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ABSTRACT

The definition of the design parameters of a Helicopter such as: geometrical parameters of the rotor, angular velocity, installed power etc, able to satisfy a datum flight spectrum, requires usually an iteration with the aim of reaching a compromise among contrasting requirements.
This kind of iteration, usually based on the experience, doesn't lead necessarily to a rational choice of the design parameters, but it is strongly influenced by the leader condition of the flight spectrum, so penalizing all the other ones.
So far, it is otherwise possible, by the help of the optimisation technique, to rationalise the choice of the design parameters, requiring them to minimize a prescribed function without failing some prescribed constraint equations.
One of the most natural optimum function in this field, is the energy required by the helicopter to perform the desired flight spectrum.
The paper will emphatize the above problems, suggesting different optimum strategies, checking the redundancies among the parts to be optimized and giving a particular care to the parameters that strongly reduce the total energy.
The differences between a helicopter designed in a traditional way and an optimised helicopter are presented.

1. INTRODUCTION

The design of a new helicopter starts usually after the fundamental requirements have been clearly defined by people or organization involved in this task, generally located outside the technical office.
The engineering representation of the basic data is generally summarized in a set of useful loads the new helicopter will have to carry and from a certain number of flight spectrums (F.S.), defined through flight conditions (F.C.) and associated time the new machine will, probably encounter during its life.
An alternative way to represent the F.S.'s the description of a set of missions defined through a logical sequence of F.C.'s and related times.
The single F.C. can be represented, for each altitude, both through a combination of flight velocity, load factor and time, and through a description of trajectories and velocities of the centre of gravity of the helicopter.
Another important aspect that must be considered before the design phase is a broad description of the geometrical configuration of the fuselage of the new helicopter, to comply, in the best way, with the fundamental requirements and to satisfy the primary mission to which the machine will be dedicated.
On the basis of the previous requirements, the fundamental question to be resolved, in the initial phase of the design, is the determination of the gross weight and the whole set of parameters that define globally the machine (rotors size and geometry, station and surface of stabilizers and fins, station of the tail rotor, range of controls and so on).
This phase is extremely complicated and gives results strongly dependent on the approach utilized.
The use the methodology consolidated by the experience, to search among the flight spectrum the most critical conditions to be satisfied and verifying successively if some of the remaining less critical conditions are satisfied too, requires usually the introduction of a design loop whose management is very difficult to face for the presence of a great number of parameters to be checked and for the narrow range in where some of them can vary.
What usually happens, following this methodology, is that the leader conditions of the flight spectrum associated to the utilization of a relatively small number of design parameters among all those possible, affect excessively the design.
Otherwise the human capability is unable to consider a lot of conditions all together or, in other words, is unable to work satisfactorily in the hyperspace of the flight spectrum and the design parameters.
Another further difficulty comes from the fact that, very often, it is necessary to satisfy some relations existing between the design variables.
A simple example of this situation can be represented by the need of limiting the tip speed of the blade, whose definition

is related to two possible design parameters: the rotor angular speed and the length of its radius.
Mathematically speaking, we can say that in the hyperspace of the parameters and flight spectrum exists a border, described by a hypersurface, which must not be exceeded.
The intersections of all the hypersurfaces determine a feasible region in which to choose the design parameters.
Many equations or relations are then expressly dependent on the technology adopted in the design, exactly as in the field of component weight determination. This fact can complicate further the problem, introducing extra variables in the design. This can happen, for example, when we want to limit the deflection of the blade tip at zero rotor speed to avoid any interference. In this case the stiffness distribution of the blade and its linear density are involved in the relation.
On the basis of problems previously presented, it is clear that the final configuration of the helicopter is strongly affected by the initially adopted assumption and, as a different choice, it could lead to a significant modification of the desired helicopter.
The effort necessary to investigate a single case is then considerable, generally faced with a good degree of experience, good sense and skill and requires time, people, and a satisfactory organization.
This way of working leaves, however, unsolved, without any possibility of reply, the fundamental question: what is the best helicopter able to satisfy the fundamental requirements?
During these years a lot of work has been done in our company to answer this question and to place the problem on a rational base. (Ref. 1.2)
The first step was the definition of a property of the helicopter representative of the flight spectrum or the mission that depended on the design parameters.
Such a function was found in the energy required by the helicopter to perform its flight spectrum to which the weight of important parts of the helicopter is related.
The reduction of the required energy reduces necessarily most of these weights (trasmission, fuel, engines, etc) and if we consider that the helicopter must carry the potential energy in the form of fuel, in flight, we understand immediately the advantageous effects of its reduction.
In this way we came to the conclusion that the answer to our problem was: the best helicopter able to satisfy the fundamental requirements is the helicopter able to utilize the minimum energy in all the flight spectrum.
An evolution of this function is the utilization of the concept of: "ENERGY UTILIZATION FACTOR" (EUF) introduced long time ago by Von Karman and successively developed by Gabrielli (ref. 3), whose definition is:

$$EUF = \frac{PAYLOAD * DISTANCE}{ENERGY\ UTILIZED}$$

This paper deals mainly with the energy concept to which the EUF is strongly related. In the particular case of constant payload, the two formulations become identical and the minimum of the energy corresponds to the maximum of the EUF.
According to the algorithm utilized, it is in principle possible to take care of many other requirements such as costs, noise, vibration level and so on, introducing them in a superfunction to become minimum or treating them as constraints and requiring the solution not to exceed some prescribed values.
The algorithm able to resolve this kind of problem is called: "OPTIMIZATION TECHNIQUE", which permits to compute the minimum value of any function inside the feasible region.
The theory is well known and widely applied in different engineering problems. A recent example of application to the helicopter field was presented in ref. 4 in designing an optimum blade with aeroelastic constraints.
In fig. 1 is shown, for clarification, a geometrical representation of the problem, for which we address to the wide literature for the theoretical explanation. Ref. 6.
The represented case is bidimensional for simplicity and the design variables are indicated on the two orthogonal axis (X1, X2).
The plot shows the function "E" to be minimized through the representation of its isovalues where for example E1>E2>..En.
The constraint equations (C1, C2,....) divide the plane in two regions: the feasible and the infeasible region. Scope of the technique is to compute the values of (X1, X2) in such a way that the function E reaches its minimum value inside the feasible regions. The trajectory described by points P1, P2,..Pn generated by the algorithm, is obtained utilizing the gradients of the function E and the constraint equations. It is clear that the solution (X1, X2) for the function "E" is completely dependent upon the constraints introduced.
In this work, the technique will be applied to determine the design parameters able to make the helicopter optimum for a prescribed flight spectrum through the minimization of the required energy.

2. ANALYTICAL ASPECTS

2.1 FLIGHT SPECTRUM ENERGY

The magnitude of the energy can be exactly computed for a mission of which we know exactly the sequence of the flight conditions and related times.
In this situation the amount of energy can be deduced through the knowledge of the required power at the beginning of each flight condition and of a function of time that depends on the hypothesis introduced on the fuel consumption during the flight.
So, for a mission, the total energy can be represented by means of an equation of the type:

$$E = \sum_{1}^{n} Si\ Poi*f(Ti) \tag{1}$$

where

S = Summation
n = Number of flight conditions that define the mission
Poi = Power required at the beginning of the i-th flight condition.
Ti = Global time of the i-th flight condition
f(Ti) = Function of time which can have different formulations according to the mechanisms of fuel consumption.

For a particular case of constant power throughout the flight condition, (no fuel consumption) this function becomes simply the time and the associated energy takes the formulation:

$$Ei = Poi*Ti \tag{2}$$

In the case where the instantaneous power and the gross weight are proportional and the "Specific Fuel Consumption" (S.F.C. ÷ (Kg/h)/hp) is constant, the function takes the expression.

$$f(Ti) = T*lg(1+T/T0)/(T/T0) \tag{3}$$

where "T0" is the time required to halve the initial gross weight or, in other words, the time required to consume a quantity of fuel exactly equal to half of the initial gross weight.
The corresponding power follows the same law, becoming half of the initial power according to the assumed hypothesis of proportionality.
The fig. 2 shows the behaviour of the energy with time of the above cases. It is interesting to note that the ratio between the two formulations is described by the factor:

$$H=lg(1+T/T0)/(T/T0) \tag{4}$$

that becomes lg2 (0,69) for T = T0 where the weight and the power (slope) are exactly half of the initial value.
The value of the constant "T0", in this particular case, depends simply on the knowledge of the constant ratio between power and weight "A" and the value of the specific fuel consumption "K" i.e.

$$T0=1/(A*K) \tag{5}$$

Taking for example A= 0.5 (HP/KG) and K=0.2 (KG/H)/HP we obtain: T0 = 10 hours.

Although the values of these two parameters are limited from the technological level, it is clear that the reduction of the energy for each flight condition can be achieved reducing the initial required power through a proper choice of the design parameters of the helicopter.

About the mission, in addition to the initial powers, an important role is played by the time associated to each F.C..
These times or related functions, can be considered as a weight factor for each initial power of the F.C. defining the mission.
In this way, we understand clearly that a F.C. can require a high power but its weight in the general economy of the mission could be negligible (and viceversa). Thus, according to the experience, to achieve a significant improvement in the behaviour of the helicopter, we have to operate on the critical condition of the mission from the energy point of view. The situation becomes extremely complicated when some conditions have comparable importance. In this case the reduction of the total energy will be obtained reducing proportionally the energy of the critical conditions.
It will be then necessary to find an appropriate compromise among the design parameters in such a way as to reduce globally the energy of the mission even though paying something for particular F.C..
All conditions of the mission are, in this way, treated with a proper importance, taking, therefore, particular care of the conditions whose weight is most important without disregarding all the other conditions of the flight spectrum. The algorithm able to reach the maximum compromise, increasing considerably the human capability in this field, is the "OPTIMIZATION TECHNIQUE", which takes care of all the conditions with their own weight and is able to reach the compromise inside a defined range of the parameters.

When we consider a flight spectrum in lieu of a mission, the meaning of the concept of energy is diminished, in the sense that we do not know the sequence of implementation of each flight condition from the helicopter or, in other words, we can say that a mission which includes in some way all the conditions of the spectrum does not exist. The only things we know in this case are the F.C.'s the helicopter will execute for a certain percentage of its life.
However, the energy concept can still help us in risolving the problem as we desire to design a machine which operates satisfactorily in each of the specified conditions.
As regard each separate condition we understand immediately that exist again some conditions requiring more energy than others or conditions with comparable energy.
It will be then necessary to apply again the compromise concept, in order to try to design satisfactorily the machine.
In this particular case, been reduced the significance of the function of time which appears in the energy expression, we can utilize its simpler formulation obtaining:

$$E = \sum_{1}^{n} Poi*Ti \qquad (6)$$

This solution allows to interpreter the energy of the spectrum in a different way. Noting that the expression is equivalent to:

$$P = \sum_{1}^{n} Poi*Ti/T \qquad (7)$$

where T is the total time of the flight spectrum, we obtain that the energy concept leads to the definition of the "mean power of the F.S.", built up through the power of each F.C. weighed by means of the percentage of its total time. The scope will be then to reduce this mean power of the F.S. through a rational choice of the design parameters based on the relative importance of the single F.C..

2.2 POWER OF THE FLIGHT CONDITION

Whatever approach we will follow, it will be necessary to compute the power required in each F.C..
The F.C. can be subdivided, for instance, in stabilized conditions, where the C.G. velocity of the helicopter has constant modulus (level flight, descent, climb, turns, etc.) and transient; in conditions to pass from a stabilized condition to another or in conditions where the machine is continuously subjected to pilot inputs.
In the first group the computation of the power is achieved through the search of the trimmed conditions where the external forces are balanced by weight and stabilized inertial forces. (centrifugal forces in stabilized turn flight).
For the second group, it is necessary to compute the power step by step, during an integration phase of which we know either time histories of controls or the shape of the trajectory and its tangent velocity.
The algorithms able to solve these problems can have a different level of sophistication, according to degrees of freedom utilized to describe the motion of the helicopter.
During these years a lot of new programs have been developed and practically each company or university involved in the helicopter field has available its own program.
The important aspect which is to be highlighted is the fact that: more sophisticated the program is more parameters can be managed during the optimisation phase. It is also necessary to remember that a limit to the difficulties faced by the code can be the computer time and the level of knowledge of the input data at the beginning of the design.
The limit in the computer time is clearly related to the countless times during which the algorithm is utilized for

the generation of the function and the computation of its gradients. As regards to the input data, it is evident that at the beginning of the design most of them are approximately evaluated both statistically and analytically. Therefore, a sophisticated code is generally useless and could lead to wrong solutions.
The requirements for a proper code covering most of the F.C.'s of the spectrum (steady conditions), can be summarized in the following statements:

1) Fast enough to avoid troubles in computer time
2) Able to contain a significant part of the design parameters of rotors, fuselage and aerodynamic surfaces.
3) Able to represent satisfactorily the gradients of the energy/power of the mission or the flight spectrum, in relation to the design variables.

Such an algorithm was found re-writing the equations of ref. 5 taking into account, through a proper integration along the blade span, all the possible aerodynamic and geometrical parameters. It was so possible to consider any distribution of twist, chord, dCp/d(alfa), Cd as a function of Mach number.
The stations along the blade, where there is a change in profiles characteristics, are retained as a further design variable, to leave the optimization technique free to choose their best extension along the span. Any geometrical parameters of fins and stabilizer can be utilized as a design variable during the optimization steps.
At present, the aerodynamic characteristics of the basic fuselage are considered constant and evaluated by means of theoretical considerations or the results from wind tunnel tests as soon as the scaled model is developed.
In appendix 1 the most significant equations of the algorithm are specified, compared with the basic ones included in ref.5.

2.3 GROSS WEIGHT DETERMINATION

As mentioned briefly in the introduction, at the beginning of a new design, what we know as basic requirements are a set of useful loads, few missions or the flight spectrum and some generic limitations on the design variables. Then, we do not know what type of gross weight (G.W.) will have the new helicopter able to carry the desired payload.
The algorithm presented in the previous chapter to compute the power, works, on the contrary, on the basis of a known G.W.. It is, then, necessary to introduce an iterative procedure to reach the desired convergence. Each step of this iterative procedure requires the possibility to know, in some way, the relationship existing between the payload and the gross weight. Knowing, for example, the required power, it is possible to trace back, in a statistical way, to the weight

of each component of the helicopter depending upon his parameter (transmission, fuel, engines, etc); or through the G.W., to predict, by means of coefficients affected by the technological level utilized, the weight of the fuselage and many other components of the helicopter.
These kinds of methodologies constitute the background of each company; the validity of the utilized approaches has been tested and refined everytime a new prototype has been built. Therefore, without examining thoroughly this particular and difficult problem, we are aware that starting from the knowledge of the G.W. it is possible to predict, according to the power required, the weight of each component of the helicopter and then the value of the useful load.
However, as explained throughout the work, what can connect in a rational way the two weights, is the optimization technique, which, applied to the F.S. energy, permits automatically to have all the information necessary to perform this prediction, minimizing, in the same time, the weight associated to the power, or rather, to the energy.
Thus it is possible to compute for each G.W. an associated optimum useful load. The intersection of this function with the desired useful load will give the desired G.W..
Fig. 3 shows the converging procedure to obtain the solution, which take advantage from the NEWTON RAPHSON technique where, starting with any G.W. close to the solution, it is possible to reach automatically the convergence value.
The design parameters featuring the converged solution, will give the desired optimum design of the helicopter.

2.4 STRATEGIES

The sequence of operations described in the previous chapter constitutes the basic algorithm to face a series of problems connected to the research of the best helicopter, in the sense explained throughout this work.
What can be, for example, the strategy to be adopted for a multirole helicopter?
The presented methodology is able to predict the best helicopter for each mission, but what is the best compromise among all the desired missions?
We think that this question could have different answers according to the strategy adopted to solve the problem. We could, for example, determine all the G.W. optimizing each mission and choosing the most critical; we could also built up an energy function weighed on the mission, defining some factors based on the relative importance of a mission as to the others, writing:

$$E=\sum_{1}^{m} Ei(\bar{D})*wi \qquad (8)$$

where

m = Number of missions
Ei = Energy associated to the i-th mission
wi = Weight factor
$\bar{D}$ = Vector of design variables

The gross weight to be utilized could be determined on the basis of the critical useful load and kept constant for all the missions or computed for each mission through the procedure presented in the previous chapter.
At present, we can not say what strategy leads to the best result, as we are lack of sufficient information to answer this question. The problem needs still some further considerations on the basic hypothesis and on the methodology to judge the results obtained from all the possible strategies utilized.

3. APPLICATION OF THE METHOD

3.1 OBJECTIVE FUNCTION

The concepts introduced in the previous chapters, with all the problems and difficulties described, have been applied to the definition of the optimum design parameters of a helicopter, whose initial configuration was designed in the traditional way, utilizing experience, good sense and ability, as explained in the introduction. The result was achieved minimizing the energy of three weighted flight spectrum, each of them executed with a defined G.W. at three different locations of the centre of gravity of the helicopter.
In this particular example the three F.S.'s, although of different importance, consist of the same F.C., and their respective G.W.'s were deduced optimizing separately the design variables of the helicopter for each F.S. The table -1A- shows, for each F.S., the related G.W., the weight factor (wi) to define its relative importance, subdivided among three C.G. locations according to the specified fraction (wj).
The table -1B- shows the F.C., of the three equal F.S.'s in matrix form, where the columns represent the velocities in percent of the "VH", the rows the load factors, while the elements of the matrix, the time associated to each F.C.
The energy function to be minimized in this application, can be deduced noting that each F.S., for a defined G.W. and C.G. location, will require the energy:

$$Ej = \overset{ni}{\underset{1}{Ss}} \; Pos(\bar{D},\ G.W.i,\ C.G.j)*Ts \qquad (9)$$

where

ni = Number of F.C.'s defining the i-th F.S. (in this case common to the three F.S.'s)

Pos = Power required at the beginning of the s-th F.C. of the j-th C.G. position, in correspondence of the i-th F.S.

$\bar{D}$ = Vector of the design variable defined in the next paragraph

C.G.j = j-th location of the C.G. of the helicopter in the i-th F.S.

Ts = Time associated to the s-th F.C. described in table -1B-

The energy of each F.S., for the three given positions of the C.G. of the helicopter described in table -1A-, will become:

$$Ei=S_{j=1}^{3}\ Ej*wj \tag{10}$$

where

wj = weight factor associated to the fraction of time into which the i-th F.S. has been subdivided.

Then, the energy of the three F.S.'s will be:

$$E=S_{i=1}^{3}\ Ei*wi \tag{11}$$

where

wi = relative importance of the i-th F.S., shown in table -1A-

Assembling the single contribution, the final expression of the global energy will be then:

$$E= S_{i=1}^{3}\ wi*S_{j=1}^{3}\ wj*S_{s=1}^{ni}\ Pos(\bar{D},\ G.W.i,\ C.G.j)*Ts \tag{12}$$

From the equation -7-, the mean power of all the weighed F.S.'s is then:

$$P=E/T \tag{13}$$

where T is the global time of all the F.S.'s.

The strategy adopted in this application can be extended introducing the weight of other parameters such as the altitude, as indicated at the top of table -1A-.

5.2 DESIGN VARIABLES/DESIGN CONSTANTS

The algorithm presented in APP. 1, to compute the power of any desired F.C. permits to use, as design variables, parameters related to rotors, fuselage and aerodynamic surfaces.
In the presented application, the design variables utilized with their starting value are shown in table 2, while the design constants are shown in table 3.

5.3 CONSTRAINTS

The constraint equations imposed on the design variables can be defined "geometrical", when a design variable is constrained to vary inside a defined range:

$$d_i^{(L)} \le d_i \le d_i^{(U)} \qquad (14)$$

and "analytical" when the border between feasible and unfeasible region is represented by an hypersurface in the space of the design variables. From the mathematical viewpoint this hypersurface is represented by a generic equation among the design variables:

$$C_i(\bar{D}) \ge 0 \qquad (i=1,2,...n) \qquad (15)$$

According to the optimization algorithm utilized, ref. 6, the constraint equations must be introduced through a user's subroutine. In our application the geometrical constraints are applied to all the design variables to control the manufacturing and geometrical requirements. Their limits are indicated in table 2.
The analytical constraints cover the limitations on:

1) Stall on the retreating blade
2) Interference between main rotor and tail rotor
3) Mach number on the advancing blade

In addition to the described constraint equations, it has been necessary to introduce a further constraint on the objective function, requiring that all the F.C.'s utilized were resolved by the "trim" subroutine in each optimization step.
It can happen, for example, that the optimization step predicts a set of new design variables too far from the starting values, physically unacceptable to describe the new configuration. In this situation the objective function would have a different formulation from the initial one as some F.C. are missing and the problem can not be controlled. It is then necessary to ensure that the energy function is correctly computed in each optimization step, reducing, where necessary, the predicted design variables along the known gradients.

5.4 RESULTS AND DISCUSSION

Figs. 4/5/6 show the results of our application. The behaviour of the objective function is indicated in fig. 4, where it is possible to understand the limit of the human capability in managing a lot of variables. The difference in this case between the human and the automatic design is about 25%.

Fig. 6 shows the result of different optimization steps identified by the initials "Dn" where "n" represents the number of the optimization cycle.

In the presented sketches, it is possible to follow the evolution of the blade geometry and twist, showing a clear tendency to taper deeply the tip as a consequence of the Mach number effects on the drag coefficient. The parameter that seems to play an important role is the angular velocity of the main rotor, which reduces to its minimum value just at the first iteration. The M.R. radius shows a contrasting behaviour reducing at the first step and increasing continuously during the next ones. The reason could be due to the particular distribution of the F.C. as regards the flight velocity.

The aerodynamic surfaces reach their critical value at the first step. For the fin, instead, its increase reduces the power required by the tail rotor to trim the helicopter around the "yaw" axis, for the stabilizer exists a complicated tie among the aerodynamic coefficients of the fuselage, the M.R. mast tilt as to the fuselage and the M.R. pitch moment to maintain the fuselage at an average minimum drag attitude for the given F.S. To confirm the validity of the optimized results, the "speed power polar" for each optimization step has been computed and compared as in fig. 5, by tmeans of a inhouse sophisticated code.

The diagram proves the ability of the optimization technique to solve rationally this class of problems.

6. CONCLUSIONS

The optimization technique is an useful means to start the design of a new helicopter, involving all the fundamental parameters of the helicopter. The energy function permits to average the design among different flight conditions that the flight spectrum or the mission introduces with the desired importance.

For multimission or different F.S. exist many possible optimization strategies based on a pseudo energy that needs some further consideration to understand which of them leads to the best solution; however, whatever way is selected, it gives an acceptable solution, generally better than the starting design. Another important point to be remembered, although contained in optimization concept, is that the final solution satisfies all the desired constraint conditions: the second fundamental ingredient of the design.

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APPENDIX I

Nomenclature

a_k = lift curve slope
b = number of blades
c_k = blade section chord
δ_k = blade profile drag coefficient
x_k = dimensionless coordinate of the Kth blade station
R = rotor radius
Ω = rotor angular velocity
ρ = air density
n = number of sections
λ = inflow ratio
T = rotor thrust
μ = advancing ratio
θ = collective pitch $= (\theta_0 + m_k x + q_k)$
a1 = longitudinal flapping coefficient
b1 = lateral flapping coefficient
ψ = azimuth
β = $a_0 - a1 \cos\psi - b1 \sin\psi$ = flapping equation

The following are the most expressive equations used in the algorithm. On the right side there are the basic ones published in ref. 5, on the left side the developed ones.

Rotor Thrust

The elementary thrust is

$$dT = \frac{1}{2} a \rho \Omega^2 R^2 \left[\theta (x + \mu \sin\psi)^2 + (\lambda - x\, d\beta/d\psi - \mu\beta\cos\psi)(x + \mu\sin\psi)\right] c\, dr$$

That integrated over the azimuth range and along the single blade section gives, adding the contributions of every section:

$$T = \frac{1}{2}\rho\Omega^2 R^3 b \sum_{1}^{n}{}_k a_k c_k \left\{ (\theta_0 + q)\left[\frac{1}{2}\mu^2 (x_{k+1} - x_k) + \frac{1}{3}(x_{k+1}^3 - x_k^3) - \frac{2}{3}\frac{4\mu^2}{2+3\mu^2}(x_{k+1}^2 - x_k^2)\right] + mR\left[\frac{1}{4}(x_{k+1}^4 - x_k^4) - \frac{1}{4}\mu^2 (x_{k+1}^2 - x_k^2) - \frac{16\mu^2}{9(2+3\mu^2)}(x_{k+1}^3 - x_k^3)\right] + \frac{\lambda}{2}\left(1 - \frac{4\mu^2}{2+3\mu^2}\right)(x_{k+1}^2 - x_k)^2 \right\}$$

$$T = \frac{1}{2} b a c \Omega^2 R^3 \left[\frac{2}{3}\theta_0 \left(1 + \frac{2}{3}\mu^2\right) + \lambda\right]$$

$\Uparrow$

REF. 5

Using the same method it's possible to write:
the expression of the in-plane, H force:

$$H = \frac{b}{4}\rho\Omega^2R^3\Big\{\mu\sum_{1}^{n}{}_k\,\delta_k c_k\left(x_{k+1}^2 - x_k^2\right) + \sum_{1}^{n}{}_k\,a_k c_k\Big[\left(x_{k+1}^4 - x_k^4\right)\frac{1}{2}a_1 m_k R + \left(x_{k+1}^3 - x_k^3\right)\left(\frac{2}{3}a_1(q_k + \theta_0) - \frac{b_1 a_0}{3}\right) + \left(x_{k+1}^2 - x_k^2\right)\left(\frac{a_1^2\mu}{2} + \frac{3}{2}a_1\lambda + \frac{\mu a_0^2}{2} - \frac{m_k R\mu\lambda}{2}\right) + \left(x_{k+1} - x_k\right)\left(q_k + \theta_0\right)\lambda\mu\Big]\Big\}$$

$$H = \frac{1}{2}\rho a b c \Omega^2 R^3\left[\frac{\mu\delta}{2a} + \frac{1}{3}a_1\theta_0 + \frac{3}{4}\lambda a_1 - \frac{1}{2}\mu\lambda\theta_0 + \frac{1}{4}\mu a_1^2 - \frac{1}{6}a_0 b_1 + \frac{1}{4}\mu a_0^2\right]$$

The expression of the coning angle a_0 :

$$a_0 = \frac{1}{8}\frac{\rho R^4 a}{I_{cf}}\sum_{1}^{n}{}_k\,c_k\Big\{\frac{2}{3}m_k R\mu\left(x_{k+1}^3 - x_k^3\right) + \frac{4}{5}m_k R\left(x_{k+1}^5 - x_k^5\right) + \left(q_k + \theta_0\right)\left[\left(x_{k+1}^2 - x_k^2\right)\mu^2 + \left(x_{k+1}^4 - x^4\right)\right] + \frac{4}{3}\lambda\left(x_{k+1}^3 - x_k^3\right)\Big\}$$

$$a_0 = \frac{\gamma}{8}\left[\theta_0\left(1 + \mu^2\right) + \frac{4}{3}\lambda\right]$$

And the expressions of the flapping coefficients al, bl:

$$a_1 = \sum_{1}^{n}{}_k\Big\{2\mu\Big[m_k R\left(x_{k+1}^4 - x_k^4\right) + \frac{4}{3}\left(q_k + \theta_0\right)\left(x_{k+1}^3 - x_k^3\right) + \lambda\left(x_{k+1}^2 - x_k^2\right)\Big]\Big\}$$

$$a_1 = \frac{2\mu\left(4\theta_0/3 + \lambda\right)}{1 - \mu^2/2}$$

$$b_1 = \frac{4}{3}\mu a_0 \Big/ \left(1 + \frac{\mu^2}{2}\right)$$

$$b_1 = \frac{\left(4\mu a_0/3 + 1.1\,\nu^{1/2}\cdot\lambda\right)}{1 + \mu^2/2}$$

REF. 5

****** S P E T T R O D E L L E C O N D I Z I O N I D I V O L O ******

*** TABELLA DELLE QUOTE E DEI RELATIVI TEMPI ***

QUOTA (M) :	0.00	1000.00	2500.00	4000.00
PERC. DI TEMPO :	100.	0.	0.	0.

*** TABELLA DEI PESI E DEI CENTRAMENTI ***

WEIGHT PESO (KG)	Wi PERC. DEL TEMPO	STAZ. C.G.	LATL. C.G.	W.L. C.G.	Wj PERC. DEL TEMPO
3170.00	20.	4.30[illegible]	0.000	1.500	0.
		4.422	0.000	1.500	100.
		4.585	0.000	1.500	0.
3500.00	50.	4.335	0.000	1.500	0.
		4.422	0.000	1.500	100.
		4.585	0.000	1.500	0.
3750.00	30.	4.37[illegible]	0.000	1.500	0.
		4.422	0.000	1.500	100.
		4.585	0.000	1.500	0.

TABLE - 1 A -

*** TABELLA DEI TEMPI DEI VOLI ***

FATTORE DI CARICO	UNITA' DI MISURA DEL TEMPO	P E R C E N T U A L E D E L L A V H					
		0.000	15.000	40.000	60.000	80.000	100.000
1.000	PERC.	0.000	0.000	0.000	0.000	0.000	0.000
1.000	PERC.	0.000	0.000	0.000	0.000	0.000	0.000
1.000	PERC.	16.000	27.000	4.000	27.000	19.000	7.000
1.200	PERC.	0.000	0.000	0.000	0.000	0.000	0.000
1.500	PERC.	0.000	0.000	0.000	0.000	0.000	0.000

TABLE - 1 B -

***** DESIGN VARIABLES *****

VARIABLE	LOWER LIMIT	STARTING VALUE	UPPER LIMIT
Hor. Stab. area	1.000	1.737	2.000
Vert. Stab. area	1.500	1.430	2.500
Radius T.R.	0.800	1.120	1.200
Chord T.R.	0.200	0.280	0.350
Radius M.R.	5.500	5.950	6.500
Ang. velocity M.R.	-40.000	-36.233	-32.000
Mast M.R. tilt	4.500	5.000	7.000
Blade chord St. 4	0.300	0.350	0.500
" " " 5	0.300	0.391	0.500
" " " 6	0.300	0.391	0.500
" " " 7	0.300	0.391	0.600
" " " 8	0.300	0.391	0.600
" " " 9	0.300	0.391	0.600
" " " 10	0.270	0.391	0.600
" " " 11	0.240	0.300	0.500
Blade twist St. 3	-6.000	6.000	6.000
" " " 4	-6.000	-1.490	2.000
" " " 5	-6.000	-1.490	2.000
" " " 6	-5.000	-1.490	1.000
" " " 7	-5.000	-1.490	1.000
" " " 8	-4.000	-1.490	1.000
" " " 9	-3.000	-1.490	1.000
" " " 10	-3.000	-1.490	1.000
" " " 11	-3.000	-0.100	1.000

TABLE - 2 -

..... DATI INIZIALI .....

PROVA DI OTTIMIZZAZIONE DEI PARAMETRI DI PROGETTO

ELICOTTERO

M A I N   R O T O R

NUMERO DELLE PALE .... =	4	RAGGIO MAIN ROTOR .... =	5.950 M
MAIN ROTOR SOLIDITY RATIO =	0.071	ECCENTRICITA' FLAP HINGE . =	0.335 M
BARICENTRO PALA (FLAP H.) =	2.349 M	PESO DI UNA PALA .... =	43.836 KG
INERZIA AL FLAPPEGGIO .... =	39.118 KG*M*S**2		

M A S T

INCLINAZIONE LONGITUDINALE =	5.000 GRADI	INCLINAZIONE LATERALE .... =	0.000 GRAD
STAZIONE MAST .... =	4.422 M	BUTLINE MAST .... =	0.000 M
WATERLINE MAST .... =	3.100 M		

T A I L   R O T O R

RAGGIO TAIL ROTOR .... =	1.120 M	CORDA TAIL ROTOR .... =	0.280 M
STAZIONE TAIL ROTOR ...... =	11.643 M	BUTLINE TAIL ROTOR .... =	-0.540 M
WATERLINE TAIL ROTOR ..... =	3.245 M		

F U S O L I E R A

AREA EQUIV. FUSOLIERA .... =	2.250 MQ	SUP. RIFERIMENTO MOMENTI . =	111.200 MQ
DISTANZA RIF. MOMENTI .... =	5.950 M	COEFF. MOMENTO COSTANTE .. =	-0.003
FUSOL.+PIANETTO (SPERIM.) =	0	FUSOL.+DERIVA (SPERIM.) .. =	0
STAZIONE FUSOLIERA .... =	4.422 M	BUTLINE FUSOLIERA .... =	0.000 M
WATERLINE FUSOLIERA ...... =	1.561 M		

P I A N E T T O

SUPERFICIE PIANETTO ...... =	1.737 MQ	INCIDENZA GEOM. INIZIALE . =	-2.600 GRADI
LEGAME INCIDENZA-TETA .... =	0.000	INCIDENZA DI CP MAX ...... =	22.000 GRADI
INFLUENZA IN HOVERING .... =	1	STAZIONE PIANETTO .... =	9.680 M
BUTLINE PIANETTO .... =	0.000 M	WATERLINE PIANETTO .... =	1.625 M

D E R I V A

SUPERFICIE DERIVA .... =	1.930 MQ	STAZIONE DERIVA .... =	10.450 M
BUTLINE DERIVA .... =	0.000 M	WATERLINE DERIVA .... =	2.700 M
CP DELLA DERIVA .... =	0.480	CR DELLA DERIVA .... =	0.012

C O E F F .   A E R O D I N A M I C I

CR PROFILO MAIN ROTOR .... =	0.010	SLOPE CP-ALFA MAIN ROTOR . =	6.500
CR PROFILO TAIL ROTOR .... =	0.011	SLOPE CP-ALFA TAIL ROTOR . =	6.100
CR PROFILO PIANETTO ...... =	0.008	SLOPE CP-ALFA PIANETTO ... =	3.290

TABLE - 3 -

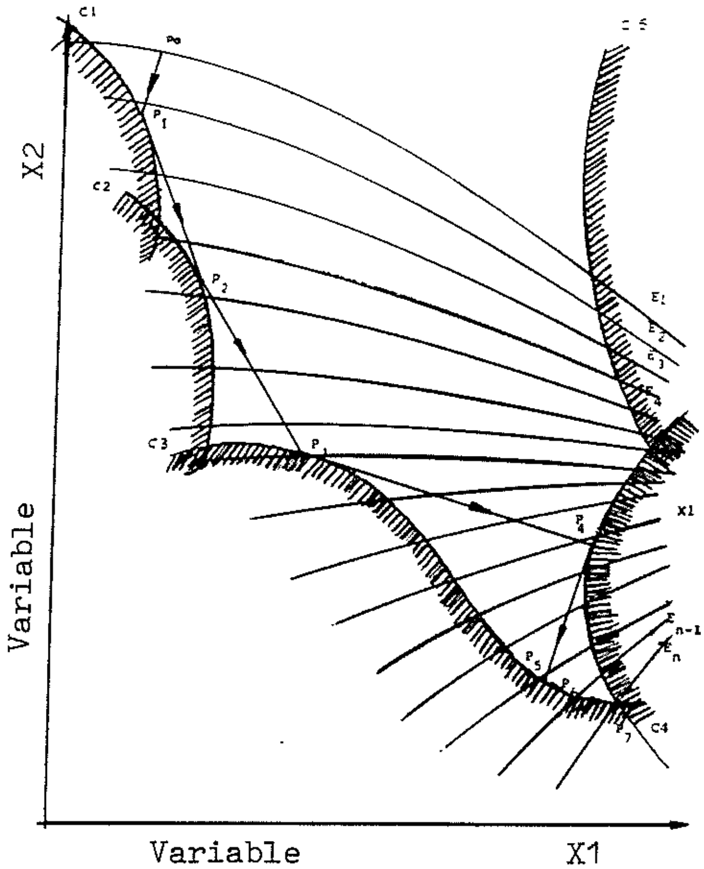


FIG. 1 - GEOMETRICAL EXAMPLE OF OPTIMIZATION

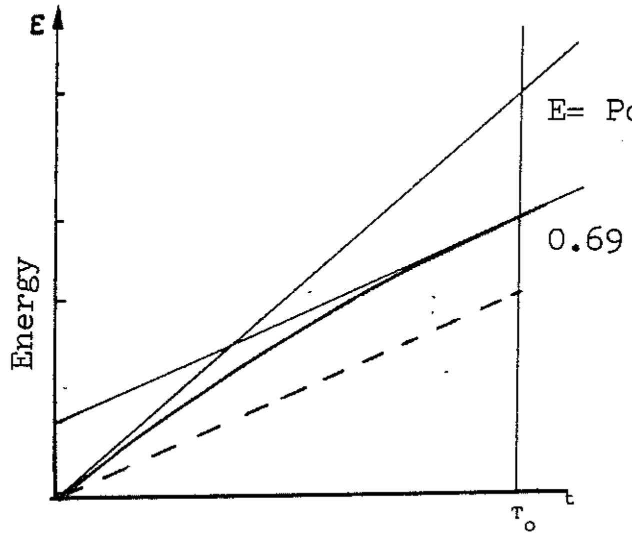


FIG. 2 A

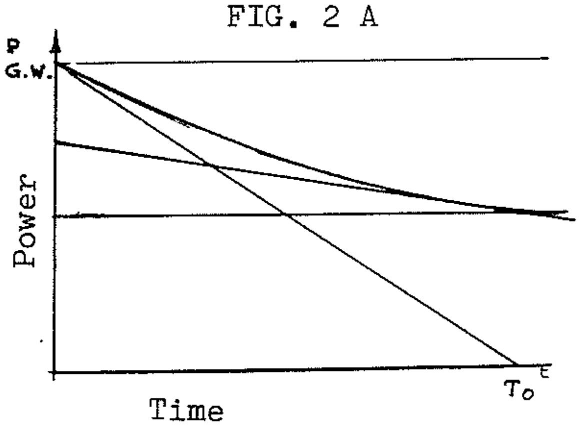


FIG. 2 B

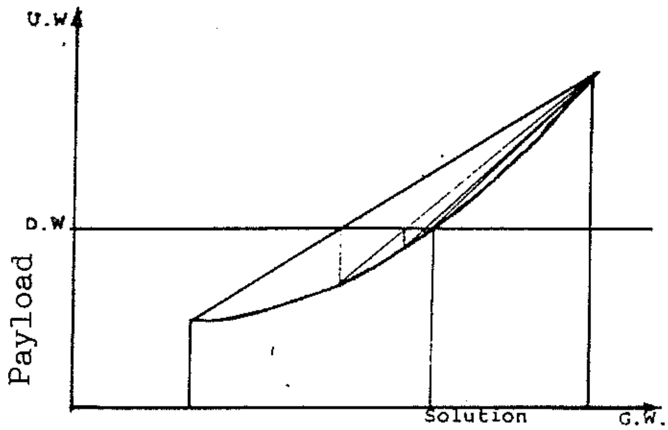


FIG. 3 - ITERATION PROCEDURE

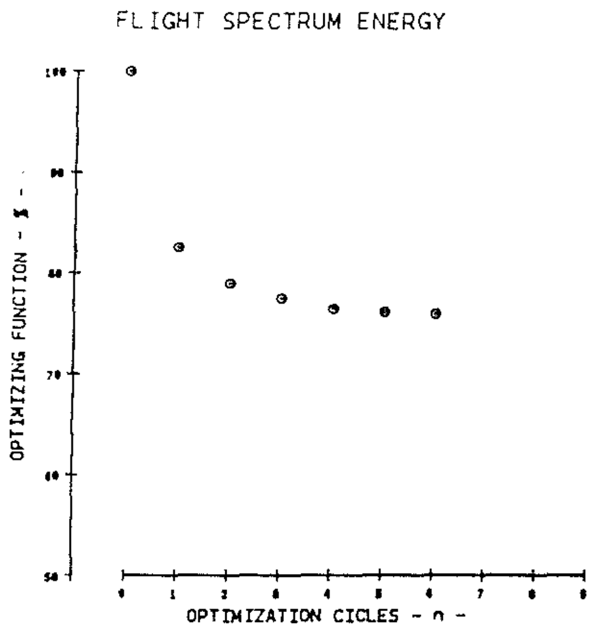


FIG. 4

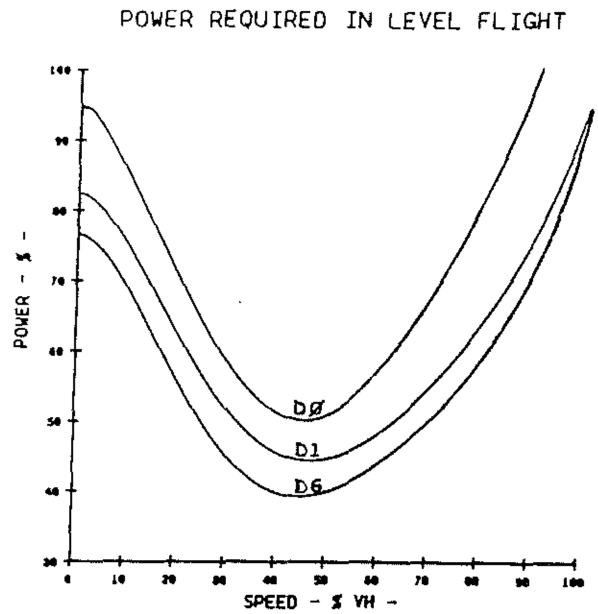
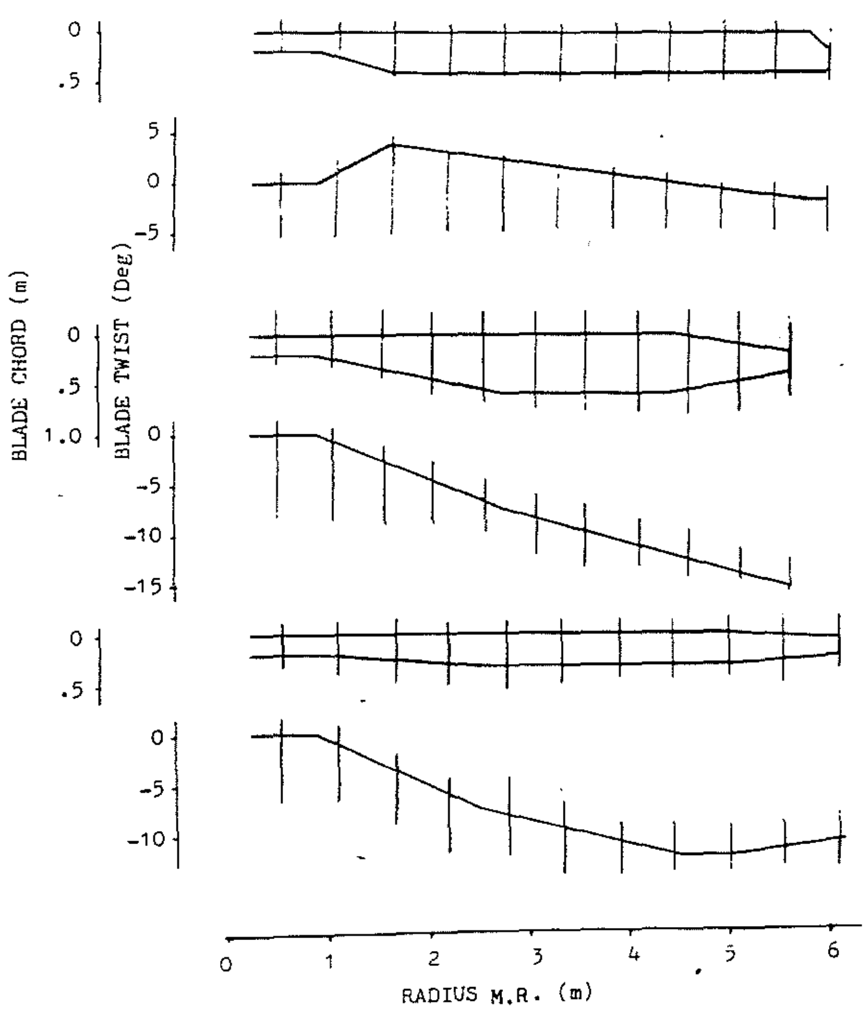


FIG. 5

OPTIMIZATION RESULTS



D0

Rotor RPM = 346
Mast M.R. tilt = 5. (Deg)
Chord T.R. = .280 (m)
Radius T.R. = 1,12 (m)
Hor. Stab. area= 1.74 (m^2)
Vert. Fin area = 1.93 (m^2)

D1

Rotor RPM = 306
Mast M.R. tilt = 5.3 (Deg)
Chord T.R. = .200 (m)
Radius T.R. = .800 (m)
Hor. Stab. area= 1.00 (m^2)
Vert. Fin area = 2.50 (m^2)

D6

Rotor RPM = 306
Mast M.R. tilt = 5.5 (Deg)
Chord T.R. = .200 (m)
Radius T.R. = 1.18 (m)
Hor. Stab. area= 1.00 (m^2)
Vert. Fin area = 2.50 (m^2)

FIG. 6