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SUPERSONIC ROTOR NOISE CALCULATION WITH SONIC BOOM PREDICTION METHODS

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Abstract

The nonlinear wave propagation from the blade tip of an open supersonic rotor is dealt with by the analytical method of characteristics using bicharacteristics as independent variables. Shocks are found from the condition that they are in first order bisectors of the characteristic slopes in front of and behind the shock. The increases in pulse duration are given by strained coordinates. It is shown that dissipation in the shock waves accounts for the differences between noise levels found from measurements and those predicted from linear theory. An arbitrary three-dimensional configuration and a length distribution of lift and drag can be described with the equivalent body of revolution. An explicit formula for the shock pressure is presented and the applicability of sonic boom minimization concepts to the design of a supersonic rotor blade tip is discussed.

1. Introduction

More than a decade ago much work has been done on the sonic boom generated by a Supersonic Transport. The Third Conference on Sonic Boom Research [1] in 1970 was one of the conferences at the end of the era of intense sonic boom research. In 1971 some of the literature was reviewed by Hayes [2]. In 1973 a method for sonic boom reduction through aircraft design and operation was developed by Seebass and George [3],[4]. Based on this work Mack and Darden [5] showed in 1980 that sonic boom minimization methods can guide the design team choices toward a low boom configuration while permitting sufficient freedom and flexibility to satisfy other design criteria. An open supersonic rotor noise theory has been presented by Hawkings and Lowson [6] in 1974. They combined Whitham's [7] sonic boom prediction method with the Lighthill aerodynamic sound theory by applying the acoustic far field solution as initial solution to Whitham's method of strained coordinates. Due to the matching conditions the nonlinear distortion of the signal is supposed to start at a finite distance from the blade. However, nonlinear effects may be important near the blade and Whitham's sonic boom prediction method is for a projectile in straight flight.

In the present paper the analytical characteristics method [8],[9] is modified to give the sonic boom of an axisymmetric body in a helical motion. Separation of variables leads to the Whitham F-function and a decay function. The Whitham F-function has to be determined from the cross sectional area distribution and the decay function is affected by the curvature of the motion. Blade thickness and blade force distribution may be accounted for by the equivalent body of revolution. Including the near field, dissipation and dissipation rate are calculated. The present paper is an extension of an earlier paper by the author [10]. Blade slap due to rotor-vortex interaction is not included in this paper.

2. Analytical Characteristics Method and Geometric Acoustics

In the analytical characteristics method characteristic manifolds are used as independent variables. The physical coordinates including the time as well as the velocity and the thermodynamic quantities are expressed by power series expansions. The characteristic space indicated by the coordinates x_0, y_0, z_0 and t_0 coincides with the physical space if there is no perturbation. The geometry of wavefronts and rays in the characteristic space has to be known before applying the analytical characteristics method. The geometry of waves generated by the blade tip is given by an expanding sphere. This sphere representing the surface of influence is centered around its point of generation (x_p, y_p, z_p, t_p) on the blade tip path.

Figure 1 depicts the helicoidal path of an advancing and rotating blade tip. x_0, y_0, z_0 are the Cartesian coordinates where the z_0-axis coincides with the rotor axis. The blade tip path may be given by $x_p(t_p)$, $y_p(t_p)$, $z_p(t_p)$ and t_p. For convenience, the sphere of influence is described by a coordinate system with its origin at the sphere origin and a longitudinal coordinate in the direction of the helicoidal path. With R as radius of rotation and M_A as advance Mach number the wave origin at time t_p is given by

$$(1) \qquad x_p = R \sin\theta \; ; \quad y_p = R \cos\theta \; ; \quad z_p = M_A t_p \quad ,$$

where

$$(2) \qquad \theta = \Omega t_p \quad \text{and} \quad \Omega = \frac{\omega_{true}}{a_0} \ell \quad .$$

The coordinates are dimensionless by the blade chord ℓ and the sound velocity a_0 in undisturbed flow. With $M_R = \Omega R$ as rotational Mach number the velocity components are obtained by differentiation of eq.(1) with respect to t_p:

(3) $\quad M_{px} = M_R \cos\theta \; ; \quad M_{py} = - M_R \sin\theta \; ; \quad M_{px} = M_A \quad .$

The advance ratio is

(4) $\quad \lambda = \dfrac{M_A}{M_R} = \mathrm{tg}\,\alpha \quad .$

With the above definitions the longitudinal coordinate is given by

(5) $\quad \mathrm{Lon.} = \cos\alpha\,[(x_0 - x_p)\cos\theta - (y_0 - y_p)\sin\theta] + \sin\alpha\,(z_0 - z_p) \quad .$

The lateral coordinate in the plane of rotation then is:

(6) $\quad \mathrm{Latr.} = (y_0 - y_p)\cos\theta + (x_0 - x_p)\sin\theta \quad .$

The other lateral coordinate is:

(7) $\quad \mathrm{Latn.} = -\sin\alpha\,[(x_0 - x_p)\cos\theta - (y_0 - y_p)\sin\theta] + \cos\alpha\,(z_0 - z_p) \quad .$

The equation for the sphere of influence now reads

(8) $\quad \mathrm{Lon}^2 + \mathrm{Latr}^2 + \mathrm{Latn}^2 - (t_0 - t_p)^2 = 0 \quad .$

Since the blade tip speed is supersonic all spheres of influence generated at different times t_p form an enveloping surface which may be understood as a characteristic surface or acoustic perturbation front.

Due to the mathematical theory the enveloping surface is given by equation (8) and its differentiation with respect to t_p :

(9) $\quad \mathrm{Lon}\, M_H - (t_0 - t_p) = 0 \quad ,$

where $M_H = \sqrt{M_R^2 + M_A^2}$ is the helical Mach number. The three-dimensional unsteady wave propagation may now be observed in planes of constant azimuthal angle (φ = const), see Fig.1, and thus may be treated as a quasi-twodimensional unsteady case. Instead of Latr and Latn there is only one lateral coordinate given by

(10) $\quad \mathrm{Lat} = \mathrm{Latr}\cos\varphi + \mathrm{Latn}\sin\varphi \quad .$

At a constant time (t_0 = const) the geometry of the enveloping surface, i.e. the acoustic perturbation front is given by a line in planes of constant azimuthal angle and may be derived from eqs.(8) and (9). The slope of the envelope then is:

$$(11) \qquad \frac{d(Lat)}{d(Lon)} = \frac{\pm 1}{\sqrt{M_H^2 - 1}} \quad .$$

The dimensionless curvature of the envelope is:

$$(12) \qquad K_r = \frac{\cos\alpha \, \Omega \, \sqrt{M_H^2 - 1} \; Latr}{Lat \, (M_H^2 - 1 + M_R \, \Omega \, Latr)} \quad .$$

From equation (12) it can be seen that for

$$(13) \qquad M_H^2 - 1 = - M_R \, \Omega \, Latr$$

the denominator and thus the radius of curvature vanishes. The envelope has a cusp at this position on the sonic cylinder where the helical Mach number is one. For zero forward speed ($M_H = M_R$) and restriction to the plane of rotation ($\cos\varphi = 1$) the geometry of the envelope has been developed numerically by Lowson and Jupe [11]. The tangent of any expanding sphere with the envelope is the bicharacteristic and its projection on the plane of constant azimuth angle is also denoted as ray, N_0. The relationships between the coordinates N_0 and Lon, Lat and Latr are given by

$$(14) \qquad Lon = \frac{N_0}{M_H} \quad ,$$

$$(15) \qquad Lat = N_0 \frac{\sqrt{M_H^2 - 1}}{M_H} \quad ,$$

$$(16) \qquad Latr = N_0 \cos\varphi \frac{\sqrt{M_H^2 - 1}}{M_H} \quad .$$

In first order shocks are propagating along the ray N_0. Thus, the problem is reduced to a quasi-onedimensional unsteady case depending on the coordinates $(t_0 - t_p)$ and N_0 and the parameter t_p, see Fig. 2. Let the tangent of the expanding sphere with the envelope be the bicharacteristic $\nu = 0$ and its parallels in the $(t_0 - t_p)$, No-plane the bicharacteristics $\nu = \text{const}$. The bicharacteristics $\mu = \text{const}$ are chosen so that they are perpendicular to the bicharacteristics $\nu =$ const

$$(17) \qquad \begin{aligned} \mu + \nu &= t_0 - t_p \quad , \\ \mu - \nu &= N_0 \quad . \end{aligned}$$

μ and ν are the independent variables.

After Oswatitsch [12] the first order coordinate perturbations of the normal coordinate $(N = N_0 + N_1 + \ldots)$ and the time $(t = t_0 + t_1 + \ldots)$ are given by

$$
(18) \qquad
\begin{aligned}
N_1 - t_1 &= \int_{\mu_0}^{\mu} [U_{1N}(\bar{\mu},\nu) + a_1(\bar{\mu},\nu)]\, d\bar{\mu} + C_1(\nu) \quad , \\
N_1 + t_1 &= \int_{\nu_0}^{\nu} [U_{1N}(\mu,\bar{\nu}) - a_1(\mu,\bar{\nu})]\, d\nu + C_2(\bar{\nu}) \quad .
\end{aligned}
$$

where a_1 and U_{1N} are the first order perturbations of the sound velocity and the velocity in the direction of N_0. U_{1N} and a_1 have to be evaluated from the compatibility conditions, i.e. from the solution of the wave equation.

3. Evaluation of the First Order Velocity Perturbations U_{1N} and a_1

The velocity potential is derived for an axisymmetric body. The source distribution then approximately is a strip in the Lon, Lat, t_0-t_p-space. This strip interacts the surface of dependence for an observer in a line. With ξ as the source position the retarded time τ of the source is given by

$$(19) \qquad \tau = t_0 - \sqrt{(\mathrm{Lon} - \xi)^2 + \mathrm{Lat}^2} \quad .$$

The derivation of the following equations for the velocity potential ϕ is similar to a corresponding derivation in a report by Stuff [13].

$$(20) \qquad \phi = - \frac{a_0 \ell}{4\pi} \int_{-\infty}^{+\infty} \frac{M(\tau)\ S'(\xi,\tau)}{\pm \sqrt{(t_0 - \tau)^2 - \mathrm{Lat}^2}}\, d\tau \quad ,$$

where S' is the derivative of the cross sectional area distribution of the axisymmetric body. Weak shock waves are located at values for μ and ν satisfying the condition

$$(21) \qquad \frac{\nu}{\mu} << 1 \quad .$$

Under this condition and with the aid of the bicharacteristics (equation (17)), one obtains, to the first order, for the upper and lower bounds of the integral (equation (20)):

$$(22) \qquad (\tau_{1,2} - t_p) = \pm \sqrt{\frac{4\mu\nu}{\sqrt{M_H^2 - 1}\left(\cos\alpha \cos\varphi\, \Omega\, \mu \pm \sqrt{M_H^2 - 1}\right)}}$$

where the plus sign in the denominator is for propagation without a cusp and the minus sign for propagation with a cusp. The first order velocity perturbations may be calculated finally from

$$(23) \qquad U_{1N} = \frac{2}{\gamma - 1}\, a_1 = -\frac{1}{2 a_0 \ell} \frac{\partial \phi}{\partial \nu}$$

with

$$(24) \qquad \frac{\partial \phi}{\partial \nu} = - M_H^2\, a_0 \ell\, 2\; F(2\nu M_H)\; G'(\mu) \quad ,$$

where

$$(25) \qquad F(2\nu M_H) = \frac{1}{2\pi} \int_0^{2\nu M_H} \frac{S''(\xi_a - \xi)\, d(\xi_a - \xi)}{\sqrt{2\nu M_H - (\xi_a - \xi)}}$$

is the Whitham [7] F-function with $(\xi_a - \xi)$ as the distance of the source from the blade tip path ξ_a and

$$(26) \qquad G'(\mu) = \frac{\sqrt{M_H/2}}{\sqrt{\mu \sqrt{M_H^2 - 1}\left(\cos\alpha \cos\varphi\, \Omega\, \mu \pm \sqrt{M_H^2 - 1}\right)}}$$

is the derivative of the damping function.

With the aid of equations (18),(21),(23) and (24) the perturbation coordinates can be obtained by an integration over the bicharacteristic μ

$$(27) \qquad N_1 = - t_1 = \frac{\gamma + 1}{4} \int_{\mu_0}^{\mu} U_{1N}(\bar{\mu}, \nu)\, d\bar{\mu} + C_1(\nu) \quad ,$$

so that

$$(28) \qquad N_1 = \frac{\gamma + 1}{4} M_H^2\; F(2\nu M_H)\; G(\mu) \quad .$$

The damping function $G(\mu)$ is

$$(29) \qquad G(\mu) = \frac{\sqrt{2 M_H}}{\sqrt{\Omega \cos\alpha \cos\varphi \sqrt{M_H^2 - 1}}} \sinh^{-1} \sqrt{\frac{\Omega\, \mu \cos\alpha \cos\varphi}{\sqrt{M_H^2 - 1}}}$$

for propagation without a cusp and

$$G(\mu) = \frac{\sqrt{2M_H}}{\sqrt{\Omega \cos\alpha \cos\varphi \sqrt{M_H^2 - 1}}} \cosh^{-1} \sqrt{\frac{\Omega \mu \cos\alpha \cos\varphi}{\sqrt{M_H^2 - 1}}} \tag{30}$$

for propagation with a cusp.

4. Shock Waves

Neglecting second order terms in the pressure jump across the shock wave the shock slope is given by the bisector of the characteristic slopes in front of and behind the shock. A second shock fitting method results from the fact that some field quantities are continuous across shock fronts, such as the perturbation potential and the streamfunction, see also Lighthill [14] and Kluwick and Horvat [15]. Of course, the displacement of the particle is also continuous across shock fronts. The geometrical representation of the second method results in the equal area rule which is called Whitham's area rule [16]. Ahead of the bow shock there are only very small perturbations from the subsonic portions of the blade. Behind the trailing shock the perturbations are also very small at large distances from the blade. Therefore, in the present paper it is assumed that the shock waves propagate approximately into still air. Then the differential equation for the shock wave may be taken from Oswatitsch [8]:

$$\frac{d\nu}{d\mu} = \frac{\gamma + 1}{8} \frac{U_{1N}}{1 + \partial t_1/\partial\nu} \quad . \tag{31}$$

With the aid of equations (23),(25),(26),(27) and (28) the solution of equation (31) may be found to be

$$\frac{\gamma + 1}{4} M_H^3 \quad G(\mu) = \frac{\int_0^{\nu} F(2\bar{\nu} M_H)\, d\bar{\nu}}{F^2(2\nu M_H)} \quad . \tag{32}$$

In order to find an explicit formula, Whitham substituted the neutral Mach line ν_n for the upper limit ν in the integral of equation (32). It should be noted that the F-function is zero on the neutral Mach line ν_n. The asymptotic formula of Whitham can be easily improved by taking into account the distance function $G(\mu)$ for the characteristic ν^*, intersecting the shock wave at a distance given by μ. The Whitham assumption is used as a first step for calculating ν^*.

$$F^2(2\nu^* M_H) = \frac{\int_0^{\nu_n} F(2\bar{\nu} M_H)\, d\bar{\nu}}{\frac{\gamma+1}{4} M_H^3 \; G(\mu)} \quad . \tag{33}$$

By comparison with iterative results of equation (32) the values for the bow shock calculated with ν^* are sufficiently accurate for distances of about 20 body lengths or larger, see also Fig. 3. The pressure jump Δp across the shock wave is given by

$$\frac{\Delta p}{p_0} = -\frac{\gamma}{a_0}\left[\frac{\partial \phi}{\partial t_0}\right]_{\text{Shock}} \quad . \tag{34}$$

With the resulting explicit formula

$$\frac{\Delta p}{p_0} = \gamma M_H^2 \sqrt{M_H/2}\; G'(\mu) \left\{ \frac{\int_0^{\nu^*} F(2\bar{\nu} M_H)\, d\bar{\nu}}{\frac{\gamma+1}{8} M_H^2 \sqrt{M_H/2}\; G(\mu)} \right\}^{\frac{1}{2}} \quad . \tag{35}$$

Apart from the fact that the characteristic ν^* depends on the distance μ (equation (33)) the shock waves are damped with

$$\frac{G'(\mu)}{\sqrt{G(\mu)}} = \frac{\left[\dfrac{M_H\, \Omega \cos\alpha \cos\varphi}{8\sqrt{M_H^2 - 1}}\right]^{\frac{1}{4}}}{\left[\mu\left[\Omega \cos\alpha \cos\varphi\, \mu + \sqrt{M_H^2 - 1}\right] \sinh^{-1} \sqrt{\dfrac{\Omega \cos\alpha \cos\varphi\, \mu}{\sqrt{M_H^2 - 1}}}\right]^{\frac{1}{2}}} \tag{36}$$

for propagation without a cusp. For propagation with a cusp $\sinh^{-1}$ and $+\sqrt{M_H^2 - 1}$ have to be replaced by $\cosh^{-1}$ and $-\sqrt{M_H^2 - 1}$, respectively. In the limit $\Omega \to 0$ Whitham's $\mu^{3/4}$ asymptotic decay law for straight flight at constant speed is reproduced. Thus, it can be concluded that equation (36) holds also in the case of in plane components of forward speed with M_H as the resultant Mach number. The increase in pulse duration is shown in Fig. 4.

5. Wave Drag, Dissipation and Acoustic Intensity

For weak shocks the specific entropy change ΔS at the shock is proportional to the cube of the shock pressure jump [17]:

$$\Delta S = c_p \frac{(\gamma^2 - 1)}{12} (U_{1N})^3 \quad . \tag{37}$$

c_p is the specific heat at constant pressure. The energy dissipated in the shock wave per unit time now may be obtained by an integration of the entropy losses $a_0 M_H \rho_0 T \Delta S$ over the shock surface $\mu \frac{d\nu}{dt_p} d\varphi\, d\mu$. With eqs.(1) to (17)

$$(38) \qquad \frac{d\nu}{dt_p} = \frac{\sqrt{M_H^2 - 1}}{M_H^2} \left(\Omega \cos\alpha \cos\varphi\mu \pm \sqrt{M_H^2 - 1} \right) .$$

Using (31) the variable of integration is changed from μ to $2\nu M_H$ so that at a distance given by ν^* the dissipation is

$$(39) \qquad \mathrm{Diss} = \pi\rho_0 a_0^3 M_H^3 \left\{ \int_0^{2\nu^* M_H} F^2(2\bar{\nu}M_H)\, d(2\bar{\nu}M_H) - \right.$$
$$\left. - \frac{2}{3} F(2\nu^* M_H) \int_0^{2\nu^* M_H} F(2\bar{\nu}M_H)\, d(2\bar{\nu}M_H) \right\}$$

At infinite distances the kinetic energy is converted completely into heat. The power of the wave drag [17] therefore is:

$$(40) \qquad D = \pi\rho_0 a_0^3 M_H^3 \int_0^{2\nu_n M_H} F^2(2\bar{\nu}M_H)\, d(2\bar{\nu}M_H) \quad .$$

However, at a finite distance the difference of (39) and (40) is left as kinetic acoustic energy per unit time:

$$(41) \qquad L_{ak} = \pi\rho_0 a_0^3 M_H^3 \left\{ \int_{2\nu^* M_H}^{2\nu_n M_H} F^2(2\bar{\nu}M_H)\, d(2\bar{\nu}M_H) + \right.$$
$$\left. + \frac{2}{3} F(2\nu^* M_H) \int_0^{2\nu^* M_H} F(2\bar{\nu}M_H)\, d(2\bar{\nu}M_H) \right\} \quad .$$

Linear theory is unable of predicting dissipation and this accounts for the differences between measured and predicted noise levels [6],[18]. The local acoustic intensity is reduced through dissipation by the same rate as the acoustic power. With the wave drag as subsidiary condition cross-sectional area distributions for minimum boom have been found [19],[20].

6. Equivalent Body of Revolution

Hayes [21] first described the effect of an arbitrary nonlifting three-dimensional configuration in an alternative representation by an equivalent body of revolution. Walkden [22] and Morris [23] found that the sonic boom due to a length distribution of lift may also be described by an equivalent body of revolution. Using the concept of the equivalent body of revolution Seebass and George [4] developed a method for the reduction of sonic boom due to lift and volume including both, the front and the rear shock. In the present case the cross-sectional area of the equivalent body of revolution is given by the helical area cut off from the blade by the Mach plane and projected on a plane normal to the direction of motion. The Mach plane is a tangential plane to the Mach conoid of dependence, i.e. a helically deformed Mach cone of dependence. Thus, the F-function for a supersonic blade tip may be found. The decay function (equation (36)) remains unchanged.

7. Comparison with Experiment

There are only a few published studies of supersonic propeller noise. The experimental results of Hubbard and Lassiter [18], though thirty years old, include measurements of the propeller performance and its acoustic field as well. They also were taken for comparison with theoretical results by Hawkings and Lowson [6]. The detailed thickness and force distribution of the Hubbard and Lassiter rotor blades [18] across the chord are not known. Only the blade chord and the maximum thickness d are given, and so the blade section is assumed to be a parabolic arc. For the local thrust and drag components the assumption is made that the local thrust and drag coefficients, c_T and c_D, are constant over the whole blade.

The Hubbard and Lassiter measurements [18] are restricted to the rotor plane. Thrust does not radiate noise in this plane. The Whitham F-function of the equivalent body of revolution due to thickness and torque force distribution then is

$$F(2\nu M_H) = \frac{\sqrt{2\nu M_H}}{2\pi} \left\{ c_D + 4d \left[\frac{1}{\sqrt{M_H^2 - 1}} - 2\nu M_H \frac{2}{3} \left(\frac{2}{\sqrt{M_H^2 - 1}} + \frac{M_H K}{\left(M_H^2 - 1\right)^2} \right) + 4\nu^2 M_H^2 \frac{8}{15} \frac{M_H K}{\left(M_H^2 - 1\right)^2} \right] \right\} , \tag{42}$$

where K is the change of helical Mach number in the radial direction. The neutral Mach line is given by

$$(43) \qquad 2\nu_n M_H = \frac{3}{4} - \frac{3}{20}\frac{M_H K}{\left(M_H^2 - 1\right)^{3/2}} + \frac{3}{16}\sqrt{M_H^2 - 1}\ \frac{c_D}{d} \quad .$$

The acoustic intensity reduction due to dissipation is calculated from equations (39) and (40). In Fig.5 the results are plotted for supersonic Mach numbers. The reduction is twice as much as proposed by Hawkings and Lowson [6]. This is because the present theory includes the near field at the blade tip, for which the dissipation rate is much higher than in the far field, see Fig.6. The calculated reduction is in satisfactory agreement with the measurements [18].

8. Conclusions

Supersonic flow is an acoustic problem in first instance. Using the analytical characteristics method formulas for the wave signal and the leading shock are derived. The shock "eats up" part of the acoustic energy by converting it into heat. These nonlinear effects are important near the blade. The shock wave rapidly becomes weaker away from the blade, and soon becomes quasi-linear. The present method may be generalized to include the trailing shock. Similar to sonic boom methods [4] then the signal may be optimized for minimum possible pressure jump or minimum possible impulse in the positive part of the signal. The rear part of the signal may be optimized by interference of blade lift and drag with blade thickness in such a way that their effects on the pressure approximately cancel each other. By introducing the wave drag as a subsidiary condition other design criteria may be met.

9. Referrences

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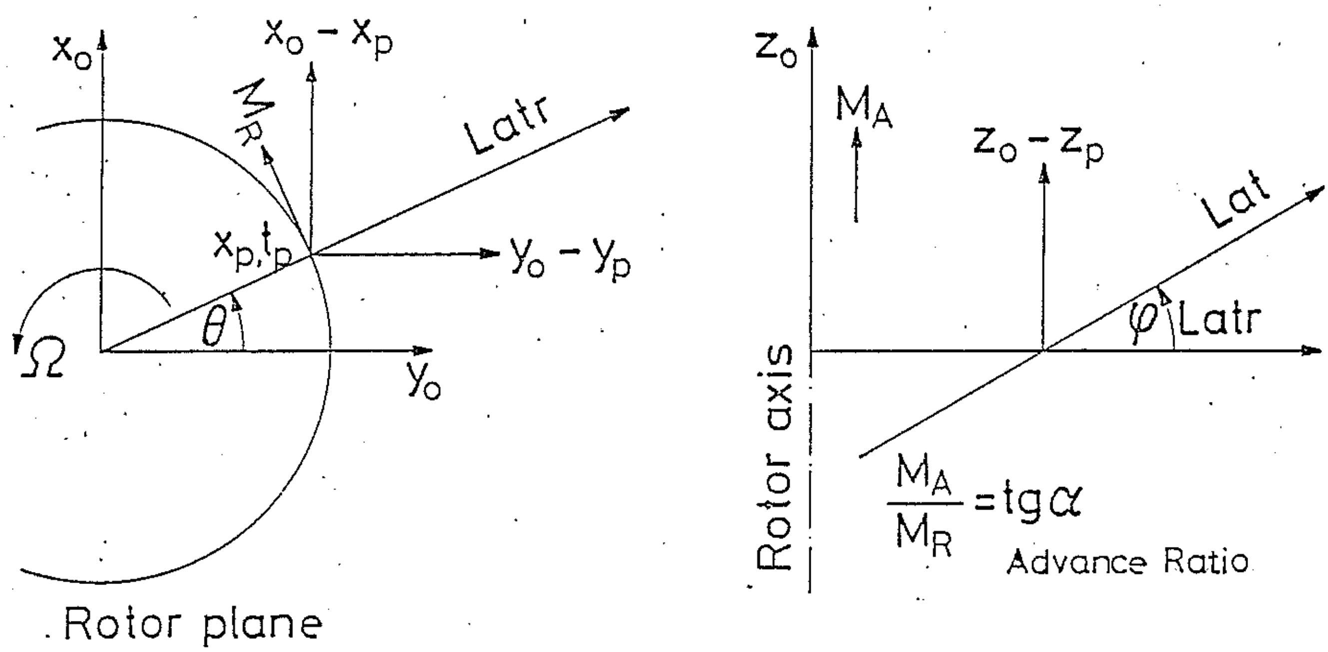

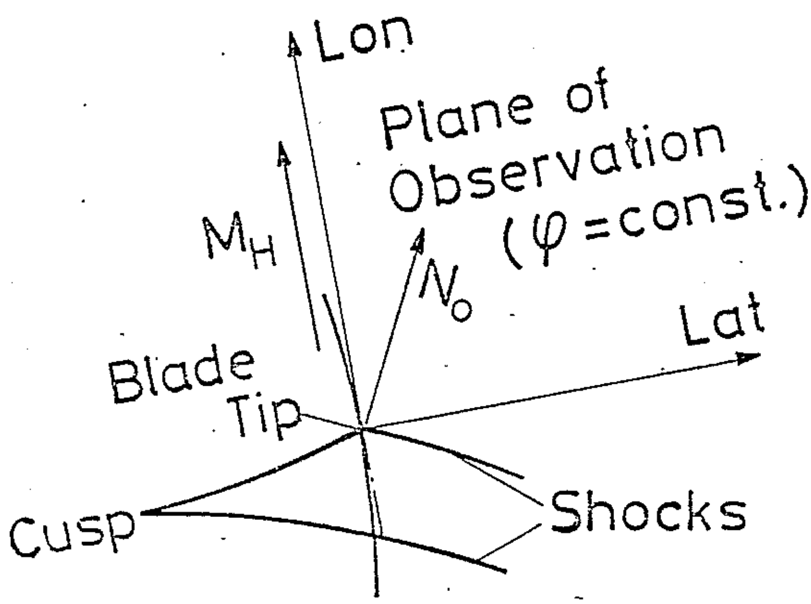

<u>Fig.1</u>: Helicoidal path of advancing blade tip and planes of observation.

M_A → advance Mach number,

M_R → rotational Mach number.

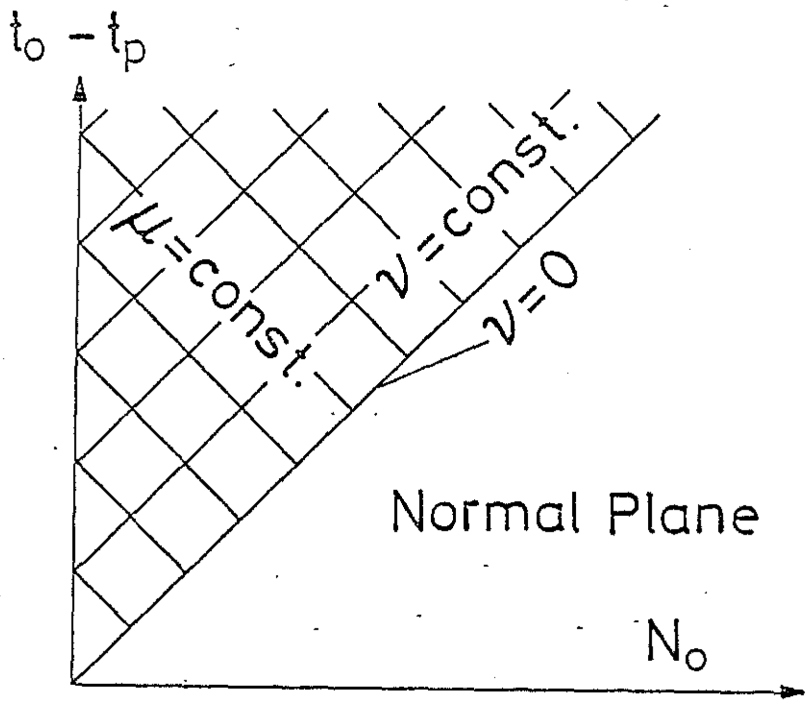

Fig. 2: Bicharacteristics μ = const and ν = const in tl normal plane to the shock front.

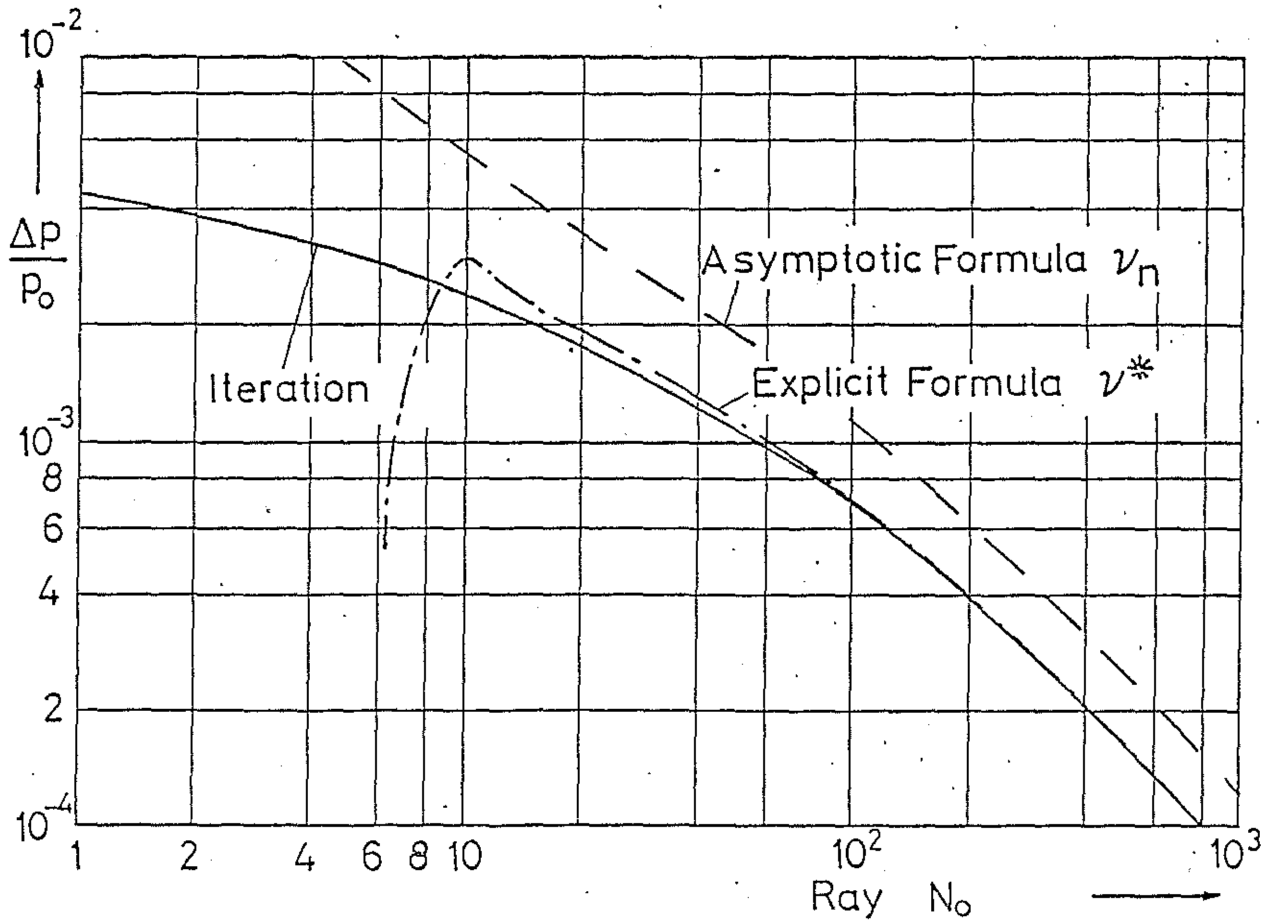

Fig. 3: Attenuation of the pressure jump in direction of propagation for $\varphi = 0$ without a cusp for M_H = 1,5.

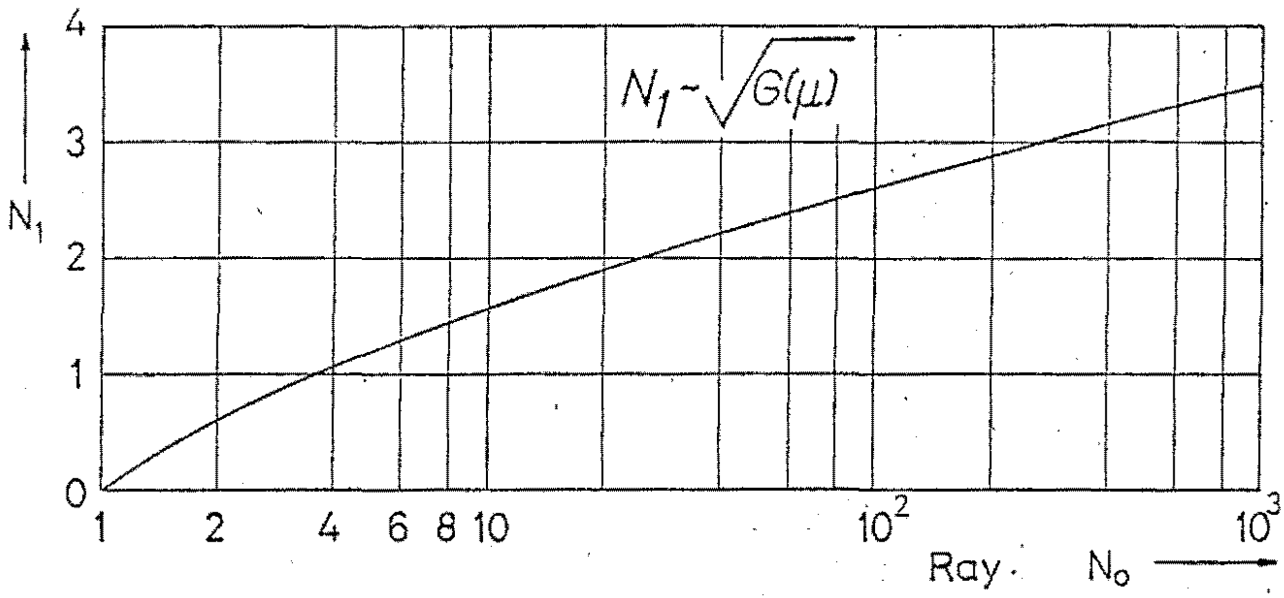

Fig. 4: Increase N_1 in pulse duration in direction of propagation for $\varphi = 0$ without a cusp. For large distances $N_1 \sim \sqrt{G(\mu)}$.

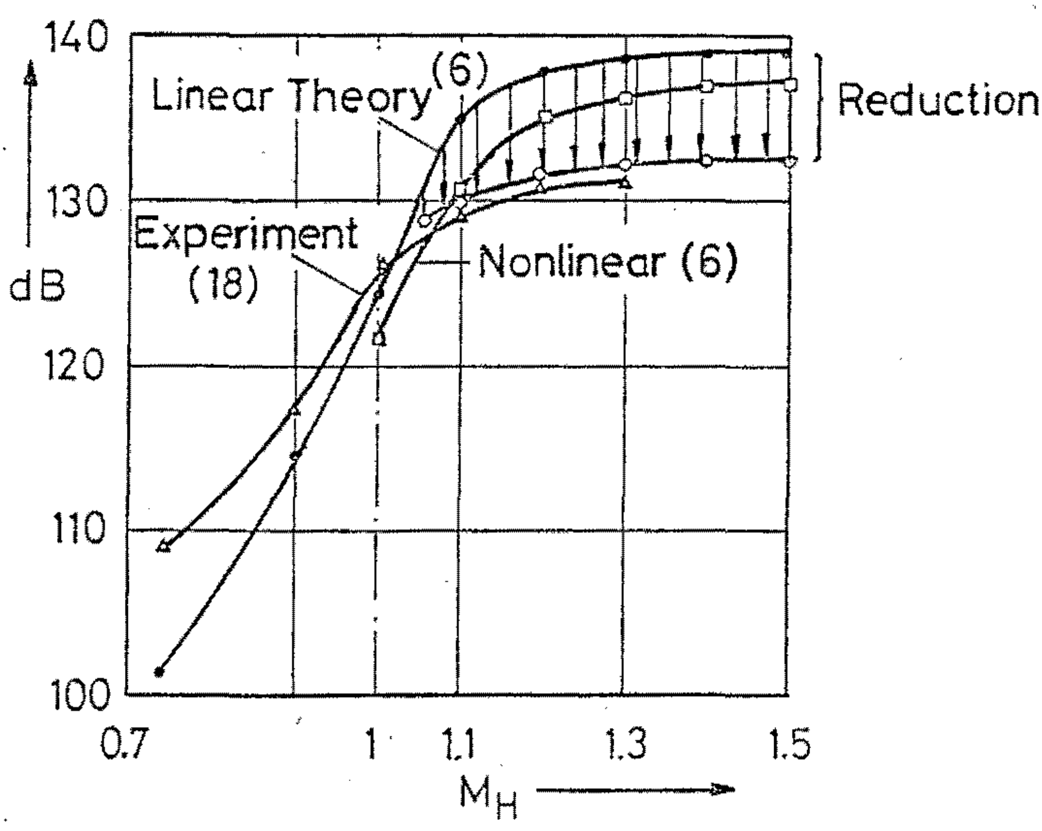

Fig. 5: Reduction in OASPL at 30 feet from the hub calculated from equation (39) with (42) and (43) for $\varphi = \alpha = 0$.

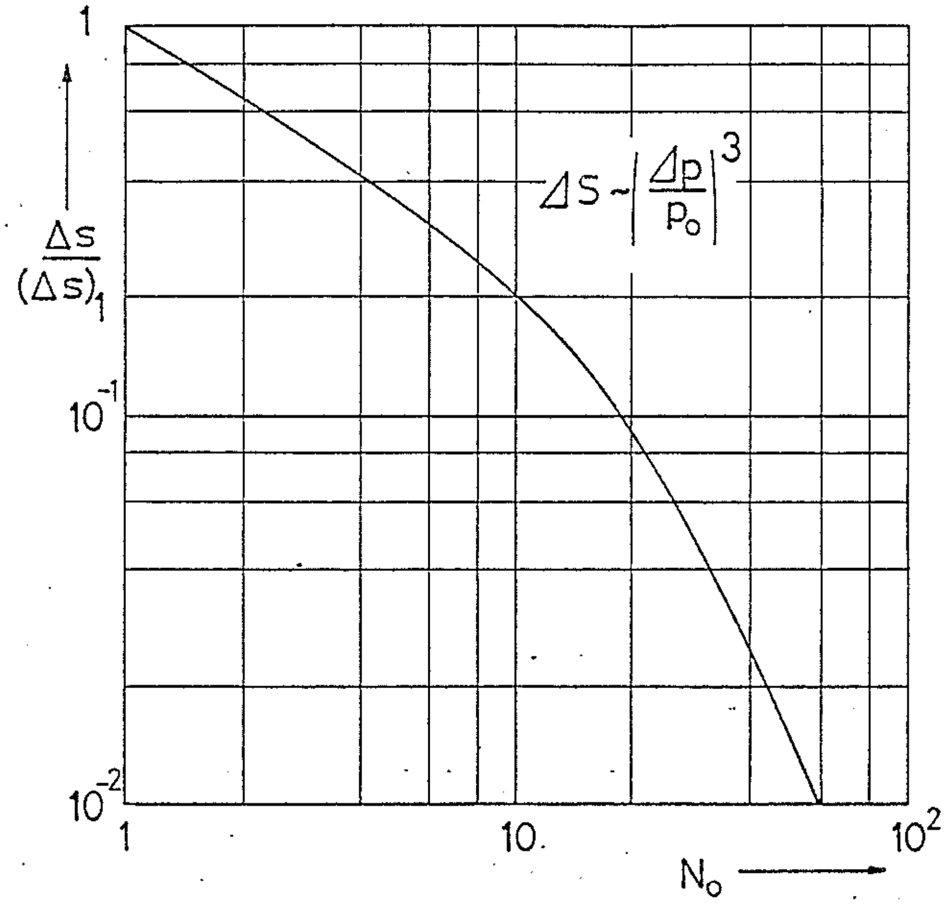

Fig. 6: Specific entropy change ΔS at the shock in direction of propagation for $M_H = 1.3$.