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ROBUST CONTROL SYNTHESIS FOR AN UNMANNED HELICOPTER

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Abstract

Controller synthesis for unmanned helicopters with minimum initial information about their parameters of their mathematical models is considered in this paper. The unknown parameters and system nonlinearity are considered as external disturbances. Two methods are proposed to solve this problem: design of the controller using feedback with compensation for disturbances estimated using observers, or design of fuzzy controllers based on the approach of Mamdani, and results of the controller of the first method. The paper presents a comparative study of the Raptor helicopter dynamics with the proposed control laws and with wind disturbances.

Nomenclature

θ,ϕ,ψ	pitch, roll and yaw angles (deg)	PID	the proportional-integral-derivative controller
p,q,r	roll, pitch and yaw rates (deg/s)	MR, TR	main rotor, tail rotor
u,v,w	longitudinal, lateral and normal velocity components (m/s)	NED	north-east-down coordinate system
$\delta_{lon}, \delta_{lat}$	longitudinal and lateral cyclic angles	LQR	linear quadratic regulator
$\delta_{col}, \delta_{ped}$	main and tail rotor collective	UAV	an unmanned aerial vehicle
W	wind actions in the body coordinate system (m/s)	GPS	global positioning system
a_s, b_s	longitudinal and lateral flapping angle of main rotor (rad)		

1. INTRODUCTION

The study of UAV control systems is motivated by the complexity of their mathematical model, their large number of experimentally determined parameters, as well as the demanding requirements for their operation.

There are various approaches for the design of UAV control systems, however, the ones preferred for practical implementation employ parameters that are easy to estimate. Such control laws include traditional PID controllers that are not effective in conditions of uncertainty and in the presence of interconnected control channels [1]. Therefore, autopilots are offered based on modified control laws like PID with tuned coefficients and control constraints [1], robust controllers with feedback on the state vector H∞ [2], LQR [3], μ - controllers [4] with disturbance compensation [5, 6], non-linear control laws using backstepping [7], dynamic inversion [8] and adaptive controllers with adjustable regulation coefficients [9].

It should be noted that the adjustment of the coefficients of the PID controller requires effort, and does not eliminate the problem of helicopter control with added wind disturbances. Robust controllers H∞, LQR and μ - controllers reduce the effect of external disturbances on the controlled output, but do not eliminate their influence, and, usually, require measurements of the state vector of the system. To improve the stability of the helicopter, a controller with disturbance compensation is used in this paper. The assessment of the disturbance is carried out using an observer [5].

Non-linear control laws, built using backstepping, do not require adjustment of controller parameters, but depend on the adopted mathematical models. In addition, they are complex to implement and sensitive to parameters and external disturbances. A method of dynamic inversion is free of these shortcomings [8], and does not require an exact model of the helicopter. A model of the torque of the main and tail rotors, however, are needed, for the implementation of the controller and measurement of accelerations are also required.

Adaptive controllers use, in addition to the main control laws, varying parameters under changing external conditions. The efficiency of such algorithms depends on the speed of convergence of the adaptation processes that finally depends on the employed mathematical model of the helicopter and uncontrollable dynamics.

Recently, fuzzy control algorithms have gained popular. This is due to the fact that their use for complex system does not require accurate mathematical description. In addition, such systems are able to maintain their performance despite the varying parameters of the system and the effect on it of external disturbances.

There are two main approaches to the construction of a helicopter fuzzy controller as in the Takagi-Sugeno [10] and Mamdani [11] works. In the Takagi-Sugeno approach knowledge of the model helicopter [10] is required, while Mamdani's approach uses only information about the input and output signals, which can be obtained during flight experiments [11]. Further, to improve the quality of the controller use of neural and neuro-fuzzy concepts, as well as various combinations of these are proposed.

In this paper, use of the Raptor helicopter offers an example for the synthesis of control laws based on its dynamic model and minimum initial information about its parameters. At the same time the unrecorded dynamics of the helicopter, the unknown parameters and nonlinearities are considered as external disturbances, and the control law is formed using observers of disturbances. To simplify the control algorithm, a fuzzy controller based on the Mamdani approach is also designed. A comparative analysis of the helicopter dynamics for the developed control laws under the influence of wind disturbances is presented in the results section of the paper.

2. FORMULATION OF THE PROBLEM

The unmanned Raptor helicopter is considered here. Its non-linear mathematical model and its parameters are identified in [2,12]. The helicopter dynamic equations take the form:

$$\begin{aligned} \dot{V} &= -\omega \times V + \frac{F}{m} + \frac{F_g}{m}, \\ \dot{\eta} &= S^{-1}\omega, \\ \dot{\omega} &= J^{-1}\left[M - \omega \times \left(J\omega\right)\right], \end{aligned} \qquad (1)$$

where $V=[u\ v\ w]^T$ and $\omega=[q\ p\ r]^T$ are the linear and angular velocity vectors in the body coordinate system; $\eta=\left[\theta\ \phi\ \psi\right]^T$ are the Euler angles; F is the aerodynamic force vector; $F_g = mg\left[-\sin\theta \;\; \cos\theta\sin\gamma \;\; \cos\theta\cos\gamma\right]^T$ is the gravity force vector; m is the helicopter mass; S is the transformation matrix; $J = diag\{J_{xx}, J_{yy}, J_{zz}\}$ is the moment of inertia matrix. M is the aerodynamic moment vector:

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \left(K_\beta + T_{mr}H_{mr}\right)\sin b_s + L_{vf} - T_{tr}H_{tr} \\ \left(K_\beta + T_{mr}H_{mr}\right)\sin a_s + M_{hf} \\ -\dfrac{P_{mr}}{\Omega_{mr}} - N_{vf} + T_{tr}D_{tr} \end{bmatrix}, \quad (2)$$

where T_{mr}, T_{tr} are the main rotor (MR) and tail rotor (TR) thrusts; K_β is the MR stiffness in β; H_{mr} is the MR hub location above the center of gravity (CG); P_{mr} is the total power of the MR; Ω_{mr} is the MR rotating speed; a_s, b_s are the longitudinal and lateral flapping angles of MR; D_{tr} is the TR hub location behind the CG; H_{tr} is the TR hub location above the CG; L_{vf}, M_{hf}, N_{vf} are the aerodynamic moments generated by fins.

The dependence of MR flapping angles to the control actions δ_{lon}, δ_{lat} is expressed by the following equations [12]:

$$\begin{aligned} \dot{a}_s &= -\tau^{-1}a_s - q + A_{bs}b_s + A_{lon}\delta_{lon} + A_{lat}\delta_{lat}, \\ \dot{b}_s &= -\tau^{-1}b_s - p + B_{as}a_s + B_{lon}\delta_{lon} + B_{lat}\delta_{lat}, \end{aligned} \quad (3)$$

where A_{lon} and B_{lat} are the ratios of the longitudinal and lateral cyclics of the input signal to the control displacements δ_{lon} and δ_{lat} respectively; A_{lat} and B_{lon} are the coefficients of the cross ties; τ is the time constant; A_{bs} and B_{as} are the coupling effects.

In order to simplify the procedure of the controller, design, its dynamics equations are presented in the following form:

$$\begin{aligned} \dot{X}_1 &= R(X_3)X_2, \\ \dot{X}_2 &= F_1(X_2, X_4, X_5, U_0, W), \\ \dot{X}_3 &= S^{-1}(X_{32})X_4, \\ \dot{X}_4 &= F_2(X_2, X_4, X_5, U_1, W), \\ \dot{X}_5 &= F_3(X_4, X_{52}, U_2), \end{aligned} \quad (4)$$

where $X_1 = [x \;\; y \;\; z]^T$ is the position vector in the local north-east-down (NED) coordinate system; $X_2 = V$, $X_3 = \eta$, $X_{32} = [\theta \;\; \phi]^T$, $X_4 = \omega$; $X_5 = \left[a_s \;\; b_s \;\; \delta_{\text{ped,int}}\right]^T$, $X_{52} = [a_s \;\; b_s]^T$, $\delta_{\text{ped,int}}$ is the intermediate state of yaw rate feedback controller; $U_0 = \left[U_0^T \;\; \delta_{\text{ped}} \;\; \delta_{col}\right]^T$, $U_0 = diag\{-g, g\cos\theta\}u_{01}$, $u_{01} = [\sin\theta \;\; \sin\phi]^T$; $U_2 = \left[\delta_{\text{lon}} \;\; \delta_{\text{lat}} \;\; \delta_{\text{ped}}\right]^T$, $U_1 = \left[\delta_{\text{ped}} \;\; \delta_{\text{col}}\right]^T$, δ_{ped} and δ_{col} are the normalized rudder servo and collective pitch inputs respectively; $R(X_3)$ and $S(X_{32})$ are the rotation and kinematic transformations matrices respectively, $W = [u_{\text{wind}} \;\; v_{\text{wind}} \;\; w_{\text{wind}}]^T$ are the wind actions in the body coordinate system, which is defined as [2]:

$$W_i = 0.5V_{p\max}\left[1 - \cos\left(\frac{2\pi}{\Delta t_p}t\right)\right], \; i = \overline{1,3}, \quad (5)$$

where $V_{p\max}$ is the maximum amplitude of the gust of wind during the time interval Δt_p.

We introduce the state vector $X = \left[X_1^T \;\; X_2^T \;\; X_3^T \;\; X_4^T \;\; X_5^T\right]^T$ and control signals vector $U = \left[\delta_{\text{lon}} \;\; \delta_{\text{lat}} \;\; \delta_{\text{ped}} \;\; \delta_{\text{col}}\right]^T$, each of which lies in the range from -1 to 1 [2].

When balancing the helicopter in hover condition we assume: $U^* = \left[\delta_{\text{lon}}^* \;\; \delta_{\text{lat}}^* \;\; \delta_{\text{ped}}^* \;\; \delta_{\text{col}}^*\right]^T$, $X^* = \left[x_0 \;\; y_0 \;\; z_0 \;\; 0 \;\; 0 \;\; 0 \;\; \theta^* \;\; \phi^* \;\; 0 \;\; 0 \;\; 0 \;\; 0 \; a_s^* \;\; b_s^* \;\; 0\right]^T$.

Equation (4) can be rewritten in terms of deviations from the trimmer condition as:

$$x = X - X^* = \left[x_1^T \ x_2^T \ x_3^T \ x_4^T \ x_5^T \right]^T,$$
$$u = U - U^* = \left[\Delta\delta_{\text{lon}} \ \Delta\delta_{\text{lat}} \ \Delta\delta_{\text{ped}} \ \Delta\delta_{\text{col}} \right]^T,$$
$$x_1 = [\Delta x \ \Delta y \ \Delta z]^T, x_2 = [u \ v \ w]^T,$$
$$x_3 = [\Delta\theta \ \Delta\phi \ \psi]^T, x_4 = [q \ p \ r]^T,$$
$$x_5 = \left[\Delta a_s \ \Delta b_s \ \delta_{\text{ped,int}} \right]^T.$$

Then the resulting system of equations represented in a form suitable for the synthesis of control:

(6) $\dot{x}_1 = R(X_3)x_2$,

(7) $\dot{x}_2 = A_1(X_{31})u_0 + f_1$,

(8) $\dot{x}_3 = S^{-1}(X_{32})x_4$,

(9) $\dot{x}_4 = A_2 x_5 + f_2$,

(10) $\dot{x}_5 = B_0 u_1 + f_3$.

Here $X_3 = X_3^* + x_3$, $X_{31} = \theta^* + \Delta\theta$, $X_{32} = X_{32}^* + x_{32}$, $x_{32} = [\Delta\theta \ \Delta\phi]^T$; $u_0 = \left[u_{01}^T \ \Delta\delta_{col} \right]^T$, u_{01} is the virtual control for the outer loop to move the helicopter relative to the earth coordinate system; $u_1 = \left[\Delta\delta_{\text{lon}} \ \Delta\delta_{\text{lat}} \ \Delta\delta_{\text{ped}} \right]^T$; f_i, $i = \overline{1,3}$ are the vectors of generalized disturbances derived from the original equations (4) after isolation the terms $A_1(X_{31})u_0$, $A_2 x_5$, $B_0 u_1$, where $A_1(X_{31}) = diag(-g, g\cos\theta, b_1)$, b_1 is the model parameter; A_2, B_0 are the diagonal matrices of the model parameter.

3. SYNTHESIS OF CONTROL LAW WITH OBSERVER

To achieve the desired helicopter stability it is necessary to provide compensation for the generalized disturbances f_i, $i = \overline{1,3}$. However, as follows from the equations (6)-(10), the f_1 and f_2 cannot be fully compensated. Therefore the control law is constructed so that it suppress the generalized disturbances which affect the dynamics of the state vectors x_i, $i = \overline{1,3}$. To do this we use an observer to construct estimates of the generalized disturbances, previously considered in [6].

For the original system, represented by the equation

(11)
$$\dot{x} = Ax + B\text{u} + D\text{w},$$
$$y = Cx + \text{v},$$

where $x \in R^n$ is the state vector; $\text{u} \in R^m$ is the control vector, $\text{w} \in R^s$ and $\text{v} \in R^l$ are the vectors of disturbances and a noise in measurements, respectively, the observer has the form:

(12) $$\dot{\hat{x}} = A\hat{x} + B\text{u} + D\hat{\text{w}} + L_1(y - C\hat{x}),$$

(13) $$\dot{\hat{\text{w}}} = \mu^{-1}\left(D^+ L_1 + L_2\right)(y - C\hat{x})$$

where $D^+ = \left(D^T D\right)^{-1} D^T$; μ is an adjustable parameter and L_1, L_2 are the matrices of coefficients, which have to be determined.

Unlike known observers [13-15], using the observer of equations (12), (13) it is possible to achieve the desired accuracy in the estimate of the state vector coordinates and disturbances without significant increase of the coefficients of the observer matrices. This is important in the presence of noise in measurement.

To simplify the control law we will hold its synthesis separately for each subsystem.
First, consider the first subsystem (6), (7), for which we write:

$$\ddot{x}_1 = R(X_3)\dot{x}_2 + \frac{d}{dt}\left(R(X_3)\right)x_2 = \\ = R(X_3)A_1\left(X_{31}\right)u_0 + \text{w}_1,$$

or

$$\dot{x}_1 = \tilde{x}_2,$$
$$\dot{\tilde{x}}_2 = R(X_3)A_1\left(X_{31}\right)u_0 + \text{w}_1,$$

where $\tilde{x}_2 = R(X_3)x_2$,

$w_1 = R(X_3) f_1 + \frac{d}{dt}(R(X_3))x_2$ are the generalized disturbances.

1) When measuring vectors x_1 and x_2, for example, with using GPS and airspeed sensors, the control law is adopted in the form:

$$
\begin{aligned}
&u_0 = A_1^{-1}(X_{31})\bar{u}_0, \\
&\bar{u}_0 = -R^T(X_3)\big[K_1(x_1 - x_{1r}) + \\
&+K_2\big(R(X_3)x_2 - \dot{x}_{1r}\big) + \hat{w}_1\big],
\end{aligned}
\tag{14}
$$

where x_{1r} is the move commands vector in the earth coordinate system; K_1, K_2 are the diagonal matrices, which are given analytically by direct indicators of quality of transients, $\hat{w}_1$ is the disturbance estimate, which is determined by the observer.
Thus, the control law is found $\Delta\delta_{\text{col}} = b_1^{-1}\Delta\bar{\delta}_{\text{col}}, \Delta\bar{\delta}_{\text{col}} = [0\ 0\ 1]\bar{u}_0$, and desired change θ_r, ϕ_r of angles: $[\sin\theta_r\ \ \sin\phi_r]^T = [I_2\ \ 0_{2\times1}]u_0$.

Taking into account the adopted notation, we obtain the observer to evaluate the disturbance $\hat{w}_1$:

$$
\begin{aligned}
&\frac{d}{dt}\hat{\tilde{x}}_2 = \tilde{u}_0 + \hat{w}_1 + L_1(\tilde{x}_2 - \hat{\tilde{x}}_2), \\
&\frac{d}{dt}\hat{w}_1 = \mu^{-1}(L_1 + L_2)(\tilde{x}_2 - \hat{\tilde{x}}_2),
\end{aligned}
\tag{15}
$$

where

$\tilde{u}_0 = R(X_3)diag\{-g, g\cos\theta, 1\}\big[\sin\theta\ \ \sin\phi\ \ \Delta\bar{\delta}_{\text{col}}\big]^T$;

L_1, L_2 are the diagonal matrices with positive elements.

Given that the vector $\tilde{x}_2$ can be measured, easy to obtain a reduced observer of third order:

$$
\begin{aligned}
&\dot{\xi} = P\xi + P(-P\tilde{x}_2 + \tilde{u}_0), \\
&\hat{w}_1 = \xi - P\tilde{x}_2,
\end{aligned}
\tag{16}
$$

where $P = -\mu^{-1}(I_3 + L_2L_1^{-1})$ is the diagonal matrix.

2) If the vector x_2 is not measured, then in the control law (14) instead of a vector $R(X_3)x_2$ its estimate $\hat{\tilde{x}}_2$ is uses, which together with an estimate $\hat{w}_1$ are constructed using the observer:

$$
\begin{aligned}
&\dot{\hat{x}}_{ext} = A_{ext}\hat{x}_{ext} + B_{ext}u_r + \\
&+H_{ext}L_{ext}(y_r - C_{ext}\hat{x}_{ext}),
\end{aligned}
\tag{17}
$$

where $y_r = x_1$ и $u_r = \tilde{u}_0$ are the observer inputs; $\hat{x}_{ext} = \big[\hat{x}_1^T\ \ \hat{\tilde{x}}_2^T\ \ \hat{w}_1^T\big]^T$ is the estimate vector; $A_{ext} = \begin{bmatrix} 0_3 & I_3 & 0_3 \\ 0_3 & 0_3 & I_3 \\ 0_3 & 0_3 & 0_3 \end{bmatrix}$, $B_{ext} = \begin{bmatrix} 0_3 \\ I_3 \\ 0_3 \end{bmatrix}$,

$H_{ext} = \begin{bmatrix} I_6 & 0_{6\times3} \\ \mu^{-1}[0_3\ I_3] & \mu^{-1}I_3 \end{bmatrix}$, $L_{ext} = \begin{bmatrix} L_{11} \\ L_{12} \\ L_2 \end{bmatrix}$,

$C_{ext} = [I_3\ \ 0_3\ \ 0_3]$, L_{11}, L_{12}, L_2 are the diagonal matrices with positive elements.

Considering the measurements of vector x_1 the dimension of the observer (17) can be lowered to a 6th order:

$$
\begin{aligned}
&\dot{\zeta} = A_r\zeta + B_r u_r + L_r y_r, \\
&h = \zeta + C_r y_r,
\end{aligned}
\tag{18}
$$

where $h = \big[\hat{\tilde{x}}_2^T\ \ w_1^T\big]^T$, $A_r = \begin{bmatrix} P & I_3 \\ G & 0_3 \end{bmatrix}$,

$B_r = \begin{bmatrix} I_3 \\ 0_3 \end{bmatrix}$, $L_r = -\begin{bmatrix} P^2 + G \\ GP \end{bmatrix}$, $C_r = -\begin{bmatrix} P \\ G \end{bmatrix}$,

$P = -L_{12}L_{11}^{-1}$, $G = -\mu^{-1}(L_{12} + L_2)L_{11}^{-1}$.

Note that observers (15) - (18) do not depend on b_1.

Now consider the second subsystem (8) - (10), which similarly can be rewritten as:

$$
\begin{aligned}
&\dot{x}_3 = \tilde{x}_4, \\
&\dot{\tilde{x}}_4 = \tilde{x}_5,
\end{aligned}
$$

$$\dot{\hat{x}}_5 = S^{-1}(X_{32})B_1u_1 + \mathrm{w}_2,$$

where w_2 are the generalized disturbances; $\tilde{x}_4 = S^{-1}(X_{32})x_4$; $B_1 = A_2B_0 = diag(b_2, b_3, b_4)$, b_i, $i = \overline{2,4}$ are the pre-unknown model parameters.

When measuring vectors x_3, x_4 the control law is of the form:

$$u_1 = B_1^{-1}S(X_{32})\bar{u}_1,$$

$$\bar{u}_1 = -\Big[\bar{K}_1\Delta x_3 + \bar{K}_2\Big(S^{-1}(X_{32})x_4 - \dot{x}_{3r}\Big) + \bar{K}_3\hat{\tilde{x}}_5 + \hat{\mathrm{w}}_2\Big], \tag{19}$$

where $\Delta x_3 = x_3 - k_r[\theta_r \ \ \phi_r \ \ 0]^T - x_{3r}$; x_{3r} is the command vector of the angles; $\bar{K}_1$, $\bar{K}_2$, $\bar{K}_3$ are the diagonal matrices with positive elements.

Thus, using the control law (19) the desired angles θ_r, ϕ_r or x_{3r} can be tracked.

The vectors $\hat{\tilde{x}}_5$ and $\hat{\mathrm{w}}_2$ can be determined using the observer (17) and (18) at $y_r = S^{-1}(X_{32})x_4$, $u_r = \bar{u}_1$. In this case $\hat{x}_{ext} = \big[\hat{\tilde{x}}_4^T \ \ \hat{\tilde{x}}_5^T \ \ \hat{\mathrm{w}}_2^T\big]^T$, $h = \big[\hat{\tilde{x}}_5^T \ \ \mathrm{w}_2^T\big]^T$ and the observer does not depend on the parameters b_i, $i = \overline{2,4}$.

Thus, for implementing the control laws (14), (19) is required determine the 4 parameters b_i, $i = \overline{1,4}$. Note that in [2] for the robust controller synthesis used 29 pre-unknown model parameters.

Parameters b_i, $i = \overline{1,4}$ can be identified based on the results of the measured speeds values w, q, p, r and set test signals $\Delta\delta_{col}$, $\Delta\delta_{\mathrm{lon}}$, $\Delta\delta_{\mathrm{lat}}$, $\Delta\delta_{\mathrm{ped}}$ using observers (16), (18).

For this purpose we assume $C = I_n$, $A = A_m + \Delta A$, $B = B_m + \Delta B$ for a subsystem of the form (11), where A_m, B_m are the initial values of the matrices. Then the subsystem can be rewritten as

$$\dot{x} = A_m x + B_m \mathrm{u} + \varphi,$$

where $\varphi(t) = \Delta A x(t) + \Delta B u(t) + D w(t)$ are the generalized disturbances is estimated using an observer of the form (12), (13). Consequently, if $t > t_n$ then equation is true

$$\hat{\varphi}^T(t) \approx \begin{bmatrix} x^T(t) & u^T(t) & 1 \end{bmatrix} \begin{bmatrix} \Delta A^T \\ \Delta B^T \\ w^T(t)D^T \end{bmatrix} \tag{20}$$

Using equation (20) a system of equations is built for discrete time points t_i

$$\underbrace{\begin{bmatrix} \hat{\varphi}^T(t_i) \\ \hat{\varphi}^T(t_{i-k}) \\ \hat{\varphi}^T(t_{i-2k}) \\ \dots \\ \hat{\varphi}^T(t_{i-Nk}) \end{bmatrix}}_{\Phi_i} = \underbrace{\begin{bmatrix} x^T(t_i) & u^T(t_i) & 1 \\ x^T(t_{i-k}) & u^T(t_{i-k}) & 1 \\ x^T(t_{i-2k}) & u^T(t_{i-2k}) & 1 \\ \dots & \dots & \dots \\ x^T(t_{i-Nk}) & u^T(t_{i-Nk}) & 1 \end{bmatrix}}_{H_i} \underbrace{\begin{bmatrix} \Delta A^T \\ \Delta B^T \\ w^T(t)D^T \end{bmatrix}}_{\Lambda_i}$$

where $N = n + m$. The vector of parameters is given by:

$$\Lambda_j = \frac{1}{j}\sum_{i=1}^{j} H_i^{-1}\Phi_i, \qquad j = 1, 2, 3, \ldots$$

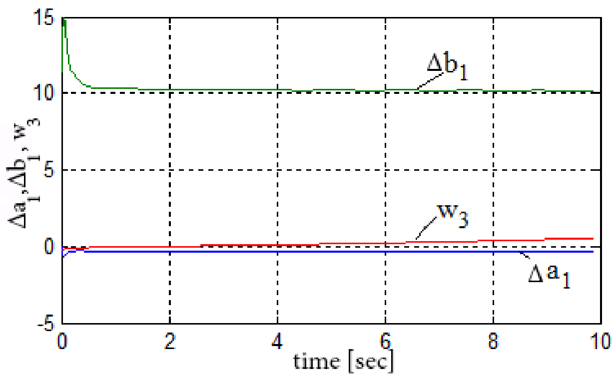
when applying test signals $u(t)$.

For example, we can write the equation for the channel of vertical velocity as:

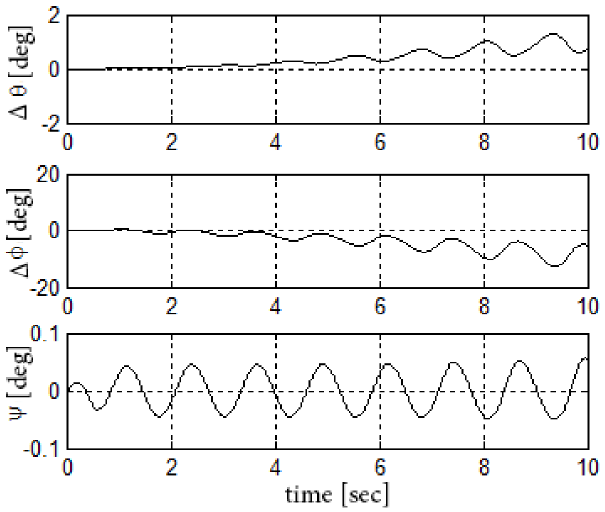
$$\dot{w}(t) = a_1 w(t) + b_1 u_4(t) + w_3(t),$$

where $a_1 = a_1^* + \Delta a_1$, $b_1 = b_1^* + \Delta b_1$, $a_1^* = 0$, $b_1^* = 20.9$; Δa_1, Δb_1 are the unknown parameters, $u_4(t)$ is the main rotor collective pitch control, $w_3(t)$ is a generalized disturbance.

As a test the reference velocity $w^*(t)=0.5\sin(5t)$ under wind gusts (5) with $V_{p\max}=10$ m/s and the absence of noise in measurements is used. If the parameters $\Delta a_1=-0.7$, $\Delta b_1=10$ deviate, then identification of parameter a_1 is carried out. Figure 1 shows the process of parameter identification in deviations from the trim values. If noise in the measurement exists, then the accuracy of b_1 parameter estimation is enhanced by filtering the signals.



a)



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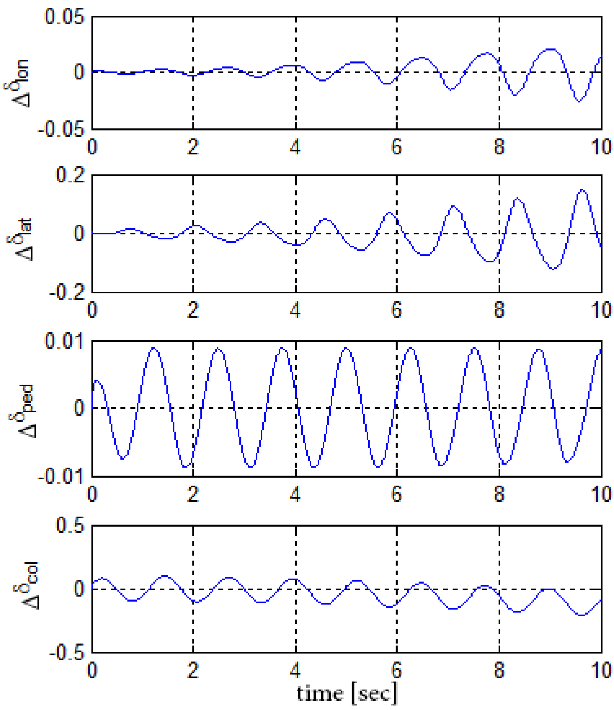


c)

Figure 1. Processes of parameter identification in deviations from the balancing values.

Trim values and parameters were found for each subsystem of (1) - (3) in hover mode of helicopter [2]: $b_1^{-1}=0.048$, $b_2^{-1}=0.0015$, $b_3^{-1}=0.00066$, $b_4^{-1}=0.0023$.

If the vector x_1 is not measured, then in the control law (14) $K_1=0_3$ is used and the control is done by the velocities x_2.

In the control laws (14) and (19) the values $K_1=0_3$, $K_2=I_3$ и $\bar{K}_1=\omega_0^3 I_3$, $\bar{K}_2=3\omega_0^2 I_3$, $\bar{K}_3=3\omega_0 I_3$, $\omega_0=15$ are used. Obviously, in this case for the first isolated subsystem, roots of the characteristic equation are $s_{1,2,3}=-1$, and for the second isolated subsystem $s_{1,2,3}=-15$.

Assumed a wind gust (5) with the values $V_{p\max}=10$ m/s, $\Delta t_p=40$ s on each axis of the coordinate system projected onto the body frame [2]. The coefficients matrices of the observer are obtained using the method of [6] with $\mu = 0.5$ for the first subsystem: $L_1 = diag\{22.4, 28.4, 31.3\}$, $L_2 = diag\{138, 260.4, 333\}$.

At the same time the reduced observer (16) has $P = diag\{-14.4, -20.4, -23.3\}$.

The matrices of coefficients were obtained for the second subsystem with $\mu = 0.5$ are $L_1 = [L_{11}\ \ L_{12}]^T$, $L_{11} = diag\{98, 86.5, 73.8\}$, $L_{12} = diag\{3006, 2424, 1817\}$; $L_2 = 10^4 diag\{2.916, 2.219, 1.49\}$. At the same time the reduced observer (18) has: $P = -diag\{30.5, 28, 24.6\}$, $G = -diag\{652.4, 569.1, 452.7\}$.

4. FUZZY CONTROLLER DESIGN

Further simplification of the controller can be obtained by designing the Mamdani fuzzy controller, whose setting is performed by the results of the controller with the observers.

To increase the speed of the fuzzy controller and to achieve smoothness of the process in each channel two signals at the input are used: the error between the true value and the command and its rate of change. Control law uses seven membership functions for each input and one output.

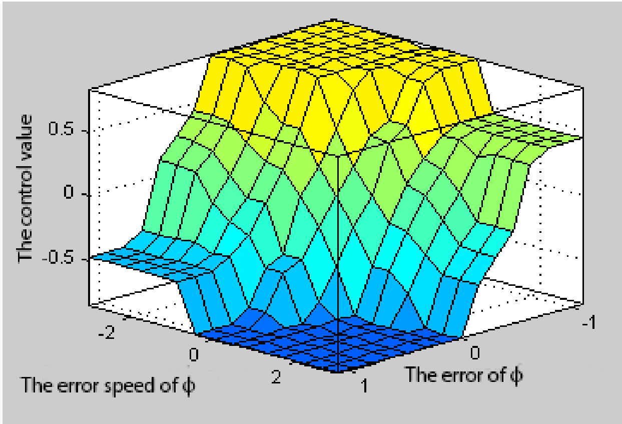
Writing rules for fuzzy controllers of Mamdani type requires to create a database of rules of the form:

$$If\ A_{i,j} = B_{i,j}\ and\ A_{i+1,j} = B_{i+1,j}\ and\ \ldots\ \ldots\ and\ A_{n,j} = B_{n,j}\ \ Then\ C_j,$$

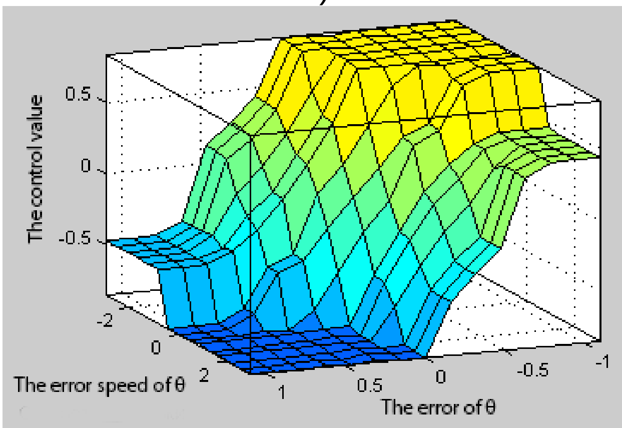
where $A_{i,j}$ - input variable, $B_{i,j}$ - compares the value, C_j - conclusion.

The rule base establishes a relationship between the level of the input signal and control output. Moreover, each controller consists of forty-nine lines.

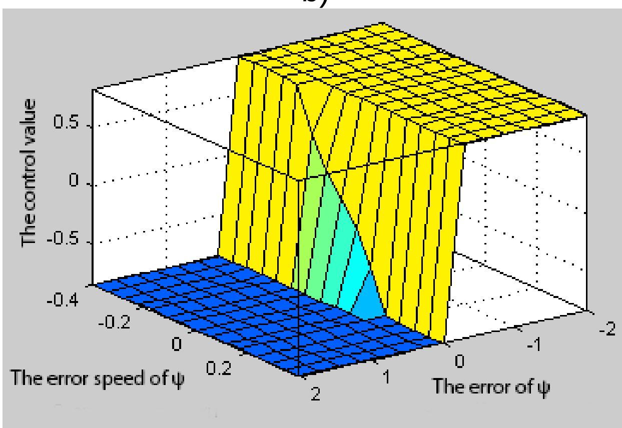
The control Surfaces for each channel are shown in Figure 2.



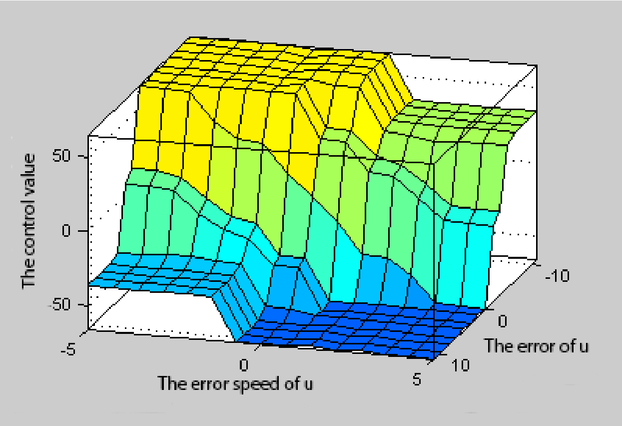
a)



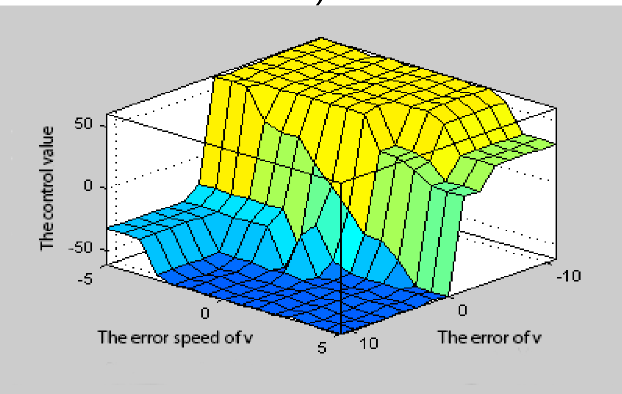
b)



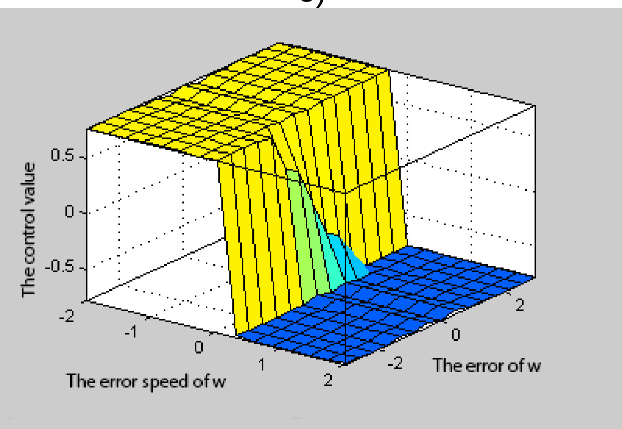
c)



d)



e)



f)

Figure 2: Control surfaces for the regulators: a) roll angle, b) pitch angle, c) yaw angle, d) longitudinal velocity, e) lateral velocity, f) vertical velocity.

5. Simulation results

A comparative analysis of the simulation of the Raptor helicopter dynamics is conducted in hover mode with and without wind gusts (5) for the control laws (14), (16), (18), (19), the fuzzy controller and the robust controller obtained in [2] using the H_∞ method. The following measurements are used: the linear helicopter velocity in the earth coordinate system, the angular velocities and angles. The simulation results are shown in Table 1, and suggests that the controller with observers and the fuzzy controller has the best dynamic properties.

Also a comparative analysis of the simulation of the helicopter dynamics is conducted for the pirouette maneuver without wind effects. The maneuver begins from hover mode at a height of 20 m. The maneuver requires movement of the helicopter on a circle with a radius of 10 m. Nose of the helicopter must be constantly sent to the center of the circle throughout the maneuver.
Table 2 presents the errors for the respective axes in the earth coordinate system and the angle of the course.

Figure 3 shows the "pirouette" maneuver for different control laws in the absence of wind. Here the controller with observers has the best dynamic properties, and also copes with wind disturbances.

Table 3 presents the simulation results for the pirouette maneuver in the wind for all channels simultaneously. It follows that the designed controllers successfully cope with the impact of the wind.

The effectiveness of the proposed control laws was also confirmed by results obtained for the simulation of a simplified model of the ANSAT helicopter, as well as by experimental results on a laboratory stand using the Raptor helicopter. This illustrated in Figure 4. Figure 5 shows the experimental processes.

Table 1: Hover mode with wind gust on each channel

The error	Maximum modulo error								
	u_{wind}			v_{wind}			w_{wind}		
	Controller with observ servers	Fuzzy controller	[2]	Controller with observers	Fuzzy controller	[2]	Controller with observ servers	Fuzzy controller	[2]
Δx (m)	0.115	0.47	0.48	7.8e-5	6e-5	0.03	6.9e-3	8.7e-3	5e-3
Δy (m)	1.0e-4	3e-4	0.12	0.8	2.76	3.24	0.079	0.047	0.19
Δz (m)	1.2e-4	6e-3	2.24	4e-3	0.305	2.60	0.016	0.382	2.24
Δu (m/s)	9e-3	0.026	0.04	0.9e-5	2e-5	0.01	5.4e-4	4.4e-4	1e-3
Δv (m/s)	7.8e-6	4e-5	0.01	0.074	0.134	0.24	6.2e-3	2.4e-3	0.01
Δw (m/s)	1.3e-5	5e-4	0.21	1.2e-3	0.02	0.23	1.3e-3	0.022	0.17

Table 2: The pirouette maneuver without wind gusts

The error	Maximum modulo error		
	The controller with observers	The fuzzy controller	[2]
Δx (m)	0.34	0.44	1.16
Δy (m)	0.24	1.79	1.41
Δz (m)	0.17	0.55	0.18
$\Delta \psi$ (deg)	1.06	1.1	4.18

Table 3: The pirouette maneuver with wind gusts on all channels simultaneously

The error	Maximum modulo error	
	The controller with observers	The fuzzy controller
Δx (m)	0.86	1.12
Δy (m)	0.25	1.98
Δz (m)	0.3	0.55
$\Delta \psi$ (deg)	1.07	1.14

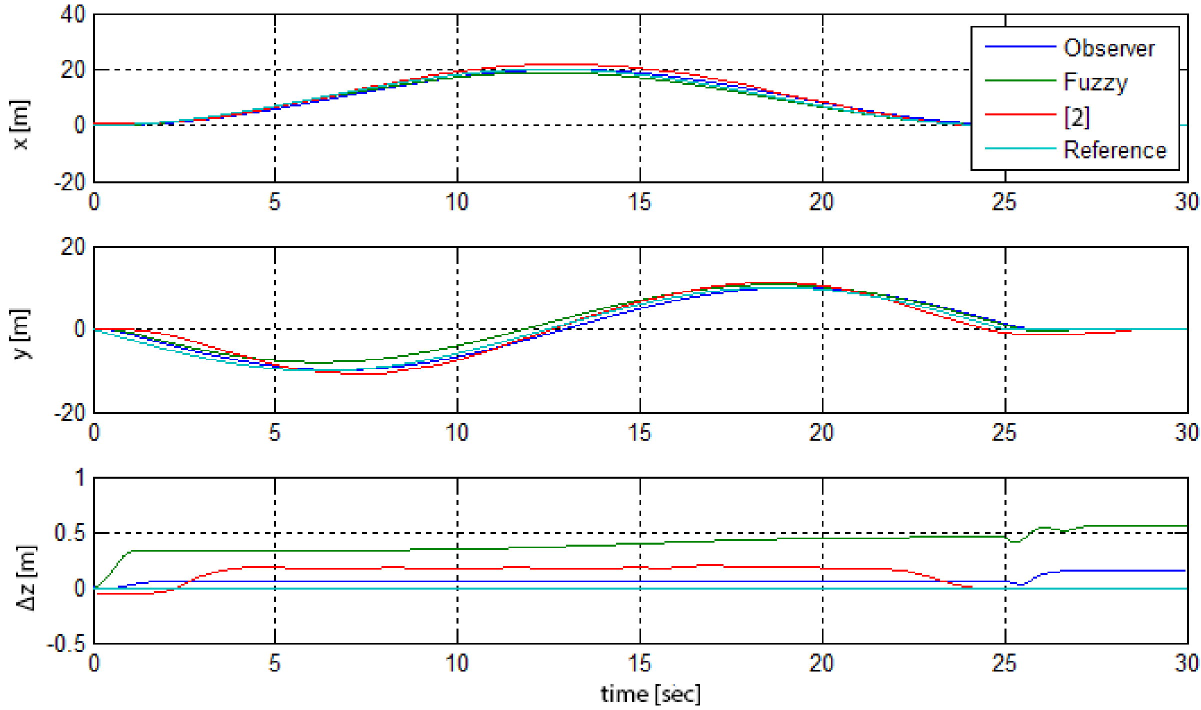


Figure 3: Simulation responses of pirouette for different control laws.

Figure 4: the 2DOF Raptor helicopter stand.

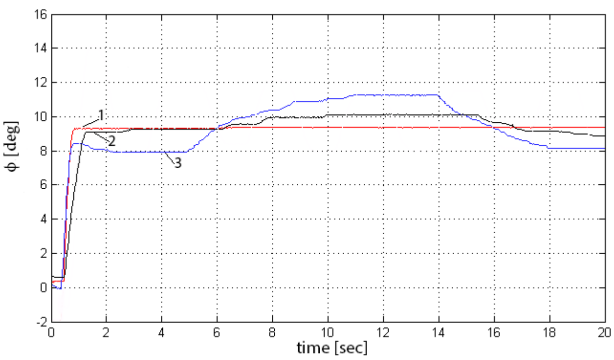


Figure 5: Stabilization of roll angle with disturbance $w = 0.4 \cdot \sin(0.4 \cdot t)$ in the control channel: 1 - PID with observer of disturbance, 2 - fuzzy controller; 3 – PID.

6. CONCLUSIONS

Using the proposed control laws it is possible to increase the stability of the Raptor helicopter compared [2] and to simplify the procedure for setting the parameters of the model helicopter and the coefficients of the controllers. The obtained control laws are recommended for use in the auto-pilot of full-size helicopter, after further refinement.

FUTURE WORK

At the next stage of the work it is planned to test the developed algorithms for the ANSAT helicopter using a non-linear model and test the control laws for the Raptor helicopter in flight.

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