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ABSTRACT

This paper presents the results of a study into the use of H^∞- optimization for the design of feedback control laws for improving the handling qualities of a combat helicopter. H^∞- optimization is a technique for the design of robust controllers. Control laws are designed for precise control of pitch and roll attitude, yaw rate and heave velocity for a combat helicopter in hover. To meet both the performance and robustness requirements of the controller we adopt a two-degree of freedom structure, where the feedback compensator is designed to have desirable robustness properties (eg. against model uncertainty and disturbances) and the pre-compensator is designed to achieve the performance objectives of tracking accuracy, speed of response, etc. A complete separation of command response and feedback properties can be achieved with this structure.

1. INTRODUCTION

The most significant change affecting handling qualities of fixed wing aircraft, has been the adoption of FBW (Fly-By-Wire) control systems with full authority control. Current helicopters on the other hand have relied on low authority stability and control augmentation systems which are mainly effective in small perturbation manoeuvres. In particular the inherent cross coupling effects between the modes controlled by the pilot cause difficulties in handling and limit the desired flight path. This is a paramount concern when considering the tasks involved[1]. The requirement, then, on any combat helicopter flight control system (FCS) is to stabilize the aircraft and decouple the controlled inputs, thus reducing pilot workload, but still to allow fast and tailored responses to pilot demands.

In an effort to reduce piloting workload, and to improve the pilot's ability to assimilate information and take decisions, helicopter handling qualities are to be improved through the use of active control technology (ACT). "Active control" signifies that a computer is directly and dynamically involved in flight control. ACT is expected to improve helicopter fighting potential by providing crisp, tailored handling qualities and carefree manoeuvring.

Handling qualities as specified in the proposed US airworthiness design standard for rotorcraft[2] are defined primarily from the aircraft transient response perspective. This assumes single input types of stimuli. Gain and phase stability margins assume decoupled loops which can be analyzed accurately one loop at a time. Classical frequency response techniques are the basis for this criteria. The introduction of multi-input coupled dynamics poses a problem for classical design techniques[3]. Multi-mode command augmentation systems with high levels of augmentation have introduced new modes of response which do not easily fit into the criteria. Dynamic coupling in multiple control loops has also made the application of classical techniques very difficult.

Our main objective will be the design of robust controllers to improve the handling qualities in high performance helicopters in a variety of trimmed and manoeuvring flight conditions. The dynamics of the helicopter are highly interactive and inherently unstable and therefore modern multivariable techniques will be needed to meet very demanding design specifications.

An important motivation for the use of feedback in control system design is the ability to provide satisfactory performance not only for operations under known conditions but also in the presence of uncertainties. Uncertainty will always be present in one form or another when trying to model an engineering process. For instance errors may be introduced by omitting high frequency dynamics from the model, uncertainty in parameter values and linearization. It is therefore important that any controller designed to give a feedback control system certain desirable properties, does so, not only to the nominal plant model, but also for any model taken from a set containing some 'neighbourhood' of the nominal model. A controller which possesses this property is called a **robust controller**.

For single-input, single-output (SISO) systems, the classical frequency domain techniques (Nyquist diagram, Bode plot, root locus, etc) give procedures that ensure that robust stability and certain desired performance levels are achieved. However these techniques do not readily extend to the multivariable case.

For multivariable systems many attempts to provide similar quantifications for these properties have been proposed over the last two decades by MacFarlane and co-workers,[4,5] Rosenbrock,[6,7] Safonov,[8] and Postlethwaite et al.[9,10] Although these contributions succeeded in extending many of the classical frequency domain concepts to the multivariable case, they failed to address satisfactorily the issue of robustness.

Since the work of Doyle and Stein[11], maximum singular value based techniques have emerged as the most popular tools for investigating robustness of linear multivariable systems. Several considerations contribute to the effectiveness of this technique. First is the direct relationship between the size of the appropriate singular values and such properties as bandwidth. Singular value functions used in statements of design conditions can be thought of as a multi-input multi-output (MIMO) generalization of the classical Bode plots. What distinguishes MIMO from SISO design conditions are the functions used to express transfer function "size". Singular values replace magnitude, however the underlying concepts governing design remain the same. Secondly, singular value analysis can provide guarantees that system properties are maintained when the plant is subject to particular types of perturbations. Finally, the singular values required for the analysis are easily obtained using widely available and reliable computer software. More discussion of singular values can be found in various texts.[12]

Over the last two decades, modern techniques have offered the promise to relieve the design problems accompanying the control of multi-input dynamically coupled systems. Optimal synthesis techniques, primarily the Linear Quadratic Gaussian (LQG) approach, are structured to directly address the MIMO design problem. Control designs resulting from LQG synthesis were also initially thought to have very attractive stability robustness properties.[13] Additional research determined that these properties held only for the "full state" measurement[14] which is difficult to achieve in practice. Attempts have been made to "recover" these robustness properties using "Loop Transfer Recovery" (LTR) methods, but these have the disadvantage of being asymptotic and the procedure is limited to minimum phase systems at present. The interested reader is referred to a recent exposition by Stein and Athans[15] for a tutorial overview of the LQG/LTR design procedure.

In recent years, the use of the H^{∞} approach to control system design has gained popularity over the traditional LQG techniques. Originating from the seminal work of Zames,[16] this approach has succeeded in generalizing several important classical concepts in SISO systems to the multivariable case.[17,18] In particular the H^{∞} approach is more effective than the linear quadratic approach in characterizing robustness with respect to plant variations.[16,19] The importance of the H^{∞}- norm stems from its natural characterization of uncertainty. The H^{∞} norm of a transfer matrix is the maximum over all frequencies of its largest singular value. Singular values provide information in terms of guaranteed bounds on system performance or tolerable perturbations. The norm is used to place an upper bound on the amount of uncertainty present in a plant. By using singular value analysis of appropriate return difference matrices it is possible to assess the degree of stability, sensitivity reduction and disturbance attenuation that is present in a given design. This paper deals with the design of feedback controllers which minimize the H^∞ norm with the constraint that the feedback system remains stable with respect to plant perturbations.

In general, every control problem possesses two natural degrees of freedom corresponding to the availability of an exogeneous input r and sensor output y (fig. 1). To utilize both degrees of freedom fully, it is necessary[20] for the controller to generate an output

$$u = K_1 r - K_2 y \tag{1}$$

where u denotes the input to the plant G, and K_1, K_2 are arbitrary transfer matrices subject solely to the constraint that the closed loop system is stable.

The compensator $K = [\ K_1\ K_2]$ is referred to as a two-degree of freedom compensator.[20] It can be shown[21] that K_1 and K_2 are independent and therefore it is possible to break down the design procedure into two stages:

(i) Synthesize K_1 for desirable command response

(ii) Synthesize K_2 for desirable feedback properties eg. disturbance rejection, stability, sensitivity and robust stability.

Thus this controller structure allows freedom to specify desirable command response for handling qualities and independently to provide "good" robustness properties. In section 2.3 we show how to synthesize this controller using a method developed by Moore et al.[22]

To illustrate the design technique a compensator is designed for a single main rotor helicopter with blade dynamics, in the hover flight condition. An attitude-command/attitude hold (ACAH) control system is chosen, since it is considered to be a requirement[2] for satisfactory handling qualities in hover and low speed NOE (nap-of-the-earth) flight in the day/visual environment. The rotor dynamics are neglected in the nominal model and are treated as uncertain. The objective is to improve handling qualities relative to the raw helicopter by meeting design requirements. The criteria used for assessing system performance are based on the proposed US handling qualities specification.[2] The proposed US specification is a major revision of the much older MIL-H-8501A helicopter handling qualities specification.[23]

2. CONTROL SYSTEM DESIGNS AS H^∞- OPTIMIZATIONS

To begin we will define the Hardy space H^∞ to consist of all complex-valued functions G(s) of a complex variable s which are analytic and bounded in the open right half-plane. We can think of this as being the space of all stable rational transfer functions. Many practical feedback design problems can be formulated as the minimization of the H^∞- norm of an appropriately weighted closed-loop transfer function matrix. The minimization of such a norm is now well understood,[24] and all the computations can be carried out using state space representations of the transfer function matrices.[25,26]

The systems are modelled as linear, time invariant, and finite dimensional and they operate in continuous time with a state-space and transfer function representation in the Laplacian s given by:

$$G = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \text{ and } \quad G(s) = C(sI - A)^{-1}B + D \qquad (2)$$

The system G is said to be stable if the state matrix A has no eigenvalues in the closed right-half plane. Suppose G is stable; then the H^∞ norm of G is defined as $||G||_\infty = \sup_\omega \bar{\sigma}[G(j\omega)]$, where $\bar{\sigma}[G(j\omega)]$ denotes the largest singular value of G at frequency ω.

The compensation configuration depicted in fig. 2 will be referred to as the Standard Compensator Configuration (SCC). The objective is to design a controller K, for the plant G such that the input/output transfer characteristics from the external input vector v to the external output vector e is desirable, according to some engineering specifications. The exogenous input v, typically consists of command signals, disturbances, and sensor noises; u is the control signal; e is the output to be controlled, its components typically being tracking errors, filtered actuator signals, etc.; and y is the measured output.

Let M denote the closed-loop transfer function matrix mapping external input v to external output e. The H^∞ approach is to design a controller K such that the H^∞ norm of M is minimized. In other words, our objective is to solve the following optimization problem.

$$\min_{K \text{ stabilizing}} ||M||_\infty , \qquad (3)$$

where the minimization is over the **whole set** of stabilizing controllers.

2.1 Weight Selections in H^∞ Design

Weighting functions are used to emphasize (de-emphasize) maximum singular values of M at various frequencies so that engineering objectives can be incorporated into the optimization procedure (3) as:

$$C := \min_{K \text{ stabilizing}} ||W_o M W_i||_\infty , \qquad (4)$$

where W_o and W_i are weights to be chosen, and C is the minimum cost. The engineering requirements are fully specified by the chosen weights. An optimal controller achieving the minimum cost C can be computed automatically within a Computer-Aided-Design (CAD) environment such as **Stable-H**,[27] an H^∞ design package developed at the University of Oxford. In general the weights we use are diagonal transfer function matrices whose diagonal elements are constants, low pass or high pass filters.

By suitable choice of weights we are able to achieve the following objectives:

1. Time-domain specifications such as actuator travel and rate limits, rise time and damping for closed loop step responses

2. Characterization of achievable performance bounds

3. Robustness to unmodelled dynamics

2.2 Classes of H^∞ Design Problems

Consider the feedback configuration depicted in fig. 3. The transfer function mapping d to y and r to e is called the **sensitivity** matrix and is represented by the symbol S, where $S = (I+GK)^{-1}$. The transfer function mapping r to y is called the **complementary sensitivity** matrix, and is represented by T where $T = I - S$. Finally, the transfer function mapping r to u is simply KS. The minimization of S can be considered to be a performance requirement, and the minimization of T and KS, as robust stability requirements.[21] A bound on the gain of KS can be thought of as a design constraint reflecting the actuator saturation limits.

2.3 Two-Degree of Freedom Controllers

The two-degree of freedom structure is conceptually illustrated in fig. 4 and can be thought of as a model-following concept. The figure is divided into the command model, feedforward and stabilization. The stabilization section provides disturbance rejection and improves the model-following performance in the presence of unmodelled dynamics (eg. rotor and actuator dynamics). The stabilization loop bandwidth is generally set as high as possible. The command model may be generated using any design technique or may contain simple transfer functions which represent the desired handling quality characteristics, as determined from simulation or flight experiments. This capability to set stabilization and command response characteristics independently is a key advantage of the two-degree of freedom structure.

The synthesis of the controller can be broken into two stages as follows.

(i) Design a controller K^* for the plant G, to have "desirable" transfer functions from r --> u and r --> y. Denote these by

$T_1^* = K^*(I + GK^*)^{-1}$ and $T_2^* = (I + GK^*)^{-1}GK^*$ respectively

K^* is designed to meet performance requirements for the nominal plant G.

(It is important to note that K^* may be realized with any design technique eg. LQG, pole placement, etc.) We wish to improve the robustness of the design while keeping T_1^* and T_2^* invariant, therefore,

(ii) Design K_2 to have "desirable" robustness properties using H^∞ optimization.

Recall we seek to design a compensator which processes r and y in an independent manner as $u = K_1 r - K_2 y$ (fig. 1). The corresponding transfer functions from r --> u and r --> y for such a controller are

$$T_1 = K_1(I + GK_2)^{-1} \quad \text{and} \quad T_2 = (I + GK_2)^{-1}GK_1 \qquad (5)$$

It can be shown[22] that necessary and sufficient conditions for model matching are:

$$T_1 = K_1(I + GK_2)^{-1} = K^*(I + GK^*)^{-1} = T_1^* \qquad (6)$$

and

$$T_2 = (I + GK_2)^{-1}GK_1 = (I + GK^*)^{-1}GK^* = T_2^* \qquad (7)$$

which implies that $K_1 = T_1^* + K_2 T_2^*$, and K_2 is stabilizing.

Thus the only requirement on K_2 is that it is stabilizing as a single-degree of freedom controller as shown in fig 5. The structure of fig 5 is verified since its transfer function is

$$(I + GK_2)^{-1}GT_1^* + (I + GK_2)^{-1}GK_2T_2^*$$

$$= (I + GK_2)^{-1}G\,(T_1^* + K_2T_2^*)$$

$$= (I + GK_2)^{-1}GK_1 = T_2^*$$

which is invariant of K_2, given that K_2 is stabilizing.

3. STABILITY ROBUSTNESS

All control system design is based on a model of the plant; however it is inevitable that the model we use is only an approximation of the true system dynamics. The difference between the model on which the design is based and the true system used in the actual control is the model uncertainty. Typical sources of uncertainty include unmodelled (high frequency) dynamics, neglected nonlinearities, effects of deliberate reduced order modelling, sensor noise and plant parameter perturbations due to environmental factors such as temperature, air speed and age. Uncertainty can be represented in the form of an additive or multiplicative perturbation, as shown in fig. 6. For the multiplicative case of fig. 6(b) the true transfer function is

$$G(s) = G_o(s,\theta)[I + \Delta(s)] \quad (8)$$

where $G_o(s,\theta)$ is a parameterized model of the plant with structured uncertainty θ, which represents the plant-parameter variations. $G_o(s,\theta)$ is a known function (known structure), but the values of the parameters θ are uncertain. The function $\Delta(s)$ is a stable unstructured uncertainty and is unknown, except that it is limited in magnitude by some function of frequency as

$$||\Delta(j\omega)|| \leq r_o(\omega) \quad (9)$$

where $r_o(\omega)$ is a known real scalar function. The bound can be viewed as a frequency dependant "radius of uncertainty" of the true plant G(s) about some model $G_o(s,\theta)$ for a given θ. In general, a good model will be well known at low frequencies resulting in small $\Delta(\omega)$ for $\omega << \omega_1$, and less well known at high frequencies where we have large $\Delta(\omega)$. Doyle[11] has derived the following stability robustness tests based on encirclements of the Nyquist diagram for additive and multiplicative plant perturbations:

for an additive plant perturbation $G_o(s) + \Delta(s)$ the system will remain stable with compensator K (designed for the the nominal plant $\Delta(s) = 0$) if

$$\bar{\sigma}(\Delta) < \underline{\sigma}(KS^{-1}) \quad (10)$$

similarly for a multiplicative plant perturbation $G_o(s)[I + \Delta(s)]$ the system will remain stable provided

$$\bar{\sigma}(\Delta) < \underline{\sigma}(T^{-1}) \quad (11)$$

where $\bar{\sigma}$ and $\underline{\sigma}$ denote the largest and smallest singular values.

Thus as long as we can satisfy the above bounds the system is guaranteed to remain stable in spite of the perturbations $\Delta(s)$. The largest singular value of a matrix is precisely the H^∞- norm. Therefore the use of this norm is natural when trying to characterize stability robustness.

4. CONTROL DESIGN

To illustrate the technique outlined in this paper, a controller has been designed for a single main rotor helicopter. The helicopter without augmentation would exhibit unsatisfactory response in hover.[28] The responses to the collective, longitudinal cyclic, lateral cyclic and pedals are highly coupled and unstable in the hover. Pilot workload is high and precise control is difficult without augmentation. Responses to gust inputs lead to poor ride qualities and difficulty in delivering weapons.

We will restrict attention to the design of the stabilization loop in the two-degree of freedom compensator since, as mentioned in section 2.3, the command model may be realized through any design technique or via flight simulations. A comprehensive nonlinear model of the helicopter was provided by the Royal Aircraft Establishment (RAE), Bedford, for control system design and simulation studies.[29,30] The model includes actuator dynamics modelled as first order lags, with rate and amplitude limits.

We will consider the helicopter in the hover flight condition. The basic linear model of the helicopter has 8 states and 4 inputs, and is unstable and non-minimum phase. If we include the rotor flapping dynamics a 14 state model is obtained, but we propose to treat the rotor dynamics as uncertain and leave them out of our nominal plant description. We will also ignore the actuator dynamics in the linear description of the model and design our controller to be robust to these neglected dynamics.

Robustness is a primary issue in the design because of model uncertainty (especially that due to the omission of high frequency rotor dynamics and actuator dynamics), sensor noise and wind gusts. The maximum achievable bandwidth is determined by the high frequency dynamic characteristics, especially those which are due to the rotor dynamics; high gain feedback may aggrevate these. This problem has been reported by Tischler[31] in the implementation of the ADOCS (Advanced Digital Optical Control System) system in which a 40% reduction of feedback gains was necessary because of unmodelled rotor lead-lag dynamics. Tischler also reports problems in implementing high-bandwidth systems whose designs are based on time-domain methods such as LQG. This has been a result of the compensators being too highly tuned to the assumed high-frequency dynamics so that when the compensators were used on the actual system with additional unmodelled dynamics, unacceptable system sensitivity and instability resulted. Frequency domain techniques such as H^∞ are able to expose the high frequency details more easily and integrate with current proposed handling quality specifications which are based on frequency domain criteria.

It is desired to have pilot longitudinal stick commands correspond to pitch attitude, lateral stick to roll attitude, collective to heave velocity and pedal position to yaw rate. The performance objectives are to minimize cross-coupling, to achieve improved stability margins, and to meet specifications on time response characteristics required for good handling.

The following 6 outputs are selected for control: height rate ($\dot{h}$), roll rate (p), pitch rate (q), heading rate ($\dot{\psi}$), pitch attitude (θ), and roll attitude (ϕ). The outputs, roll rate and pitch rate are not to be controlled directly but are included to improve control. Table 1 shows the required response types for each axis of control as specified for Hover and Low Speed.[2]

The state space description of the linearized rigid body equations of motion are expressed in the standard form as

$$\dot{x} = Ax + Bu , \qquad (12)$$
$$y = Cx$$

where the state vector x and the input vector u are :-

$$x = \begin{bmatrix} \theta \\ \phi \\ p \\ q \\ r \\ u \\ v \\ w \end{bmatrix} \begin{matrix} \text{pitch attitude} \\ \text{roll attitude} \\ \text{roll rate} \\ \text{pitch rate} \\ \text{yaw rate} \\ \text{forward velocity} \\ \text{lateral velocity} \\ \text{vertical velocity} \end{matrix} \qquad u = \begin{bmatrix} \theta_o \\ \theta_{1s} \\ \theta_{1c} \\ \theta_{ot} \end{bmatrix} \begin{matrix} \text{collective} \\ \text{longitudinal cyclic} \\ \text{lateral cyclic} \\ \text{tail rotor collective} \end{matrix}$$

$$y = \begin{bmatrix} \dot{h} \\ p \\ q \\ \dot{\psi} \\ \theta \\ \phi \end{bmatrix} \begin{matrix} \text{height rate} \\ \text{roll rate} \\ \text{pitch rate} \\ \text{heading rate} \\ \text{pitch attitude} \\ \text{roll attitude} \end{matrix}$$

The A, B, and C matrices were obtained by numerical linearization of the nonlinear analytical model of the helicopter[29] and are given in table 2.

The specification for the closed-loop system is to design a compensator for the controlled outputs in such a way as to satisfy the handling qualities criteria outlined in the proposed airworthiness standard.[2] Some typical requirements for the pitch and roll axes are given in table 3.

4.1 Problem Formulation and Weighting Function Selection

Let the helicopter transfer function be G and define the sensitivity function S as $S := (I + GK)^{-1}$. We propose to find a stabilizing controller K that minimizes:

$$\left\| \begin{matrix} W_1 S W_3 \\ W_2 K S W_3 \end{matrix} \right\|_\infty \tag{13}$$

where the diagonal weights W_1, W_2, W_3 are chosen to meet the design objectives. The problem is illustrated in fig. 7. By defining v = r and $e = (e_1, e_2)^T$ we see how the problem can be transformed into the SCC of fig. 2.

Selection of W_1

The Bode magnitude plots of the weighting function W_1 are shown in fig. 8. First order high gain low-pass filters are used on $\dot{h}$, $\dot{\psi}$, θ and ϕ to ensure that these outputs can be controlled accurately (d.c. sensitivity of

0.2%) with good disturbance attenuation up to about 6 rad/s. With only 4 plant outputs, no attempt is made to control directly the extra rate outputs p and q at low frequencies, but second order band-pass filters are used on each of these variables to reject disturbances and cross-coupling effects in the frequency range 5 to 8 rad/s. The frequency response of W_1 approximates integral action in each of the four controlled loops; hence it forces the resulting closed loop system to have almost perfect steady-state disturbance rejection. The low-pass filters on $\dot{h}$, $\dot{\psi}$, θ and ϕ are given a finite attenuation which has the effect of reducing overshoot in these channels.

Selection of W_2

First-order high pass filters are used on each of the plant inputs and are shown in the Bode diagram of fig 9. A cut off frequency of about 10 rad/s is used on each of the heave, pitch and roll loops to limit the system bandwidth, and also to limit the magnitude of the poles of the controller. A higher cut off frequency is used on the directional loop to allow tighter control of this loop. A low frequency gain of -100dB was used so that W_1SW_3 clearly dominates the cost function at low frequencies.

Selection of W_3

The diagonal weighting function W_3 is chosen to be a constant matrix with a weight of 0.1 on each of the rates and 1 on each of the other output demands. The reduced weighting on the rates (which are not directly controlled) is chosen so that some disturbance rejection is obtained on these outputs without them significantly affecting the cost. That is the primary aim of W_3 is to force good tracking in $\dot{h}$, $\dot{\psi}$, θ, and ϕ.

4.2 Analysis

This section will summarize the analyses of the helicopter control system and will concentrate on linear analysis, although extensive nonlinear simulation was also used to verify the control laws.

4.2.1 Robustness to unmodelled rotor dynamics

Since the collective, longitudinal cyclic, and lateral cyclic controls are all implemented with the main rotor, model uncertainty can be parameterized as a multiplicative perturbation at the four inputs. The structure of the perturbation rules out uncertain couplings between tail rotor and the other three inputs but does not account for bounded uncertain couplings between collective, longitudinal cyclic and lateral cyclic.

Fig. 10 shows the uncertainty Δ modelled as a multiplicative plant perturbation and superimposed is the frequency response of $\underline{\sigma}(T^{-1})$. From this we can see that our robustness test is satisfied for frequencies greater than 0.1 rad/s. Although the test fails for frequencies less than 0.1 rad/s this does not mean that the system is unstable at low frequencies, as in general the test is conservative. The multiplicative uncertainty has structure that is not accounted for in comparing the maximum singular value to the desired upper bound. Structured perturbations can be dealt with by the small μ-test of Doyle,[32] but at present there is no synthesis technique for this.

4.2.2 Sensitivity

A plot of the singular values of $(I + GK)^{-1}$ is shown in fig 11. These four singular values correspond to the sensitivity between the four inputs: collective, longitudinal and lateral cyclic, and tail rotor collective to the four directly controlled outputs $\dot{h}$, θ, ϕ, and $\dot{\psi}$ respectively. Note from this that we have forced the sensitivity function to be small up to a frequency of about 6 rad/s consistent with our choice of W_1. This simultaneously allows good disturbance attenuation and tracking performance.

4.2.3 Complementary Sensitivity

A plot of the singular values of $GK(I + GK)^{-1}$ is shown in fig. 12. The two largest singular values, corresponding to the pitch and roll loops, exceed our desired bandwidth of 10 rad/s for stability robustness. The other two loops, the heave and directional loop satisfy our constraint as evident from the plot. This violation of the bandwidth requirements is not drastic, because our constraint is more stringent than is required due to the multiplicative uncertainty having structure that is not accounted for. This was confirmed by extensive time simulations. To reduce the bandwidth of these loops, all that is required is to reduce the weighting on these loops in the sensitivity minimization part. However this will lead to a reduction in performance which was considered to be a less desirable option.

4.2.4 Time Simulations

The response of the helicopter to the four pilot commands was simulated in the TSIM[33] environment using the nonlinear model, HELISIM3,[29] provided by the RAE. This nonlinear model contained blade flapping dynamics and actuator dynamics which were initially omitted from the nominal linearized description of the plant. These simulations showed that the responses to pilot commands were significantly more decoupled compared to the unaugmented system. Fig. 13 shows a step command of 30° of bank angle. It can be seen from the plot that the desired response is achieved in 1·5 secs consistent with the requirement given in table 3. A slight drift in the roll angle is seen due to the directional stability of the helicopter trying to turn the craft from its original heading and causing the nose to drop. The increase of speed from 0 to 40 kts represents a considerable change in the dynamic characteristics of the helicopter. This change in the characteristics of the helicopter represents unmodelled dynamics with change in flight envelope. Recall that our nominal model is linearized about the hover. It has been shown that the controller has been able to maintain performance throughout without the need for gain scheduling.

Fig. 14 shows a corresponding 15^{o} step demand in pitch angle. The desired response is achieved in less than 1·5 secs with little coupling in the other channels. Figures 15 and 16 show the response to a step command of 10 ft/s in heave velocity and 0.2 rad/s step command in yaw rate repectively.

The pitch-to-roll and roll-to-pitch coupling are well within the limits of table 3. The addition of the actuator dynamics did not cause the nonlinear time responses to differ greatly from the linear time responses and stability was not affected.

5. CONCLUSIONS AND RECOMMENDATIONS

Control laws for a typical combat helicopter in the hover flight condition have been presented. The control laws form the stabilization loop of a control structure for tailoring pilot commands to meet specified criteria. A new multivariable design algorithm based on the minimization of H^{∞} norms has been used to confer robustness to the resulting closed loop system. The performance and stability robustness of the control laws were presented in terms of singular values of specific frequency reponses. The loop shaping procedure for designing the control laws using H^{∞} optimization was presented and then analyzed with respect to the singular values of the sensitivity and complementary sensitivity functions.

It is well known that control in the hover is difficult due to the rapid and significant changes in dynamics when manoeuvring from this position. Throughout the design procedure a nominal model of the helicopter trimmed about the hover position has been used. The controller designed has fixed gains relative to the nominal model. The nonlinear time simulations show the design to be robustly stable to the unmodelled rotor and actuator dynamics and to unmodelled dynamics due to changes in the flight envelope. By constraining the allowable actuator response (by minimizing KS), nonlinearities due to limiting have not arisen with the limits modelled.

Areas in which further research will need to be aimed are the effects of structural dynamic modes and digital implementation of the control laws.

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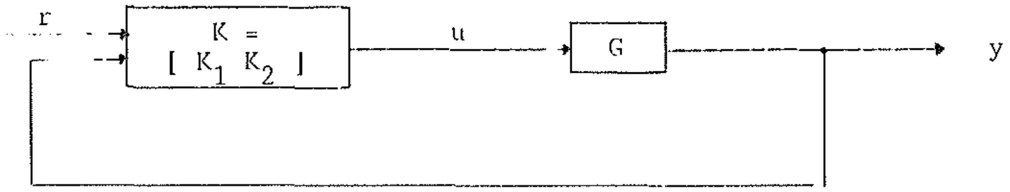


Fig. 1 Two Degree-of-Freedom Controller

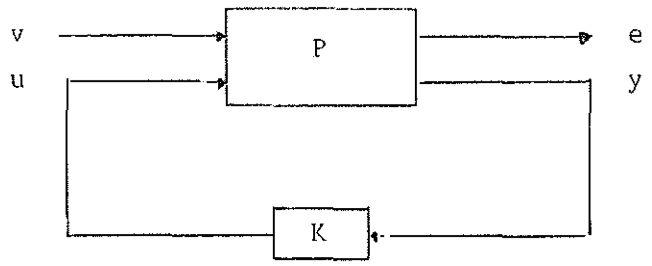


Fig. 2 Standard Compensation Configuration

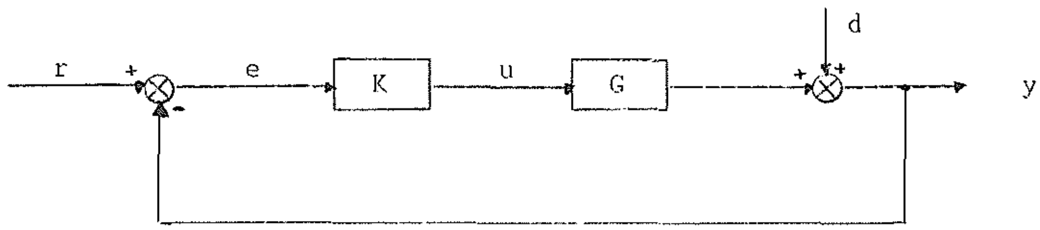


Fig. 3 Feedback Configuration

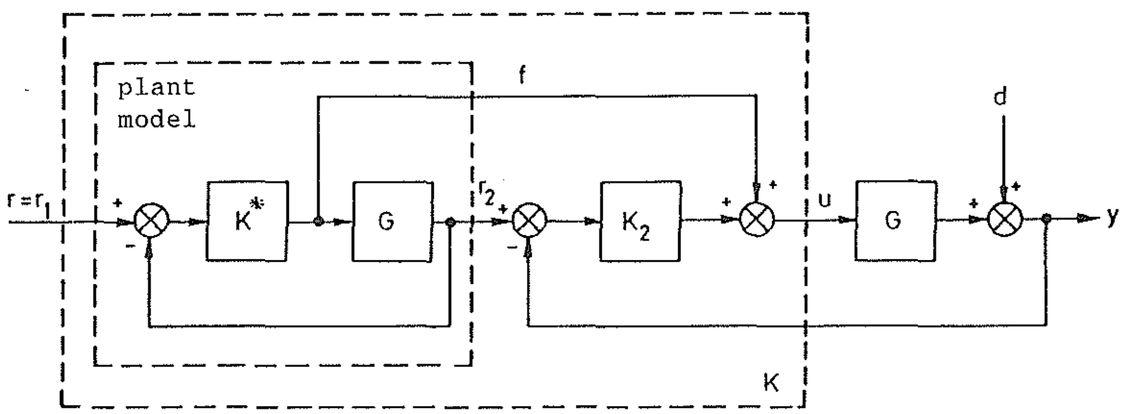


Fig. 4 Conceptual Configuation for a TDF Controller

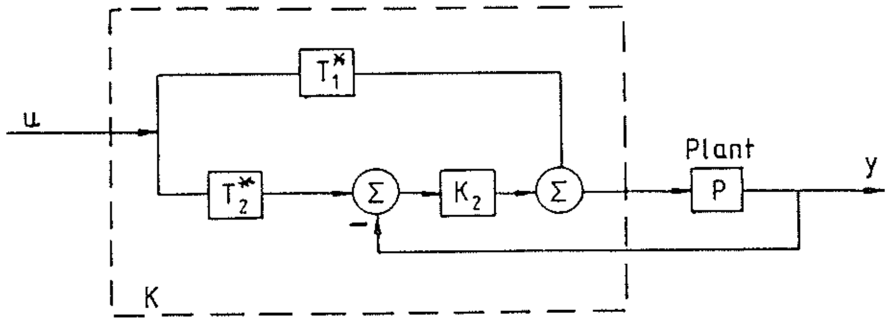
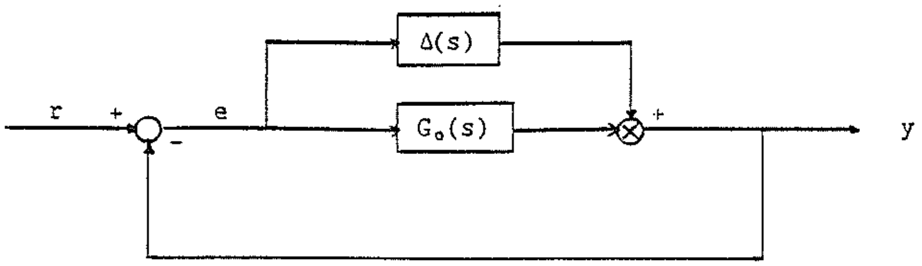
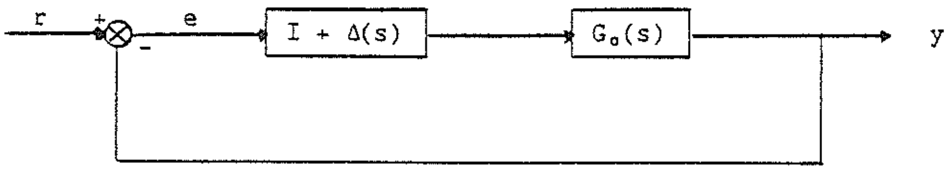


Fig. 5. Model-matching controllers.



(a) Additive Perturbation



(b) Multiplicative Perturbation

Fig. 6 Representation of Uncertainty

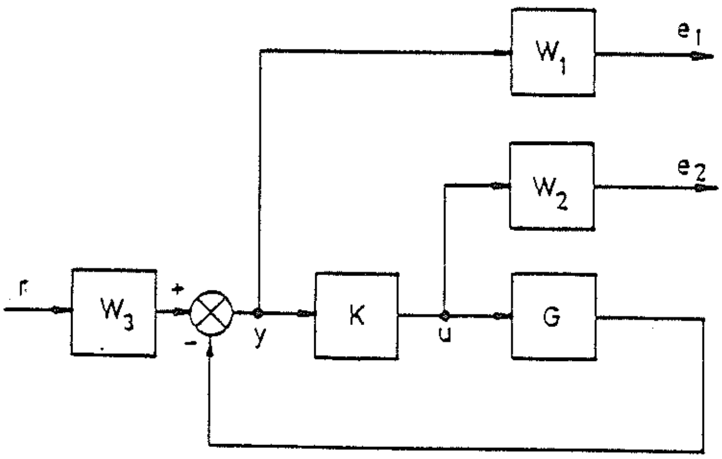


Fig. 7 Helicopter Control Problem

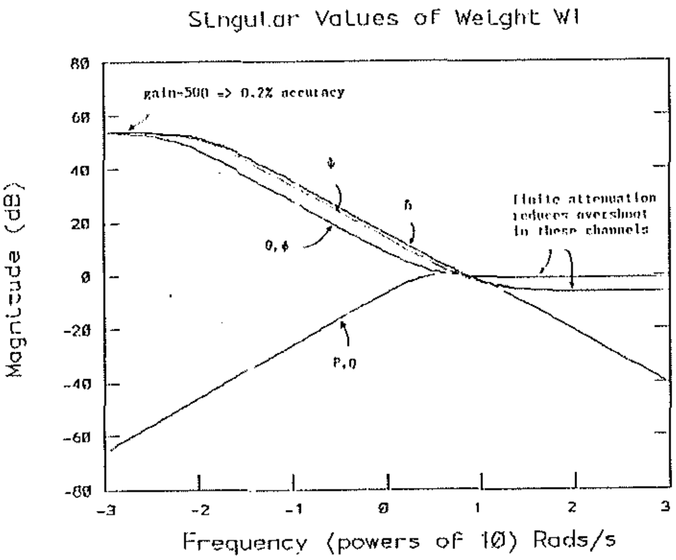


Fig 8

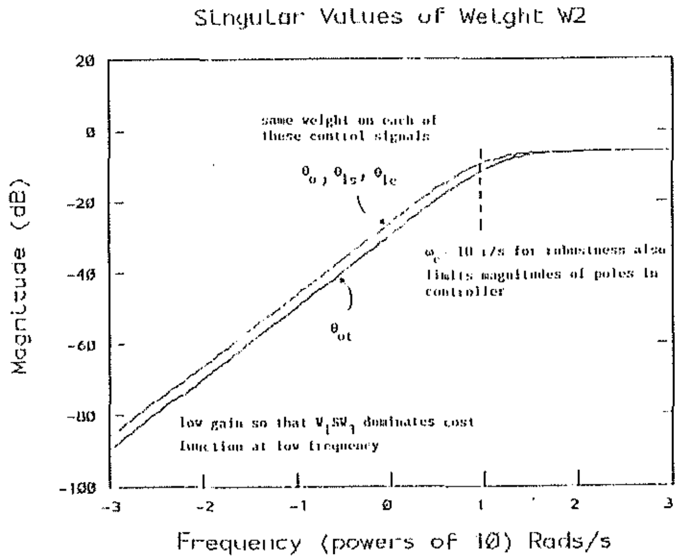


Fig 9

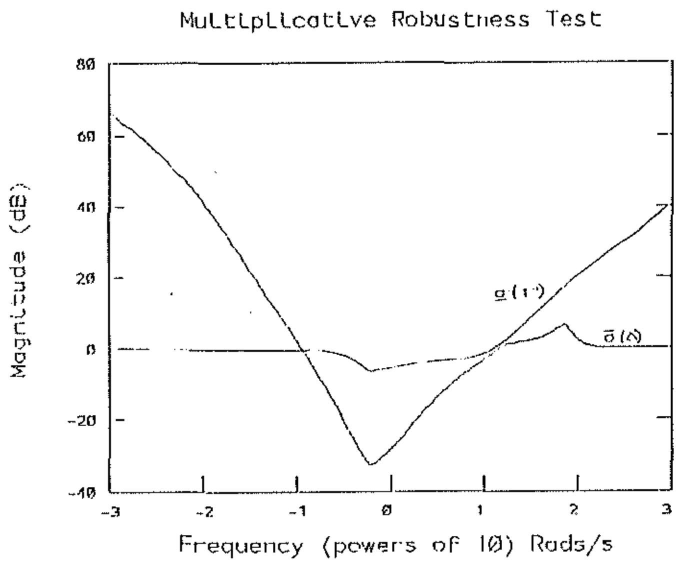


Fig 10

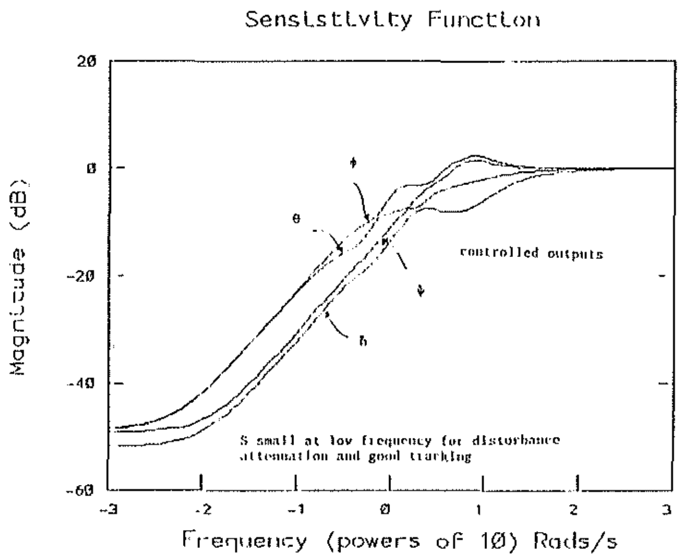


Fig 11

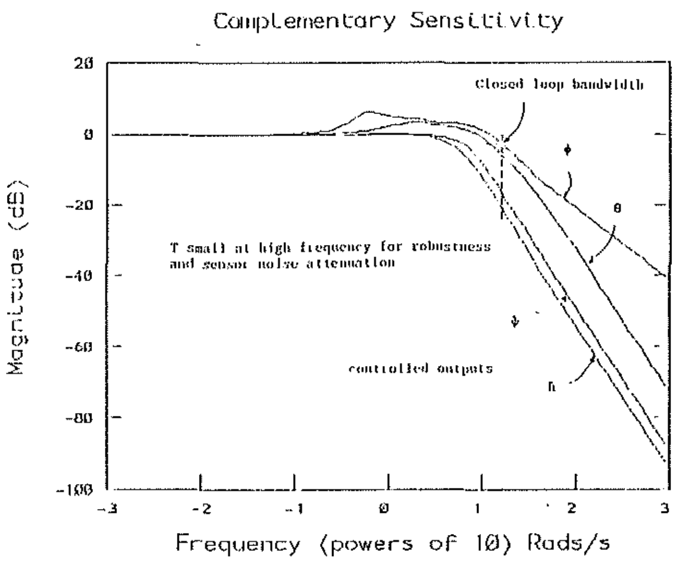


Fig 12

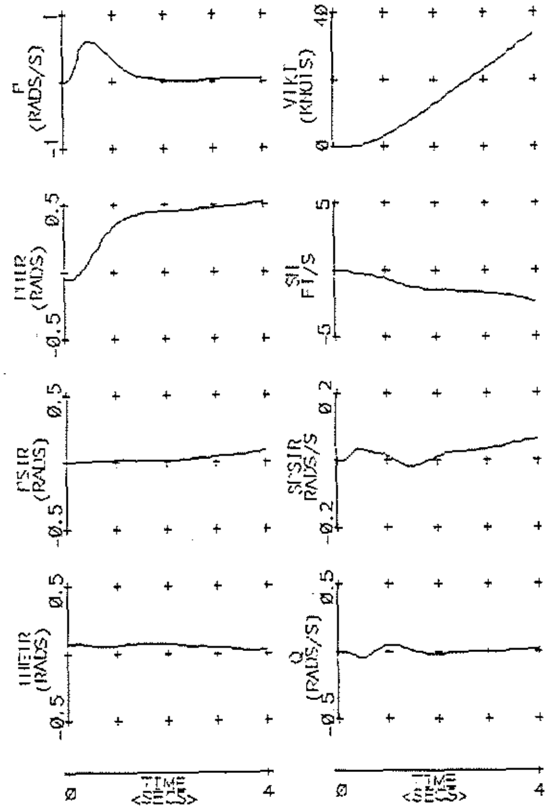


Fig 13. Roll response to a 30° step command in bank angle

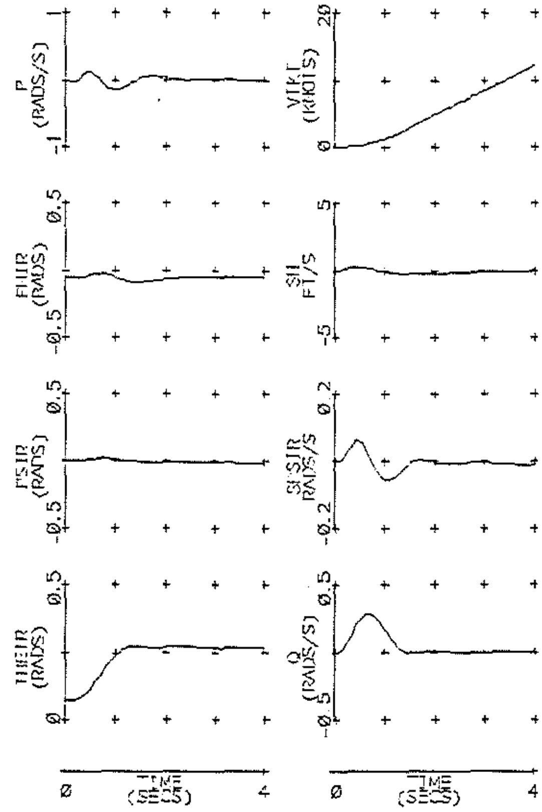


Fig 14. Pitch response to a 15° step command in pitch angle

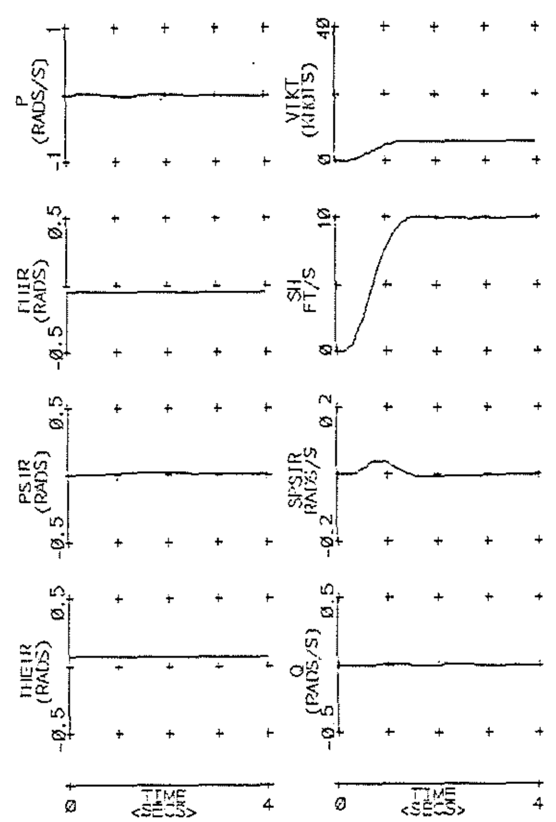


Fig 15. Height response to a 10 ft/s step command in heave velocity

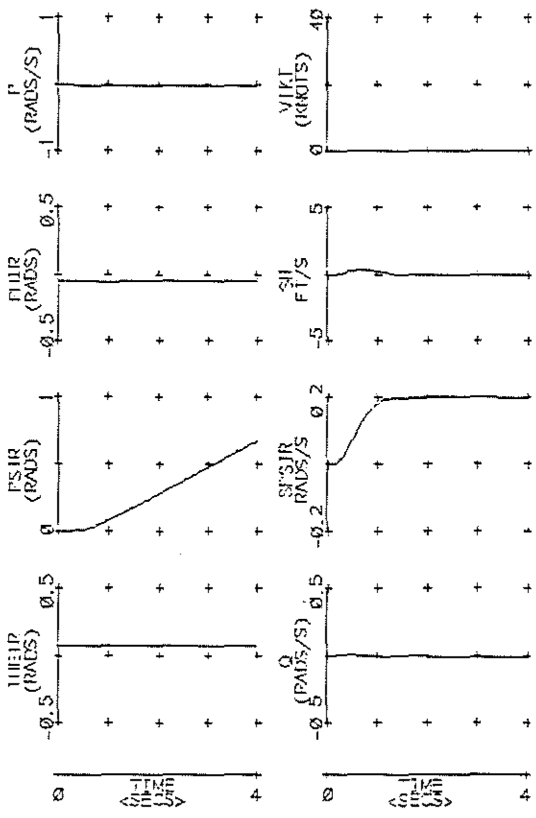


Fig 16. Yaw response to a 11° step command in yaw rate

TABLE 1. REQUIRED RESPONSE-TYPE FOR HOVER AND LOW SPEED

AXIS OF CONTROL	REQUIRED RESPONSE-TYPE
Pitch and Roll	ACAH
Yaw	Rate
Height	Rate

Note: ACAH -- Attitude command/Attitude Hold

TABLE 2. System matrix A, Control matrix B and Measurement matrix C

SYSTEM MATRIX

THETR	PHIR	P	Q	R	U	V	W
0.0000000	0.0000000	0.0000000	0.9985791	0.05328893	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	-0.05343404	1.001298	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	1.000000	-0.003935174	0.07374105	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	-10.98004	-2.643282	-0.06251947	0.1035012	-0.08205526	0.007183709
0.0000000	0.0000000	0.4558556	-1.896806	0.007408097	0.01426707	0.01792734	0.001814035
0.0000000	0.0000000	-1.923158	-0.4810042	-0.3766521	0.01859729	0.000561662	0.002725934
-32.07327	0.0000000	-0.5238533	2.182126	-0.01127720	-0.01983032	-0.02005057	0.02149119
0.1262140	32.02769	-2.261519	-0.5253553	0.4419744	0.02055577	-0.04019620	0.0002727032
2.365131	1.709175	-0.02288818	0.1413345	0.3604889	0.02363319	-0.003047180	-0.3103424
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.07364545	0.05314422	-0.9958676

CONTROL MATRIX

THO	TH1S	TH1C	THOTR
0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000
0.1227945	0.1312919	-2.741718	-0.01958942
0.01663534	0.4732456	0.02266005	0.0000000
0.3018110	0.02372016	-0.4945787	-0.2638639
0.3972411	-0.5457401	-0.02614260	0.0000000
-0.02058744	0.02613068	-0.5458474	0.3211498
-5.371761	-0.001144409	0.0000000	0.0000000

MEASUREMENT MATRIX

THETR	PHIR	P	Q	R	U	V	W
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	1.472909E-02	1.0628844E-02	-0.19917352
0.0000000	0.0000000	2.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	5.000000	0.0000000	0.0000000	0.0000000	0.0000000
7.16085E-05	3.0701775E-04	0.0000000	-2.671702	5.00649	0.0000000	0.0000000	0.0000000
5.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	2.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

TABLE 3. HOVER AND LOW SPEED REQUIREMENTS FOR THE ROLL AND PITCH AXES

Large-Amplitude Attitude Changes

It shall be possible to:

i) Achieve a pitch attitude of -30° and 10° in less than 1.5 secs from the hover flight condition.

ii) Achieve a roll attitude of $\pm 30^{\circ}$ in less than 1.5 secs.

There shall be no objectionable nonlinearities or overshoots in the response.

Attitude Hold

If an Attitude Hold is required, the pitch (roll) attitude response to a pulse longitudinal (lateral) controller shall return to within ± 10% of peak in less than 10 secs for Level 1. For Level 2, there shall be no tendency for pitch (roll) attitude to diverge following the pulse input.

Long-Term Pitch (Roll) Response to Longitudinal (Lateral) Controller

There shall be no noticeable divergence in pitch (roll) attitude following a rapid input of the longitudinal (lateral) controller. The input shall be a pulse for rate Response-Types and a step for Attitude Response-Types.

Interaxis Coupling

The pitch (roll) attitude response to an abrupt step change in roll (pitch) attitude response shall not exceed the limits specified below:

PARAMETER	LEVEL 1	LEVEL 2
$\theta_{peak}/\phi_{step}$ or $\phi_{peak}/\theta_{step}$	± 0.25	± 0.60

LIMITING VALUES FOR PITCH-TO-ROLL AND ROLL-TO-PITCH COUPLING