EIGHTEENTH EUROPEAN ROTORCRAFT FORUM

BU - 13

Paper N° 150

CONTRIBUTION TO PERFORMANCE ASSESSMENT OF HELICOPTER ROTOR SPEED CONTROL BY NUMERICAL OPTIMIZATION

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ONERA, FRANCE

Septembre 15-18, 1992

AVIGNON, FRANCE

ASSOCIATION AERONAUTIQUE ET ASTRONAUTIQUE DE FRANCE

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Résumé. Cet article présente une étude relative à l'amélioration des performances en manoeuvre d'un hélicoptère par l'utilisation du contrôle actif moteur. La manoeuvre considérée consiste à effectuer en temps fixé un saut dans un plan vertical et en revenant à la même altitude de départ. Le modèle étudié est du type point matériel avec une commande supplémentaire de vitesse de rotation du rotor. La résolution du problème de commande optimale est faite par deux méthodes numériques d'optimisation, respectivement un méthode du premier ordre (gradient projeté) et une méthode du second ordre (quasilinéarisation). Un accroissement de la vitesse de rotation du rotor principal de 10% par rapport à la valeur nominale permet ainsi, pour certains cas de vol, un gain de plus de 60 % sur la variation de l'altitude maximale atteinte. Ces résultats permettent d'illustrer l'intérêt d'une telle commande pour augmenter les performances de l'hélicoptère.

Abstract. This paper is concerned with the improvement to helicopter manoeuvrability and agility by control of a continuous, variable rotor speed. The manoeuver considered here consists in jumping in a vertical plane and returning to level flight at the same initial altitude, the end time is fixed. The helicopter model is a point-mass, and a new control related to rotor speed is considered in addition to the classic control variables. Optimal control strategies are obtained by two numerical optimization techniques, respectively from first-order (projected gradient) and second-order (quasilinearization) algorithm. In comparison with a constant rotor speed vehicle, a 10 % increase of rotor speed allows thus, in some flight conditions, more than 60 % improvement in the maximum altitude variation. This illustrates interest of variable rotor speed control to extend maneuverability of helicopters.

Nomenclature.

R	= rotor radius	M	= aircraft weight
σ	= rotor solidity	g	= gravitational constant
x	= x-coordinate (earth-fixed)	T	= rotor thrust
V	= aircraft velocity	Czm	= lift coefficient
h	= aircraft altitude	$Cz\alpha$	= lift gradient of the blade
Ω	= rotor speed	θ_o	= collective angle
θ	= aircraft body pitch angle	$(SCx)_f$	= drag surface coefficient of the fuselage
α_d	= rotor attack angle		
μ_d	= advance ratio	W_a	= available power
λ_z	= vertical speed ratio	W_r	= required power
λ_i	= induced velocity	W_m	= engine power
ρ	= air density		

1. Introduction

The development of a new generation of flighter helicopters in the last decade motivated an increase interest in performance improvements. Maneuverability of helicopters depends on many parameters, some of which include disk loading, blade loading, engine power, blade hinge offset, rotor speed. All these factors are more or less important, but the performance agility in all flight regions is directly related to the ability to produce high level of rotor thrust. This rotor thrust depends on many fixed factors and on rotor speed.

In the eighties, it has been demonstrated that increasing rotor speed could improve flying qualities for certain maneuvers. Research performed today suggests that using rotor speed control provides several advantages, namely an increase control power and responsiveness, lower vibration levels and additional rotor thrust, which can be used to maneuver more agressively [1].

This paper presents results of an optimization study which demonstrates the significant improvement in maneuverability obtained by using continuous, variable rotor speed control. A particular maneuver was chosen, namely a jump in a vertical plane to avoid an obstacle. The equation of motion and the rotor speed dynamic give a dynamical system described by a set of first-order differential equations. A payoff functional is then defined to formulate a optimal control problem.

This problem is then solved using two numerical optimization methods, namely the projected gradient method and the quasilinearization. The gradient method provides first an approximate optimal solution. The second method is then used, in order to improve the final convergence. Numerical results, related to a typical helicopter model, are then presented.

2. Equations of motion

It is known that a point mass model is able to provide valid results for performance evaluation [1]. The aerodynamic forces on the aircraft are defined by :

- the rotor thrust T, perpendicular to the rotor tip-path plane, the down-wash effects on the fuselage being neglected ;
- the fuselage drag, defined by the surface drag coefficient $(SCx)_f$, in opposite direction to the air-speed vector.

The equations of motion, in a vertical plane, referenced to a fixed frame, are:

$$\dot{x} = V_x \quad (1)$$

$$\dot{h} = V_z \quad (2)$$

$$\dot{V}_x = \frac{1}{M}(-T\sin\theta - \frac{1}{2}\rho(SCx)_f VV_x) \quad (3)$$

$$\dot{V}_z = \frac{1}{M}(T\cos\theta - \frac{1}{2}\rho(SCx)_f VV_z) - g \quad (4)$$

$$V = \sqrt{V_z^2 + V_x^2}$$

$$\sin\theta = \frac{V_z\cos\alpha_d + V_x\sin\alpha_d}{V}$$

$$\cos\theta = \frac{V_x\cos\alpha_d - V_z\sin\alpha_d}{V}$$

where (x, h, V_x, V_z) are respectively horizontal distance, altitude, horizontal and vertical components of airspeed V, θ and α_d are the references of the tip path plane to the earth-fixed and kinematic frame (see Fig.1).

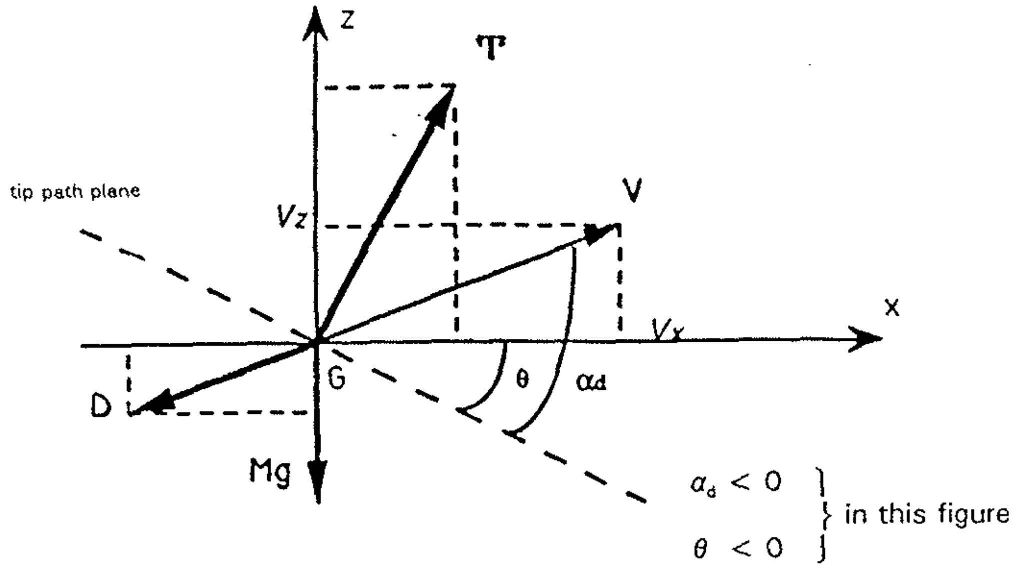


Fig. 1 - Parameter definition

The rotor thrust is calculated using the classical blade element and disk model theory [2][3]. For preliminary purposes, this rotor thrust is simplified. It will be defined by and analytical expression [4] :

$$T = \frac{1}{6}\rho S\sigma(\Omega R)^2 Czm \quad (5)$$

$$Czm \approx \frac{KCzmo}{1-\frac{\sigma . KCzm1}{12 . \mu_d}} \quad (6)$$

In these relations (5)(6), Czm is the mean lift coefficient, μ_d is the advance ratio, Kczmo and $Kczm_1$ are coefficients which depend on blade parameters (θ_0, $C_{z\alpha}$) and flight conditions :

$Kczmo = Kczmo\ (\mu_d, \lambda_z, C_{z\alpha}, \theta o)$
$Kczm_1 = Kczm_1\ (\mu_d, C_{z\alpha})$

Let us notice that the relations (5)(6) are valid when the difference (λ_z-λ_i) between vertical speed and induced speed ratio remains lower than advance ratio, i.e. $|\lambda_z-\lambda_i| \ll \mu_d$. This approximation avoids a complex implicit equation to be solved, at each integration step, to obtain the induced speed [4].

The dynamics of the rotor is described by :

$$\dot{\Omega} = \frac{W_a - W_r}{I_x\,\Omega} \quad (7)$$

where Wa is the available power on the main rotor, Wr is the required power. Wa is a function of the engine power Wm, the efficiency of the transmission system..., Wr is a function of the flight conditions, the rotor speed...

The aircraft classical controls, with a point-mass model, are the collective angle θo and the attack angle α_d of the tip-path plane, and rotor speed is assumed to be constant.

Normally, in order to evaluate interest of continuous, variable (C-V) rotor speed control, one might consider engine power control Wm, with constraints on allowable range of variation of rotor speed, and engine power.

In order to evaluate improvements to maneuverability using (C-V) rotor speed control, this equation (7) will be substitute by a first order system, for sake of simplification :

$$\dot{\Omega} = \frac{\Omega_{des}-\Omega}{\tau} \qquad (8)$$

where τ is the time constant, Ω_{des} represents a commanded rotor speed.

Aircraft controls must respect the following limitations :

$$\begin{aligned} \theta_{0m} &\leq \theta_0 \leq \theta_{0M} \\ \alpha_{dm} &\leq \alpha_d \leq \alpha_{dM} \end{aligned} \qquad (9a)$$

and in case of (C-V) rotor speed control :

$$|\Omega_{des}| \leq 1{,}10\ \Omega_0 \qquad (9b)$$

where Ω_0 is the nominal constant rotor speed.

3. The optimal control problem

3.1. Problem formulation

In order to get an objective quantification of the performance enhancement by using (C-V) rotor speed control, the first step consists to define properly the obstacle avoidance manoeuvre (o.a.m.) in terms of an optimal control problem. Insight of interest of (C-V) rotor speed control can be then obtained by comparaison of performance achieved by optimal control strategies without or with (C-V) rotor speed control. For nap-over-earth (n.o.e.) manoeuvre, an optimal (o.a.m.) could consist in passing round the obstacle as close as possible to the obstacle and returning to level flight in minimum time.

For preliminary purposes, a simpler manoeuvre is considered hereafter : it consists in jumping in a vertical plane and returning to a horizontal flight at the same initial height. The pay-off function, to be maximized, is defined as the area described by the aircraft trajectory in the x-h plane (see Fig.2).

The optimal control problem can be then stated as follows : find the control law which transfers the aircraft dynamics, described by (1)-(8), from a given initial state to some final state at fixed final time, while minimizing the criterion :

$$J = \int_{t_0}^{t_f} (h-h_0)V_x\, dt + K\int_{t_0}^{t_f} \left(\frac{\theta_0-\theta_{00}}{\theta_{00}}\right)^2 dt + K\int_{t_0}^{t_f} \left(\frac{\Omega_d-\Omega_0}{\Omega_0}\right)^2 dt \qquad (10)$$

The first term represents the surface of the trajectory in the x-h axes. Note that the other terms of this fonctional are added to alleviate the Bang-Bang solution on the control variables Ω_d and θ_0.

The optimized trajectory must fulfill the following constraints on altitude and horizontal airspeed :

$$h(t_f) = h_0$$
$$V_x(t_f) = V_x(o) = 0$$

3.2. Mesures of effectiveness

The expected trajectory of the helicopter is given in Fig.2.

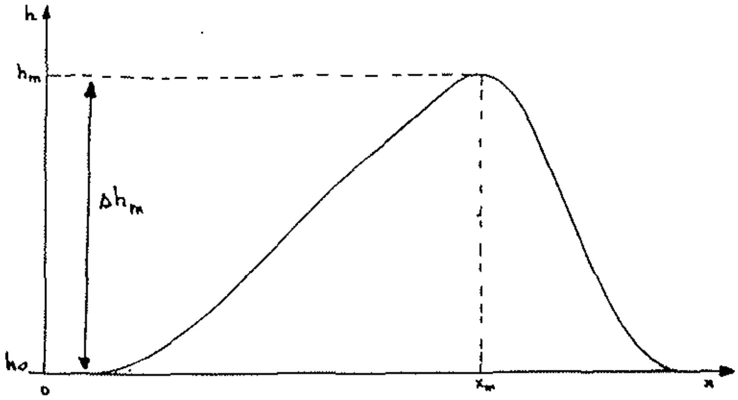


Fig.2 - Typical flight profile

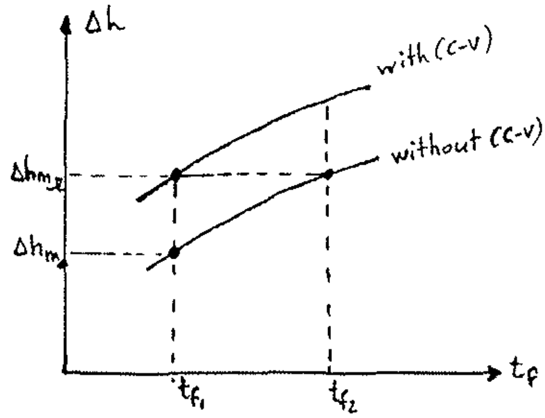


Fig.3 - Performance measures

It is characterized by coordinates (x_m, h_m) related to maximum achieved altitude. By varying final time t_f, the results obtained without, and with (C-V) rotor speed control can be presented in Fig.3. Enhancement of performance provided by rotor speed control could then be analysed from two ways :

a) With a fixed flight time t_f, (C-V) rotor speed control provides a gain on maximum altitude variation, defined by $\Delta h_{m\Omega}/\Delta h_{mo}$;
b) Given a altitude variation Δh_m, (C-V) rotor speed control provides a gain on the flight time, which could be characterized by $(t_{f2}-t_{f1})/t_{f1}$.

4. Optimal control

Optimal control theory [5] shows that the solution of the problem, formulated by equations (1) to (10), necessitates to solve a non-linear differential equations system, with conditions defined at initial and final values ; thus solution can only been obtained through numerical optimization techniques [5].

The optimal control law for control vector U* (with $U = (\theta_0, \alpha_d, \Omega_d)^T$), must satisfy the following conditions :

$$\dot{\lambda}_x = 0 \qquad \lambda_{xf} = 0 \qquad (12)$$
$$\dot{\lambda}_h = V_z \qquad \lambda_{hf} = \nu_h \qquad (13)$$
$$\dot{\lambda}_{vx} = -\frac{\partial H}{\partial V_x} \qquad \lambda_{vxf} = 0 \qquad (14)$$
$$\dot{\lambda}_{vz} = -\frac{\partial H}{\partial V_z} \qquad \lambda_{vzf} = \nu_{vz} \qquad (15)$$
$$\dot{\lambda}_{\Omega} = -\frac{\partial H}{\partial \Omega} \qquad \lambda_{\Omega f} = 0 \qquad (16)$$
$$U* = \underset{U}{Argmin}\, H \qquad (17)$$

where :

$$H = -(h-h_0)V_x+K(\frac{\theta_0-\theta_{00}}{\theta_{00}})^2+K(\frac{\Omega_d-\Omega_0}{\Omega_0})^2+\lambda_x V_x+\lambda_h V_z+\lambda_\Omega(\frac{\Omega_d-\Omega}{\tau})$$
$$+\lambda_{vx}(\frac{1}{M}(-T\sin\theta-\frac{1}{2}\rho(SCx)_f VV_x))+\lambda_{vz}(\frac{1}{M}(T\cos\theta-\frac{1}{2}\rho(SCx)_f VV_z)-g) \quad (18)$$

The adjoint vector Λ $\{\Lambda = (\Lambda_x, \Lambda_h, \Lambda_{vx}, \Lambda_{vz}, \Lambda_\Omega)^T\}$, related to the corresponding state vector x $\{x = (x, h, V_x, V_z, \Omega)^T\}$, is defined by end conditions at time t_f, ν_h and ν_{vz} are the lagrange multipliers which must be calculated such that the final constraints on state variables are verified.

With the aircraft model used in this study, optimal control solution from (17) can be derived analytically for θ_0 and Ω_d, and α_d is given by an implicit equation :

$$\Omega_d{*} = -\frac{\lambda_\Omega \Omega_0^2}{\tau 2K} + \Omega_0 \quad (9)$$

$$\theta_0{*} = -\frac{\partial T}{\partial \theta_0}\cdot\frac{-\lambda_{vx}\sin\theta+\lambda_{vz}\cos\theta}{M}\cdot\frac{\theta_{00}^2}{2K}+\theta_{00} \quad (20)$$

$$0 = \frac{1}{M}\frac{\partial T}{\partial \alpha_d}(-\lambda_{vx}\sin\theta+\lambda_{vz}\cos\theta)+\frac{T}{M}(-\lambda_{vx}\cos\theta-\lambda_{vz}\sin\theta) \quad (21)$$

where

$$\frac{\partial T}{\partial \alpha_d} = \frac{1}{6}\rho S\sigma(\Omega R)^2\cdot\frac{1}{(1-\frac{\sigma.KCzm1}{12.\mu_d})^2}.G \quad with \quad G = G(\mu_d, \lambda_z, C_{z\alpha}, \theta_0)$$

Complete analytical relations are given in [4].

If we notice that $\mu_d^2 \ll 1$, expression (21) can be simplified :

$$\frac{\frac{3}{2}\cdot\mu_d}{(\theta_0+\frac{3}{2}\lambda_z)} - \frac{\frac{\lambda_z}{\mu_d}\frac{\sigma.Cz\alpha}{8.\mu_d}}{(1+\frac{\sigma.Cz\alpha}{8.\mu_d})} = \frac{(\lambda_{vx}\cos\theta+\lambda_{vz}\sin\theta)}{(-\lambda_{vx}\sin\theta+\lambda_{vz}\cos\theta)} \quad (22)$$

5. Numerical optimization solution

The optimal control problem is solved using projected gradient method and quasilinearization. The gradient method gives a first approach of the optimal solution, the second-order method is chosen in order to verify the optimality of the solution and improve the results.

5.1. Projected gradient algorithm

Let us recall that gradient methods, which are first order type, present the main advantage to provide good robustness, which means that convergence can be achieved even if the initial trajectory remains far away from the optimal solution. Nevertheless, as all first order techniques, the final rate of convergence is rather slow.

The optimal solution uses firstly a generalized projected gradient which has been developed at ONERA [6], and well experienced, for general purpose of optimization trajectory. This method allows to take into account several kinds of constraints (controls, state, ...), by using the multiple adjoint vector technique, where a distinct adjoint vector is associated with each final constraint.

5.2. Quasilinearization method

Original algorithm of quasilinearization method [7][8], has been recently modified at ONERA [4], its description is more detailed below.

Consider the state and the adjoint differential equations (1), (2), (3), (4), (8), (12) to (16) and substitute the control variable in these equations by its optimal expression (19), (20), (22). The optimal solution is obtained when the following conditions are verified :

$$\mathring{Z} = F(Z) \quad with \quad Z = (x^T, \lambda^T) \tag{23}$$

$$G(Z_f, \nu) = \begin{Bmatrix} \lambda(t_f) - \frac{\partial \phi}{\partial x_f} \\ \psi(x_f) - \psi(\overline{x}_f) \end{Bmatrix} = O \quad with \quad \begin{cases} \phi = \psi^T . \nu \\ \psi = [h, V_z]^T, \nu = [\nu_h, \nu_{vz}] \end{cases} \qquad (24)$$

Suppose that at the N stage of the iteration, an approximate solution has been obtained. To get closer to the exact solution variations δZ will be calculated by the linear differential system :

$$\begin{cases} \mathring{Z}_N + \delta\mathring{Z} - F(Z_N) - \frac{\partial F}{\partial Z}(Z_N) . \delta Z = O & (24) \\ G(Z_N(t_f)\overline{\nu}) + \frac{\partial G}{\partial Z}(Z_N(t_f)) . \delta Z_f + \frac{\partial G}{\partial \nu} . \delta \nu = O & (25) \end{cases}$$

The solution of this system involves a linear two-point boundary value problem (TBBVB), indeed (24) must be solved from the knowledge of partly initial state Z (t_0) and partly final state Z (t_f) and unknown parameters $\delta\nu$. The complete solution is obtained by the transition matrix techniques which is described in [4].

The new functions and the control are then :

$$Z_{N+1} = Z_N + \delta Z \quad with \quad U*_{N+1} = Argmin\ H\ (Z_N + \delta Z)$$

To avoid divergence with this method, during first steps matrices D_N and E_N can be introduced which depend on errors between an estimate optimal solution and the nominal solution. These matrices have the following properties :

$$D_N = Identity \quad when \quad (\mathring{Z}_N - F(Z_N)) < \varepsilon_d$$
$$E_N = Identity \quad when \quad G(Z_N(t_f), \nu_N) < \varepsilon_f$$

and the linearized differential systems is then

$$\begin{cases} \delta\mathring{Z} = \frac{\partial F}{\partial Z}(\delta Z) - D_N . (\mathring{Z}_N - F(Z_N)) & (26) \\ E_N G(Z_{Nf}) + \frac{\partial G}{\partial Z} . (Z_{Nf}) \delta Z + \frac{\partial G}{\partial \nu} . \delta \nu = O & (27) \end{cases}$$

A step by step description of the algorithm follows :

1) Assume initial nominal state and adjoint functions Z (t),
2) At the N stage of the iteration, evaluate matrices

$$\frac{\partial F}{\partial Z}(Z_N) \quad , \quad D_N \quad and \quad E_N,$$

3) Solve the linear two-point boundary value problem described by (26),(27) and obtain $Z_{N+1} = Z_N + \delta Z$,
4) Compute U^*_{N+1} from U^*_{N+1} = Argmin H $(Z_N+\delta Z)$,
5) Compute e_d = N $(Z_{N+1}$-F $(Z_{N+1}))$ and e_f = N (G $(Z_{N+1}(t_f))$),
6) The optimal solution is obtained when $e_f < \epsilon_f$ and $e_d < \epsilon_d$. Otherwise go to 2.

6. Numerical results and discussion

Numerical simulations have been performed using a typical helicopter model, with the following conditions : one initial state value related to low-speed horizontal flight, and several fixed final times, respectively 10s, 12s and 15s. Numerical values are given in Appendix 1.

The projected gradient algorithm was initialized with a level flight trajectory and convergence was achieved, with this first order technique, after about some hundred iterations. Initialization of quasilinearization algorithm, from solution obtained by gradient method, allows a slight improvement of results, after a few iterations (less than 10).

The results are presented in Table 1, Fig.4 and 5, display control laws and trajectories related to one final time (12s), without and with rotor speed control.

Let us notice that, for this jump manoeuvre, the optimal solution, for continuous rotor speed control, is a constant maximum allowable rotor speed. Approximations used in problem formulation (§ 2-4) are fulfilled in the numerical results (see [4]).

Table 1 shows that an increase of 10 % in nominal rotor speed allows an increase in the maximum altitude variation of more than 60 %.

Table 1 - Optimal computation results

end time	with rotor speed control			without rotor speed control		
t_f(s)	Δh_m(m)	x_m(m)	x_f(m)	Δh_m(m)	x_m(m)	x_f(m)
10	20,5	140	210	12,5	126	190
12	31	185	270	18,5	165	245
15	41,5	269	380	26	225	345

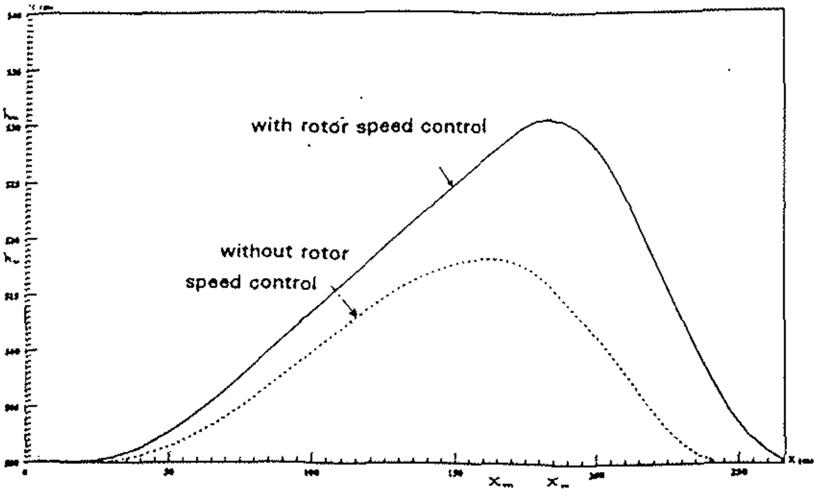
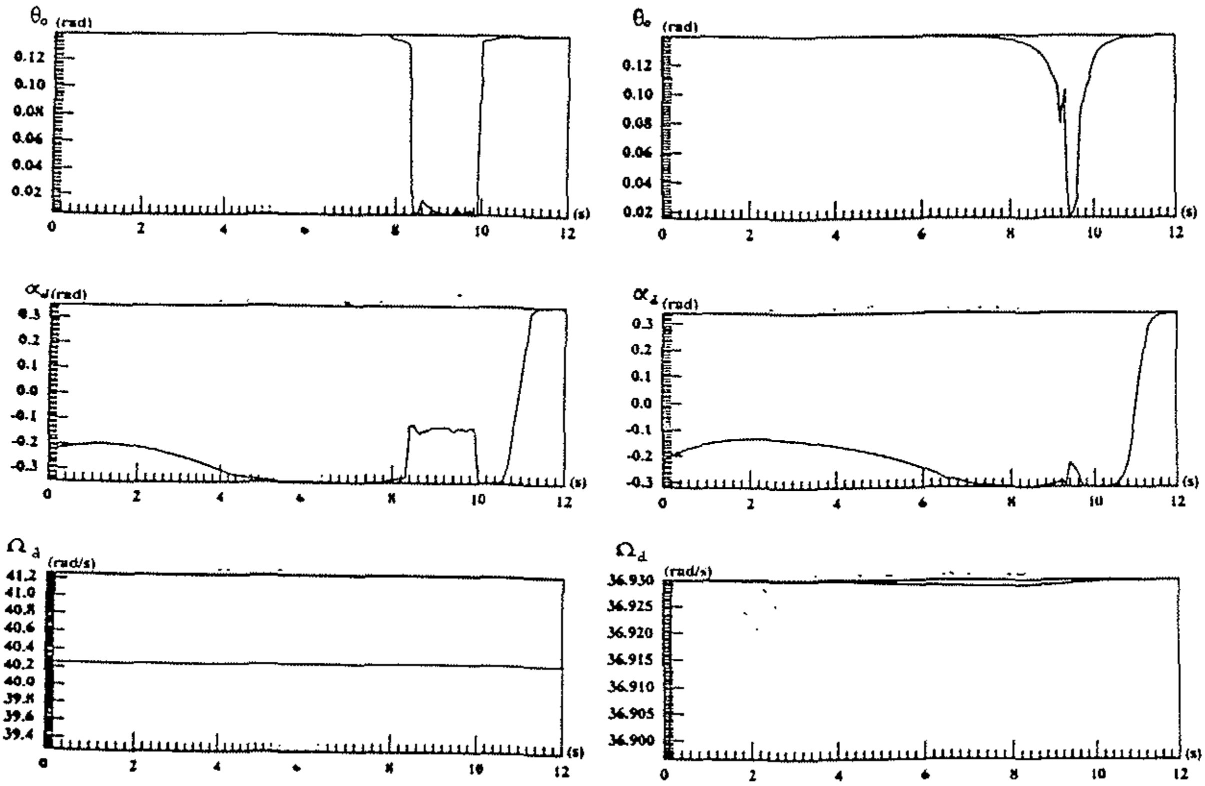


Fig.4 - Optimal trajectory



with rotor speed control　　　　without rotor speed control

Fig.5 - Control histories

7. Conclusion

Although only one maneuvre was considered, the different cases of typical "obstacle avoidance maneuvre" show that an increase of a 10 % of rotor speed control allows an increase of variation on maximum achievable height of more than 60 %. Validation of simplications adopted in this paper and of the point-mass helicopter model should be investigated by comparison with a more complete simulation model and also with flight tests. Other manoeuvres need also to be considered, in order to confirm the interest of continuous-various rotor speed control.

Nevertheless, the obtained results illustrate the great interest of rotor speed control to increase the operational performances of future helicopters. This study demonstrates also the usefullness of numerical optimization techniques for performance evaluation of helicopter in complex maneuvres.

Acknowledgement

The author wishes to thank M. Huynh ONERA, for his valuable comments and remarks.

Appendix 1

Main constants used in numerical simulations :

ρ_0	=	1,225 kg/m^3	Initial conditions / constraints
M	=	3 500 kg	
R	=	5,965 m	x_0 = 0 m
σ	=	0,086	h_0 = 500 m
α_{d0}	=	- 0,0062 rd	V_{x0} = 15 m/s
θ_{0r}	=	0,12 rd	V_{z0} = 0
Ω_0	=	36,6 rd/s	θ_{0m} = 0 ; θ_{0M} = 0,4 rd
K	=	0.001	Ω_{dm} = - 0,35 ; Ω_{dM} = 0,35 rd

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