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COMPARISONS OF THREE NUMERICAL TRIM METHODS

FOR ROTOR AIR LOADS

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ABSTRACT

Most rotor air loads programs are trimmed by an iterative process with two steps per iteration. In the first step, controls are guessed and the equations are integrated in time until all transients are decayed. In the second step, the controls are improved based upon the difference between the desired hub loads (thrust, propulsive force, side force) and the computed loads. As an alternative to numerical integration, however, recent papers have suggested a procedure called periodic shooting. The numerical shooting procedure can be used sequentially in the above, 2-step process; or it can be used in parallel with the control strategy as a unified trim method.

In this paper, these three trim methods (conventional, sequential shooting, parallel shooting) are applied to production-version rotor air loads programs. The convergence and efficiency of the methods are studied, and the converged results are compared with wind-tunnel data.

1. Introduction

Any calculation of rotor air loads requires the periodic solution to the rotor aeroelastic equations with a known set of control settings. Similarly, most dynamic stability calculations are based on perturbation equations written about a periodic equilibrium position. Therefore, calculation of rotor control settings and periodic response is a fundamental aspect of rotor analysis.

This calculation is not at all trivial, however. Even when the controls are known, it is not always easy to solve for the periodic solution. This is especially true when one or more system modes has small damping. In fact, however, the rotors controls are not known. Instead, what is known is the lift force, propulsive force, and side force desired for a flight condition. The pilot controls, therefore, also appear as unknowns in the problem.

In general, there are three categories of methods to solve for the periodic rotor response. These are: 1) Numerical Integration, 2) Periodic Shooting, and 3) Harmonic Balance. There are also three categories of methods for finding the control settings. These are: 1) Automatic Pilot, 2) Newton-Raphson, and 3) Algebraic Control Equations. Each of the three response methods (1,2,3) is particularly suited for one of the three control methods (1,2,3) in the sense that they are compatible for application in a parallel strategy. For example, numerical integration and automatic pilot are applied in Reference 1; Shooting and Newton-Raphson are applied in Reference 2; and Harmonic Balance with Algebraic Control Equations is applied in Reference 3.

Despite this compatibility, however, most production version air loads programs use numerical integration coupled with Newton-Raphson (a rather incompatible combination). The purpose of this paper is to compare three methods: 1) the conventional numerical integration with Newton-Raphson, 2) the sequential application of periodic shooting with Newton-Raphson (without capitalizing on their compatibility), and 3) the parallel application of the two methods.

2. Background

2.1 The Transition Matrix

The first step in solution of a system of linear differential equations is the determination of the transition matrix $[\phi]$. Given a set of n linear equations of the form

$$\{\dot{x}\} = [A(t)]\{x\} + b(t) \tag{1}$$

where A(t) and b(t) are periodic with period, τ, the transition matrix, $[\phi]$, is defined such that, for b(t) = 0,

$$x(t) - [\phi(t)]\{x(0)\} \qquad 0 \leq t \leq \tau \tag{2}$$

This further implies that

$$[\dot{\phi}] = [A]\,[\phi] \tag{3}$$

in practice $[\phi]$ can be found by numerical integration of equation (1) with b(t) = 0. For nonlinear systems, the equation of state will have the form

$$\{\dot{x}(t)\} = \{F(x,t)\} \tag{4}$$

It is often helpful to linearize these equations around a nominal or periodic equilibrium position $\{x_p\}$. This solution solves the equations

$$\{\dot{x}_p\} = \{F(x_p,t)\} \tag{5a}$$

$$\{x_p(0)\} = \{x_p(\tau)\} \tag{5b}$$

Now, we write equations for perturbations about $x_p(t)$.

$$x(t) = x_p(t) + \delta x(t) \tag{6}$$

where higher powers of δx are negligible compared to δx. Now if F(x,t) is smooth enough to have a Taylor series representation, then

$$\{F(x,t)\} = \{F(x_p,t_p)\} + \left[\frac{\partial f_i}{\partial x_j}\right]_{x=x_p} \{\delta x(t)\} \tag{7}$$

This leads to the equations

$$\{\delta\dot{x}\} = \left[\frac{\partial f_i}{\partial x_j}\right]_{x=x_p} \{\delta x\} \tag{8}$$

Then the transition matrix $[\phi(t)]$ can be found from sequential perturbations of each element of $\{x(0)\}$ by a small amount, say away from $\{x_p(0)\}$. The resultant perturbed initial conditions can be used in Equation (4a), and integration through one period gives a solution $\{x(t)\}$ from which $\{\delta x(t)\}$ may be obtained by

$$\{\delta x(t)\} = \{x(t)\} - \{x_p(t)\} \tag{9a}$$

or

$$\{x(t)\} = \{x_p(t)\} + \{\delta x(t)\} \tag{9b}$$

The transition matrix may be formed by dividing the $\{\delta x\}$ columns by Δ and assembling them in $[\phi]$ such that

$$\{x(t)\} = \{x_p(t)\} + [\phi(t)]\ \{\delta x(0)\} \tag{10}$$

as in Equation (7).

2.2 Periodic Shooting

The method prescribed here, periodic shooting, utilizes the transition matrix $[\phi]$ to find a periodic solution in a direct way. The first step in this procedure (once $[\phi]$ is known) is to integrate Equation (1) through one period with zero initial conditions but with $\{b(t)\}$ retained. The resultant, non-periodic solution will be called $\{x_f\}$.

It follows from linearity that the general solution to Equation (1) is

$$\{x(t)\} = \{x_f(t)\} + [\phi(t)]\ \{x(0)\} \tag{11}$$

Now a periodic solution, $\{x(0)\} = \{x(\tau)\}$ can be immediately achieved from the initial conditions

$$\{x(\tau)\} = \{x_f(\tau)\} + [\phi(\tau)]\ \{x(0)\} = \{x(0)\}$$

$$\{x(0)\}\ [I - \phi(\tau)] = \{x_f(\tau)\}$$

$$\{x(0)\} = [I - \phi(\tau)]^{-1}\ \{x_f(\tau)\} \tag{12}$$

The resultant periodic solution is obtained from substitution of Equation (12) into Equation (11). The calculation in Equation (12) is called "periodic shooting" because the initial conditions are "aimed" so as to hit the target $\{x(\tau)\} = \{x(0)\}$. We should mention here that the calculation in Equation (12) is conceptually identical (but computationally much simpler) than the method described in Reference (4).

In the case of a nonlinear system, Equation (5a), the procedure is similar to that outlined above. For example, estimated initial conditions, $\{x_E(0)\}$, can be assumed and an integrated solution found $\{x_E(t)\}$, that is not periodic but is a first estimate of $\{x_p\}$. Thus the initial conditions can be modified in an attempt to make $\{x_E\}$ periodic.

$$\{x_p(0)\} = \{x_E(0)\} + [I - \phi(\tau)]^{-1}\{x_E(\tau) - x_E(0)\} \quad (13)$$

The procedure can then be repeated with $x_p(0)$ generating a new estimate $x_E(t)$. Thus, the above algorithm can be utilized to find the periodic solution $x_p(t)$ to a nonlinear system. It should be noted that this is equivalent to a modified Newton-Raphson procedure to find the initial conditions that will result in a periodic solution.

Thus, the above method and time-wise integration (until all transients decay) stand as two alternative methods for the periodic response. The third method, harmonic balance, is not treated in this paper. Now, the complete rotor trim involves calculation of control settings and periodic response. Three possible means of effecting trim are outlined below.

CONTROL STRATEGY

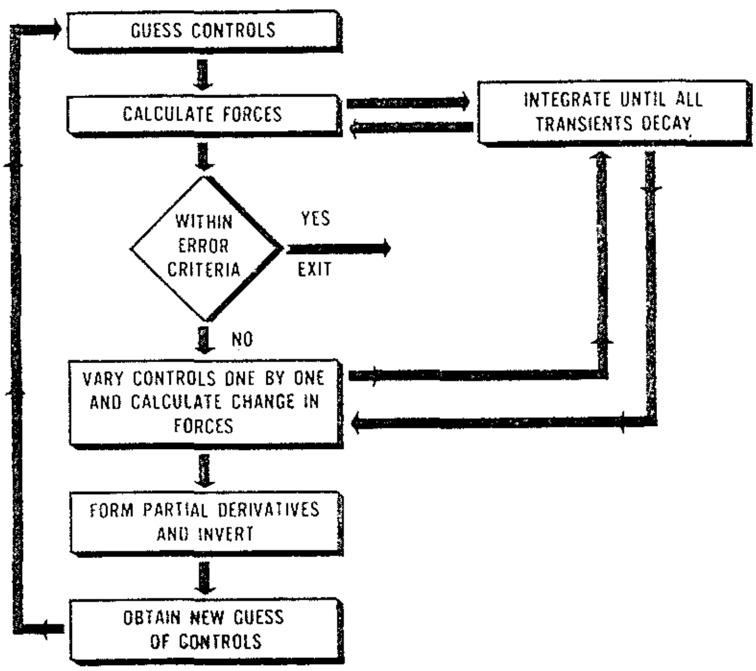

Figure 1. Flow Chart for Conventional Method

2.3 The Conventional Method

This is a method which uses a Newton-Raphson iteration procedure for convergence on controls (called control strategy) and integrates through time until a steady-state solution is found for the given initial conditions. A flow chart of this method is shown in Figure 1. It can be seen that, first, the controls are guessed. Second, the equations are integrated in time until a periodic solution is obtained (until all transients decay). Third, the forces are found. If they are within a certain error criteria, the program stops. If not, each control is perturbed; and, for each perturbation, integration in time is performed until transients decay. Fourth, a partial-derivative matrix is formed and new values for controls are found using a modified Newton-Raphson procedure.

$$\{\theta\}_{new} = \{\theta\}_{old} + \left[\frac{\partial F_i}{\partial \theta_j}\right]^{-1}\{F_{desired} - F_{actual}\} \quad (14)$$

In some airloads programs only an approximate version of $[\partial F_i/\partial\theta_i]^{-1}$ is used. Approximations include: 1) neglect of diagonal terms, 2) closed form approximations, and 3) pseudo inverses. These approximations may save computation for each iteration, but at the possible expense of requiring more total iterations.

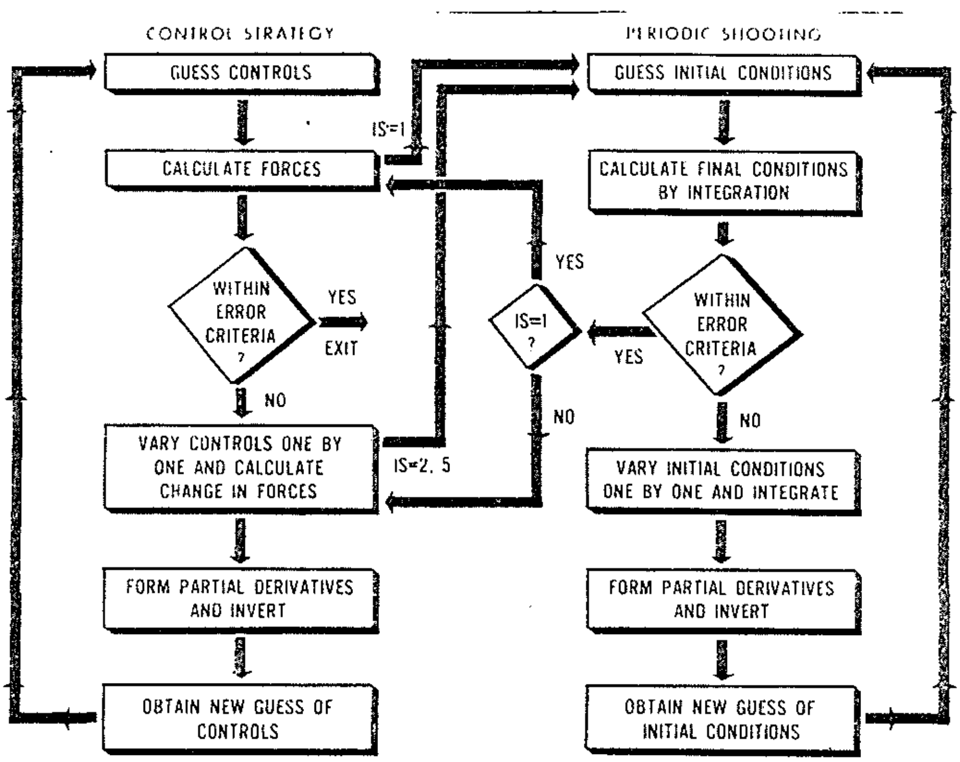

Figure 2. Flow Chart for Sequential Method

2.4 The Sequential Method

In the sequential method of periodic shooting, the right block in Figure 1 is replaced by the shooting algorithm described previously. This is depicted in Figure 2. A convergence criteria must be applied to the shooting algorithm. This is done as follows. A solution is considered to be converged when the error between each of the state variables at $\Psi = 0$ and $\Psi = 2\pi$ is less than some chosen value. Thus every time the block diagram calls for a periodic solution (i.e., at every control perturbation), a new convergence is required on initial conditions.

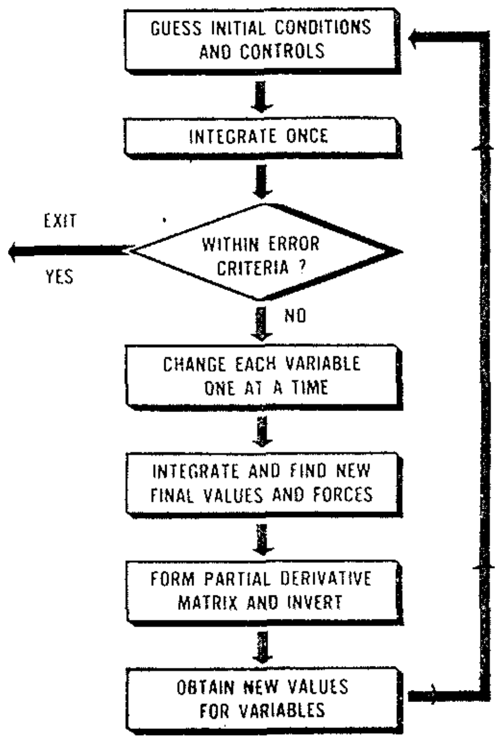

Figure 3. Flow Chart for Parallel Method

2.5 The Parallel Method

In this method the iterations on control variables and initial conditions in Figure 2 are combined into one scheme that iterates simultaneously on controls and periodicity. The procedure is similar to the former algorithm except that a single partial derivative matrix is obtained that includes the changes in forces and periodicity with respect to control settings and initial conditions. The flow chart for this refined method is given in Figure 3. Here, we have capitalized on the fact that both strategies (controls and periodic solution) are Newton-Raphson procedures. Thus, it makes sense to combine these into a single Newton-Raphson scheme with both controls and initial conditions as unknowns.

3. Application to Production Program

3.1 Discussion of Rotor Loads Basis

A test of the above described methods is provided by application to a rotor loads and performance analysis that has been developed as a subrouting for use within the AVRADCOM, Applied Technology Laboratory (ATL) V/STOL Preliminary Design Program. The importance of an efficient iteration method within a preliminary design process becomes evident as the analysis is permitted to allow more and more design variables to be considered. The basis for the applied rotor analysis is documented in the cited References 5, 6.

The basic equations are for a rigid hinged blade with hinge offset. Only flapping dynamics are considered. Calculation of the rotor loads requires the airfoil section lift and drag characteristics as well as the resultant velocity. The airfoil section characteristics are provided for section angles of attack from -180° to 180° for Mach numbers from 0. to 1.0. The use of basic steady airfoil lift and drag measured as a result of 2- dimensional transonic wind

tunnel tests is done with confidence along most of the rotor blade span. However, three separate adjustments are required to account for air flow and blade motion which can become significant depending upon the rotor operating regime. The first of these adjustments is a so-called tip relief model, derived in Reference 7, which accounts for the reduced compressibility existing in the 3-dimensional flow near the tip of a lifting surface. The tip relief model is based upon the potential representation of the thickness effect of an airfoil by a source-sink distribution. The thickness effect can be thought of as a qualitative explanation for tip relief in the sense that 2-dimensional flow requires greater displacement in a perpendicular direction than 3-dimensional flow about a finite tip. Therefore, there results a relief in the flow about the tip as compared to the 2-dimensional flow. The potential function is formulated for a finite wing by subtracting the functions for complementary wings on both sides from the function for an infinite wing (2-dimensional airfoil). Formulation in this manner relates the velocity on the 2-dimensional airfoil to that on the finite wing.

The second adjustment to the 2-dimensional airfoil data is intended to account for the radial flow conditions that exist on a rotor blade due to its yawed position present for much of the azimuthal circuit. The significant features of this method (Reference 8) include an estimate for the increased skin friction drag due to the use of the resultant velocity acting at a yaw angle to the blade element and a stall delay due to an increased lift capability evidenced in yawed flow experiments on various wings.

The third adjustment to the basic wind tunnel tested airfoil data is an approximation of the stall hysteresis with lift overshoot that occurs as a result of an airfoil oscillating near stall. The cyclic pitch variation required by a conventional rotor system causes this unsteady airfoil characteristic to have a significant effect upon calculations when the operating condition allows appreciable stall. The formulation, detailed in Reference 8, is based upon tests of four airfoil sections from 6% to 12% thick. Derived from these tests are the stall delay angles as a function of a dimensionless parameter, $\sqrt{|C\dot{\alpha}/2V|}$ (analogous to the reduced frequency parameter) where C = blade chord and V = local velocity. Linear functions have been developed for a stall delay parameter which depend on the airfoil thickness, Mach number, $\sqrt{|C\dot{\alpha}/2V|}$ parameter, and whether it is lift or moment stall which is being examined. The moment stall formulation is used to determine the unsteady drag coefficient. Reference 9 shows this to be a good approximation.

The non-dimensional integral expressions for the three rotor forces (thrust along rotor shaft, and propulsive force and side force, perpendicular to each other and the rotor shaft) are derived from the resolution of the airfoil force coefficients as they vary along the rotor blade. The integral spans the distance from the root cutout $\bar{r}_c$ to the tip (1). Tip losses, or the approximation of the loss of lift due to the finite blade, are approximated by setting lift = 0.0 at $\bar{r} = 1$ and assuming a linear variation in the lift from $\bar{r} = .97$ to $\bar{r} = 1$. The drag force coefficient used at $\bar{r} = 1$ is that which has been calculated as a result of applying the above summarized tip relief method at lift = 0.0.

3.2 Application of Iteration Methods

The application of the procedure summarized above requires an iteration method to solve for the required rotor forces and the accompanying steady state rotor blade motion. Specifically, the ATL V/STOL Preliminary Design Program requires rotor torque and tip path-plane inclination when given the rotor forces. The iteration method must provide convergence of magnitude and direction upon the resultant of rotor lift, propulsive force and side force (Figure 4). This is done by adjusting collective pitch (θ_o), longitudinal cyclic angle (θ_s), and lateral cyclic (θ_c). Steady state blade flapping magnitude and velocity must be attained. Three methods of iteration have been applied to investigate the relative efficiency of each in achieving convergence through the variation of the five variables.

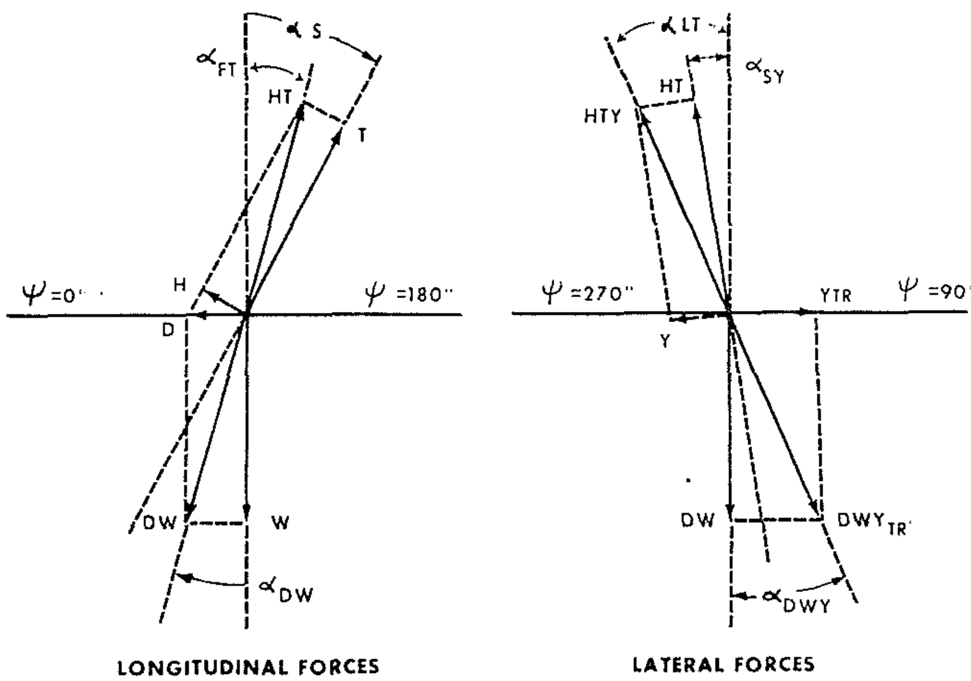

Figure 4. Force Vectors

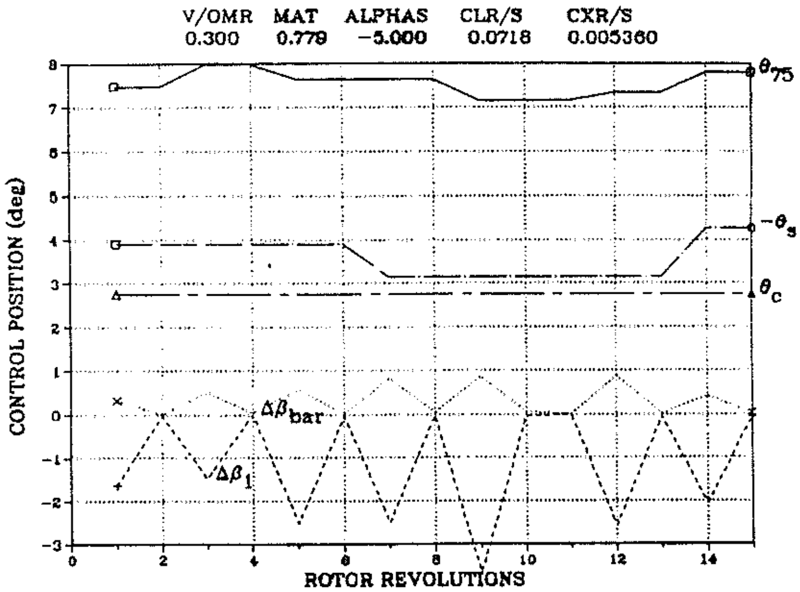

Figure 5. Step by Step Conventional Method

1) Step by step conventional method: This first method (Figure 5) steps through each one of the variables, insuring convergence within a specified tolerance before proceeding to the next variable. Steady State flapping is calculated first and then $_o$ is incremented toward the resultant vector magnitude convergence. Total force is not integrated until a steady state flapping is achieved. When the force magnitude is converged, $_s$ is incremented toward the vector direction required. The vector magnitude is then checked and then reiterated until it again is converged. This procedure is repeated until both magnitude and direction are correct. At this point, $_c$ is incremented and when the results of this perturbation are available, tests are made to check the previously achieved convergence on $_o$ and $_s$. If this test shows non-convergence the procedure is repeated from the point of non-convergence. Extrapolation and interpolation are accomplished in small enough linear steps so as to approximate the non-linearities of the problem. When enough consistent perturbations have been accomplished, the step by step procedure is deviated upon, in that when one control is incremented, enough is known about the sensitivities so that the other controls can be changed at the same time. Thus the off-diagonal terms (coupling terms) of the inverse matrix can be included. Upon changing $_s$:

$$\theta_o = \theta_{o_1} + (\alpha_{DW} - \alpha_{FT_1})(\theta_{o_2} - \theta_{o_1})/(\alpha_{FT_1} - \alpha_{FT_2}) \tag{15}$$

Upon changing θ_c:

$$\theta_o = \theta_{o_1} + (\alpha_{DWY} - \alpha_{LT_1})(\theta_{o_2} - \theta_{o_1})/(\alpha_{LT_2} - \alpha_{LT_1}) \tag{16}$$

$$\theta_s = \theta_{s_1} + (\alpha_{DWY} - \alpha_{LT_1})(\theta_{s_2} - \theta_{s_1})/(\alpha_{LT_2} - \alpha_{LT_1}) \qquad (17)$$

where 1 and 2 are from magnitude or direction converged conditions.

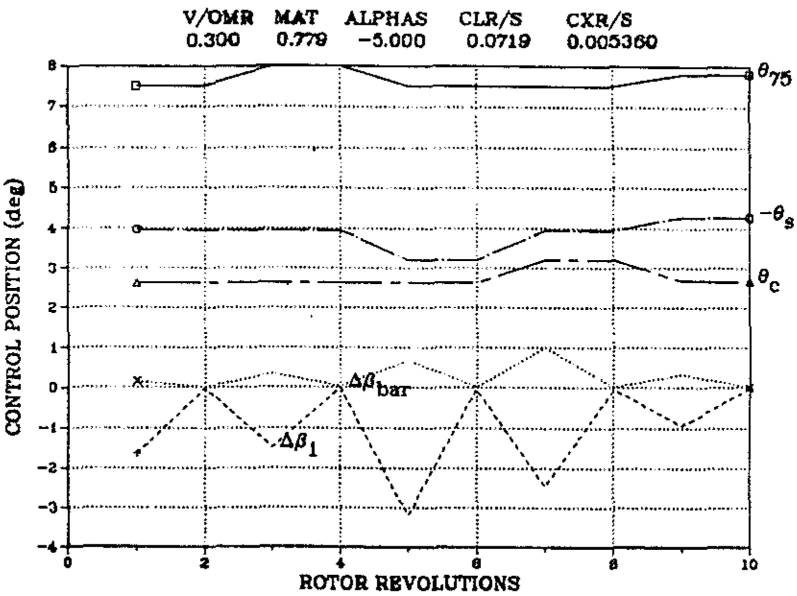

Figure 6. Newton-Raphson Sequential Method

2) Sequential Newton-Raphson: This method (Figure 6) requires the establishment of a matrix of slopes representing the sensitivity of the rotor forces to the isolated perturbation of each of the control variables. This matrix is applied, through inversion, to the Newton-Raphson equation to achieve simultaneous convergence. The method is termed Sequential because each perturbation requires first the convergence of flapping displacement and velocity. Flapping convergence is achieved through periodic shooting with the first perturbation being the value found from numerical integration. If an accurate first estimate is possible for the flap motion, numerical integration will yield a periodic solution within two revolutions for a practical articulated rotor. For the two variables included in this problem, sequential periodic shooting would require four rotor revolutions to establish the required matrix and then another (minimum) to converge, for a total of five revolutions. Because convergence is tested for every perturbation, the Newton-Raphson sequential integration (conventional method) is superior, in this application, to the Newton-Raphson sequential shooting technique. Sequential shooting would be advantageous for more blade motion degrees of freedom. When all the control variables are perturbed and the simultaneous solution of all the variables does not result in convergence, two variations of matrix update are possible. The first variation checks to see which parameter is furthest from convergence and then allows a perturbation of this single variable in order to update only the one affected matrix column. A new estimation is then made for all the variables and convergence is retested. The second variation requires perturbation of each of the variables when convergence is

not achieved, thus resulting in a completely updated matrix. This second variation is the procedure for which results are presented.

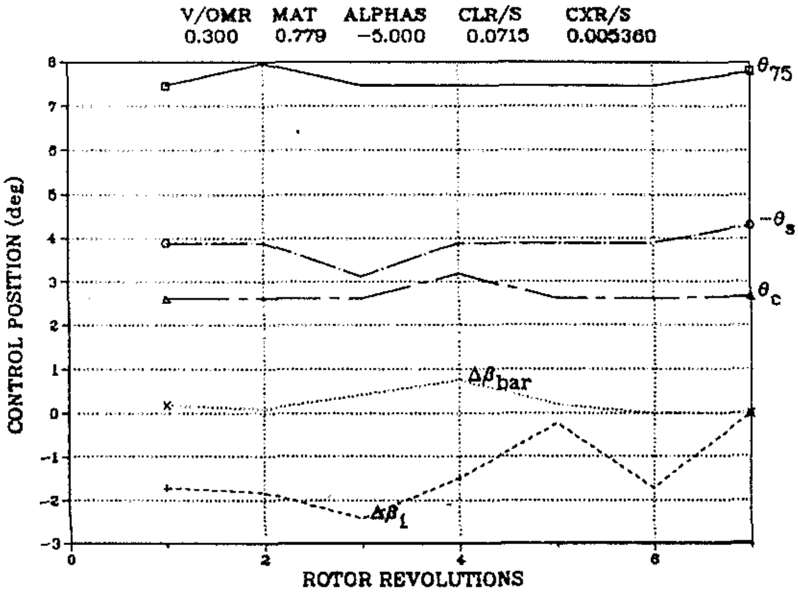

Figure 7. Newton-Raphson Parallel Shooting

3) Parallel Newton-Raphson shooting: This method (Figure 7) extends the sequential method to include in the Newton-Raphson formulation, the periodic blade motion variables. Isolated perturbation of each of the variables is used to construct a combined matrix of slopes representing the sensitivity of both the forces and periodic blade motion. The term "parallel" then, refers to the fact that the periodic blade motion is being iterated upon at the same time as the integrated force magnitude and direction.

3.3 Method of Application

Each of the above three methods has been used, on an equal basis, in conjunction with the rotor loads and performance method summarized above. An equal basis of comparison is assured by the use of the same estimates for the starting values of the control variables. For each series of calculations (each rotor shaft angle at a particular advance ratio) the first point uses the estimates for control angles and blade motion based on a closed form solution. The subsequent points use this same closed form solution for the angles, but the estimate is modified based on the differences between the initial values and the converged values for the previous point. Improvements to this scheme would include extrapolating the converged controls (angles and blade motion) based on the previous two values. The convergence criteria is the same for all cases: 1% of resultant force magnitude; 1% of resultant force direction; 10% of rotor side force; 1% of blade flapping angle and velocity (except for small angles, the tolerances for which is .001 radians). These convergence criteria are small enough such that a consistent set of data can be calculated. Overall rotor performance is relatively insensitive to side force, so the larger tolerance is acceptable.

Each perturbation includes a tolerance test on every required variable, so each time a perturbation is required, the control is perturbed in the direction toward convergence. Each time the controls are calculated as a result of the complete matrix update, they are saved to be used with their previous counterparts to predict the next value used during the subsequent perturbations. An important consideration during calculation for cases near the analytical lift limit for the rotor is to limit the extrapolated control predictions in order to avoid a condition too far into stall (beyond the required condition). The two Newton-Raphson methods which produced the results shown here do not include specific tests to contend with predictions which overshoot the target and end up too far into stall. This would be a problem only if the predicted controls required calculations in the area of the second lift rise at very high angles of attack, since the slopes would indicate iterations to even higher angles of attack. The overshoot at conditions near "maximum" lift is most critical for the step-by-step conventional method since convergence is accomplished for each individual control variable (its related force, direction, or motion) while the remaining variables are held constant. This means that, if, for some reason, the combination of controls becomes unreasonable, a false indication of lift required being greater than lift available will result. For this reason some checks are required which result in a restart at a more reasonable value for the step-by-step method.

4. Results

4.1 Preliminary Investigations

Before proceeding to the results for a production airloads program, it is interesting to compare results for a research oriented response problem as given in Reference 2. Three separate assumptions can be established for a comparison of periodic shooting with numerical integration (solution of equations of motion until tranients decay). Figure 8 illustrates the boundaries established when these assumptions are coupled to a knowledge of the stability of problem. For the sake of comparison, it is assumed that each method starts with a first guess of the initial conditions (often zero), having an error, E_o. Each method must then be pursued until a desired error, E is reached. It is also assumed that the equations are nonlinear so that the shooting method requires several iterations. For the controls known case, Figure 8 shows that for 10% damping, such as is typical of articulated rotors with dampers, direct numerical integration is always preferred, even for only one degree of freedom. For hingeless rotors, however, for which as little as 1% damping is typical, direct integration is superior only when more than 16 degrees of freedom are present. For damping less than 0.1%, as is typical in stability work, direct integration is generally inferior to the present method of periodic shooting.

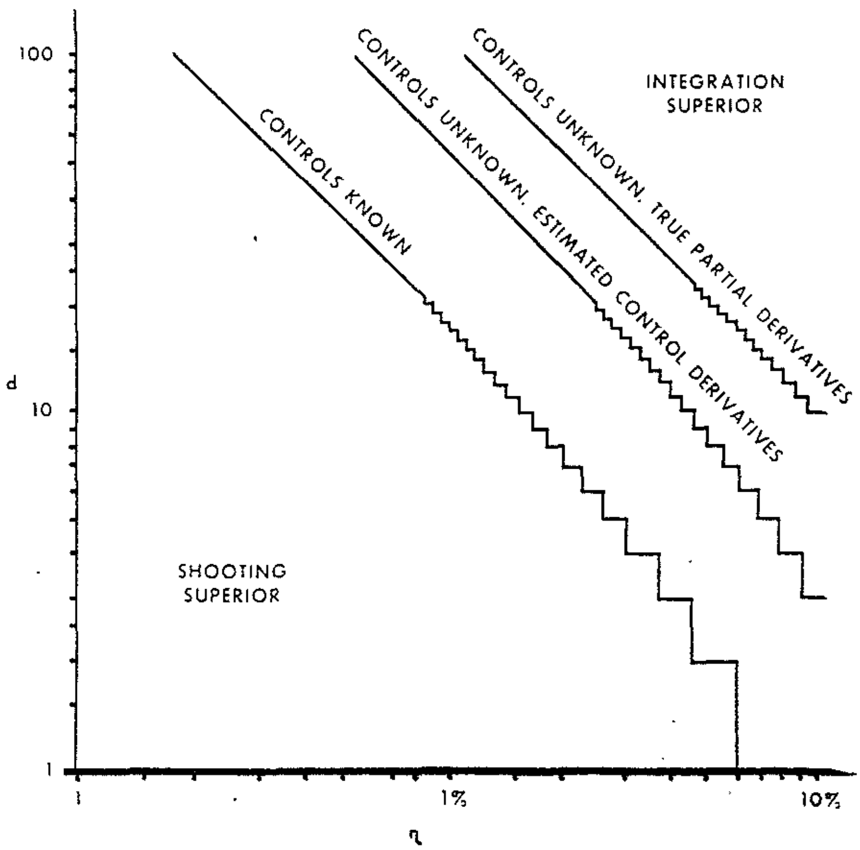

Figure 8. Comparison of Numerical Integration and Periodic Shooting

For the case when the controls are not known, the above comparison must be modified to include the fact that controls and initial conditions are found simultaneously by the combined Newton-Raphson shooting method, but are found sequentially when Newton-Raphson is coupled with direct integration. Figure 8 shows for the comparison between shooting and direct integration becomes more favorable for periodic shooting when the controls must be found. For typical articulated rotors (damping 10%) shooting is superior for less than 10 degrees of freedom and for typical hingeless rotors (damping 1%) shooting is superior for less than 100 degrees of freedom. Thus there is a great potential advantage of the shooting method over numerical integration even for large problems.

Finally, it might be argued that the potential advantage of direct integration would increase if direct integration were used with only an estimated set of partial derivatives. However, as seen in Figure 8, even for estimated derivatives, it has been found that there is still a favorable trade-off between shooting and integration.

Thus, the relative advantage of shooting is enhanced for systems with low damping. For unstable systems, (damping less than 0.0) direct integration cannot be used and so, periodic shooting (or some other method) is necessary.

4.2 Direct Applications

In order to provide a comparison of the relative efficiencies of the iteration procedures which would be indicative of what is required to undertake a complete rotor loads and performance analysis, calculations have been made of an advanced rotor design for which wind tunnel data is available (References 10, 11, 12). The simulation of the rectangular planform rotor (baseline) requires the ability to include three airfoil data tables with interpolation between adjacent ones to represent transition section characteristics (Figure 9). The airfoil data tables consist of data measured in a transonic wind tunnel at Reynolds numbers which are representative of the full scale rotor test.

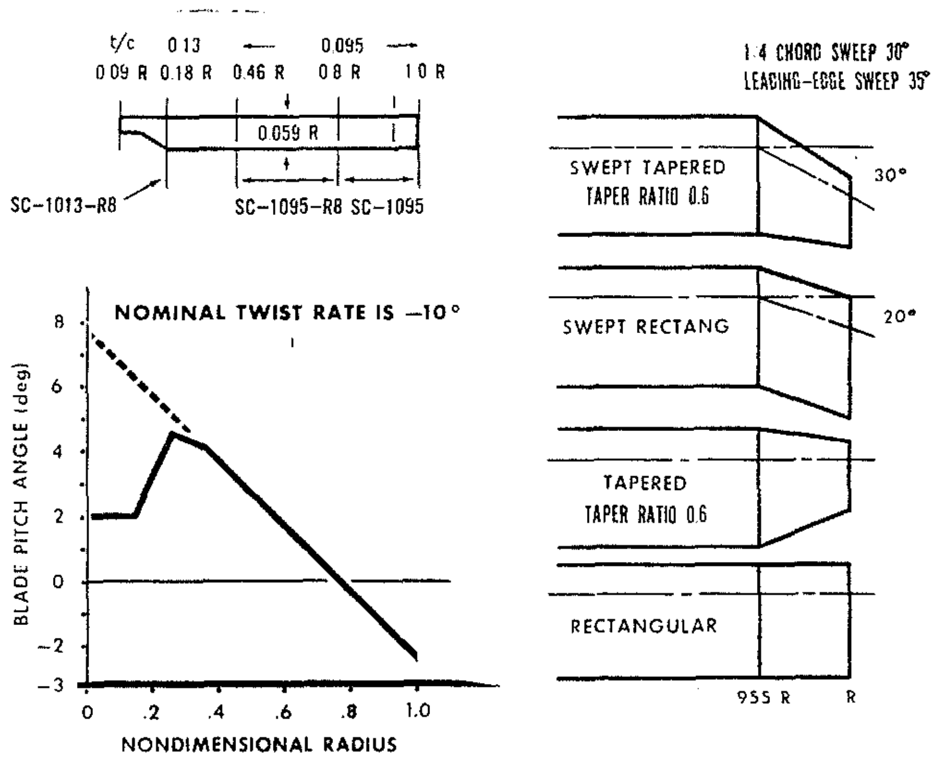

Figure 9. Blade and Tip Geometry

Figures 10, 11 and 12 illustrate the range of rotor test conditions and the number of points which were calculated to provide a comparison. The actual reported test point values (lift, drag, side force and shaft angle) are used as the calculated trim valves. This not only exercises the trim procedures to the maximum possible extent for this analysis, but also insures the calculation of the actual rotor condition as measured in the test. Although the calculated value of the relative rotor power does not, in all cases, correlate well with the test value, the trends are quite representative for the range from the autorotative to the propulsive state of the rotor.

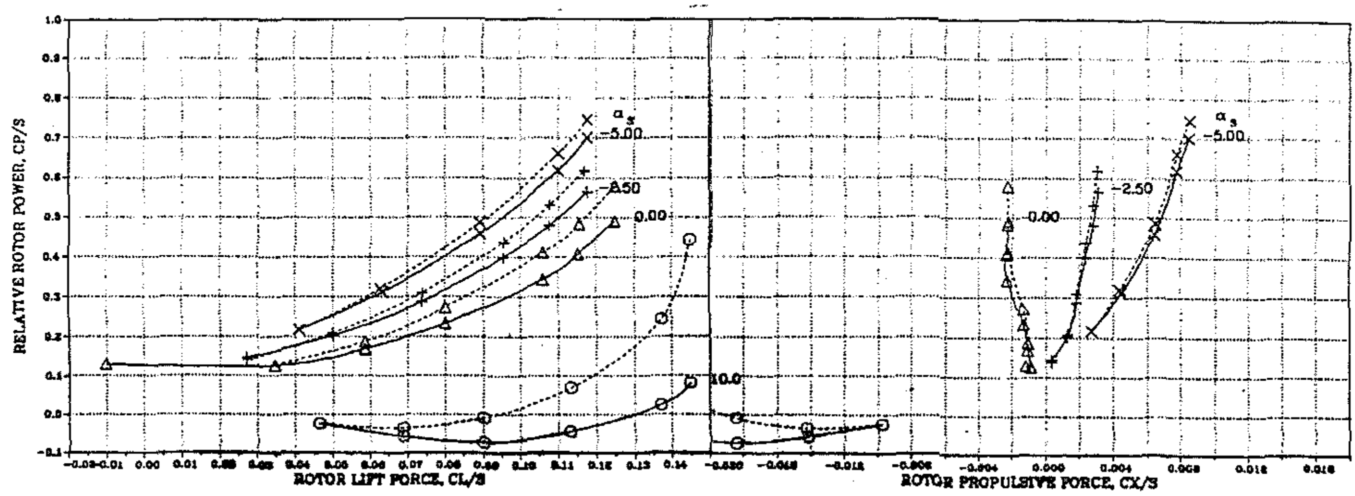

Figure 10. Advance Ratio = .2, Advancing Tip Mach Number = .72

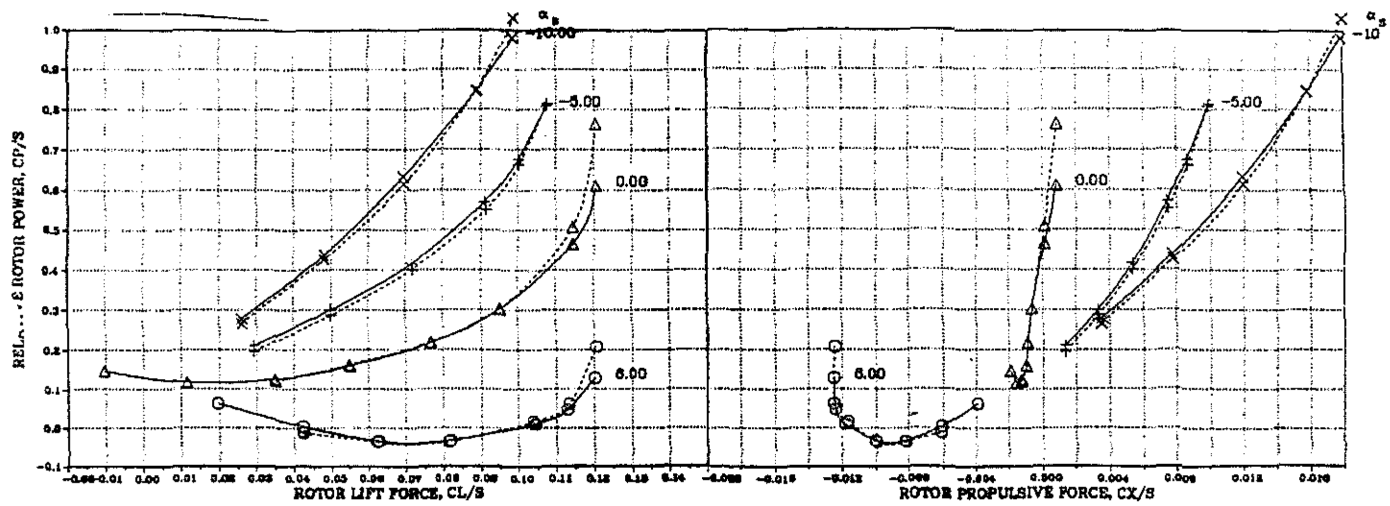

Figure 11. Advance Ratio = .30, Advancing Tip Mach Number = .78

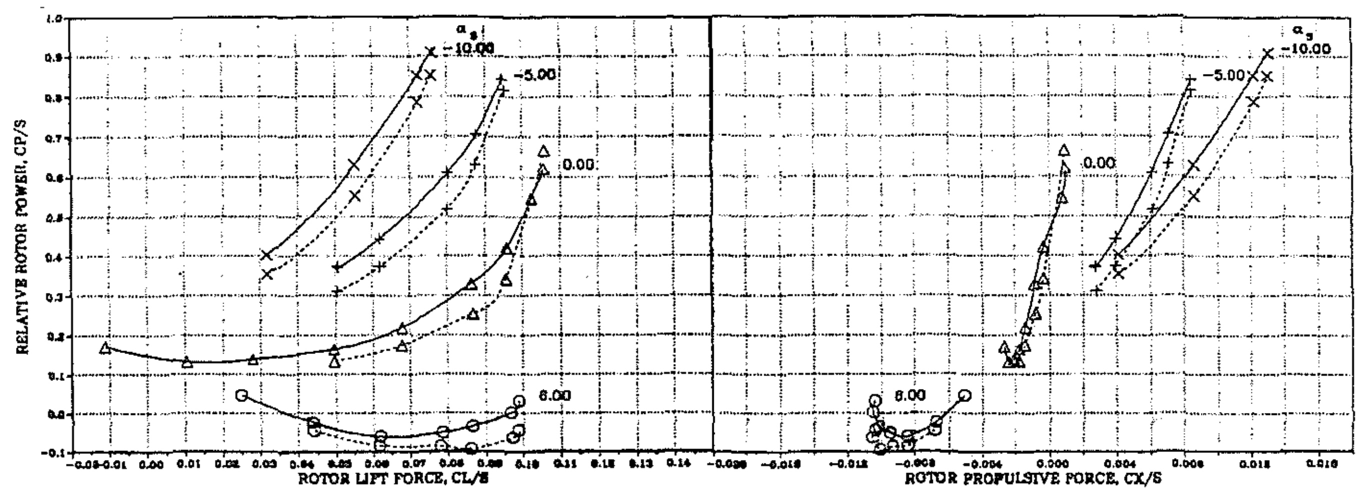

Figure 12. Advance Ratio = .375, Advancing Tip Mach Number = .825

Correlation can be obtained through the adjustment of profile drag and inflow velocity. The profile drag increment would account for the differences between the wind tunnel airfoil and the full scale section (small imperfections). The inflow velocity varible adjustment can be used (References 13, 14) to adjust the slope of the variation of relative rotor power with advance ratio. Through the comparison of the analysis with the full range of the test results, it is insured that the trim iteration methods are exercised to their useful limits. The test results represent a helicopter rotor at its maximum lift and propulsive force limits (within the power required limit of the test facility) for a wide range of inflow conditions.

A summary of the rotor revolutions required for each case is shown in Figure 13 for a comparison of each of the trim iteration methods. It is apparent that for this analysis, the Newton-Raphson and Shooting methods are superior in an overall reliability and efficiency sense. However, it is interesting to note that the parallel shooting method fails to converge at some isolated cases for a very high lift condition, where the other two methods are successful.

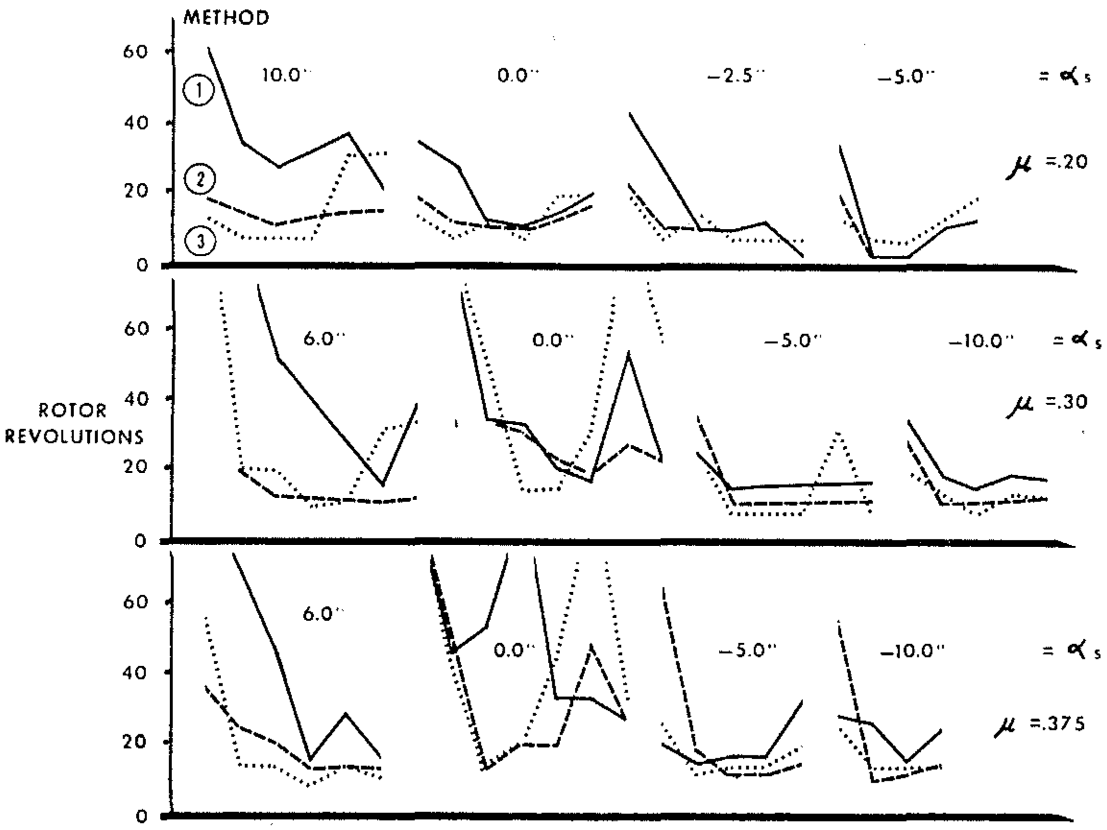

Figure 13. Summary of Convergence

5. Conclusions

The shooting method can be applied to an existing airloads program with a moderate amount of program changes. Many airloads programs are set up to remain in the azimuth loop until blade motion transients decay. This loop must be interrupted to allow only one revolution per perturbation and the resulting partial derivative then added to the matrix for the Newton-Raphson method.

For this example, parallel shooting is superior to the conventional method for about 50% of the cases. This is consistent with earlier estimates for a system with 1 Degree of Freedom and 14% damping.

The use of an approximate Partial Derivative Matrix is not satisfactory and requires an average of 2 - 8 times as many iterations as when using the full matrix.

No unusual convergence problems were encountered. The parallel shooting method and the conventional method generally failed to converge for the same cases (about 6% of the time).

The sequential application of shooting is generally much less efficient than the parallel application, requiring 3 to 4 times as many rotor revolutions.

The periodic shooting technique can be successfully applied to a rotor airloads program which includes detailed aerodynamics and dynamic stall, to calculate the full range of performance of an advanced technology operational helicopter rotor.

6. References

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