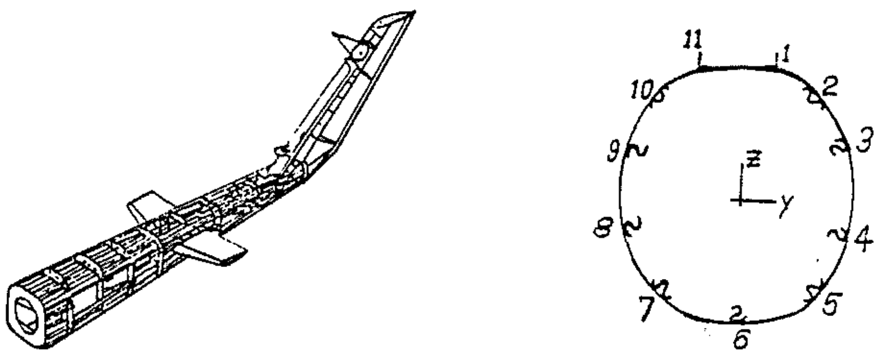


Figure 4. Tail boom

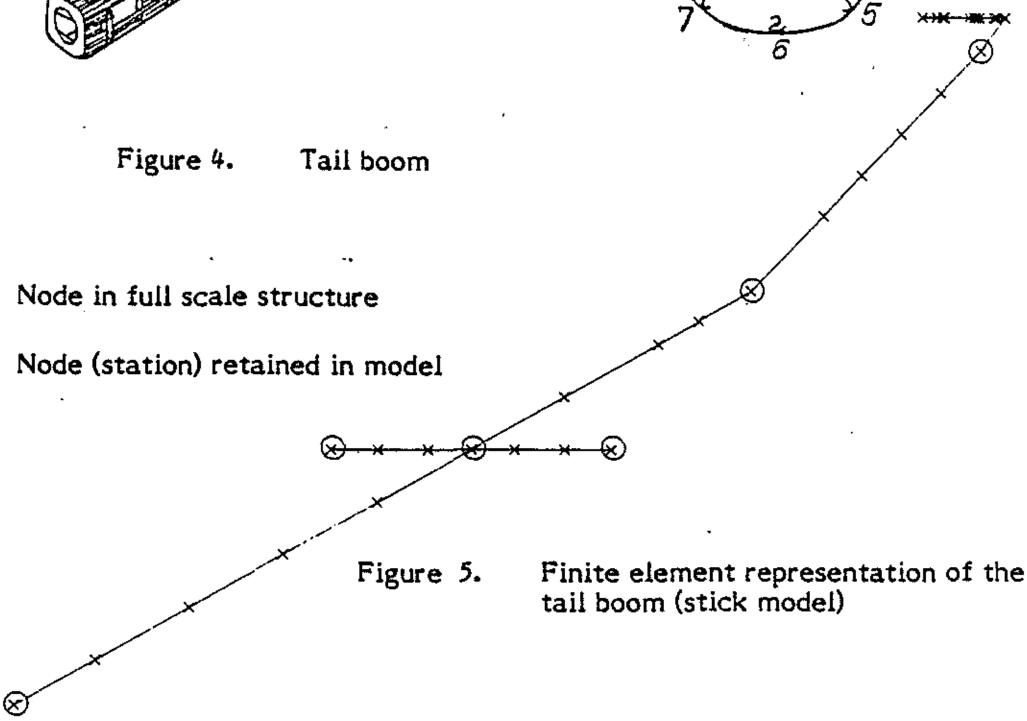


Figure 5. Finite element representation of the tail boom (stick model)

the modelling procedure and the presentation of the results. The selected tailboom and the vertical fin are illustrated in Figure 4. An elastic line NASTRAN model of this structure, has been used as the initial analytical model from which the properties of a simplified physical model are to be determined. The full scale NASTRAN model consists of 32 nodes and 192 degrees of freedom and is made up of beam elements only. This model is next reduced to a 6 node / 36 d.o.f. model through the Guyan's reduction technique. Once the condensed stiffness matrix is obtained, it can be decomposed into component stiffness matrices corresponding to elements 1 to 6 in Figure 5.

The mass for each element is determined by summing the masses at all the internal nodes along with a fraction of the mass at the common nodes where more than one element meet. For this purpose, the mass at the common node is distributed among the different elements incident on it according to their lengths. The sum of the masses due to the truncated portion of the vertical fin and the tail rotor mast is considered as a concentrated mass to be lumped at node 6.

It is to be noted that at this stage, the element stiffness matrices obtained by decomposition will correspond to the global coordinate system. It is necessary to rotate each of these matrices to the element coordinate system before proceeding to determine the element cross-section properties. It is also essential to know the form of the stiffness matrix for a general beam element with varying cross-sectional properties. The derivation of such a stiffness matrix is discussed in the Appendix for a general beam element (Figure 6).

5. DESIGN OF THE BEAM ELEMENTS

For the tailboom model, the element stiffness matrices are extracted from the condensed global stiffness matrix according to the procedure outlined in section 4. The next step in scale modelling involves designing beam elements that possess these element stiffness matrices and also the required mass. The design process should yield elements that are simpler than the original structure and easy to construct in practice. In the present case, it is assumed that the element stiffness matrices can be reproduced by beam segments with single steps across which the cross-sectional properties change. If x_s denotes the distance at which the step occurs and Le denotes the element length, the cross-sectional properties remain constant (denoted by subscript 1) from 0 to x_s, and they assume different constant values (denoted by subscript 2) from x_s to L_e. For this case, the equations for integrals α, β, γ and δ that are needed to obtain the element stiffness matrix can be obtained. The definition of these integrals and the description of the element stiffness matrices are discussed in the Appendix.

$$\delta_A/L_e = [EA]_1 \; \bar{x}_s + [EA]_2 \, [1 - \bar{x}_s] \qquad (5.1)$$

$$\delta_J/L_e = [GJ]_1 \; \bar{x}_s + [GJ]_2 \, [1 - \bar{x}_s] \qquad (5.2)$$

$$[1/L_e] \, \alpha_{\substack{y\\z}} = [EI_{1\,\substack{y\\z}}] \; \bar{x}_s + [EI_{2\,\substack{y\\z}}][1 - \bar{x}_s] \qquad (5.3, 5.4)$$

$$[1/L_e^2] \, \beta_{\substack{y\\z}} = [EI_{1\,\substack{y\\z}}] \; \bar{x}_s^2/2 + [EI_{2\,\substack{y\\z}}] \frac{[1 - \bar{x}_s^2]}{2} \qquad (5.5, 5.6)$$

$$[1/L_e^3] \, \gamma_{\substack{y\\z}} = [EI_{1\,\substack{y\\z}}] \; \bar{x}_s^3/3 + [EI_{2\,\substack{y\\z}}] \frac{[1 - \bar{x}_s^3]}{3} \qquad (5.7, 5.8)$$

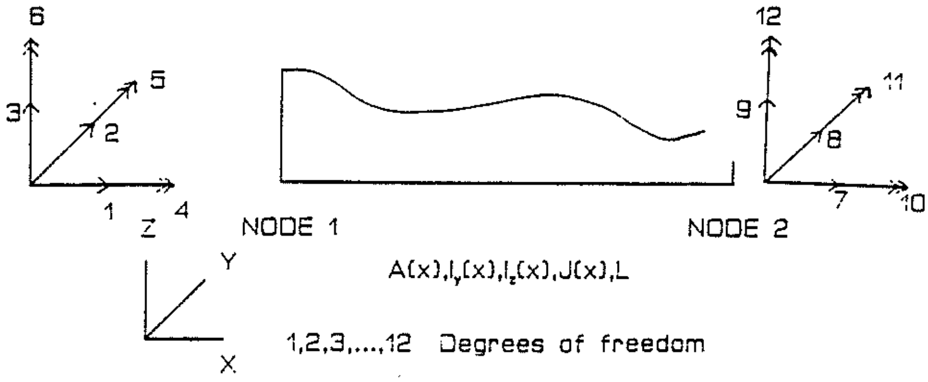


Figure 6. General beam element

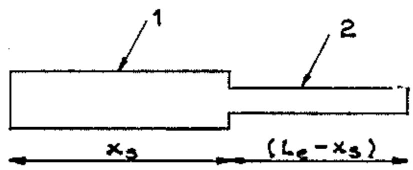


Figure 7. Typical element in model design

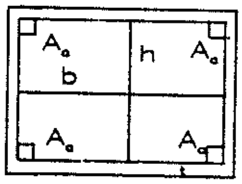
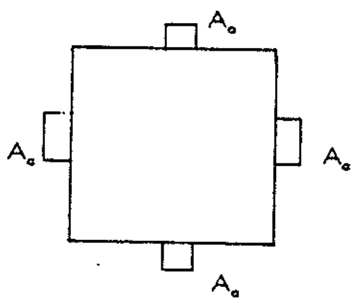
Element No.	Shape	Description
1,4,5		Hollow rectangular tube with concentrated areas
2,3		Solid square with concentrated areas

Figure 8. Model details

The element length Le for each element is taken to be the same as in the original structure. In order to preserve accuracy, it is necessary to consider many internal nodes in a given element.

The design of the element is accomplished by choosing a suitable cross-sectional shape and computing the cross-sectional dimensions for the chosen shape before and after changing the step at x_s, where x_s also is an unknown quantity. The material properties are assumed to remain the same across the step.

The design process can now be formulated as an optimization problem in which the objective function to be minimized is given by

$$\phi = \sum_{i=1}^{8} e_i^2 \tag{5.9}$$

where e_i is the error in the i^{th} component in equation 4 for any set of assumed cross-sectional dimensions. It is necessary to implement linear constraints during optimization (such as all the cross-sectional properties must be positive) to obtain physically meaningful results. Incorporating these constraints through penalty functions, the modified objective function is $\hat{\phi}$.

$$\hat{\phi} = \sum_{i=1}^{8} e_i^2 + \left\{ \sum_{j=1}^{n} \left[\frac{1}{r - r_l} + \frac{1}{r_u - r} \right] \right\} \tag{5.10}$$

where

p = penalty parameter

r = cross-sectional property or x_s

r_l = lower limit for r

r_u = upper limit for r

n = no. of parameters to be determined.

The unknown parameters which are varied to minimize $\hat{\phi}$ include the cross-sectional dimensions or properties and the step length x_s. Optimal estimates for these parameters are determined by employing the sequential unconstrained minimization technique (SUMT) in which $\hat{\phi}$ is optimized for successively decreasing values of the penalty parameter with the final minimization having been accomplished by setting p equal to zero. The IMSL library subroutine ZXMIN was used for the unconstrained minimization at each step.

In the design procedure discussed heretofore, attention has been focused mainly on reproducing the element stiffness properties by choosing appropriate cross-sectional dimensions. But in order to preserve the dynamic behavior of the original structure, it is of considerable importance to ensure that the mass distribution is also simulated as accurately as possible.

Rewriting equation (5.1),

$$\frac{\delta_A}{E} = A_1 x_s + A_2 (L_e - x_s) = V_e \tag{5.11}$$

where V_e = volume of the element. The element volume can also be determined as

$$V_e = \frac{M_e}{\rho}$$

where

M_e = mass of the element

ρ = mass density.

If the volume computed from δ_A does not agree with the volume computed from the mass requirements, equation (5.1) is replaced with

$$A_1 \overline{x_s} + A_2 (1 - \overline{x_s}) = \frac{M_e}{\rho L_e} \qquad (5.13)$$

The effect of this operation is that the total mass of the element is preserved while the axial stiffness of the element may or may not be reproduced accurately depending on the agreement of the two independent volume requirements.

6. SCALING

After a geometrically simple model of the original structure has been determined, it should be scaled down so that a smaller model can be built and employed in dynamic response studies. In the conventional replication technique, the reduced model is obtained by dividing all the dimensions of the full-scale structure by the scaling factor. But it is easy to show that such a scaling does not reproduce the modal parameters of the original structure even though they can be extrapolated from certain nondimensional parameters. In the form of scaling proposed here, efforts are made to preserve certain modal parameters. The scaling procedure is best illustrated through a beam example.

Consider the differential equation for the free vibration problem.

$$EI \frac{\delta^4 w}{\delta x^4} + \rho A \frac{\delta^2 w}{\delta t^2} = \qquad (6.1)$$

By defining the nondimensional cordinate $\bar{x} = {}^x/L$

Where L is the beam length

$$\left[\frac{EI}{\rho A L^4}\right] \frac{\delta^4 w}{\delta^4} + \frac{\delta^2 w}{\delta t^2} = 0 \qquad (6.2)$$

If the beam is scaled so that the quantity $[EI / \rho A L^4]$ remains the same, the dynamic response of the original and the reduced beam will be the same. It is to be noted that the boundary conditions are applied at the non domensional values of $\overline{x}$. Thus,

$$[EI / \rho A L^4]_s = [EI / \rho A L^4]_m \qquad (6.3)$$

where the subscripts s and m denote the original structure and the scaled model respectively.

If the model is made of the same material as the original structure,

$$[I / A L^4]_s = [I / A L^4]_m \qquad (6.4)$$

The length of the beam is assumed to be reduced by a factor,

$$\frac{L_s}{L_m} = \ell \qquad (6.5)$$

Freq. 35.90 Hz

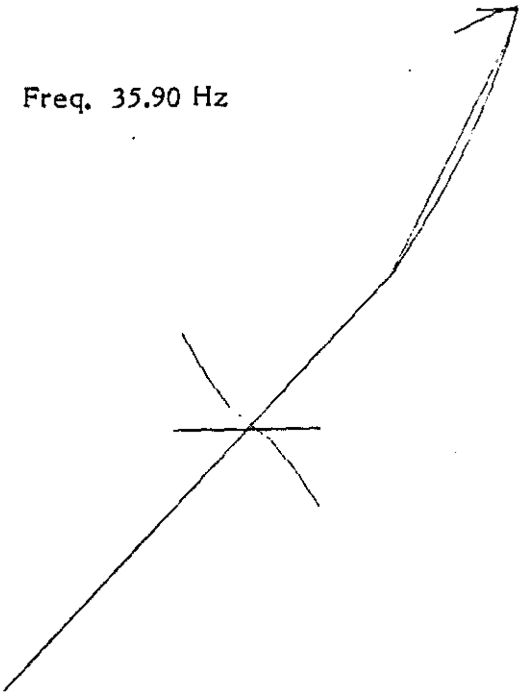


Figure 9a. Full scale structure (8th mode)

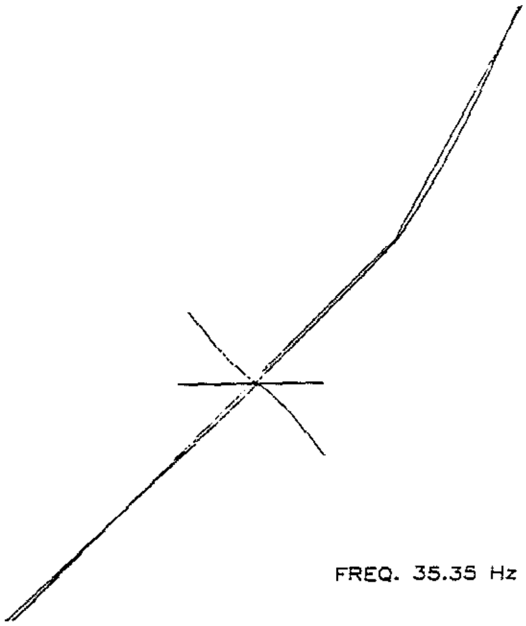


Figure 9b. Half-scale model (8th mode)

Freq. 55.22 Hz

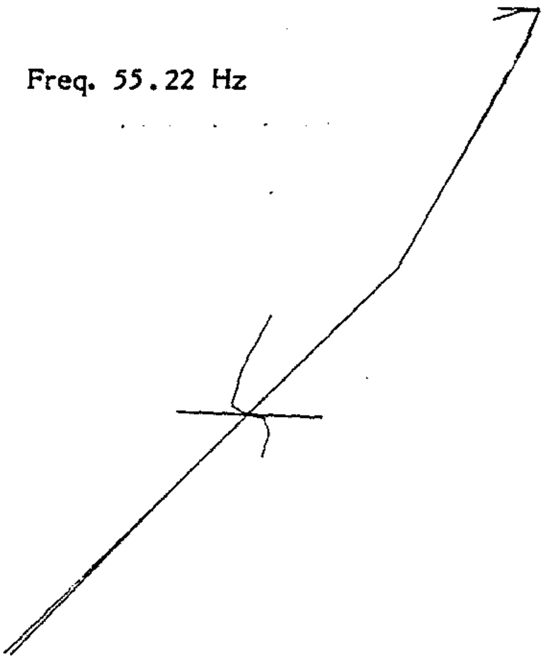


Figure 10a. Full scale structure (11th mode)

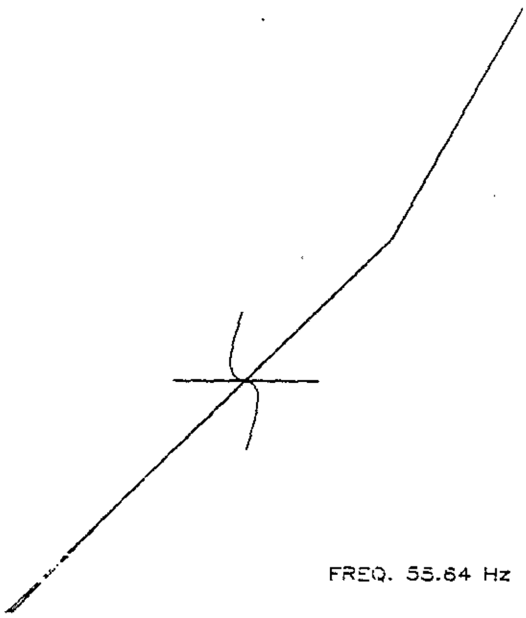


Figure 10b. Half scale model (9th mode; Note: 2 local modes have disappeared in the model)

Table I: Cross Sectional Properties

Element No. / Property	1 Full Scale	1 Half Scale	4 Full Scale	4 Half Scale	5 Full Scale	5 Half Scale
A_1 (in^2)	.188	.012	.157	.01	.822	.051
b_1 (in)	11.21	2.8	9.49	2.37	3.21	.805
h_1 (in)	13.35	3.34	9.06	2.27	11.42	2.855
t_1 (in)	.119	.03	.077	.019	.042	.0156
A_2 (in^2)	.768	.048	.237	.015	.481	.03
b_2 (in)	7.34	1.84	4.75	1.19	3.22	.805
h_2 (in)	7.27	1.82	6.40	1.60	7.88	1.97
t_2 (in)	.128	.032	.111	.028	.042	.0156
L_e (in)	101.96	50.98	62.89	31.45	71.41	35.71
x_g/L_e	.615	.615	.821	.821	.509	.509

Table II.: COMPARISON OF FREQUENCIES

MODE NO.	FREQUENCIES (Hz) STRUCTURE	FULL SCALE MODEL	1/2 SCALE MODEL
7	25.95	26.10	26.11
8	35.90	35.30	35.35
9	44.27	–	–
10	52.82	–	–
11	55.22	55.62	55.64
12	57.24	55.66	55.73
13	59.28	58.72	58.73
14	77.98	70.26	70.69
15	109.47	91.10	92.09
16	124.61	139.97	142.67
17	185.11	229.84	229.88
18	208.71	240.41	240.93
19	232.56	258.86	260.17
20	233.22	266.34	289.18

it is easy to verify that equation (5) will be satisfied if the cross-sectional dimensions are scaled down by the factor ℓ^2 . In the case of beams with concentrated masses, the scaling for the masses is according to the following relationship.

$$\left[\frac{M}{\rho A L} \right]_s = \left[\frac{M}{\rho A L} \right]_m \qquad (6.6)$$

Since all the elements used in the model are beam elements, each element diminsions and the nonstructured masses have been scaled according to the described procedure.

7. RESULTS FOR A SPECIFIC CASE

The selected tail boom full scale structure consisted of details as shown in figure 4. A typical detail at a particular section is also shown in figure 4. A proposed structural dynamic model has simpler cross sections as illustrated in Figure 5 for individual members. After optimization the appropriate cross-sectional properties have been obtained.

The resulting design has yielded certain natural frequencies and mode shapes. These have been compared with those of the full scale structure. The mode shapes of interest agreed reasonably well. However some of the frequencies were higher. A structural dynamic system identification procedure (13) has been used to adjust these frequencies. The identification algorithm employed to adjust the non-structured mass involves using only the frequencies of the full scale structure (13). By expressing the eigenvalue problem as

$$M\, x_{m_i} = \bar{\lambda}_{m_i}\, K\, x_{m_i} \qquad (7.1)$$

and the mass matrix as

$$M = M_o + p\, M_1 \qquad (7.2)$$

one obtains

$$\bar{\lambda}_{m_i} = x_{m_i}^T \left[M_o + p\, M_1 \right] x_{m_i} \qquad (7.3)$$

where

λ_{m_i} , x_{m_i} = i^{th} eigenpair for the model
M = model mass matrix
K = model stiffness matrix
M_o = mass matrix corresponding to the beam elements
M_1 = mass matrix corresponding to the concentrated tip mass
p = parameter to be estimated

and

$$x_{mi}^T\, K\, x_{mi} = 1.$$

Parameter p is obtained by minimizing

$$\psi = \sum_{j=1}^{\eta_f}\left[\bar{\lambda}_{mj} - \bar{\lambda}_{fj}\right]^2 \qquad (7.4)$$

where

$\bar{\lambda}_{fj}$ = j^{th} full scale eigenvalue corresponding to $\bar{\lambda}_{mj}$

η_f = no. of eigeenvalues used in estimation

From eq. (7.4)

$$\sum_{j=1}^{\eta_f}\left[\bar{\lambda}_{mj} - \bar{\lambda}_{fj}\right]\frac{\delta\bar{\lambda}_{mj}}{\delta p} = 0 \qquad (7.5)$$

The derivative of the j^{th} model eigenvalue with respect to can be obtained from eqs. (7.1) and (7.2) as

$$\frac{\delta\bar{\lambda}_{mj}}{\delta} = x_{mj}^T M_1 x_{mj} \qquad (7.6)$$

Equations (7.3) and (7.6) are substituted into eq. (7.5) which is then solved for p . This procedure is repeated again until sufficient convergence is achieved.

8. RESULTS

For the selected tail boom the cross sectional properties (figure 8.) are tabulated in table I. In table II, the natural frequencies have been compared. In this table the title "structure" refers to the full scale structure. The term "full scale model" refers to the model where over all dimensions have not been scaled, but the geometry has been simplified by using appropriate optimization procedures. It can be seen that the half-scale model frequencies agree with the full scale structure to a reasonable degree of tolerance. The results of the optimization procedure indicated the necessity of a nonstructured mass at station 6 to match the first bending frequency. It is also to be noted that 2 local modes have disappeared when condensed. In the analysis no attempt has been made to reproduce the axial vibration modes. The mode shapes have been compared in figures 9 and 10. In table III, a different design has been illustrated. In this design all sheet metals have been constrained to have a thickness of 1/64 inch.

9. DISCUSSIONS

A comparison of the natural frequencies and mode shapes indicate that the designed physical scale model can reproduce selected natural frequencies and mode shapes of the full scale structure to a resonable degree of accuracy. The accuracy decreases with modes of higher order. The specific design discussed in this paper is only one of the many possible solution. A postulation of different types of cross-sections, imposition of constraints on aerodynamic surfaces result in different type of models. Similarly the design can be based on assumptions other than the availability of some experimental results in addition to finite element models.

TABLE III FREQUENCIES AFTER IDENTIFICATION

Full Scale Mode No.	Full Scale Freq. (Hz)	Corresponding Model Freq. (Hz)
7	25.95	26.09
8	35.90	35.78
9	44.27	-----
10	52.82	------
11	55.22	55.64
12	57.24	55.70
13	59.28	58.66
14	77.98	69.31
15	109.47	91.99
16	124.61	139.41

Mass added at the tip = 024225 lb. $sec^2/in.$

10. SUMMARY

In this paper the design of structural dynamic physical models has been discussed. The modelling philosophy has been to obtain a scale model that can be used for structural dynamic tests. It is further required that the model actually reproduce selected number of natural frequencies and mode shapes of the full scale structure to a reasonable degree of accuracy. Other requirements result in different models. The design is based on the best available structural dynamic information. These are the finite element models for the desired full scale structure or the component of interest. It is also required that either the solution of these finite element equations or some experimental results on the dynamic characteristics of the full scale structure be available in the form of the natural frequencies and mode shapes.

The design procedure involves the use of optimization procedures and structural dynamic identification techniques. The procedure has been illustrated by considering the specific example of the tail boom of an existing helicopter. The natural frequencies and the mode shapes of the half-scale model (calculated) have been compared with those of the full scale tail boom. The designed model is only one of the many models that can be selected.

The next logical step should be to fabricate the model and compare the model results with those of the full scale structure. One problem that is likely to be encountered is the simulation of damping.

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APPENDIX: STIFFNESS MATRIX FOR A GENERAL BEAM ELEMENT.

For a general beam element with six degrees of freedom at each node, the stiffness matrix for the element can be expressed as

$$[Ke] = \left[\begin{array}{c|c} K_{UL} & K_{UR} \\ \hline K_{UR}^T & K_{LR} \end{array}\right]$$

where the upper left partition $[K_{UL}]$, which is the only component needed to define $[Ke]$ completely, is given by

$$[K_{UL}] = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ & b_z & 0 & 0 & 0 & C_z \\ & & b_y & 0 & -C_y & 0 \\ & & & e & 0 & 0 \\ & \text{Symmetric} & & & d_y & 0 \\ & & & & - & d_z \end{bmatrix}$$

with

$$a = \frac{1}{L^2}\,\delta_A \quad , \quad e = \frac{1}{L^2}\,\delta_J$$

$$b_{\substack{y\\z}} = \frac{36}{L^4}\,\alpha_{\substack{y\\z}} - \frac{144}{L^5}\,\beta_{\substack{y\\z}} + \frac{144}{L^6}\,\gamma_{\substack{y\\z}}$$

$$c_{\substack{y\\z}} = \frac{24}{L^3}\,\alpha_{\substack{y\\z}} - \frac{84}{L^4}\,\beta_{\substack{y\\z}} + \frac{72}{L^5}\,\gamma_{\substack{y\\z}}$$

$$d_{\substack{y\\z}} = \frac{16}{L^2}\,\alpha_{\substack{y\\z}} - \frac{48}{L^3}\,\beta_{\substack{y\\z}} + \frac{36}{L^4}\,\gamma_{\substack{y\\z}}$$

$$\delta_A = \int_0^L EA\,dx \quad , \quad \delta_J = \int_0^L GJ\,dx$$

$$\alpha_{\substack{y\\z}} = \int_0^L EI_{\substack{y\\z}}\,dx \quad , \quad \beta_{\substack{y\\z}} = \int_0^L EI_{\substack{y\\z}}\,x\,dx \quad , \quad \gamma_{\substack{y\\z}} = \int_0^L EI_{\substack{y\\z}}\,x^2 dx$$

A = Cross sectional area

J = Torsional constant

E = Young's modules

G = Shear modules

I_y, I_z = Moment of inertia about the y and the z axis

In the event that the element stiffness matrix is known, one can determine α, β, γ and δ from the stiffness coefficients a, b, c, d and e.