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SOLUTION OF THE COMPRESSIBLE EULER EQUATIONS FOR STEADY FLOWS AROUND HELICOPTER ROTOR BLADES BY AN IMPLICIT SPACE-CENTERED METHOD

by

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ABSTRACT

An implicit space-centered Euler solver due to A. Lerat, and previously developed, at ONERA, for the calculation of steady transonic flows around airfoils and wings, is used here for the numerical simulation of steady compressible flows around helicopter rotor blades.

For an extension to the problem of a multibladed rotor in hovering-flight, the method is formulated in a rotating frame attached to the blades, using the pseudo-unsteady system of the Euler equations derived by assuming that the total rothalpy is constant.

The basic 3D wing code is first applied to the calculation of the steady flow past a wall-mounted half wing with swept tip shape, simulating the transonic flow occuring at the tip of a helicopter rotor blade.

Then, preliminary numerical results concerning the two-bladed model rotor of US-Army in hovering-flight, and obtained in the framework of a global rotor calculation without wake modelling, are presented and compared with experimental data.

1. INTRODUCTION

This study is devoted to the numerical solution of steady flows around helicopter rotor blades by solving the compressible Euler equations, following pioneering works in U.S.A. [1-8], and more recently in Europe [9-13].

The Euler equations describe the flow of an inviscid and non heat-conducting fluid. The inviscid assumption is generally well adapted to model high Reynolds number flows when the viscous effects are not too important. In addition, contrary to the full potential equation, the Euler equations contain the exact discontinuities of the perfect fluid model : shock waves verifying the Rankine-Hugoniot relations and also vortex sheets. The capability to calculate rotational phenomena is of major interest for rotor blade flow simulations.

Although we are only interested, here, on a steady-state solution, the unsteady form of the Euler equations is considered. This is to take advantage of their hyperbolic nature in time and to use a time-iterative method starting from given initial data. However, in order to improve the computational efficiency, the full unsteady Euler equations can be replaced by pseudo-unsteady systems giving consistent steady solutions. For steady airfoil or wing applications, a classical pseudo-unsteady system is derived assuming constant total enthalpy. In this pseudo-unsteady system, only the mass and momentum conservation equations are considered, but the energy equation is replaced by the Bernoulli relation. Similarly, for extension to the problem of a multibladed rotor in hovering-flight, the method is formulated, here, in a rotating frame attached to the blades, using a pseudo-unsteady system of the Euler equations. This pseudo-unsteady system, obtained with the assumption that the total rothalpy is constant in the whole flow domain, is described in section 2.

In this work, the method used to solve the Euler equations for helicopter applications, is based on a space-centered implicit solver of second order of accuracy introduced by Lerat [14-16] and developed at ONERA, in the curvilinear finite-volume formulation, for the solution of steady transonic flows around airfoils [17,18] and more recently around wings [19]. We recall that this original space-centered solver works without artificial viscosity for steady transonic flow calculations. This feature is very important for the accuracy of the solutions. In another way, the efficiency of the algorithm results from its implicit character. Details on the theoretical bases and properties of the Lerat's Euler solver, can be found in the previous references.

The implicit solver is slightly modified in order to take into account the additional source terms due to the driving forces, in the pseudo-unsteady formulation developed for the hovering-flight problem. These modifications are similar to those introduced in the work of reference [16] concerning the extension of the basic implicit scheme to the solution of a quasi-one dimensional nozzle Euler flow. The resulting numerical method is summarized in section 3.

Section 4 is devoted to the numerical applications. The basic 3D wing code, introduced in reference [19] and called "WAVES" (Without Artificial Viscosity Euler Solver), is first applied to the calculation of the steady flow past a wall-mounted half wing with swept tip shape simulating the transonic flow occuring at the tip of a helicopter rotor blade. The numerical results are compared with those of experiments performed at ONERA.

Finally, the Euler code "WAVES" extended to rotating blades, is applied to the simulation of the flow around the two-bladed model rotor of US-Army in hovering flight. As in previous works [7-13], we try to take into account the blade vortex interaction, without wake modelling, through a global calculation of the rotor. However, the flow is periodic and the computational domain is restricted around one blade. Preliminary results, obtained with a simplified treatment of the boundary conditions in the far-field and in a rather coarse mesh, are compared with other Euler computations and with available experimental data, for both non-lifting and lifting flow configurations.

2. GOVERNING EQUATIONS

To solve the problem of a multibladed rotor in hovering-flight, the Euler equations are formulated in a rotating frame attached to the blades. Several formulations are possible. A pseudo-unsteady system of the Euler equations, introduced previously for turbomachinery applications [20], is considered here. This system is obtained by choosing, as unknowns of the problem, the components of the projection, in the relative frame, of the relative flow velocity $\vec{U}_r$ given by :

$$\vec{U}_r = \vec{U} - \vec{U}_e ,$$

where $\vec{U}$ is the absolute velocity and where $\vec{U}_e = \vec{\Omega} \times \vec{r}$ is the driven velocity of the blades with respect to the absolute frame; $\vec{\Omega}$ being the rotation velocity of the rotor and $\vec{r}$ the local radial position. The absolute velocity is equal to zero at infinity in the hovering-flight problem. It is possible to integrate the relative energy equation which expresses that the total rothalpy is constant in the whole fluid domain at the steady-state, and we have only to discretize the mass and momentum conservation equations, the pressure being obtained directly from the relation for the total rothalpy.

For a computational domain V, of boundary S with outer normal $\vec{n}$, attached to the blades, the conservation equations of mass and momentum, written in the relative frame, are the following :

$$\int_V \frac{\partial \rho}{\partial t} dV + \int_S \rho \vec{U}_r . \vec{n} dS = 0,$$

$$\int_V \frac{\partial \rho \vec{U}_r}{\partial t} dV + \int_S [\, \rho \vec{U}_r (\vec{U}_r . \vec{n}) + p \vec{n} \,]\, dS = \int_V (\vec{F}_1 + \vec{F}_2) dV, \qquad (1)$$

with the relation for the total rothalpy :

$$I = \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{\vec{U}_r^{\,2}}{2} - \frac{(\vec{\Omega} X \vec{r})^2}{2} = I_0 , \qquad (2)$$

where $I_0 = \frac{\gamma}{\gamma-1} \frac{p_0}{\rho_0}$ is the constant total rothalpy in the unperturbed freestream.

The additional forces $\vec{F}_1$ and $\vec{F}_2$ due to the rotation appear explicitly as source terms in the momentum equation :

$$\vec{F}_1 = - \rho \, \vec{\Omega} X \; (\vec{\Omega} X \vec{r} \,) \qquad \text{(centrifugal force)},$$

and

$$\vec{F}_2 = -2 \, \rho \, \vec{\Omega} X \vec{U}_r \qquad \text{(Coriolis force)}.$$

By using cartesian coordinates in the relative frame (ξ,η,ζ) and setting $\vec{\Omega} = (0, 0, \Omega)^T$, we can write the system (1) in the following condensed form :

$$\int_V \frac{\partial W}{\partial t} d\xi d\eta d\zeta + \int_S [\, f \, n^{\xi} + g \, n^{\eta} + h \, n^{\zeta} \,]\, dS = \int_V T d\xi d\eta d\zeta . \qquad (3)$$

where W, $f = f(W, \vec{r})$, $g = g(W, \vec{r})$, $h = h(W, \vec{r})$ and $T = T(W, \vec{r})$ are given by :

$$W = \begin{pmatrix} \rho \\ \rho u_r \\ \rho v_r \\ \rho w_r \end{pmatrix}, \qquad T = \rho \begin{pmatrix} 0 \\ \Omega^2 \xi + 2\Omega v_r \\ \Omega^2 \eta - 2\Omega u_r \\ 0 \end{pmatrix},$$

$$f = \begin{bmatrix} \rho u_r \\ \rho u_r^2 + p \\ \rho u_r v_r \\ \rho u_r w_r \end{bmatrix}, \quad g = \begin{bmatrix} \rho v_r \\ \rho v_r u_r \\ \rho v_r^2 + p \\ \rho v_r w_r \end{bmatrix}, \quad h = \begin{bmatrix} \rho w_r \\ \rho w_r u_r \\ \rho w_r v_r \\ \rho w_r^2 + p \end{bmatrix},$$

and where u_r, v_r and w_r are the scalar components of the relative velocity $\vec{U}_r$ projected in the relative frame and p is the pressure given by :

$$p = \frac{\gamma-1}{\gamma}\rho\,[I_0 - \frac{\vec{U}_r^2}{2} + \frac{(\vec{\Omega}\, X\, \vec{r})^2}{2}]$$

System (3) can also be written under a local form using the relative coordinates (ξ,η,ζ) as follows :

$$W_t + f_\xi + g_\eta + h_\zeta = T, \qquad (4)$$

with the associated quasi-conservative form :

$$W_t + A\ W_\xi + B\ W_\eta + C\ W_\zeta = T, \qquad (5)$$

where :

$$A = df/dW\ ,\ B = dg/dW\ ,\ C = dh/dW$$

are the Jacobian matrices of the Euler fluxes. We also define :

$$D = dT/dW.$$

The formulation has to be completed by initial data to start with the time-iterative process and also by boundary conditions on the frontiers of the computational domain V. By example on the surface of the blades we apply the slip boundary condition :

$$\vec{U}_r.\vec{n} = 0. \qquad (6)$$

3. NUMERICAL ALGORITHM

The extension of the Lerat's implicit solver to the solution of the Euler equations for transonic wing applications is described in details in [19] where the complete 3D finite-volume curvilinear numerical algorithm is given. We recall that the method can be implemented by means of two successive stages at each time-step :

- an explicit stage of second order of accuracy which is an original multidimensional version of the Lax-Wendroff method (fourth order linearly dissipative) involving one predictor in each space direction,
- an implicit stage of the order of the truncation error of the global scheme allowing the use of large CFL numbers.

The implicit stage is split into each space-direction using the ADI method. This leads to solve only algebraic block-tridiagonal linear systems. It is also possible to simplify the implicit stage, as it was introduced for the first time in reference [21], by replacing the block matrices by their spectral radii ("implicit residual smoothing technique"). In any case, the linear systems are solved efficiently using the LU factorization method.

For application to the hovering-flight problem, the explicit stage of the numerical method is slightly modified, due to the presence of source terms in the equations (3). The modifications introduced are similar to those developed in a previous work by V. Daru and A. Lerat [16] for the application of the method to the problem of a quasi-one dimensional nozzle Euler flow (inhomogeneous term due to the variation of the cross section of the nozzle). In a first version of the hovering-flight code, we have used the "implicit residual smoothing technique" for the implicit stage. In order to summarize the resulting numerical algorithm, it is written below in a semi-discretized form (only the time-differencing) for the solution of system (4).

Explicit stage :

- *Predictor*

$$\Delta\tilde{W}^n = -\Delta t\,(f^n_\xi + g^n_\eta + h^n_\zeta - T^n)$$

- *Corrector :*

$$\Delta W^{expl} = -\Delta t\,(\tilde{f}^{n+1/2}_\xi + \tilde{g}^{n+1/2}_\eta + \tilde{h}^{n+1/2}_\zeta - \tilde{T}^{n+1/2})$$

with :

$$\tilde{f}^{n+1/2} = f^n + \frac{1}{2} A^n\,\Delta\tilde{W}^n, \quad \tilde{g}^{n+1/2} = g^n + \frac{1}{2} B^n\,\Delta\tilde{W}^n,$$

$$\tilde{h}^{n+1/2} = h^n + \frac{1}{2} C^n\,\Delta\tilde{W}^n, \quad \tilde{T}^{n+1/2} = T^n + \frac{1}{2} D^n\,\Delta\tilde{W}^n$$

Implicit stage :

$$\Delta W^* - \frac{\Delta t^2}{2}[(\rho^n_A)^2(\Delta W^*)_\xi]_\xi = \Delta W^{expl} ,$$

$$\Delta W^{**} - \frac{\Delta t^2}{2}[(\rho^n_B)^2(\Delta W^{**})_\eta]_\eta = \Delta W^* ,$$

$$\Delta W - \frac{\Delta t^2}{2}[(\rho^n_C)^2(\Delta W)_\zeta]_\zeta = \Delta W^{**} ,$$

where :

$$f^n = f\,(W^n, \vec{r}) ,$$

(and the same notations holds for $g^n, h^n, T^n, A^n, B^n, C^n, D^n, \rho^n_A, \rho^n_B, \rho^n_C$),

$$\Delta W = W^{n+1} - W^n ,$$

W^n being the numerical solution at time $t = n\,\Delta t$, and ρ^n_A, ρ^n_B, ρ^n_C being respectively the spectral radii of the matrices A^n, B^n, and C^n.

Space-discretization is performed through centered operators leading to second order of accuracy with the use of 3x3x3 points.

4. APPLICATIONS

4.1 Transonic flow around a wall-mounted half-wing

In connection with helicopter problems, applications of the basic Euler wing code "WAVES" described in [19], are presented for the calculation of steady transonic flows around a wall-mounted half-wing model with straight or swept tip shape. The wing is composed of NAC0012 airfoil sections. It is untwisted and has a geometric aspect ratio equal to 3. The sweep angle is of 30^o.

The computational "C-H" mesh used is composed of 102 x 21 x 40 = 85680 cells (with 40 "C" mesh planes in the spanwise direction). A partial view of the mesh on the wall and on the wing surface is represented in figure 1.

Figures 2a and 2b show the iso-Mach lines on the upper surface of the wing in the cases respectively of a straight and of a swept tip and for a flow configuration with a Mach number at upstream infinity equal to 0.85 and with an angle of attack of zero degree. As it is well known, we can see that the presence of the sweep reduces the extension of the supersonic pocket at the tip of the wing and also decreases the intensity of the shock wave. The good numerical representation of the shock waves shows the capability of the method to compute the flow discontinuities without adding any dissipative correction. See also [19] for a complete analysis of the solution obtained in a fine mesh for the reference wing ONERA M6 in transonic regime.

In addition, the robustness of the wing code is tested for a flow configuration with an angle of attack of 6 degrees and with a Mach number equal to 0.60. We can appreciate, in figure 3, the good agreement, between the implicit Euler calculations and the experimental results obtained at ONERA [22], in spite of the Euler model neglecting the viscous effects.

4.2 Two-bladed model rotor of US-Army in hovering-flight

Construction of the grid

To simulate the steady flow around a complete multibladed rotor in hovering-flight, in the framework of a global Euler calculation with pure capture of vortex sheets, the computational domain may be restricted around one blade by applying a periodicity condition. For the specific case of a two-bladed rotor, the grid has to be constructed only for a half rotor disk. Like previously for the wing application, we use a "C-H" mesh topology (figure 4a) with "C" mesh planes distributed along the blade, which is well-adapted to an accurate calculation of the rounded leading edge of the blade sections. However, with this kind of mesh topology, the implementation of the periodicity boundary condition needs to design the blade, in the vicinity of the hub, with symetrical airfoils with respect to the axis of rotation. It also requires the introduction of a fictitious mesh plane outside of the hub (figure 4a). To solve this problem, we have introduced an elliptic profile, with zero incidence, at the hub (figure 4b), as it is made in the work [24]. In addition, other elliptic profiles are distributed along the blade near the hub, and are gradually modified to fit the real blade profiles (figure 4c).

A global view of the mesh for NACA0012 untwisted untapered rotor blade of US-

Army, with aspect ratio equal to 6 and a collective pitch of 5 degrees, is presented in figure 4d. The grid, constructed for the first numerical experiments presented here is rather coarse for a helicopter problem. It is composed of 102 x 21 x 40 = 85680 cells, with 40 mesh planes in the radial direction, and with only 25 planes along the blade. The grid extension is the following : three times the blade length (R) in the radial direction, twice the blade length in the direction normal to the blade, and twice the blade length above and below the rotor disk.

boundary conditions

We have considered two kinds of boundary conditions with the hover code in the computational domain of figure 4a. In the first one, called "isolated-blade" simulation, the periodicity boundary condition is replaced by entry/exit unperturbed flow conditions applied in the cells of the fictitious mesh-plane. This simulation, allows a code validation without the implementation of the interpolation routines needed for the periodicity condition. In another way it can be used for the non-lifting calculations. However, in a classical way [1-6], the blade vortex interaction could be simulated by the introduction of wake corrections. In the second kind of problem, called "interacting-blades" simulation, the blade vortex interaction is taken into account through the periodicity boundary condition, without particular treatment.

In both the "isolated-blade" and "interacting-blades" simulations, the conditions of unperturbed freestream are applied in the far field. At infinity the absolute velocity $\vec{U}_0$ is equal to zero and then we prescribe : $\vec{U}_r = -\vec{\Omega} X \vec{r}$. This treatment, not consistent with the characteristic theory generates spurious waves reflexions on the frontiers, and has to be improved in further calculations.

The slip condition (6) is applied on the blade surface with the conservative treatment used in the wing code [19] which does not require any extrapolation procedure to determine the pressure at the wall.

The time-iterative process is started by prescribing the conditions of the unpertubed freestream in all the cells of the computational domain that is :

$$\rho = \rho_0 \, , p = p_0 \text{ and } \vec{U}_r = -\vec{\Omega} X \vec{r} \, .$$

Non-lifting calculation

A first validation of the hovering-flight code is proposed in the case of a classical hover configuration, for the two-bladed model rotor of US-Army [23], with a tip Mach number M_t equal to 0.52 and a collective pitch angle θ_c equal to 0^0.

There is no wake in this configuration and, as expected, the solutions obtained are quite similar for the isolated-blade and the interacting-blades simulation. The lines of constant relative Mach number calculated on the blade surface are presented in figure 5a. In addition, the variation of the local pressure coefficient :

$$[Cp_l = (p - p_0) / (1/2.\rho_0(|\vec{\Omega}|.|\vec{r}|)^2]$$

is plotted in figure 5b around various blade sections. The comparison shows a good agreement with an other Euler solution [4] and with the experiment of US-Army [23].

For this application with no wake, the code converges very fastly to a steady solution using a CFL number equal to 10. The computing cost of the hover code is, at present time, of 12.10^{-6} sec per iteration and per mesh-cell on the CRAY 2 computer. 1000 iterations are needed to obtain a converged solution, with a mean quadratic residual on density lower than 10^{-5}, leading to a global CPU time of 18 minutes.

Lifting calculation

We have calculated the lifting hovering-flight configuration corresponding to a tip Mach number $M_t = 0.815$, and to a collective pitch angle $\theta_c = 5^o$.

As in the previous application, an "isolated-blade" calculation converges fastly to a steady-state. The real two-bladed calculation requires three times more iterations. (3200 iterations are needed to reach a mean residual of 5.10^{-5}). We have observed that the maximum of the residuals is reached in the planes where the periodicity condition is applied and more precisely near the hub. The total computing cost is 55 minutes on the CRAY 2 computer.

In figure 6 we compare the implicit Euler solutions obtained respectively in the case of an isolated blade calculation (figure 6a), and in the case of a real two-bladed computation (figure 6b) with introduction of the periodicity boundary condition. More precisely, the iso-Mach lines for the relative Mach number on the upper surface of the blade are compared for the two calculations. As expected, it is observed that the transonic pocket is reduced when the blade vortex interaction is taken into account. We can observe the good representation of the numerical shock structure. We recall that all the calculations work without artificial viscosity. Nevertheless, the comparison with experimental data [23] for the local pressure coefficient at some spanwise locations near the tip of the blades (figure 7), is not so good than the comparisons presented in references [11-13] for Euler solutions obtained also in the framework of a global rotor calculation. These preliminary results will be problably improved by using a finer mesh resolution with remoted frontiers, and also by applying a more accurate treatment for the implementation of boundary conditions in the far field.

5. CONCLUSIONS

A validation of the Euler wing code "WAVES" is first presented for the calculation of the steady flow past a wall-mounted half wing with swept tip shape, simulating the transonic flow occuring at the tip of a helicopter rotor blade. The comparison with experiments performed at ONERA is very good.

Then, the "WAVES" code, extended to a multibladed rotor in hovering-flight using a pseudo-unsteady system of the Euler equations, is applied to the calculation of the two bladed model rotor of US-Army. Both non-lifting and lifting configurations are considered. In subsonic non-lifting case the implicit method converges fastly to steady state. The agreement with experiment is good. For the lifting configuration, in transonic regime, the speed of convergence is reduced due to the presence of the wake and also probably due to the pure explicit treatment of the periodicity condition used. However, the computing time is reduced in an important way with respect to classical explicit methods. Non oscillating shock structure are obtained without artificial viscosity. Preliminary results for lifting case show some discrepancy with the experiment. So, it seems to be necessary to follow on the development of the code with particular care for the implementation of the boundary conditions especially in the far field. Another way to improve the calculation is to use finer grids. Finally a more general mesh topology [25], suitable for rotors with more than two blades, could be considered. It is also a "C-H" mesh topology, but the "C" meshes are projected on circular cylinders.

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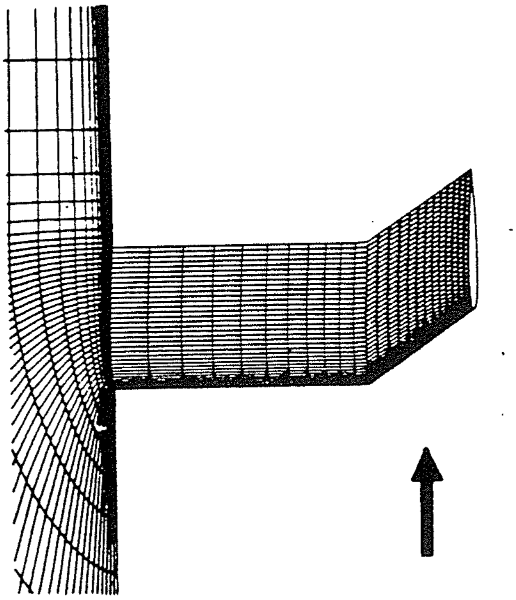
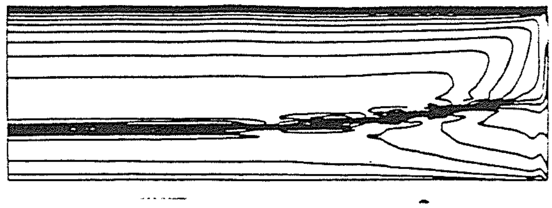
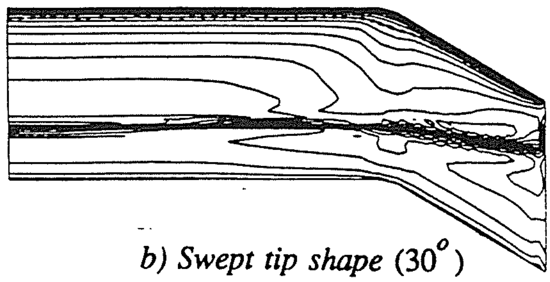


Figure 1

Partial view of the grid on the wall and on the wing surface



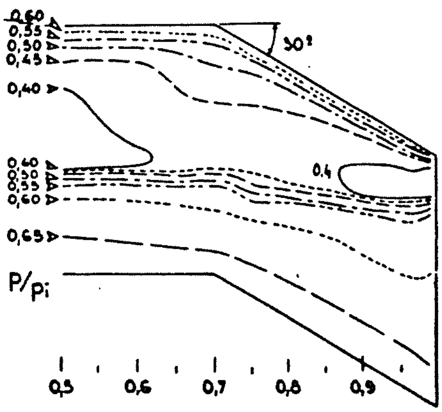
a) Straight tip shape

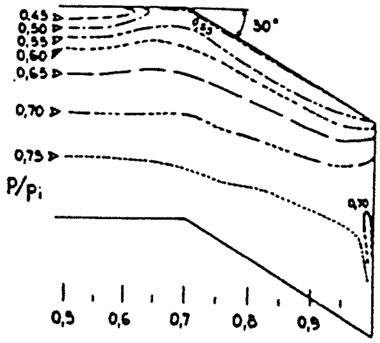


b) Swept tip shape (30°)

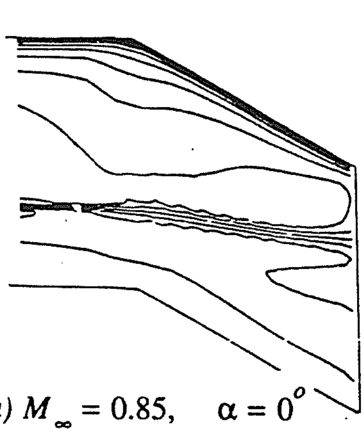
Figure 2

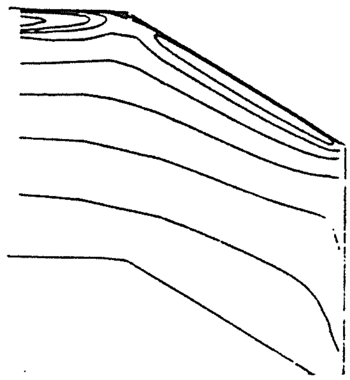
Iso-Mach lines on the upper surface of the wing calculated by the implicit Euler method of [19] for $M_\infty = 0.85, \quad \alpha = 0^o$.





experiment [22]





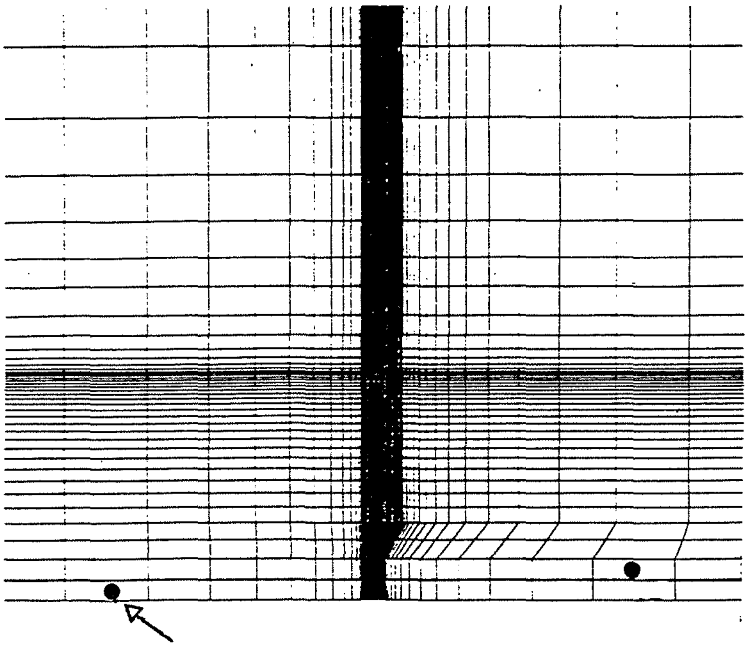
Euler calculation

a) $M_\infty = 0.85, \quad \alpha = 0^o$

b) $M_\infty = 0.60, \quad \alpha = 6^o$.

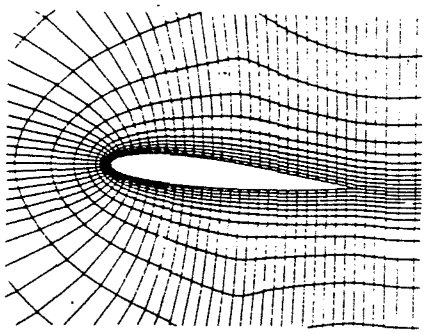
Figure 3

Comparison Euler calculation - experiment for the pressure distribution on the upper surface of the wing.

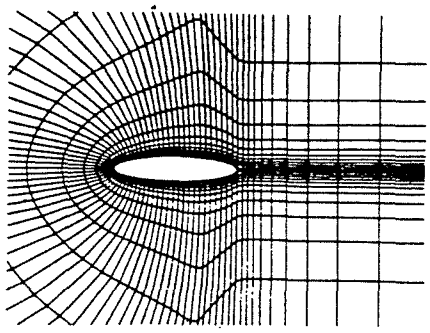


Fictitious point for the implementation
of the periodicity condition

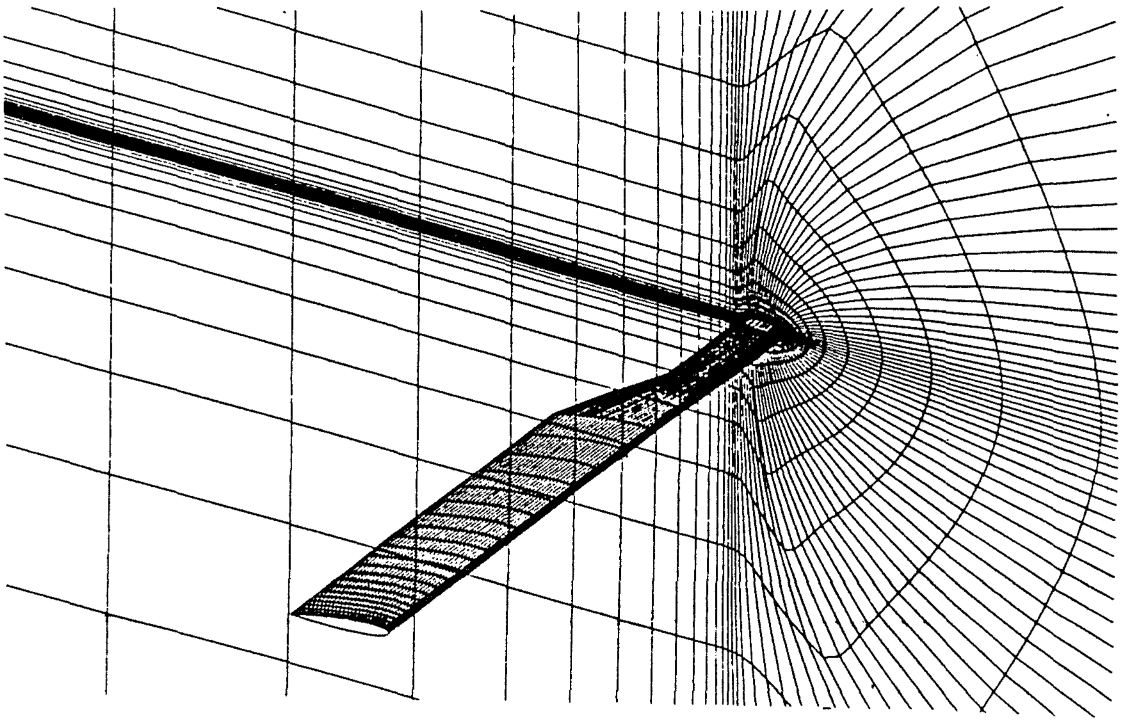
a) "C-H" grid topology



b) grid for NACA0012 profile



c) grid for elliptic profile near the hub



d) global view of the grid

Figure 4

Views of the "C-H" grid for the model two-bladed rotor of US-Army in hovering-flight.