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KINEMATIC INVERSE SOLUTION OF HELICOPTER TURRETED GUN

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Abstract

The combat helicopter has evolved into one of the most powerful weapon system on the modern batterfield. The employment of all kinds of helicopter in Gulf War is unprecedented. There are many factors which influence a combat helicopter's success in the air combat role. Among those factors, the helicopter must have the maneuverability and agility to gain a firing position first and weapon systems must be able to get first round hits for successful air combat. In an attempt to address these questions, a general analytical model of helicopter maneuvering flight and projectile's path is established in this paper. Taking the helicopter air combat at short ranges as an example, the solution technique can be described with the following three stages. This method calculates first the kinematic parameters of a attack helicopter and then the flight attitude and control law of the attack helicopter. At the final stage, the target position、 projectile's path、 hit time and the firing position of turreted gun are calculated. The development and results here may provide referable basis for maximizing hit probability of gun in air firing.
Key Words: helicopters, turreted gun, air combat, maneuvering flight, projectile ballistics.

1. Introduction

Gulf war is a local war of unprecedented size since the Second World War. In this war of high technology、 strength and attrition, the employment of all kinds of helicopter is unprecedented. The fighter helicopter, as a modern weaponry, is a new、 concentratedly technical weapon. Whether the armed helicopter is described as a "tank killer" or "low and lethal" attack power, in a word, its might impels many countries more and more to develop and counter it. In order to really counter helicopter, the most effective weapon system is yet helicopter. So, helicopter air combat is unavoidable. Also it is inevitable trend of scrambling for air domination of low level.

Helicopter air–to–air weapons include often the guns, rockets and missiles. But, in close range fight, helicopter uses mainly the guns to attack enemy helicopter. So it is necessary to analyse the combat effectiveness of air–to–air gun. As for these questions, the conventional method calculates first the projectile's path based on a given firing position of gun and then the projectile's dispersion, target vulnerability as well as kill probability[1] according to all kinds of bias (including target motion). On the contrary in this paper, air–to–air gun firing position is calculated based on every round hitting the mass centre of target. This idea is the so–called kinematic inverse solution developed in this paper.

2. Mathematical Model of Air Combat

In order to get the favourable firing conditions, the attack helicopter must have good maneuverability and agility, one of many factors influencing a helicopter's success in the air combat is superior maneuvering capability of attack helicopters over target helicopters. As theoretical basis for research of air combat, helicopter maneuvering flight is the advancing condition of hitting target of helicopter weapon and then the performance of weapon is the following prediction of capability of killing target. The air combat model in this paper includes in general the simulation and analysis of helicopter maneuvering flight dynamics and projectile's path.

2.1 Dynamic equations of helicopter maneuvering flight

2.1.1 Euler equations

Helicopter motion state in a body fixed frame (see Fig.1) may be described with v_x, v_y, v_z the translational velocities , and ω_x, ω_y, ω_z the angular velocities. The Euler equations are as follows.

$$\left.\begin{aligned}
&F_x - mgsin\Theta_s - m(\dot{V}_x - \omega_z V_y + \omega_y V_z) = 0\\
&F_y - mgcos\Theta_s cos\gamma_s - m(\dot{V}_y - \omega_x V_z + \omega_z V_x) = 0\\
&F_z + mgcos\Theta_s sin\gamma_s - m(\dot{V}_z - \omega_y V_x + \omega_x V_y) = 0\\
&M_x - I_x \frac{d\omega_x}{dt} - (I_z - I_y)\omega_y\omega_z - I_{yz}(\omega_z^2 - \omega_y^2) - I_{xy}\omega_x\omega_z\\
&\quad + I_{zx}\omega_x\omega_y + I_{xy}\frac{d\omega_y}{dt} + I_{zx}\frac{d\omega_z}{dt} = 0\\
&M_y - I_y \frac{d\omega_y}{dt} - (I_x - I_z)\omega_z\omega_x - I_{zx}(\omega_x^2 - \omega_z^2) - I_{yz}\omega_x\omega_y\\
&\quad + I_{xy}\omega_y\omega_z + I_{yz}\frac{d\omega_z}{dt} + I_{xy}\frac{d\omega_x}{dt} = 0\\
&M_z - I_z \frac{d\omega_z}{dt} - (I_y - I_x)\omega_x\omega_y - I_{xy}(\omega_y^2 - \omega_x^2) - I_{zx}\omega_y\omega_z\\
&\quad + I_{zy}\omega_x\omega_z + I_{zx}\frac{d\omega_x}{dt} + I_{zy}\frac{d\omega_y}{dt} = 0
\end{aligned}\right\} \qquad (1)$$

where the helicopter attitude angles (Θ_s, γ_s, ψ_s) are the Euler angles relating the body fixed frame to the earth fixed frame of reference.

2.1.2 Rotor tip–path–plane equations

For the flapping motion of blades, only the zero and first harmonic are concerned. The elastic deflection of the blade can be written as

$$Y_1 \approx \eta_1(r)q_1(\psi) = \eta_1(r)[a_0(t) - a_1(t)cos\psi - b_1(t)sin\psi]$$

Starting from the force on the blade section, taking the method of separation of variables and the property of orthogonality of the mode shape, the partial differential equation of the blade elastic axis can be changed into the following set of tip–path–plane equations:

$$\begin{bmatrix}\ddot{a}_0\\ \ddot{a}_1\\ \ddot{b}_1\end{bmatrix} + [A]_{3\times 3}\begin{bmatrix}\dot{a}_0\\ \dot{a}_1\\ \dot{b}_1\end{bmatrix} + [B]_{3\times 3}\begin{bmatrix}a_0\\ a_1\\ b_1\end{bmatrix} = [C]_{3\times 1} \qquad (2)$$

2.1.3 Velocity equations

From Fig.2, the velocity components in the earth fixed frame are

$$\left.\begin{aligned}
&\dot{X}_E = Vcos\theta_P cos\psi_h\\
&\dot{Y}_E = Vsin\theta_P\\
&\dot{Z}_E = Vcos\theta_P sin(-\psi_h)
\end{aligned}\right\} \qquad (3)$$

2.1.4 Angular velocity equations

According to the projection relations in the body fixed frame, the following relations of angular velocities can be derived

$$\left.\begin{aligned} \dot{\gamma}_s &= \omega_x - tg\Theta_s(\omega_y cos\gamma_s - \omega_z sin\gamma_s) \\ \dot{\Theta}_s &= \omega_y sin\gamma_s + \omega_z cos\gamma_s \\ \dot{\psi}_s &= (1/cos\Theta_s)(\omega_y cos\gamma_s - \omega_z sin\gamma_s) \end{aligned}\right\} \quad (4)$$

2.2 Ballistic equations of a projectile

2.2.1 Initial motion conditions

Turret location relative to a helicopter centre of gravity is shown in Fig.3. Ballistic initial motion conditions of the projectile is determined by the helicopter kinematic parameters. When the helicopter maneuvers with velocity $\vec{V}_c(t)$ and angular velocity $\vec{\omega}$, velocity and acceleration at any point P of gun barrel are respectively given by

$$\left.\begin{aligned} \vec{V}_p &= \vec{V}_c + \vec{\omega} \times \vec{C}_p + \vec{V}_{pr} \\ \vec{a}_p &= \vec{a}_c + \dot{\vec{\omega}} \times \vec{C}_p + \vec{\omega} \times (\vec{\omega} \times \vec{C}_p) + 2\vec{\omega} \times \vec{V}_{pr} + \vec{a}_{pr} \end{aligned}\right\} \quad (5)$$

where $\vec{V}_{pr}$, $\vec{a}_{pr}$ is separately relative velocity and acceleration of the projectile relative to gun barrel. Similarly, the initial kinematic conditions of the point at which projectile flies into air from gun barrel is determined by formula (5).

2.2.2 Ballistic equations

If the angle of attack of the projectile is assumed to remain zero over the time of flight , the kinematic equations of the projectile as a mass point in wind axis frame can be expressed as :

$$\left.\begin{aligned} \frac{dV_p}{dt} &= \frac{R_x}{m_p} - gsin\gamma_w \\ V_p\frac{d\gamma_w}{dt} &= -gcos\gamma_w \end{aligned}\right\} \quad (6)$$

where R_x is aerodynamic drag, γ_w is wind–axis Euler pitch angle.

3. Inverse Solution Technique

Taking the basic air combat motion as an example, the method of inverse solution is described as follows.

3.1 Elementary motion in air combat

3.1.1 Attack helicopter (Blue helicopter)

The initial point of attack helicopter is taken as the original point of earth fixed frame. After finding the enemy targets in the upper left (north west corner), the attack helicopter makes at once 90° left turn maneuver so as to gain the best firing position (see Fig.4). The turn rate of the attack helicopter are given by following equations.

$$\left.\begin{aligned} \dot\psi_{h1} &= a_1 t_{BH}^3 + b_1 t_{BH}^2 + c_1 t_{BH} + d_1 & 0 \leqslant t_{BH} < t_1 \\ \dot\psi_{h2} &= V_c(t) / R_c & t_1 \leqslant t_{BH} < t_2 \\ \dot\psi_{h3} &= a_2 t_{BH}^3 + b_2 t_{BH}^2 + c_2 t_{BH} + d_2 & t_2 \leqslant t_{BH} \leqslant t_3 \end{aligned}\right\} \quad (7)$$

From simultaneous equations (7) and (3), the flight path of attack helicopter is calculated

3.1.2 Target helicopter (Red helicopter)

If the target helicopter does not have counter attack threat to the attack helicopter, it performs level acceleration to quickly flee. The future position of the target helicopter at any time can be calculated by the following equations:

$$\vec{S}_{RH} = 0.5\vec{A}_{RH} t^2 + \vec{V}_{RH} t + \vec{S}_{RH0} \quad (8)$$

3.1.3 Projectile

The attack helicopter is chosen as BO−105 helicopter. Turreted gun is similar to the AH−64 full−traverse or limited−traverse turreted gun. As for the yaw flow of the projectile, the Euler wind−axis yaw rate is given as a linear approximation of projectile time−of−flight by

$$\frac{d\dot\psi_w}{dt} = -(0.00135 + 0.00005t) \quad (9)$$

3.2 Inverse solution process

3.2.1 Calculation of kinematic parameters of the attack helicopter

According to the combat effective turn radius and the transient turn proportional factor of the attack helicopter, all kinematic parameters of equation (7) and flight path of the attack helicopter can be calculated[2].

3.2.2 The flight attitude and control deflections of the attack helicopter

From the equations(1) through (4), the flight attitude and control law of the attack helicopter can be calculated[3]. The rotor tip−path−plane equations are solved by using implicit Adams method[4].

3.2.3 Calculation of projectile's path and target helicopter's position

If time of flight of the round to the target, elevation angle and azimuth angle of turreted gun are respectively denoted by t_{hit}, Θ_{tu} and ψ_{tu}, from equations (6) (9) and (8) the projectile's path and target position can be calculated. At t_{hit} time when modulus of vector $\vec{S}_{rd}$ minus vector $\vec{S}_{RH}$ is equal to or less than 0.1×10^{-3}, it is assumed that the round hits the target helicopter.

4. A Sample Calculation and Result Analysis

4.1 The simulating conditions of helicopter air combat

After finding the target helicopter in the upper left(northwest corner), the attack helicopter performs quickly 90 ° left turn maneuver at constant velocity V = 157 km / h(μ = 0.2), with the effective radius Re = 295m and the transient proportional factor K_m = 0.1. After the flight time of the attack helicopter is past 1.2s,its turret gun start running fire within 1 second. the firing rate is 1000rd / min and the muzzle velocity of the projectile relative to gun barrel is 1100 m / s. At the same time, after feeling the threat of the first round from the attack helicopter, the target helicopter flees at once in right west direction.

Based on above conditions, we make theoretically every round hit centre of gravity of the target helicopter so as to improve the combat effectiveness. Then the corresponding attitude of turreted gun is required. Such a calculating work is just the research background of this paper.

4.2 The calculating results

4.2.1 The initial condition of running air–firing

According to above defined conditions, the kinematic parameters, control law and flight attitude of the attack helicopter in air combat can be calculated. Then the following results can be obtained as t_1 = 1.8s, t_2 = 9s, t_3 = 10.8s i.e. the transient turn in the first or last part takes 1.8 seconds and the steady turn part takes 7.2 seconds.

The turn radius in steady turn section is R_c = 251.16m and the turn rate (r / s) in three sections are respectively

entry section 0–1, $\dot{\psi}_{h1} = -0.058489t_{BH}^3 + 0.158833t_{BH}^2 \qquad 0 < t_{BH} < 1.8$

steady section 1–2, $\dot{\psi}_{h2} = V / R_c = 0.173528 \qquad 1.8 < t_{BH} < 9$

exit section 2–3, $\dot{\psi}_{h3} = 0.058489t_{BH}^3 - 1.747169t_{BH}^2 + 17.2531t_{BH} - 56.2233 \qquad 9 < t_{BH} \leqslant 10.8$

The flight path of the attack helicopter is shown in Fig.5. The time history of attitude angle and angular velocity of the attack helicopter are shown in Fig.6 through Fig.11. From the flight history of the attack helicopter, it is found that the fire range of running firing is located in the second half of transient turn 0–1 and the first half of steady turn 1–2.

4.2.2 Ballistics, target position and kinematic law of turreted gun

The calculation proceeds according to recurrence steps. The calculation begins with the initial values of hit time t_{hit0}. elevation angle Θ_{tu0} and azimuth angle ψ_{tu0}. Then projectile's path, target position and attitude of turreted gun are calculated. In every next step, by calculating the value of $|\Delta\vec{S}| = |\vec{S}_{rd} - \vec{S}_{RH}|$ and new t_{hitj}, Θ_{tuj}, ψ_{tuj}, the iterative calculation proceeds repeatedly until the value of $|\Delta\vec{S}|$ equals or is less than 0.1×10^{-3}.

Fig.12 through Fig.14 show respectively the variation of the elevation angle, azimuth angle of turret and hit time of the projectile with running firing process. From Fig.12 it is found that when target is at rest, the elevation angle of turreted gun and its change rate is small. Also elevation angle get larger and larger with the increase of target velocity and acceleration. At $V_{RH} = 22m/s$ and $A_{RH} = 0.1g$, the maximum value of increasing elevation angle is up to $0.26 \times 57.3 = 14.8°$ or so. The smoothless turn point of elevation angle curve happens in the transition period of the attack helicopter from transient turn to steady turn.

Similarly, the same conclusion of azimuth angle in Fig.13 can be drawn. As for hit time, because of target motion (such as having velocity and acceleration), it is apparent that the hit time t_{hit} becomes larger (see Fig.14). Fig.15 through Fig.17 show the effect of target initial position on the elevation angle and azimuth angle of turreted gun and hit time of the projectile. It is apparently found in Fig.15 that when target is located under the attack helicopter (such as $Y_{ERH} = -285m$), the elevation angle of turreted gun becomes smaller. With the increase of Y_{ERH} value, the elevation angle of turreted gun gets larger (such as maximum value $\Theta_{tu} = 25.21°$ for $Y_{ERH} = 0$).

Because of the attack helicopter's roll motion of following, the larger elevation angle brings the larger drift towards target zone in firing azimuth. So, when target is more beyond the attack helicopter (i. e. Y_{ERH} is larger), elevation angle Θ_{tu} gets larger, azimuth angle and hit time becomes smaller.

Finally, the influence of sudden acceleration on motion of turreted gun and hit time of the projectile is investigated. Seeing Fig.18 through Fig.20, when target flees with the acceleration 2.2g, the elevation angle, azimuth angle of turreted gun and hit time of the projectile all increase as compared with target at rest. Also the amplitude of increasing is not small.

5. Conclusion

The following conclusion can be made from above air combat circumstance.

(1) The turreted gun should have as possible as large variation range of elevation and azimuth angle and their change frequencies should be large.

(2) Because the natural construction of a helicopter limits the up eleva-

tion angle within 20 ° . So a helicopter must have good vertical acceleration performance so as to gain upper firing zone beyond the target.

(3) When the target helicopter has counterattack threat, the variation of the attack helicopter and turreted gun motion with time would be very sharp. This is also an important factor which needs to be considered in design

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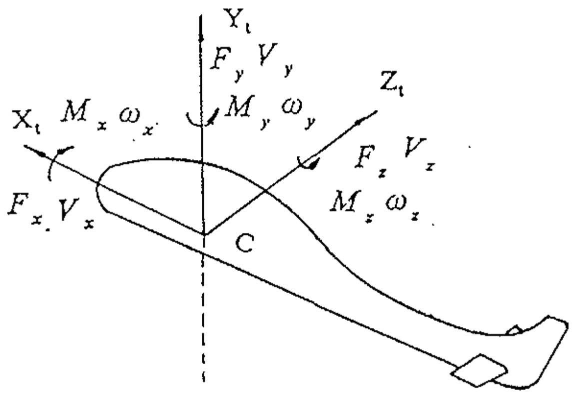

Fig.1 Body fixed axes system

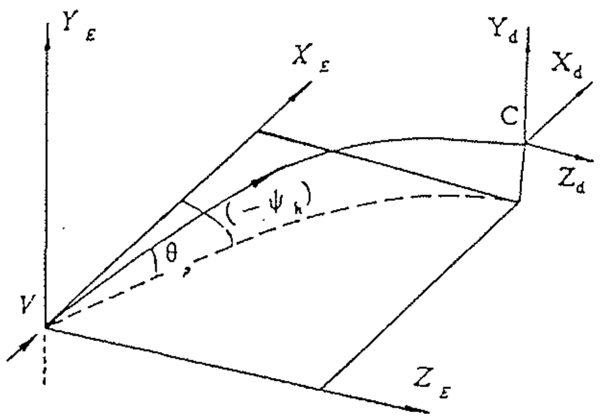

Fig.2 Description of the maneuvering flight path

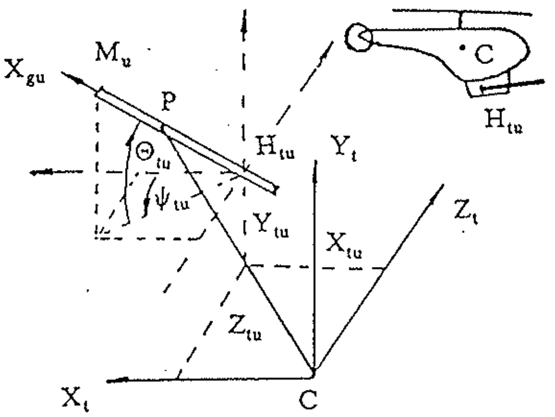

Fig.3 Turret location relative to helicopter

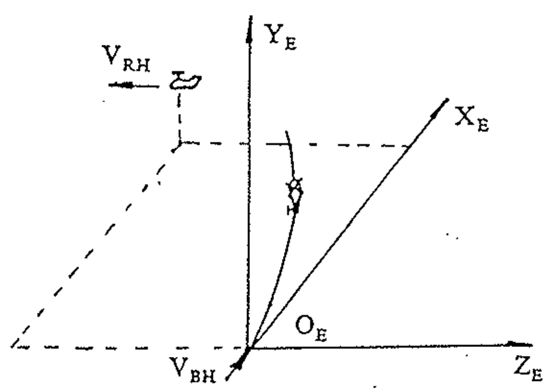

Fig.4 Initial position of air combat

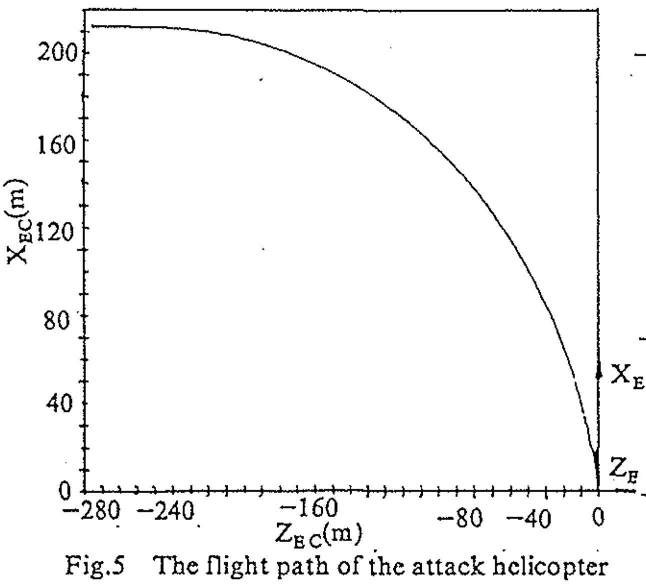

Fig.5 The flight path of the attack helicopter

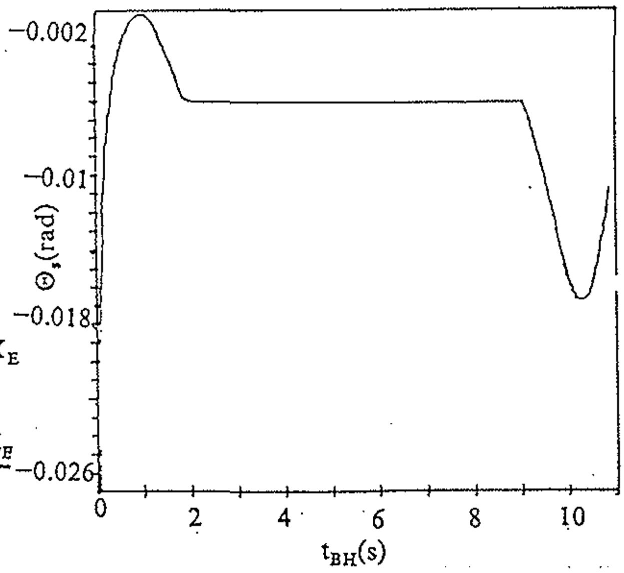

Fig.6 The pitch angle of the attack helicopter

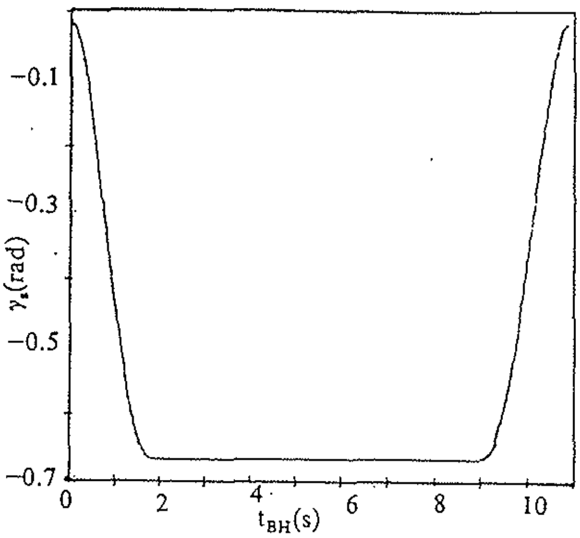

Fig.7 The bank angle of the attack helicopter

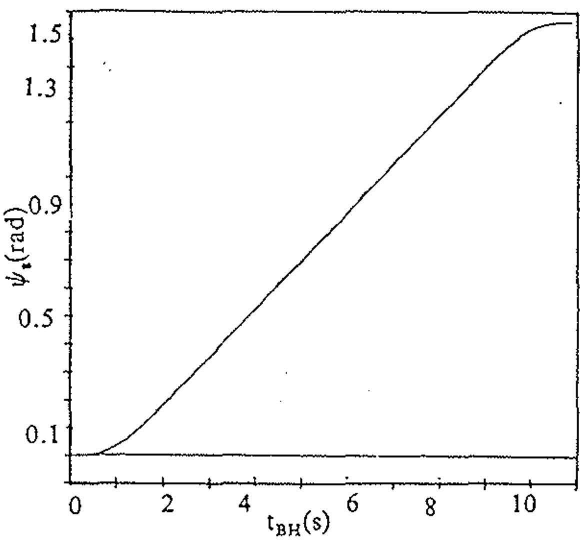

Fig.8 The yaw angle of the attack helicoper

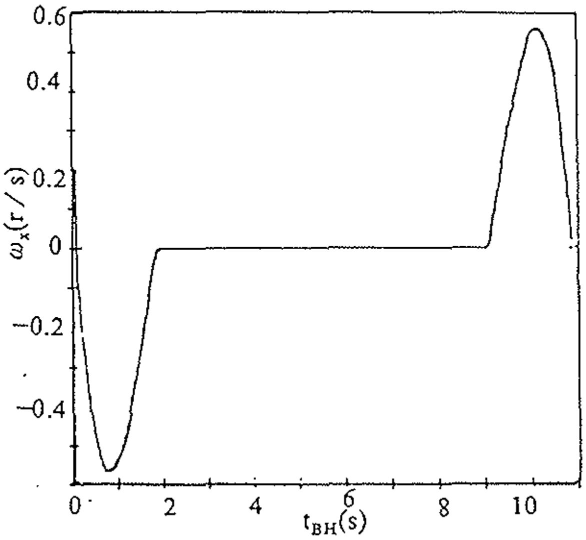

Fig.9 The roll rate of the attack helicopter

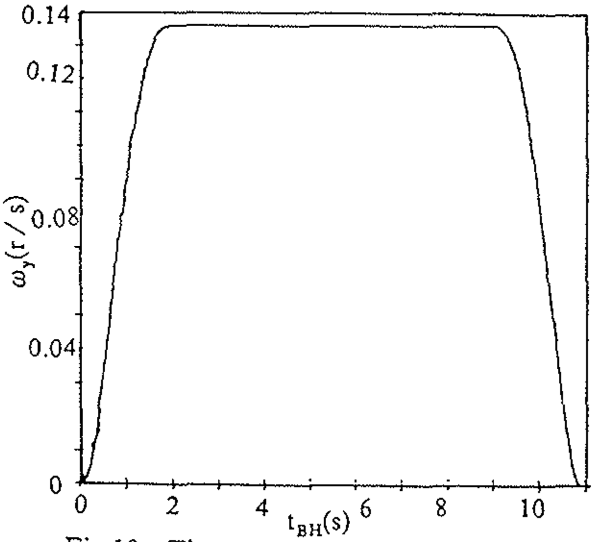

Fig.10 The yaw rate of the attack helicopter

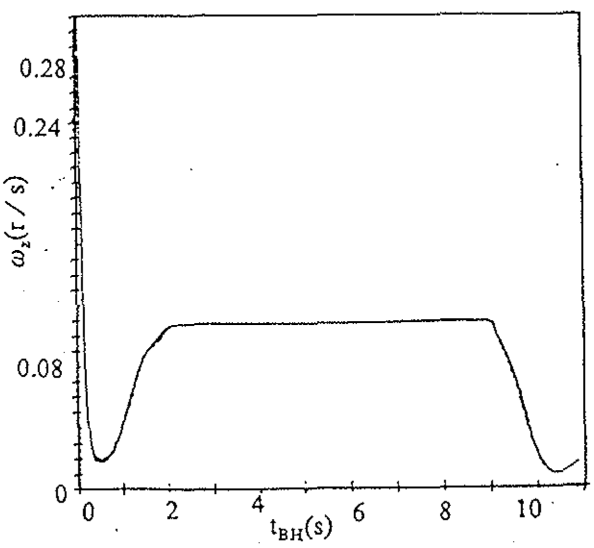

Fig.11 The pitch angle of the attack helicopter

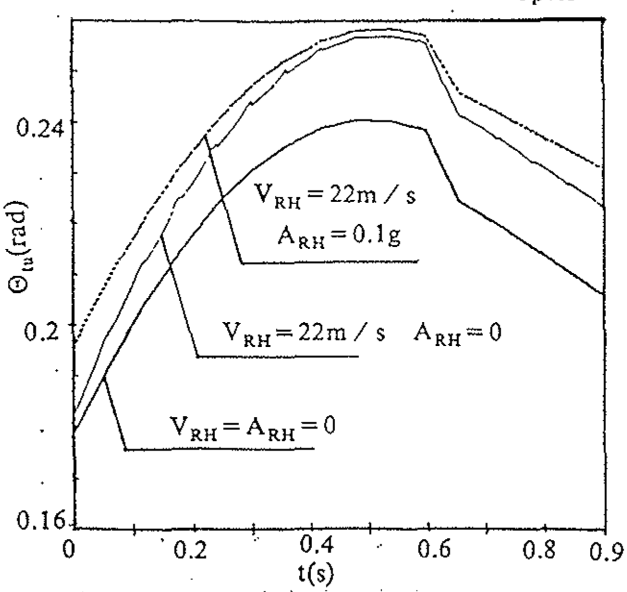

Fig.12 The variation of elevation angle of the turreted gun

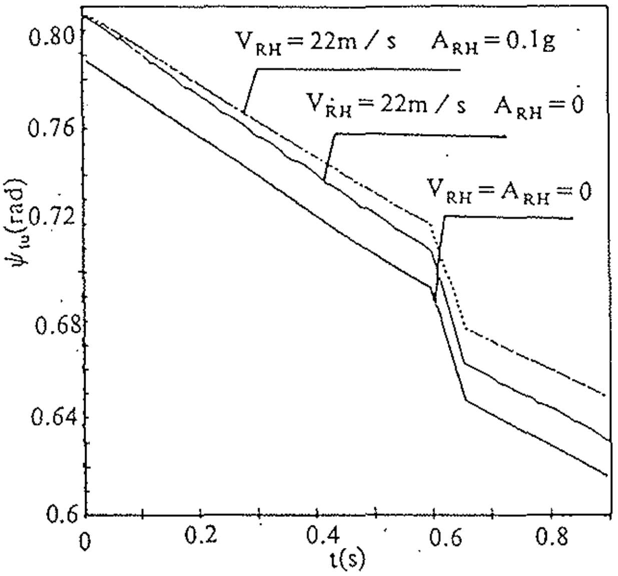

Fig.13 The variation of azimuth angle of the turreted gun

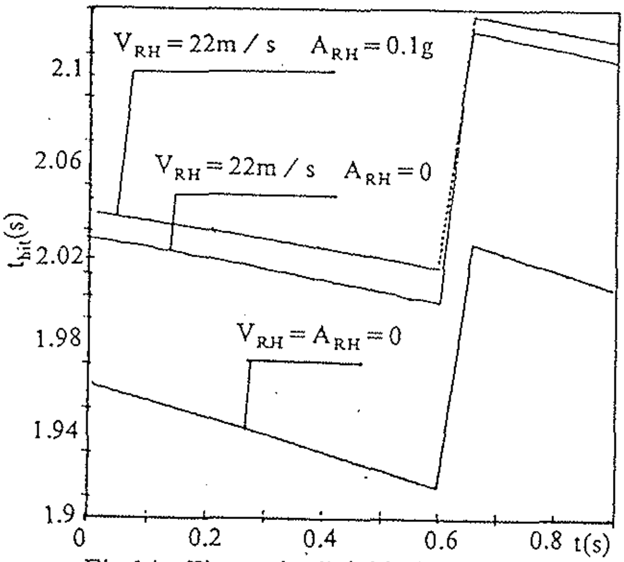

Fig.14 The projectile's hit time of the running firing in one second

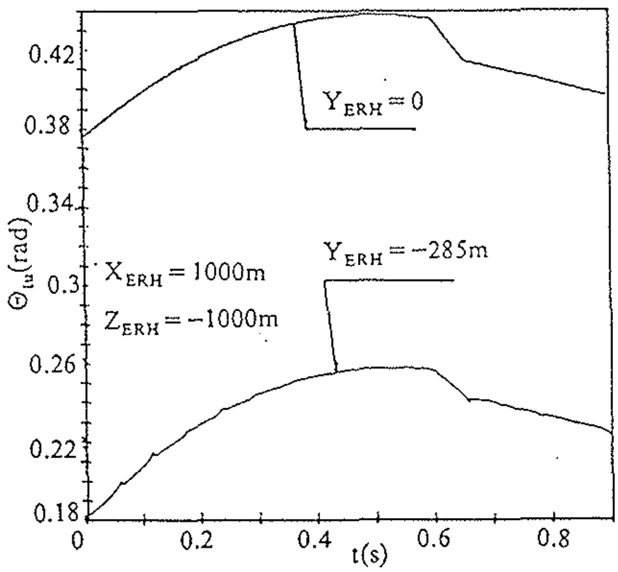

Fig.15 The effect of the target position on elevation angle

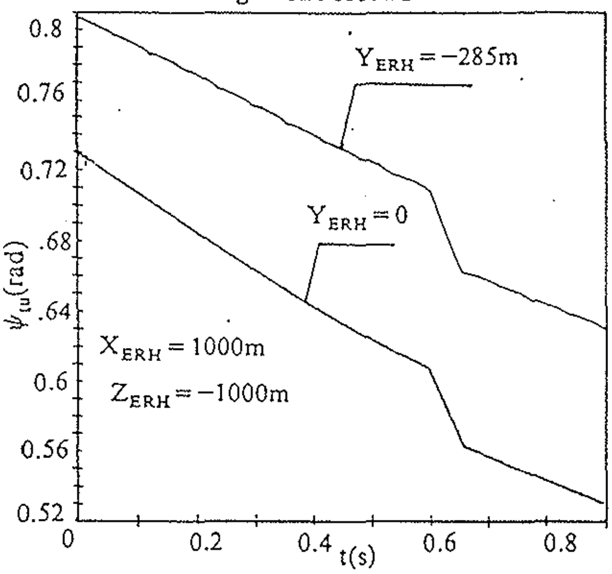

Fig.16 The effect of the target position on azimuth angle

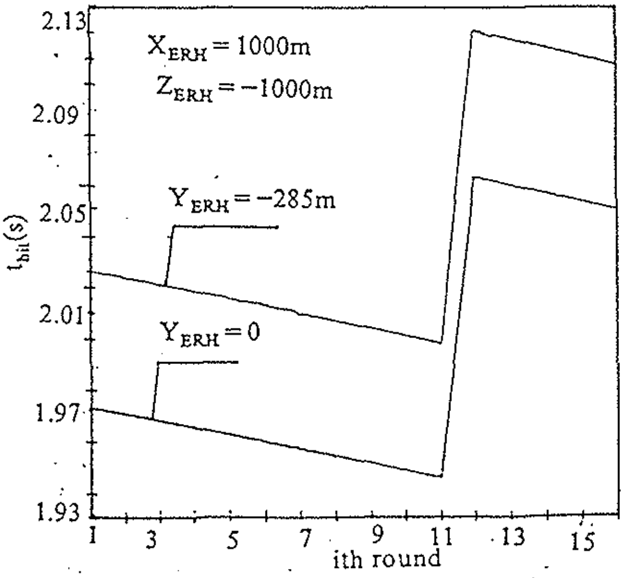

Fig.17 The effect of the target position on hit time of the projectile

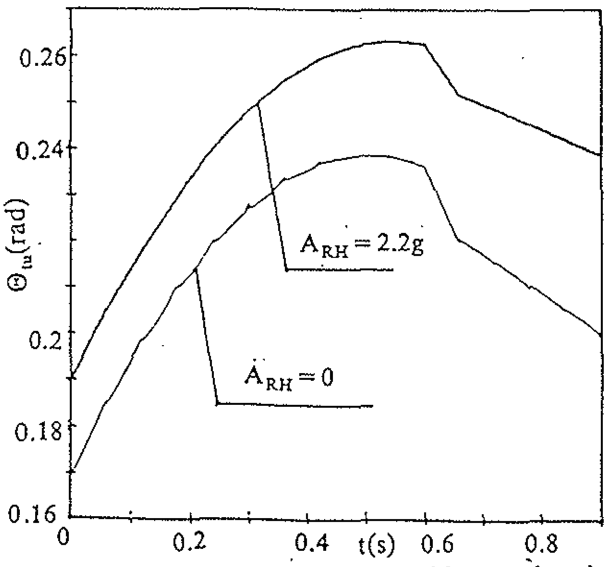

Fig.18 The effect of target sudden acceleration on elevation angle

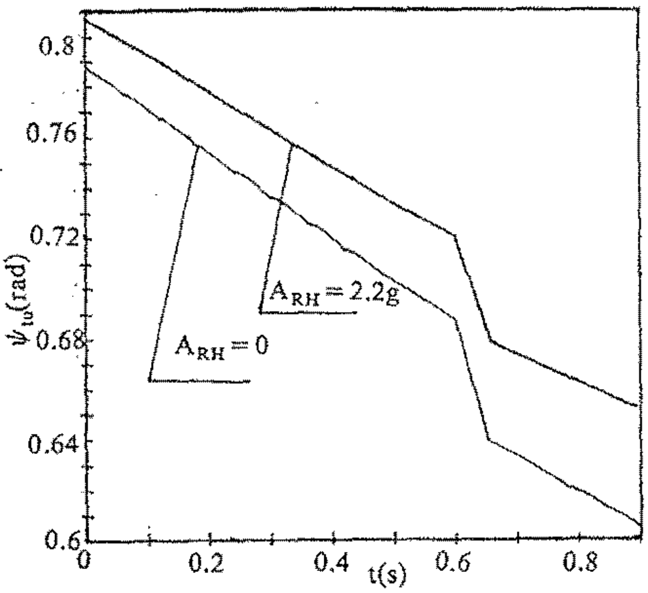

Fig.19 The effect of target sudden acceleration on azimuth angle

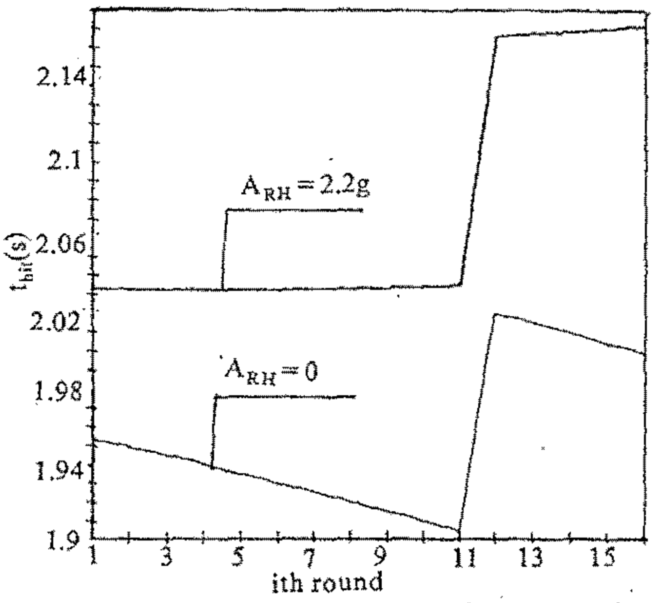

Fig.20 The effect of target sudden acceleration on hit time of the projectile