APPROXIMATIONS FOR INCLUSION OF ROTOR LAG DYNAMICS IN HELICOPTER FLIGHT DYNAMICS MODELS

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Abstract

Approximate forms are suggested for augmenting linear rotor/body response models to include rotor lag dynamics. Use of an analytically linearized rotor/body model has shown that the primary affect comes from the additional angular rate contributions of the lag inertial response. Addition of lag dynamics may be made assuming these dynamics are represented by an isolated rotor with no shaft motion. Implications of such an approximation are indicated through comparison with flight test data and sensitivity of stability levels with body rate feedback.

Introduction

Development of high performance flight control systems for rotorcraft requires good definition of the rotor dynamics and their influence on the dynamic characteristics of the aircraft. System identification techniques often need to be employed to verify the important parameters governing the dynamic response of the complete rotor-body system. While the flapping dynamics can be included in many system identification studies, such that the coupled body/flap motion can be identified, little success has been obtained in identifying the lag contributions. It has been shown that inclusion of the lag dynamics is important in the design of high performance flight control systems [1], and there has been considerable investigation recently of the possibility of increasing the lag damping with feedback [2,3]. Successful studies of this sort require good models of the coupled body/flap/lag motions. The lag motion, while clearly a contributor to the response of the helicopter, is more difficult to identify because of the increasing order of the dynamic model required to describe the complete system response.

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While the full motion of the combined helicopter fuselage and rotor is highly nonlinear due to both rotor wake interactions and aeroelastic couplings, linearized models of rotor/body response are useful in both stability assessment and flight control system design. To this end, the development of linear rotorcraft flight dynamics models continues to be an active area of research. Linear models have been developed directly from system identification studies on flight test data [4], numerical linearization of general-purpose simulation programs [5,6], as part of comprehensive helicopter analysis codes [7], and from analytically linearizing symbolically generated rotor/body equations of motion [8]. This paper will outline the continuing development of a linearized rotor/body dynamic model, validate its capability to accurately predict helicopter response to control inputs, and use it to illustrate the inclusion of lag dynamics in body/flap dynamic models.

Linear Model Development

Development of a linearized rotor/body dynamic model at Princeton has been underway for a number of years. Initially started as a means of analyzing flexible shaft couplings in ground resonance problems, the dynamic equations are formulated using a Lagrangian approach in order to capture all the important inertial coupling terms in the analysis. Early extensions to the model allowed for rigid body motion to be treated as a special case of a generalized hub displacement, resulting in a model for hovering flight that includes rigid blade cyclic lag and flap multiblade coordinates, cyclic dynamic inflow, and fuselage translation and rotation degrees of freedom [9]. This program was extended to include forward flight aerodynamics, and collective modes for flap (coning), lag and dynamic inflow through a re-derivation using both REDUCE and MACSYMA symbolic manipulation programs [8]. This process required that the complete energy expressions for the coupled helicopter body/rotor system be computed in stages, resulting in an unwieldy process to generate each term in the final dynamic equations. In addition, the formulation incorporated programming shortcuts that required adherence to a particular combination of rigid body modes to represent the previously generalized hub motion. Linearization was achieved through symbolic manipulation, and a simplified flat wake model was used to represent tail rotor and tailplane interactional aerodynamics. Correlation with flight test was excellent for on-axis responses and fair to good for off-axis responses.

In order to improve the predictive capability for off-axis response to pilot inputs, the model was investigated in detail for sensitivities of the results to modest parametric variations. This study led to improvements in the model, including incorporation of more modeling detail in the proper phasing of the swashplate inputs with steady lag angles and particular hub geometries [10]. Since the code for generating the system matrices of the rotor/body model had been the product of several faculty and graduate student efforts, the most recent improvements have been to organize the input data to provide a consistent and logical set of information to allow ease of analysis for a variety of helicopter types [11]. Extensive correlation exercises with this improved model show it to predict the response to pilot input quite well, as can be seen in Fig. 1.

Because of this checkered development history, however, the current rotor/body model is difficult to extend to include additional degrees of freedom such as fuselage elastic modes, or drive train and blade flexibility, and thus a program is underway to re-derive the model yet again in a more consistent fashion that allows direct user interaction in the equation generation process. Such a feature allows treatment of specialized hub geometries, added flexibility in the specification of desired degrees of freedom considered, and output of matrix coefficients for a linearized model in a form suitable for direct compilation into Fortran. All of this functionality is accomplished through the use of the Mathematica programming environment [12], with its symbolic manipulation capability and Fortran output features.

Such computer-aided equation generation is not new, but previous researchers have either concentrated upon production of complete nonlinear models through symbolic computation [13] or linear models via cleverly organized numerical integration techniques [14,15]. The method adopted at Princeton is based upon a symbolic version of the approach outlined in [14], as it leads to the smallest memory requirements during the generation of the inertial terms in the equations, and direct linearization about a nominal trim point. Sample Mathematica code segments to generate inertial terms for the re-derived model are included in this paper as an Appendix.

Inclusion of Lag Dynamics

Since the primary interest in the use of this linearized model in this paper is to investigate the effect of adding lag dynamics, it is worthwhile to validate the capability of the model to predict the lag response using flight data. Tests conducted in 1989 on a highly instrumented Blackhawk helicopter included some flights specifically designed to provide high quality data for frequency-domain identification exercises. One such run from this data base is shown in Fig. 2, where a sine sweep (chirp) input was applied in the lateral cyclic stick at hover in order to excite the aircraft roll dynamics. This test point will be used to discuss the effects of lag dynamics on overall aircraft response, since horizontal tail aerodynamic contributions will be minimal for this flight condition.

A smoothed frequency response function can be estimated from this forced response record through the ratio of the cross power spectrum of the windowed input and output signals to the power spectrum of the lateral stick input. This estimate is presented as circled data in Fig. 3, which includes the curves representing the linearized rotor/body model in hover for roll rate response to lateral stick input. Matching of the transfer function magnitude response near both the regressing and advancing lag modes is excellent, but some variation exists in the phase angle correlation with theory. Despite this latter anomaly, partially due to a decrease of coherence near rotor rotational speed, sufficient confidence in the magnitude predictions justifies the use of the linearized model in the discussions that follow.

As discussed in [16], classical quasi-static flight dynamics models for helicopters are inadequate to describe the high frequency response behavior of contemporary helicopters, due to the manner in which rotor moments are generated from cyclic pitch inputs at these frequencies. Significant departures in magnitude and phase characteristics in roll rate due to lateral cyclic inputs begin to appear above frequencies as low as one-tenth of rotor rpm. While inclusion of the regressing flap mode recaptures the magnitude and phase of the full system response out to approximately 0.3 times rotor frequency, addition of the flap advancing mode (resulting in a fourth-order flap model) is still not sufficient to significantly match the dynamic response of the full model.

This is due to the fact that without the inclusion of lag dynamics, the in-plane shears generated from cyclic inputs are modeled as direct control effectiveness terms in the expression for rolling moment. Inclusion of lag dynamics provides "inertial relief" at high frequencies, such that direct control terms and lag acceleration almost cancel, resulting in marked reductions in the magnitude response of the lateral cyclic to roll rate response transfer functions beyond the lag advancing mode. The lag dynamics also introduce a zero pair near the regressing lag mode that gives rise to significant reductions in phase response, adversely affecting overall stability at moderate gain levels of roll rate feedback (see [1]).

Since the lag dynamic character is so closely coupled to the generation of rotor moments as a function of cyclic inputs, it would seem that a reasonable approximation to augment body/flap flight dynamics models would be the inclusion of lag acceleration effects <u>assuming the body is fixed in space</u>. This approach is reasonable, as the primary contribution to the overall dynamic behavior will be at relatively high frequencies, near the rotor rotation speed. In order to confirm this theory, the lateral multiblade lag response rate for the complete body/flap/lag dynamic model is compared with that calculated assuming no body motion in Fig. 4. As can be seen, very little difference between the two responses is evident, suggesting that inclusion of the lag acceleration effect from such an approximation should provide the necessary improvement in a body/flap dynamic model. That it still represents an approximate correction may be seen in Fig. 5, where the lateral lag displacement is compared for the full system and the rotor-only model. Offsets in the lag displacement show that the "trim" values for the two lag systems (one having the body rolling, one with a fixed shaft) accounts for the differences in responses, indicating a mismatch at low frequencies.

Finally, if the lag acceleration is a dominant effect at high frequency in the rolling moment equation, then the expression for roll acceleration will include a term proportional to lag acceleration as well. That is, since:

$$I_x \dot{p} = L$$

and

$$L = [\text{ (other terms) }] - \varepsilon \ddot{\zeta}_{1c}$$

then:

$$\dot{p} = [\ldots] - \left\{ \frac{b}{2} \frac{S_B}{I_X} \right\} \ddot{\zeta}_{1c}$$

so a reasonable approximation to the roll response transfer function would be:

$$\left. \frac{p(j\omega)}{A_{1s}(j\omega)} \right|_{\text{full system}} = \left. \frac{p(j\omega)}{A_{1s}(j\omega)} \right|_{\text{no lag}} - \varepsilon \left. \frac{\dot{\zeta}_{1c}(j\omega)}{A_{1s}(j\omega)} \right|_{\text{body fixed}}$$

Figure 6 compares the difference between the full and the "no-lag" transfer function from lateral cyclic to roll rate, with the transfer function from lateral cyclic to lateral lag rate. The pronounced similarity of these two frequency responses near the lag regressing and advancing modes suggests that this approximation is a good one, suitable for use in system identification applications using flight data.

<u>Conclusions</u>

Further refinements of a linearized rotor/body model have permitted its extension to include additional aeroelastic effects in a consistent fashion, aiding investigation of additional configurations. The model correlates well with both time and frequency response flight test data, and can be used for investigations into approximations for simplified dynamic model sets. One such approximation has shown that the lag dynamics, of importance in capturing high frequency aircraft response behavior, may be added to an existing rotorcraft model by assuming the lag response to be effected only by control inputs. This permits the addition of a lag model that assumes the rotor shaft is fixed in space, simplifying the coupling necessary to add to an existing flight dynamics model.

Acknowledgements

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Appendix

Sample Mathematica code for generation of inertial terms in a linearized model are included for illustration of the approach used in the updated rotor/body mathematical model.

```
(* Mathematica use for linearized rotor/body
model in hover:
    development of inertial terms ONLY *)

(* Read in the file containing the functions for
automatic equation generation *)
<< tools

(* Define position of rotor blade in space *)
beta[t] = beta0[t] + beta1c[t]*Cos[psi[t]] +
beta1s[t]*Sin[psi[t]];
zeta[t] = zeta0[t] + zeta1c[t]*Cos[psi[t]] +
zeta1s[t]*Sin[psi[t]];
pos = translate[x,0,0];
pos = rotateY[beta[t]].pos;
pos = rotateZ[zeta[t]].pos;
pos = pos + translate[e,0,0];
pos = rotateZ[ -psi[t] ].pos;

(* Re-orient blade axes <x-aft,y-stbd,z-up> to
fuselage body
 axes <x-fwd,y-stbd,z-down> *)
pos = {{-1,0,0},{0,1,0},{0,0,-1}}.pos;

(* Position body in inertial space via Euler angles
and displacements
*)
pos = rotateX[-phix[t]].pos;
pos = rotateY[-phiy[t]].pos;
pos = rotateZ[-phiz[t]].pos;
pos = pos + translate[xh[t],yh[t],zh[t]];

(* Velocity and accelerations *)
vel = D[pos,t];
acc = D[pos,{t,2}];

(* Assign dof's *)
ndof = 12;
q[1]  = beta0[t];
q[2]  = beta1c[t];
q[3]  = beta1s[t];
q[4]  = zeta0[t];
q[5]  = zeta1c[t];
q[6]  = zeta1s[t];
q[7]  = phix[t];
q[8]  = phiy[t];
q[9]  = phiz[t];
q[10] = xh[t];
q[11] = yh[t];
q[12] = zh[t];
setdof[12];

(* Control Inputs *)
ninputs = 1;
u[1] = theta[t];
u0[1] = 0;

(* Assign initial conditions *)
q0[1] = beta00;
q0[2] = beta1c0;
q0[3] = beta1s0;
q0[4] = zeta00;
q0[5] = zeta1c0;
q0[6] = zeta1s0;
q0[7] = 0;
q0[8] = 0;
q0[9] = 0;
q0[10] = 0;
q0[11] = 0;
q0[12] = 0;

(* Get inertial matrices *)
inertial[ndof,ninputs];
```

"tools" file containing utility functions for equation generation:

```
(*******************************
Mathematica procedure for generation of
linearized mass, damping, control and stiffness
matrices from inertial, aerodynamic and
structural contributions, as per Gibbons-Done
approach (Vertica,8,n.3,1984,pp.229-241,
"Automatic Generation of Helicopter Rotor
Aeroelastic Equations of Motion", M.P. Gibbons,
G.T.S. Done).

This routine assumes the existence of 3x1
symbolic vector quantities "pos","vel", and
"acc", and returns associated elements of the P,
Q, and R matrices (a.k.a. M, C and K matrices)

The linearized equations are thus:

Pm qddot + { Qm + Qa } qdot + { Rm + Ra + Rs } q
= Ba u

***********************)
```

```
(*********************************)
(*  Initialization Functions  *)
(*********************************)

(* Assign name of output file for results *)
f77[mat_,i_,j_] := SequenceForm["
",mat,"(",i,",",j,")  =  "];
OpenWrite["math.out",FormatType-
>FortranForm,PageWidth->58];

(* Assign generalized dof names to problem
dof's. *)
setdof[n_]:=
      Block[{i,j},
        Do[
             qdot[i]  = D[ q[i],t ];
             qddot[i] = D[ q[i],{t,2} ],
          {i,n} ];
    ];

(* Initial condition evaluation *)
zero[exp_,val_] := exp /.val->0;
icset[exp_,var_,val_] := exp /. var->val;
icees[exp_,n_,m_] :=
      Block[{i,tmp},
         tmp = exp;
        Do[
              tmp = icset[tmp,q[i],q0[i]];
             tmp = zero[tmp,qdot[i]];
             tmp = zero[tmp,qddot[i]],
          {i,n}];
        Do[
              tmp = icset[tmp,u[i],u0[i]],
          {i,m}];
          tmp = tmp /. psi'[t]->1;
          tmp = zero[tmp,psi''[t]];
         Return[ tmp ]
    ];

(*********************************
Integral Definitions and Integration Operators
*********************************)
(* Azimuthal averaging for integrating over 1
revolution *)
azave[exp_,v_,n_] := Sum[(exp /. v->2 Pi ia /
n),  {ia,0,n-1}]/n;

(* Assign names to various mass integrals over
blade span *)
massint[exp_] :=
     Block[ {tmp},
        tmp = 0;
           tmp = tmp + ib*Coefficient[
Expand[scalar[exp]],x,2];
           tmp = tmp + sb*Coefficient[
Expand[scalar[exp]],x,1];
           tmp = tmp + mb*Coefficient[
Expand[scalar[exp]],x,0];
            tmp = azave[tmp,psi[t],4];
          Return[tmp]
     ];

(* Fast polynomial integration *)
pintg[exp_,x_] :=
      Block[ {tmp},
         tmp = Expand[exp];
        Sum[
((x^(n+1))/(n+1))*Coefficient[tmp,x,n],
           {n,0,Exponent[tmp,x]}]
     ];

(* Polynomial integration with limits *)
pintg2[exp_,x_,lo_,hi_] :=
      Block[ {zz},
          zz = pintg[ exp, x];
           ( zz /. x->hi ) - ( zz /. x->lo )
     ];

(* Define span integral operator *)
spanint[exp_] := azave[ pintg2[
scalar[exp],x,0,1] ,  psi[t],  4];

(***************************)
(*  Inertial Contributions  *)
(***************************)

(* Inertial contribution to linearized matrices *)
pmat[qi_,qj_] := massint[ Transpose[ D[pos,qi]
].D[ pos,qj ] ];
qmat[qi_,qj_] := massint[ 2*Transpose[
D[pos,qi] ].D[ vel,qj ]];
rmat[qi_,qj_] := massint[ Transpose[ D[pos,qi]
].D[ acc,qj ] +
             Transpose[ D[pos,qi,qj] ].acc ];

(* Inertial contributions *)
inertial[n_,m_]:=
      Block[ {i,j,text},
        Do[
              WriteString["math.out",f77[pm,i,j]];
           Write["math.out",icees[
pmat[q[i],q[j]],n,m  ]  ],
          {j,n},{i,n}];
        Do[
              WriteString["math.out",f77[qm,i,j]];
```

```
            Write["math.out",icees[
qmat[q[i],q[j]],n,m  ]  ],
          {j,n},{i,n}];
     Do[
              WriteString["math.out",f77[rm,i,j]];
            Write["math.out",icees[
rmat[q[i],q[j]],n,m  ]  ],
          {j,n},{i,n}]
   ];


(******************************)
(* Linear Algebra Operators *)
(******************************)

translate[x_,y_,z_]:=  {{x},{y},{z}};
```

```
rotateX[a_]:=  {{1,0,0},{0,Cos[a],Sin[a]},{0,-
Sin[a],Cos[a]}};
rotateY[a_]:=  {{Cos[a],0,-
Sin[a]},{0,1,0},{Sin[a],0,Cos[a]}};
rotateZ[a_]:=  {{Cos[a],Sin[a],0},{-
Sin[a],Cos[a],0},{0,0,1}};

smallangles[exp_,x_] := exp /. {Cos[x]->1,
Sin[x]->x,  Cos[-x]->1,
      Sin[-x]->-x};

scalar[exp_]  :=  exp[[1,1]];
```

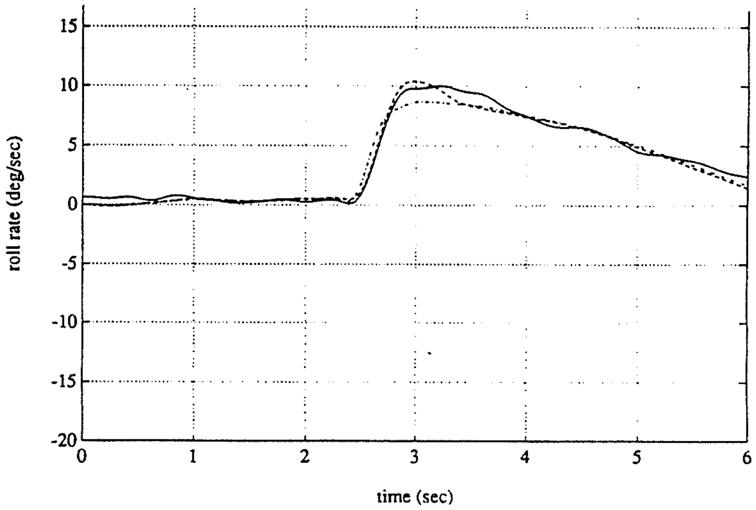

Fig. 1: UH-60 roll response to lateral cyclic step input, hover correlation.

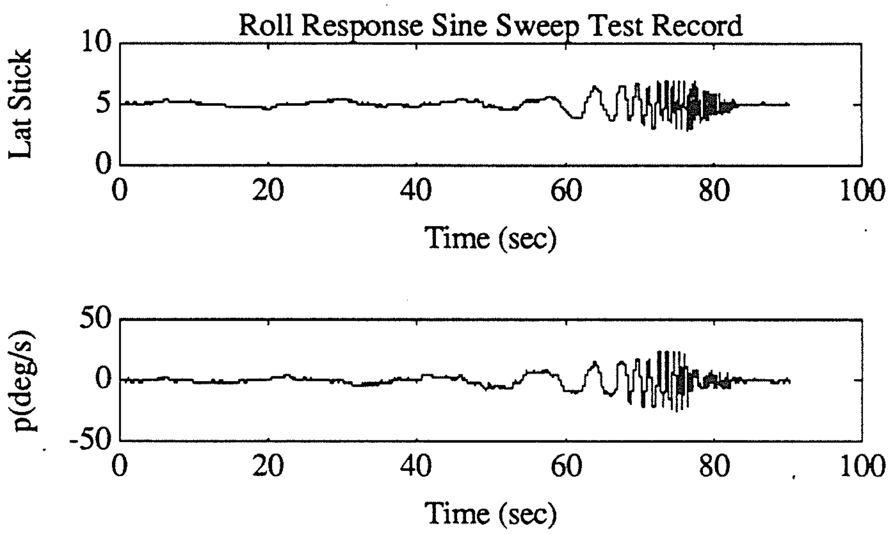

Fig. 2: "Chirp" sine sweep lateral cyclic excitation of roll response, UH-60.

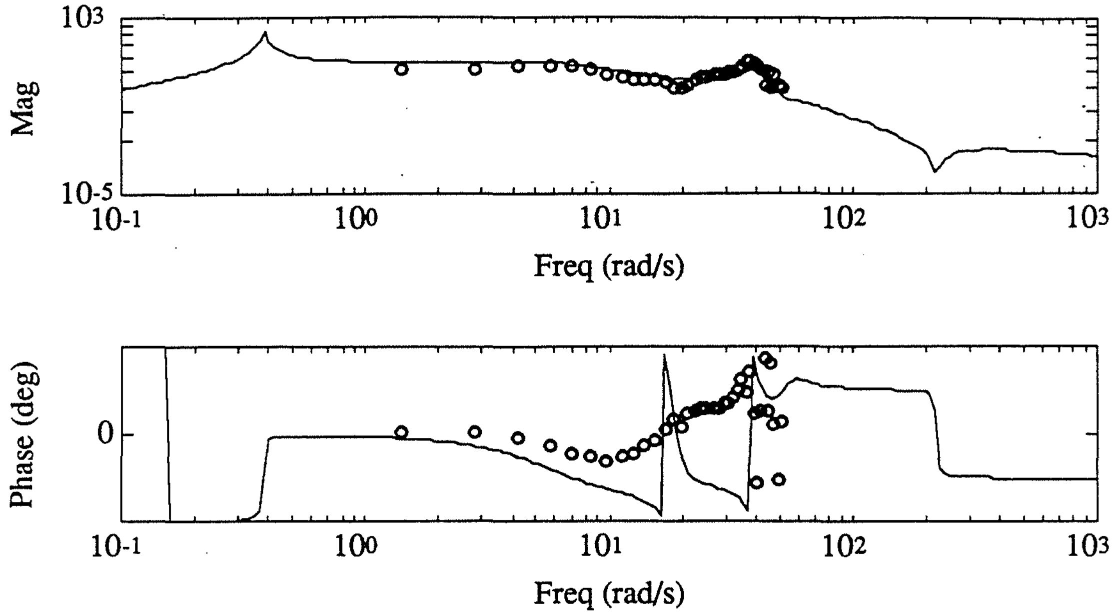

Fig. 3: Measured and predicted lateral cyclic to roll transfer function, UH-60.

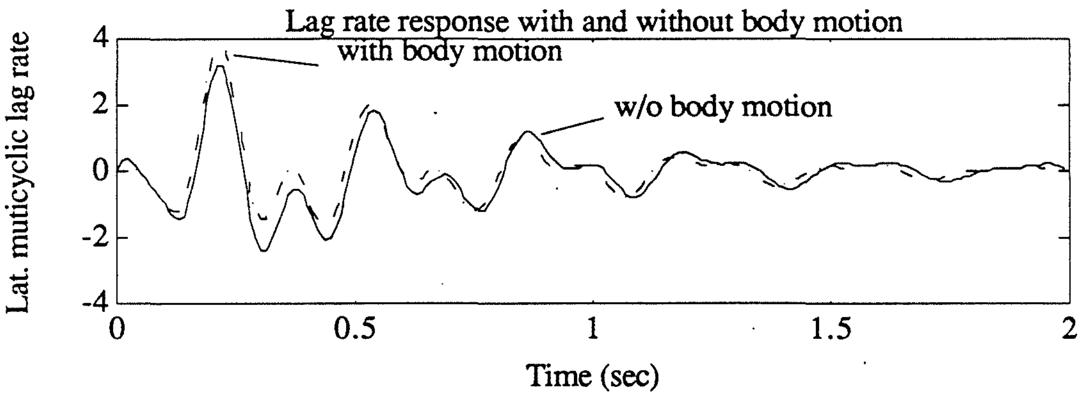

Fig. 4: Lateral multiblade lag rate response including and neglecting body motion

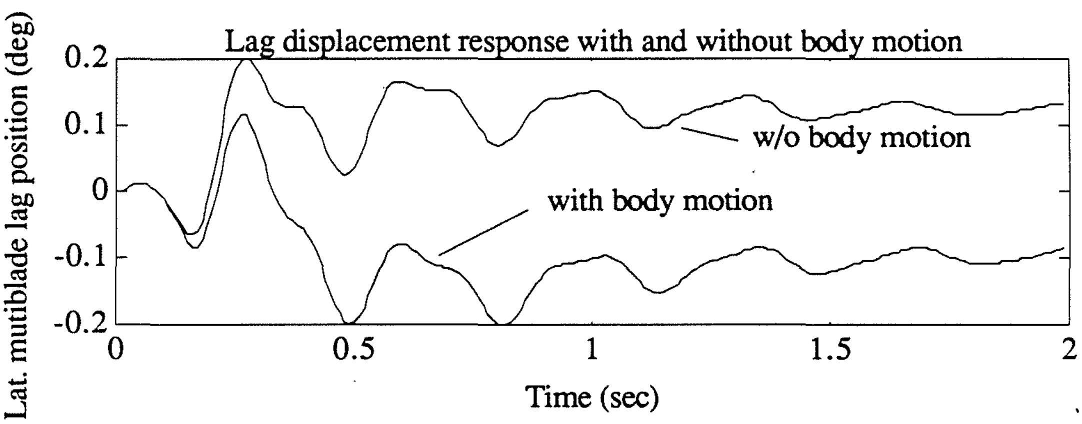

Fig. 5: Lateral multiblade lag position response including and neglecting body motion

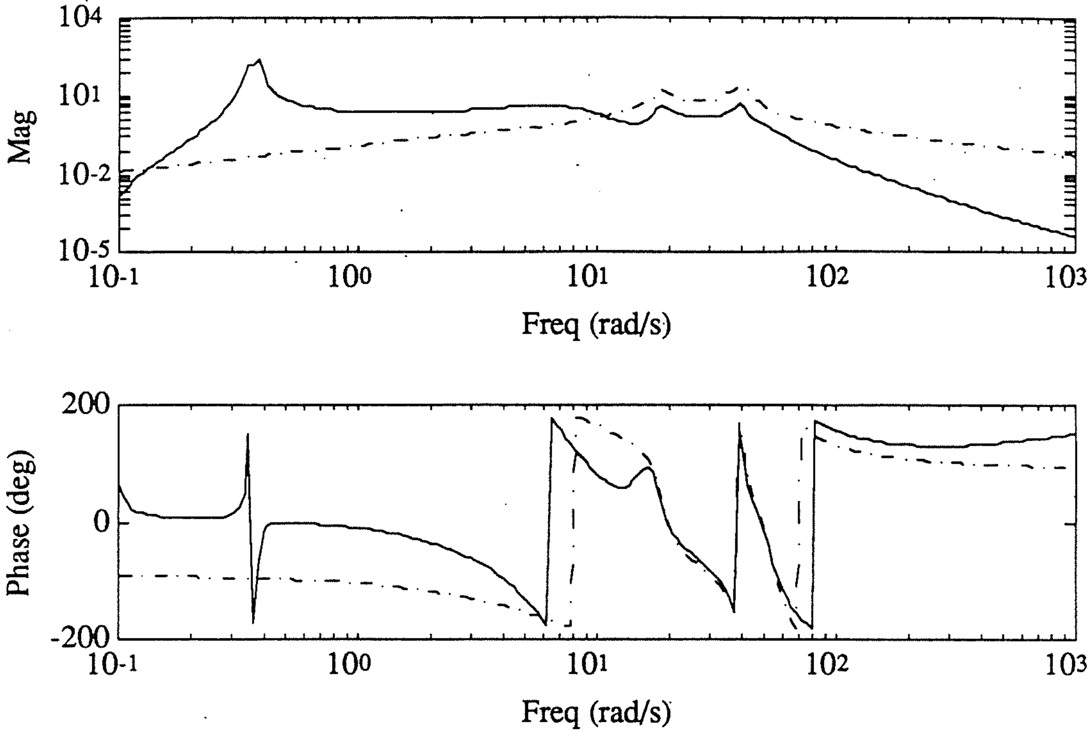

Fig. 6: Frequency response comparison of :

$$\left.\frac{p(j\omega)}{A_{1s}(j\omega)}\right|_{\text{full system}} - \left.\frac{p(j\omega)}{A_{1s}(j\omega)}\right|_{\text{no lag}} \quad \text{(solid line)}$$

$$\text{with:} \quad - \varepsilon \left.\frac{\dot{\zeta}_{1c}(j\omega)}{A_{1s}(j\omega)}\right|_{\text{body fixed}} \quad \text{(dashed line)}$$