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THE APPLICATION OF ADVANCED TECHNIQUES IN THE DESIGN OF THE FLIGHT CONTROL LAWS FOR MODERN FLY-BY-WIRE SYSTEMS

by

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THE APPLICATION OF ADVANCED TECHNIQUES IN THE DESIGN OF THE FLIGHT CONTROL LAWS FOR MODERN FLY-BY-WIRE SYSTEMS

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The definition of the laws for the dynamic stabilization and automatic flight control for a state-of-the-art helicopter presents a series of difficulties due to both the complexity of the dynamic system and the intricacies of the requirements and desired objectives. The application of modern multivariable control technologies and the advent of Fly-by-Wire systems, permitting the control laws governing the automatic flight control system (AFCS) operation to no longer be limited by the presence of mechanical connections between the pilot commands and the swashplate actuators, is today capable of revolutionizing not merely the implementation on modern autopilots but their design as well. The control laws implemented on digital on-board computers require an ever-increasing sophistication in the design process, in order that they be perfectly capable of modifying the intrinsic dynamics of the vehicle, according to the pilot and mission requirements.

The purpose of this work is to illustrate the results of the application of modern design techniques, both in the time domain and in the frequency domain, to a representative helicopter model, through the comparison of subsequent simulation results with those obtained by more traditional design methods. Appropriate numerical tools for dynamic analysis are used which, thanks to the availability of an accurate helicopter numerical model, allow the development and validation of a comprehensive design of the control laws.

1. NOTATION

A = system matrix
A_c = estimator matrix
A_{1c} = main rotor lateral power actuator output
A_{1s} = rotor disc lateral pitch
A_{1sa} = lateral cyclic actuator input (blade deg)
B = input matrix
B_{1c} = main rotor fore/aft power actuator output
B_{1s} = rotor disc fore/aft pitch
B_{1sa} = fore/aft cyclic actuator input (blade deg)
C = output matrix
C_{1sa} = collective actuator input (blade deg)
D = direct coupling matrix
D_w = system disturbances matrix
M = measurement (sensor) matrix
F = filter matrix
G = controller (gains) matrix
h = helicopter height (m)
J = scalar cost function
p = roll rate (rad/s)
q = pitch rate (rad/s)
q = integrators vector
r = yaw rate (rad/s)
u = input (control) vector
u = longitudinal velocity (X axis) (m/s)
v = lateral velocity (Y axis) (m/s)
v = sensor noise vector
w = Z-axis velocity axis (m/s)
w = input noise vector
x = state vector
x_c = estimated state vector
y = output vector
Y_{sa} = tail rotor actuator input (blade deg)
z = measurement (sensor) vector
θ = pitch angle (deg)
θ_{0c} = collective power actuator output
θ_{0Tc} = tail rotor power actutor output
φ = roll angle (deg)
ψ = heading (yaw) angle (deg)

2. INTRODUCTION

Methods for designing active Stability Augmentation Systems (SAS) and Control Augmentation Systems (CAS) for both fixed wing and rotary wing aircraft have traditionally been based on a frequency-domain approach. Well known design and analysis methods based on frequency response, and stability criteria widely known and accepted by control specialists as well as design engineers, such as the Nyquist, Bode and root-locus methods, have been satisfactorily and succesfully used in many applications in the aerospace industry. However, although through the use of these tools rather complex autopilot system have been designed, in an essentially multivariable context, the methods themselves were mainly suited for single-input single-output (SISO) control problems, and required a high degree of adaptation and physical insight to tackle multivariable systems. In recent years, increasing aircraft performance levels are posing equally increasing performance requirements on stability and control augmentation systems (SCAS) and on Automatic Flight Control Systems (AFCS) which have to cope with an ever more complex dynamic behaviour and a higher degree of interaction among the control variables.

Historically, the first flight control systems on which an essentially multivariable design was used were those of airplanes. In fact, for modern aircraft, automatic stabilization systems and systems which enhance handling qualities have with time assumed importance and an ever-increasing role, paralleling the increasing performance required of the vehicles. The first applications of optimal control were developed, based on the minimization of opportune cost functions, and led to positive results both in terms of effective realization and implementation of systems, and in terms of new and important theoretical developments.

Methods which were initially mainly justified by the relative simplicity of the solution to the problem, if opportunely formulated (i.e., LQG), were later revealed to be fertile ground for development even in apparently diverse design approaches, as might have seemed those based on the use of Laplace transforms and (multivariable) frequency response. The development of multivariable design methods in the time domain, based on the use of state variables, and in the frequency domain, based on transfer function matrices, is by now an acquired background for the majority of control systems design engineers.

In any case, even though these methods have been applied effectively, as mentioned above, in aeronautics for fixed wing vehicles, there is still difficulty in establishing their place in helicopter applications. This is only partially explained and justified by the increased complexity in the helicopter system dynamics, of a degree which can easily reveal the limitations of certain design techniques.

From any viewpoint, the helicopter is a complex system. Its dynamic motion is characterized by the presence of non-linearities, by the high degree of interaction between the control variables and the system transitory characteristics with non-minimum phase, in other words, with zeros in the right half-plane. If one excludes the typical non-linearities introduced by equations which describe the dynamics and the kinematics of the unconstrained body, which are in fact the same for each vehicle, the principal source of non-linearities and interaction which renders the helicopter motion so different from that of an airplane are the main rotor equations.

This fundamental component of the helicopter is essential both for thrust and for control moment and, if on one hand determines the agility of the vehicle, on the other hand introduces large coupling between the lateral modes and the longitudinal modes due to the the gyroscopic effect. Further complications are introduced in the dynamics due to flapping and the lead-lag effect of the blades, in addition to the aerodynamic interaction between the rotor motion and the translation of the vehicle.

Nevertheless, the advantages which can be derived from the use of advanced design techniques are far more important precisely because the control system is complex.

The purpose of this work is to illustrate the results of the application of modern design techniques, both in the time domain and in the frequency domain, to a representative helicopter model, through the comparison of subsequent simulation results with those obtained by more traditional design methods.

3. THE HELICOPTER MODEL

As in most modern control system design applications, the design process relies heavily on a good comprehension (i.e., good model) of the plant dynamics.

A brief discription of the H/C model that has been used for this design study is therefore included for the sake of completeness.

The complexity of the model is deemed adequate for designing reasonably fast control loops (e.g., a few rad/s bandwidth) for a class of single main rotor, medium-to-heavy size helicopters (9-12 tons) such as the NH90 or EH101. Although generic, the non-linear model is based on highly realistic data.

The model was developed and implemented using the TSIM dynamic analysis package [12].

Model Overview

The non-linear helicopter model adopted is capable of representing the aircraft over the complete flight envelope. Since the investigations involved the aircraft stability and control, the rigid body modes were of primary interest, together with the interactions of the dynamics of the rotor disc and the flight controls.

Equations of Motion

The motion of the helicopter is described as usual in terms of translational components along the body axes and angular components about the body axes, with origin in the c.g., X positive forward, Y positive starboard and Z positive downward. The translational velocities along the body axes (u, v, w), the angular rates about the same axes (p, q, r), the attitude angles of the body axes w.r.t. the terrestrial trihedral (θ , φ , ψ), expressed in degrees, are the state variables of the rigid-body motion equations.

Main Rotor Representation

The airframe rigid body degrees of freedom are increased with the main rotor relative motion represented in the rotor model. Only the flapping motion is considered, while the blade lag is neglected as it is usually relevant only in high frequency analysis. The blades are assumed to be rigid in bending and torsion. The flapping hinge has an offset from the rotor shaft axis. The blade flapping dynamics is represented by a Fourier series in the blade azimuth truncated at the first harmonic terms. The flapping equations, as well as the equations expressing the forces and the moments due to the rotor, are derived by applying the blade element theory and by retaining the terms considered to be important.

Aerofoil Characteristic

The aerodynamic characteristics of the blades are derived from the linearized lift and drag curves of the two-dimensional aerofoil section (steady-state). Correction factors are introduced to take into account the compressibility and tip-loss factors.

Airframe Forces and Moments

The airframe forces and moments equations are based on the results obtained at the wind tunnel. Local incidence and sideslip angles are taken into account. The rotor downwash contribution is considered retained into a cylindrical wake, and the local effective incidence is computed by including the mean rotor induced velocity.

Engine System

It is assumed that the rotor speed is maintained constant overall the simulation. Neither the transmission dynamics nor the engine response were included into the model. This is considered adequate for the FCS design task as well as for the moderate maneuvre cases.

Flight Control System

The flight control system is modeled as a FBW system driven by a full authority flight control computer. It is considered that the flight commands can be generated both by the pilot, acting on the cyclic stick, the collective lever and the rudder pedals, and by the autopilot (AFCS). These commands are processed and mixed by the FCC control laws and the adequate commands are issued to the power actuators. The hydraulic power actuator response characteristics are represented by a lag time constant.

The Linearized Model

The adopted mathematical model described in the sections above is based on real data. The non-linear model has been linearized, for the task of the autostabilization synthesis, about the nominal flight condition of 80 knots with the center of gravity in its forward position. The A, B, C, D system quadruple has been generated using the TSIM linearization facility. The state vector is

$$\mathbf{x} = [\theta, \varphi\ \psi, p, q, r, u, v, w, h, \theta_{0c}, B_{1c}, A_{1c}, \theta_{0Tc}, A_{1s}, B_{1s}]^T \quad 3.1$$

Two different control strategies have been investigated. The first one acts on the 3 channels of the cyclic controls (fore/aft and lateral) and the tail rotor,

$$\mathbf{u} = [B_{1sa}, A_{1sa}, Y_{sa}]^T \quad 3.2$$

while the second one acts also on the collective, completing the control action on all the 4 axes control.

$$\mathbf{u} = [B_{1sa}, A_{1sa}, CL_{sa}, Y_{sa}]^T \quad 3.3$$

4. OVERVIEW OF DESIGN METHODS

Frequency Domain Methods

Multivariable frequency domain design methods were pioneered in the 70's mostly by the British School, who recognized its advantages at a time when most of the academic community regarded the frequency domain as obsolete and unsuitable for multivariable problems. Sequential loop closing is the simplest approach to multivariable design in the frequency domain.

First a SISO controller is designed for one pair of input and output variables, then another SISO controller is designed for a second pair of variables, taking into account the effect of having closed the feedback loop around the first pair, and so on until all the I/O pairs have been considered. This sequential procedure suffers from a number of drawbacks and is rarely effective except in special cases.

Sensibly more successful are the so-called Nyquist Array methods (INA: Inverse Nyquist Array and DNA: Direct Nyquist Array) that operate on a square plant's transfer function matrix with a generalization of the well known SISO Nyquist and Bode methods. With these methods, the design is split into two stages. In the first stage, a dynamic compensator is designed to make the plant's return ratio (i.e. the compensated or " augmented" plant) diagonally dominant, while in the second stage, a set of separate SISO compensators are designed, one for each pair of input-output variables.

These methods are based on a generalized Nyquist stability criterion and on the mathematical concept of diagonal dominance which have an easy control design interpretation and are rather appealing for H/C flight control system design. The first stage, i.e., achieving diagonal dominance over a range of frequencies, is basically the same as what the AFCS design specialists call "eliminating cross-coupling among the control variables" and is usually the most difficult step. Its execution, however, is a basic SCAS design requirement for obtaining good helicopter handling qualities (see [13] for examples) and is therefore a must; this makes the INA/DNA methods an interesting choice.

Some automatic ways of achieving dominance have been developed in recent years, in addition to the trial-and-error methods that were originally suggested. One of these is called Pseudo-diagonalization [11] and the results of its application (or rather, an extension of it) are described later in this paper.

Another successful design method is the Characteristic-Loci (CL) method and leads to the design of so-called Approximate Commutative Compensators (ACC). The appeal of an ACC is that it allows one to manipulate the characteristic loci as if they were ordinary Nyquist loci.

This method however, while extremely valuable and powerful for the analysis, seems to be somewhat less effective in the synthesis process. This, at least was our experience in applying it to the H/C AFCS design, as described later in the paper.

Time domain methods

For a plant described by a model in state-space form, i.e. by a time domain representation of its dynamic behavior, the well established Linear Quadratic Gaussian or LQG design methodology has been available for many years, as it was in fact the first truly multivariable design approach to be developed. The solution of the LQG problem, stated as the minimization of a "cost" function,

$$J = \lim_{T \to \infty} E\,[\,(\mathbf{x}^T Q\,\mathbf{x} + \mathbf{u}^T R\,\mathbf{u})\,dt\,], \quad 4.1$$

constrained by,

$$\begin{aligned} \mathbf{x} &= A\,\mathbf{x} + B\,\mathbf{u} + \mathbf{w} \\ \mathbf{y} &= C\,\mathbf{x} + \mathbf{v} \end{aligned} \quad 4.2$$

by means of a suitable (optimal) feedback control law, relies on the so-called separation principle and leads to a control system architecture composed of a state estimator (usually a Kalman filter) and an (estimated) state variable constant feedback (compensator). Efficient numerical algorithms are widely available for calculating the estimator and compensator gains and this makes the design procedure quite straightforward.

This method allows one to shape the principal gains of the augmented plant at either the input or the output, in order to achieve required performance or robustness specifications. Stability is obtained automatically and therefore, in general, the characteristic loci need not be examined. However, in its crudest interpretation, LQG design can exhibit arbitrarily poor stability margins; therefore, many improvements have been devised over the basic LQG theory. One popular method is called Loop Transfer Recovery (LTR) and consists in a method of automatically shaping the principal gains of the augmented plant. However, the presence of slightly damped poles or zeros in the plant, and even more, the presence of right half-plane zeros (i.e., *non-minimum* phase), as in the case of helicopters (see model section), is expected to lead to difficulties in the application of this method, which, therefore, has not been considered as a candidate design technique. A relatively new approach to multivariable feedback design is the renowned $H\infty$ optimal control design method.

5. CONTROLLER DESIGN IN THE FREQUENCY DOMAIN

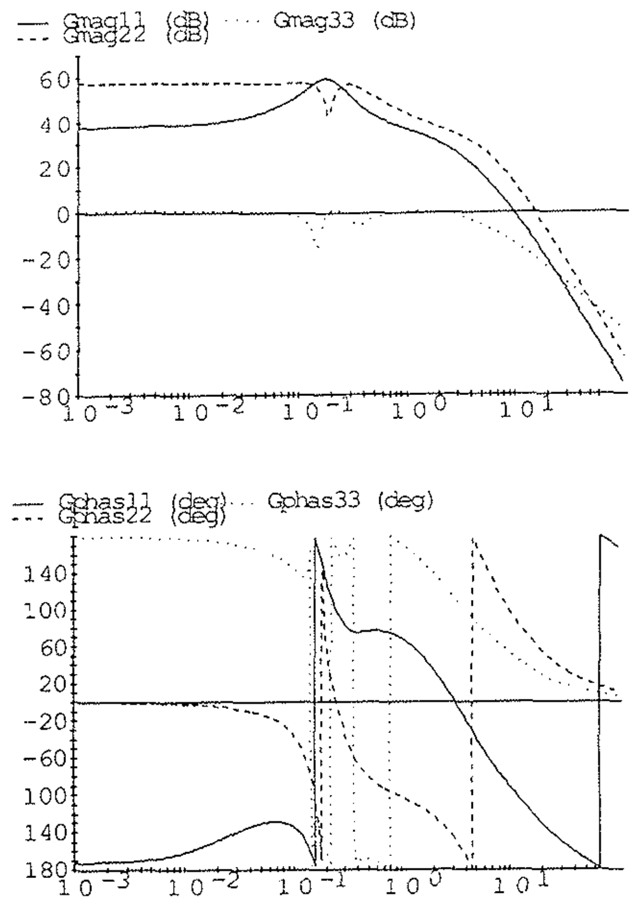
By stating the design objectives in terms of "handling qualities", one usually refers to both time and frequency domain properties, such as time constants, bandwidth, damping ratios, phase shifts.

According with the widely accepted classification provided by [13], the objective was to design an Attitude Command Attitude Hold (ACAH) control system using the linearized H/C model described in Sec. 3. However, since the basic H/C model showed an unstable low-frequency pitch oscillation and a slightly damped Dutch roll oscillation, the model was augmented with a simple SAS feedback loop in order to deal with a stable system for the ACAH design. This two-step design process is in accordance with everyday practice where SAS and Attitude Hold systems are considered separately, and does not need any special comment. Furthermore, the details of this simple SAS are not given as they are not significant to the discussion.

Starting with the linear 16 state helicopter model, three inputs (actuator command in pitch η_F, roll η_L, and yaw η_R) and three outputs (helicopter pitch angle θ, roll angle φ, and yaw rate r) were selected, corresponding to the following squared system transfer function matrix G(s):

$$G(s) = \begin{vmatrix} \theta/\eta_F & \theta/\eta_L & \theta/\eta_R \\ \varphi/\eta_F & \varphi/\eta_L & \varphi/\eta_R \\ r/\eta_F & r/\eta_L & r/\eta_R \end{vmatrix} \qquad 5.1$$

The transfer functions of interest are those on the diagonal - pitch angle θ w.r.t. fore/aft actuator, roll angle φ w.r.t. lateral actuator, and yaw rate r w.r.t. tail rotor actuator. In order to have a de-coupled system, it is desireable to reduce the off-diagonal elements to a minimum with respect to those on the diagonal. The Bode diagrams of the diagonal elements are shown in Figs. 5-1 and 5-2.



Figs.5-1 & 5-2. Bode diags. of linear H/C model

Characteristic-Locus Method with Approximate Commutative Compensators

The square transfer function matrix G(s), with three inputs and outputs, has a spectral decomposition [10]

$$G(s) = W(s)\,\Lambda(s)\,W^{-1}(s) \qquad 5.2$$

$$\Lambda(s) = diag\,[\lambda_1(s), \lambda_2(s), \lambda_3(s)] \qquad 5.3$$

where W(s) is the matrix whose columns are the eigenvectors, or characteristic directions, and $\Lambda(s)$ is a diagonal matrix composed of the eigenvalues, or characteristic functions, of G(s).

If the compensator matrix K(s) is constructed as

$$K(s) = W(s)\,M(s)\,W^{-1}(s) \qquad 5.4$$
$$M(s) = \text{diag}[\mu_1(s), \mu_2(s), \mu_3(s)]$$

then the return ratio is

$$G(s)\,K(s) = W(s)\,\Lambda(s)\,M(s)\,W^{-1}(s) = \qquad 5.5$$
$$= W(s)\,N(s)\,W^{-1}(s)$$

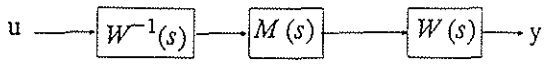
where

$$N(s) = diag[\nu_1(s), \nu_2(s), \ldots, \nu_m(s)] \qquad 5.6$$

and

$$\nu_i(s) = \lambda_i(s)\,\mu_i(s) \qquad 5.7$$

Therefore, the system "K plus G" connected in series has eigenvalues which are the product of the eigenvalues of the plant, G, and compensator, K. The next step is to then obtain a Bode diagram for each $\lambda_i(s)$ and to design a compensator $\mu_i(s)$ for each $\lambda_{i(s)}$, using single-loop techniques. The compensator then becomes the series connection of three systems,



But since the matrices $W(s)$ and $W^{-1}(s)$ are usually irrational functions, it is practical to approximate them with rational matrices. Therefore, $K(s)=W(s)M(s)W^{-1}(s)$ is approximated by:

$$K(s)=A_w(s)\,M(s)\,B_w(s) \qquad 5.8$$

where

$$A_w(s)\approx W(s) \qquad 5.9$$
$$B_w(s)\approx W^{-1}(s)$$

The simplest solution is to choose a frequency $j\omega_0$ in order that, although W(s) and its inverse are complex matrices, they can be approximated by real A_w and B_w matrices. The ALIGN algorithm [10], solves this problem giving an A_w matrix with the columns as scalar multiples of the columns of $W(j\omega_0)$.

If $V(s)=W^{-1}(s)$, then the i-th column of A_w, a_{w_i}, is solved according to the following algorithm:

$$a_{w_i} = \mathrm{argmax}\ \frac{|v_i^H a_{w_i}|^2}{\sum_{i\neq j} |v_j^H a_{w_i}|} \qquad 5.10$$

where v_j is the conjugate of the j-th row of V(s).

McFarlane and Kouvaritakis [8] showed that this effectively solves the problem of maximimizing the diagonal elements while minimizing the off diagonals by solving a generalized eigenvalue problem.

Single Frequency Approximate Commutative Compensator

The use of the approximate commutative compensator, with constant A_w and B_w matrices, relies on the eigenvectors of the plant not changing too quickly with frequency, since the compensation would be limited if its effects were predictable at only ω_0.

An attempt to design such a compensator was made, choosing a fixed frequency of ω_0 = 10 rad/sec. The product $G(s)\cdot K(s)$ was computed and a check for interaction was then made for a sweep of frequencies from 0.001 to 100. rad/sec.

The following observations are made:

a) The A_w and B_w matrices could not maintain little interaction over this range of frequencies,

b) the Bode diagrams which were produced by this compensator were rather unattractive, especially the phase diagrams.

At this stage of the investigation, the method was abandoned, and a pseudo-diagonalization technique was adopted in order to apply a DNA-type design method.

Multi-Frequency Diagonalization

Column dominance is achieved by a generalized ALIGN algorithm,

$$a_{w_i} = \mathrm{argmax}\ \frac{\sum_k p_k \left|v_i^H(j\omega_k)\,a_{w_i}\right|^2}{\sum_k p_k \left\{\sum_{j\neq i} \left|v_j^H(j\omega_k)a_{w_i}\right|^2\right\}} \qquad 5.11$$

where $p_1,\dots,p_n$ is a set of real non-negative weights which allows certain frequencies to be emphasized more than others in order to achieve diagonal dominance. This solution also failed in the case of the linearized helicopter model to produce diagonal dominance over the entire range of frequencies.

Dynamic Diagonalization

However, a possible solution to the problem is multi-frequency alignment with a dynamic compensator [9].

For the system G(s), a compensator K(s) is found, structured as $[\,K_0,\ K_1\cdot s,\ K_2\cdot s^2\,]$ which renders the system Q(s) = G(s)*K(s) diagonally dominant for a range of frequencies ($\omega=\omega_0, \omega_1,\dots, \omega_k$).

Given

$$Q(s) = G(s)\cdot K(s)\text{, or} \qquad 5.12$$
$$q_{ij}(j\omega) = g_i^T(j\omega)\cdot k_j,$$

if

$$k_j(s) = k_{0j} + k_{1j}\,s + k_{2j}\,s^2, \qquad 5.13$$

then

$$q_{ij} = g_i^{\,T}(j\omega)\cdot[k_{0j} + k_{1j}\,s + k_{2j}\,s^2] = \gamma_i^{\,T}(j\omega)\cdot\eta_j, \qquad 5.14$$

where

$$\gamma_i^T = [g_i^T(j\omega),\ j\omega\cdot g_i^T(j\omega),\ (j\omega)^2\cdot g_i^{\,T}(j\omega)], \qquad 5.15$$
$$\eta_j = [\ k_{0j}^{\,T},\ k_{1j}^{\,T},\ k_{2j}^{\,T}\ ].$$

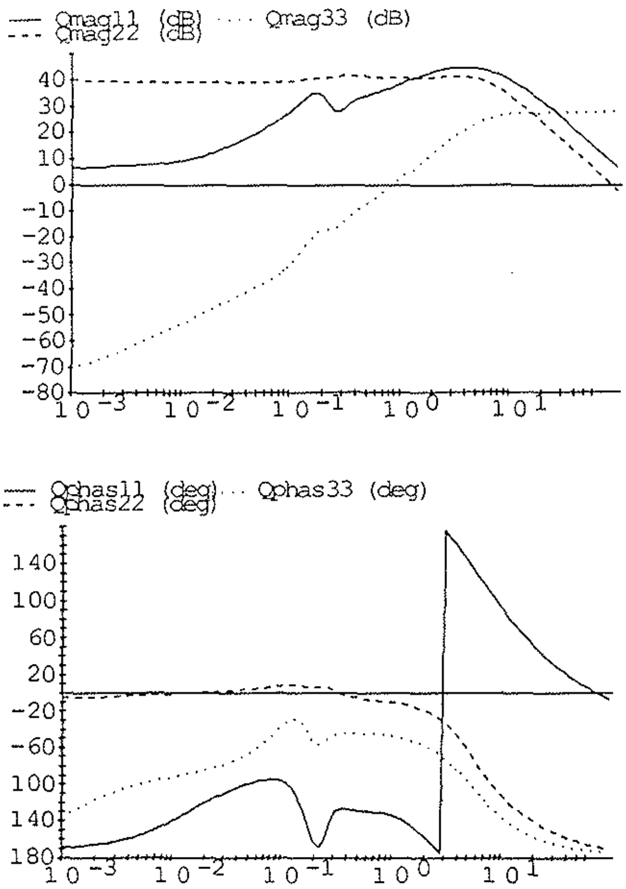
By minimizing the following expression, which is the reciprocal of the cost function maximized by ALIGN,

$$J_j = \frac{\sum_k p_k \left\{\sum_{j\neq i} \left|g_i^{\,T}(j\omega_k)k_j\right|^2\right\}}{\sum_k p_k \left|g_j^{\,T}(j\omega_k)\,k_j\right|^2} \qquad 5.16$$

the column dominance is maximized.

Unfortunately, for the current helicopter problem, diagonal dominance was still very difficult to obtain over the entire range of frequencies. Therefore, a further attempt was made to dedicate a set of weights to each column of K, so that p(k) is no longer a vector but a matrix, p(k,j). In this manner, diagonal dominance was achieved for the entire range of frequencies.

The diagonal Bode diagrams of Q(s) are shown in Figs. 5-3 and 5-4.



Figs.5-3 & 5-4. Bode mag. and phase of Q(s)

Individual Loop Shaping

In order to obtain the desired response characteristics, a second compensator, H(s) is added to the system. Therefore,

$$R(s) = G(s)\,K(s)\,H(s) \qquad 5.17$$

The matrix H(s) is constructed loop by loop, by examining each Bode diagram of $Q(s) = G(s) \cdot K(s)$ (Figs. 5-3 and 5-4) and positioning the crossover frequency and slope in a manner to render the system well behaved while maintaining stability.

Characteristic Loci

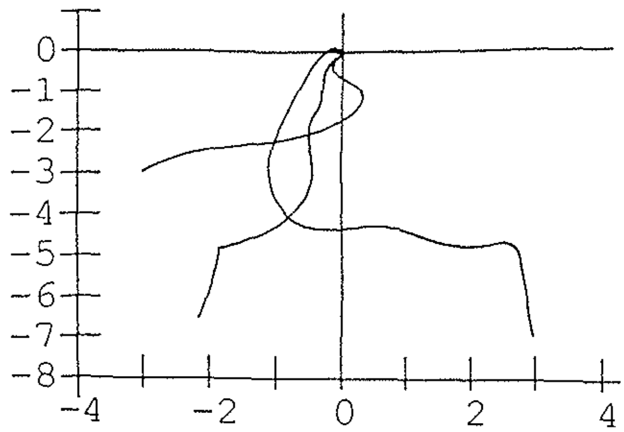
The characteristic loci of the function R(s) are computed for a sweep of frequencies from 0.1 to 100. rad/s, shown in Fig. 5-5.

They do not encircle -1 on the real axis, therefore the closed loop system will be stable.

Linear Time Response

The compensator $K(s) \cdot H(s)$ was transformed from the frequency domain into a system in state space form, with an A, B, C, & D matrix set.

The time simulation of the entire system - compensator + helicopter - was performed, and the response to a "smoothed" step is evaluated.



Figs.5-5.Characteristic Loci

The time response to a step input shows a very fast response, as expected, having chosen the crossover frenquencies to be 10 rad/sec, and therefore the time constants to be 0.1 sec. In addition, a spike occured at the output of the compensator or input to the helicopter which was rather undesireable.

Therefore, two steps were taken.

1) The crossover frequency was moved to 5 rad/sec, giving a response time of 0.2 sec.

2) A filter F(s) was placed between the compensator and input to the helicopter.

The filter F(s), is constructed as

$$F(s) = diag\left[\frac{1}{1+sT}\,;\frac{1}{1+sT}\,;\frac{1}{1+sT}\right] \qquad 5.18$$

where the time constant T = 0.1 sec (corresponding to a break frequency of 10 rad/sec, which falls to the right of the compensator crossover frequency of 5 rad/sec) The Bode diagrams are shown in Figs. 5-6 and 5-7.

This was checked to assure that stability was maintained by the examination of the characteristic loci.

6. CONTROLLER DESIGN IN THE TIME DOMAIN

The same problem of synthesizing an ACAH control system for the helicopter has been addressed in the time domain by using linear optimal control methods.

Command Decoupling

As stated before, one of the most desirable characteristics of a modern flight control system is the capability to totally eliminate the aerodynamic cross-coupling between the four control axes [13].

The implementation of this feature in todeay's control systems is not totally satisfactory, and when the pilot operates one control axis, he is forced to act on the other commands to counter the cross-coupling effects between the helicopter axes.

With the introduction of a FBW control system the pilot inputs must be processed by a flight control computer before they are issued to the power actuators. This fact suggests the possibility of adopting complex multivariable control laws, able to automatically generate the input to the control axes to counter the cross-coupling effects, thus providing a pure response of the H/C.

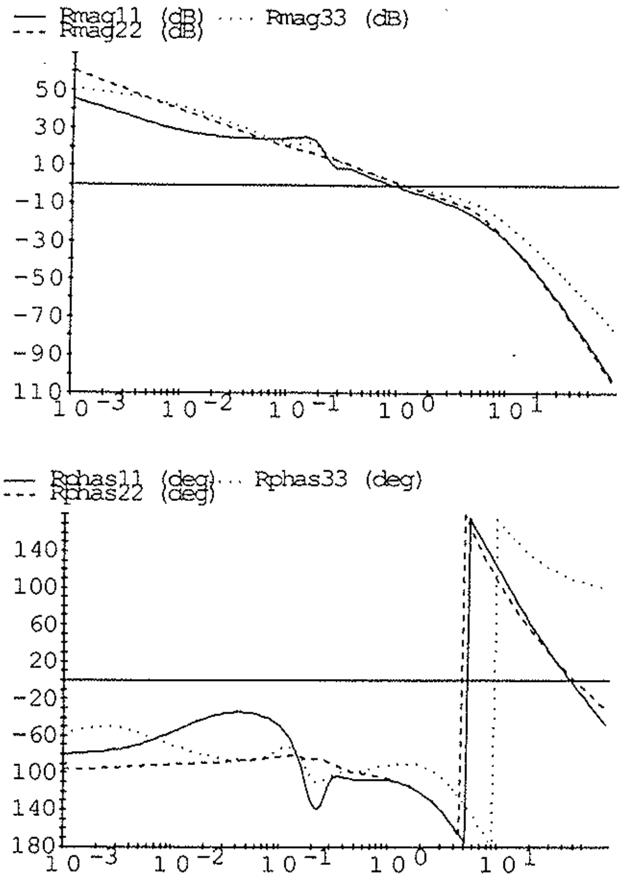


Fig.5-6 & 5-7: Bode magnitude and phase of R(s)

Integral Action

The classic LQR theory provides the designer with a straightforward time-domain control synthesis method that assures the asympthotic stability of the closed-loop system. However, it is well known that such a system is not directly capable of countering sustained disturbances and has no inherent tracking capabilities.

The introduction of an integral action on the most significant states is highly attractive [1], in order to overcome this problem. Moreover, the adopted control configuration summarized in Figure 6-1, with the presence

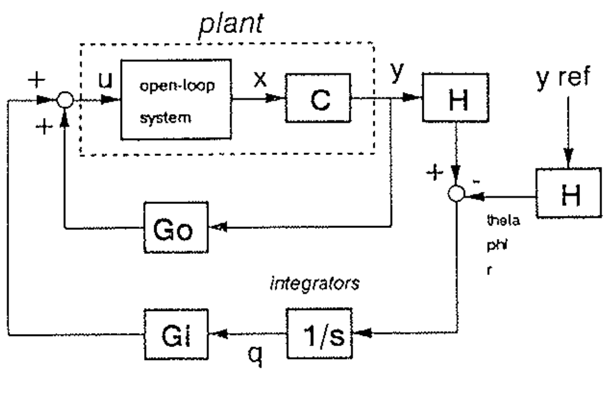


Fig.6-1 The Proportional plus Integral Scheme

of a regulation on the integral states, has been shown [3] to solve the problem of tracking of the desired set-point.

The standard linear time-invariant system

$$\dot{\mathbf{x}}(t) = A\,\mathbf{x}(t) + B\,\mathbf{u}(t) \qquad 6.1$$
$$\mathbf{y}(t) = C\,\mathbf{x}(t)$$

is then augmented with integrators on the outputs

$$\dot{\mathbf{q}} = H\,(\mathbf{y} - \mathbf{y}_{ref}). \qquad 6.2$$

Here the H matrix is composed only of 0's and 1's, and is used to select an integral effect only on a desired subset of the output variables.

The assumptions under which this control configuration assures the asymptotic regulation for a class of step references (set points) are:

$$(A, B)\ \text{is controllable} \qquad 6.3$$
$$\text{rank}\begin{bmatrix} A & C \\ B & 0 \end{bmatrix} = (n + r)$$

As a result,.the control law

$$\mathbf{u}(t) = G_o\,\mathbf{x}(t) + G_i\,\mathbf{q}(t) \qquad 6.4$$

assures both the asympthotic stability of the augmented system and the tracking of the given reference, y_{ref}. In addition, this control configuration maintains the aforementioned closed-loop characteristics also in the presence of linearization error and/or model uncertainities (robust control).

The standard LQR/LQG control synthesis methods can be easily applied in developing such an augmented controller by modifying the given system as necessary. The state vector is augmented as:

$$\mathbf{x}' = [\,\mathbf{x},\ \ \mathbf{q}\,]^T; \qquad 6.5$$

consequently the given system (A, B) is transformed into the augmented system (A', B') as follows.

$$A' = \begin{vmatrix} A & 0 \\ H & 0 \end{vmatrix},\ B' = \begin{vmatrix} B & 0 \\ 0 & -H \end{vmatrix}, \qquad 6.6$$
$$G' = \begin{vmatrix} G_o & G_i \\ 0 & 0 \end{vmatrix}$$

The optimal control synthesis generates the augmented control law (3.4) in the form [5]

$$\mathbf{u}'(t) = G\,\mathbf{x}(t)\,. \qquad 6.7$$

Effect on Decoupling

The basic implication of this control system architecture is that the pilot inputs, i.e. the inceptor sensor signals, are directly connected to the output reference values (y_{ref} in eq. 6.2). In turn, this implies that no feedforward (or CAS) correction is applied and that command decoupling is to be obtained solely by state feedback.

Although this is not a direct objective of the standard LQR/LQG design method, the method itself can be suitably tailored and adapted to obtain this result. Eigenstructure assignement via state feedback [6] and the Loop Transfer Recovery (LTR) methods are significant examples.

In this study, however, no special effort was necessary as command decoupling was simply obtained as a side product of the system augmentation with integral action.

The Integrators Selection

In the present ACAH design study, the control stick and the rudder pedals must act on the helicopter pitch angle, θ, the roll angle, φ and the yaw rate, r. Consequently, three integrators have been added to the original 16-state system, corresponding to the 1st, 2nd and 6th state, and the pilot's control effect is to set the reference on these integrators as in (6.2).

The State Estimation

The optimal linear state feedback control gains are synthesized following the basic assumption that the entire state vector can be measured accurately. In the case of a real helicopter, only the outputs of a set sensors are available to the control system, i.e. , only some linear combinations of the states can be measured. This can be expressed by

$$\mathbf{z}(t) = M\,\mathbf{x}(t). \qquad 6.8$$

Moreover, it is assumed that both the system plant equation and the measurements are affected respectively by state excitation noise, w, and measurement noise, v, described as uncorrelated white noise processes with intensity W and V

$$w(t) = W(t)\,\delta(t) \qquad 6.9$$
$$v(t) = V(t)\,\delta(t)$$

Infact, the system plant equation is considered to be affected both by parameter uncertainties, such as linearization errors, and by input noise from the real actuators, while the real sensors are physical devices also affected by noise. The evaluation system is re-written as

$$\dot{\mathbf{x}}(t) = A\,\mathbf{x}(t) + B\,\mathbf{u}(t) + D_w\,\mathbf{w}(t) \qquad 6.10$$
$$\mathbf{y}(t) = C\,\mathbf{x}(t)$$
$$\mathbf{z}(t) = M\,\mathbf{x}(t) + \mathbf{v}(t)$$

The state estimator is developed by using the optimal observer approach, following the Kalman-Bucy filter theory [1], [11]. The (optimal) state estimator scheme is summarized in the figure 6-2, and the filter gain matrix F is computed by

$$F = P\,M^{T}\,V^{-1} \qquad 6.11$$

where P is the solution of

$$0 = P\,A^{T} + A\,P + D_w\,W\,D_w^{T} - P\,M^{T}\,V^{-1}\,M\,P \qquad 6.12$$

The CSDS Optimal Control Design

The adopted tool to perform the synthesis of the ACAH control law is the Control Structure Design Sensitivity (CSDS) package, originally developed by AGUSTA Sistemi under ESA-ESTEC contract for the design of structures and control of large space systems, in presence of strong control-structure interaction effects. The control optimization facility of CSDS allows the user to design a full state dynamic estimator and controller using a set of control design algorithms such as the Output Variance Constraint (OVC) with H_2 or H_∞ constraints on the outputs, in presence of disturbances on both inputs and measurements.

The optimal gain and the state estimate feedback laws are obtained using the separation principle [2]. The continuous time invariant system is given as in equation (6-10), where A, B, C are the augmented matrices, built from the linearized helicopter system matrices by adding the columns and rows to represent the integral action on the required states. The simple assumption is made that (A, B) is controllable and (A, M) is observable.

The assumed state estimator and controller configuration is

$$\dot{\mathbf{x}}_c(t) = A_c\,\mathbf{x}_c(t) + F\,\mathbf{z}(t) \qquad 6.13$$
$$\mathbf{u}(t) = G\,\mathbf{x}_c(t)$$

where $\mathbf{x}_c$ is the estimated state and

$$A_c = A + B\,G + F\,M \qquad 6.14$$

is the dynamic estimator matrix, F the filter matrix and G the controller matrix. CSDS computes the F, G, A_c matrices by minimizing the cost function $J = E_\infty\,\mathbf{u}^{T} R\,\mathbf{u}$, representing the control energy, subject to the σ_i "soft" variance constraints or the ε_i "hard" constraints on the maximum value of the critical outputs

$$\|\,y_i\,\|_2^2 \le \sigma_i^2 \quad \text{for i=1} \ldots n_y, \text{ or} \qquad 6.15$$

$$\|\,y_i\,\|_\infty^2 \le \varepsilon_i^2 \quad \text{for i=1} \ldots n_y \qquad 6.16$$

It can be demonstrated [5], [7] that $\varepsilon_i = \sigma_i \times \mu$ for any disturbance satisfying the H_2 constraint

$$\|\,w^{T}\,v^{T}\,\|_2^2 \le \mu^2 \qquad 6.17$$

The Q matrix is tuned during an iterative process to provide a controller that satisfies the required output variance constraints. The Q_{ii} element also represents the

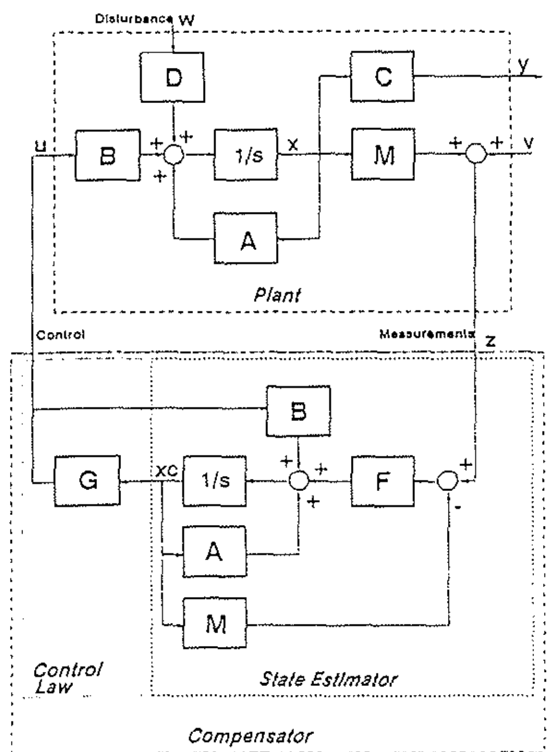


Fig.6-2 The Controlled System Scheme

sensitivity of the relevant output y_i. The update of the Q matrix is driven by

$$Q_{ii}{}^{(k+1)} = Q_{ii}{}^{(k)} \frac{y_{iRMS}{}^2}{\sigma_i^2}. \qquad 6.18$$

If the initial Q matrix is satisfactory for the designer's tasks, the Q matrix iterative tuning process can be avoided. The standard LQG control design method is applied to the system (6.1), minimizing the scalar cost function (4-1)

Eigenvalue Analysis

The uncontrolled helicopter dynamics shows an instability, according to the open-loop system eigenvalues reported in the table 1. The transmission zeros of the H/C model used for ACAH design are [-0.5822, -0.1184, -0.0177]. Once the full state dynamic controller designed using CSDS as described above, is implemented in the TSIM analysis package, the eigenvalues of the closed-loop linearized system are changed as in the table 1.

table 1: System Eigenvalues

Uncontrolled	80 knots.	146 knots.	40 knots.
-10.00	-14.64 ± 2.481 i	-15.18 ± 1.339 i	-15.18
-10.00	-11.41 ± 6.055 i	-11.04 ± 6.554 i	-13.38 ± 4.380 i
-10.00	-10.45 ± 1.024 i	-10.55 ± 0.6322 i	-10.90 ± 2.677 i
-10.00	-10.36	-9.976	-10.83
-9.879 ± 2.028 i	-10.05	-9.953	-10.01
-2.976	-9.956	-9.482	-9.958
-0.7190	-9.381	-8.958	-9.734
-0.4536	-7.838 ± 2.113 i	-8.884 ± 6.821 i	-9.595
-0.2363 ± 1.092 i	-5.223 ± 5.730 i	-7.468 ± 4.720 i	-9.041 ± 7.025 i
-0.9623 e-01	-4.301 ± 7.893 i	-6.669	-6.548
-0.2472 e-02	-3.974 ± 3.876 i	-3.383 ± 3.551 i	-4.065 ± 5.048 i
0.3159 e-03	-3.600 ± 4.304 i	-3.253 ± 8.067 i	-3.714 ± 6.531 i
0.304e-01 ± 0.3254 i	-3.379	-2.993	-3.542 ± 7.967 i
0.0000 x 22	-1.648 ± 1.862 i	-1.788	-2.805 ± 3.419 i
-	-1.503 ± 4.942 i	-1.465 ± 4.255 i	-1.860 ± 1.370 i
-	-1.030	-1.216 ± 0.7384 i	-1.104 ± 0.2166 i
-	-1.017	-1.037	-1.004
-	-0.9496 ± 0.6656 i	-0.8245	-0.8896 ± 1.467 i
-	-0.8964 ± 0.9111 i	-0.4678	-0.8345 ± 2.833 i
-	-0.6655	-0.3457	-0.6041 ± 0.5189 i
-	-0.3460	-0.116± 0.461 e-01 i	-0.3454
-	-0.1111	-0.7194 ± 0.9787 i	-0.1456
-	-0.9286 e-01	-0.4935 ± 5.472 i	-0.5135 e-01
-	-0.1818 e-01	-0.1983 e-01 ± 0.1721 e-02 i	-0.1906 e-01
-	-0.1782 e-01	–	-0.3744 e-02
-	-0.2842 e-17	-0.2270 e-15	0.2482 e-15

As is evident there is always an eigenvalue close the origin, which corresponds to the ψ state (yaw angle),

Singular Values

The LQR/LQG optimal control synthesis guarantees the asymptotic stability of the closed-loop system. An analysis of the system performances can be performed by plotting the singular values of the Loop Transfer Function Matrix (LTFM) versus the frequeny on a bode magnitude plot [12]. It is assumed for this evaluation that the state is perfectly estimeted, i.e. $\mathbf{x} \equiv \mathbf{x}_c$, and the estimator is neglected.

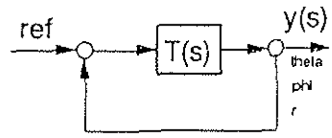


Fig.6-3 LTFM definition

The open-loop transfer function of the plant is

$$P(s) = C[sI - A]^{-1}B = \text{plant} \qquad 6.19$$

Once the inner loop (see figure 6-1) is closed, the system is described by

$$\mathbf{y}(s) = \left\{ [1 - P(s)\,G_o]^{-1} P(s) \right\} \mathbf{u}(s) \qquad 6.20$$

by adding and closing the outer loop (the integrators), the 3 by 3 LTFM $T(s)$ is computed as

$$T(s) = H(s)\,[1 - P(s)\,G_o]^{-1} P(s) \frac{G_1}{s} \qquad 6.21$$

and the singular values of $T(s)$ are

$$\text{sing}(T) = \sqrt{\text{eigen}(T) \cdot (T^H)}, \qquad 6.22$$

where T^H is the complex conjugate of T.

The figure 6-4 shows how the system has a fairly good crossover region and sensor noise response, while the performances are good for σ_1, σ_2 (correspondant to θ, φ) but not completely satisfactory for σ_3 (r).

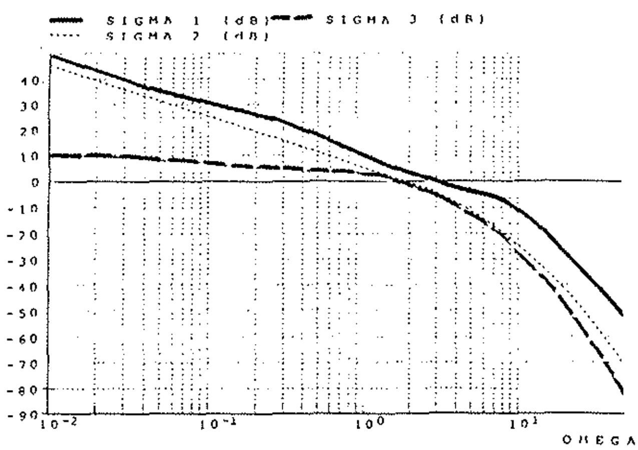


Fig.6-4 Singular Values

8-states Model Investigation

A reduced model has been developed in order to synthesize a simplified control matrices quadruple. The truncation to the classical 8 states (pitch angle, roll angle, 3 attitude rates and 3 translational velocities) implies to neglect the dynamics of the power actuators as well as the rotor. The relevant differential equations have been reduced to algebraic equations, then resolved with respect to the variables representing the states to be truncated and finally these variables have been substituted with their algebraic expressions.

The control synthesis performed on this model leads to a 11 states controller (8 states plus the 3 integrators) which apperars to be successful when tested on the linear model, but reveals its serious limits when tested within the TSIM non-linear model.

As a consequence, it is assumed that an effective controller must be designed from a linearized system that takes into account the *fundamental rotor dynamics*.

7. RESULTS

The Non-Linear Simulation

The control systems developed have been implemented into the non-linear dynamic simulation environment TSIM. A set of test cases have been performed and analyzed in order to check the control system performances when applied to the non-linear helicopter model. The robustness of the controller when used in flight conditions far from the original design condition was also a critical task of this investigation.

Pseudo-Diagonal Compensator

Case 1: A two degree step in pitch at 80 knots forward velocity (Fig. 7-1) shows good control in all three axes (θ, ϕ, and ψ) and all three angular body rates (PD, QD, and RD). The small transient in ϕ and ψ show that while the de-coupling between the axes is quite good, it is not perfect. Since control in yaw is performed on rate and not position, the residual yaw angle is not zeroed but rather maintained. This angle also causes a slight offset in roll. These angular offsets cause the helicopter translational velocities u, v, and w to wander.

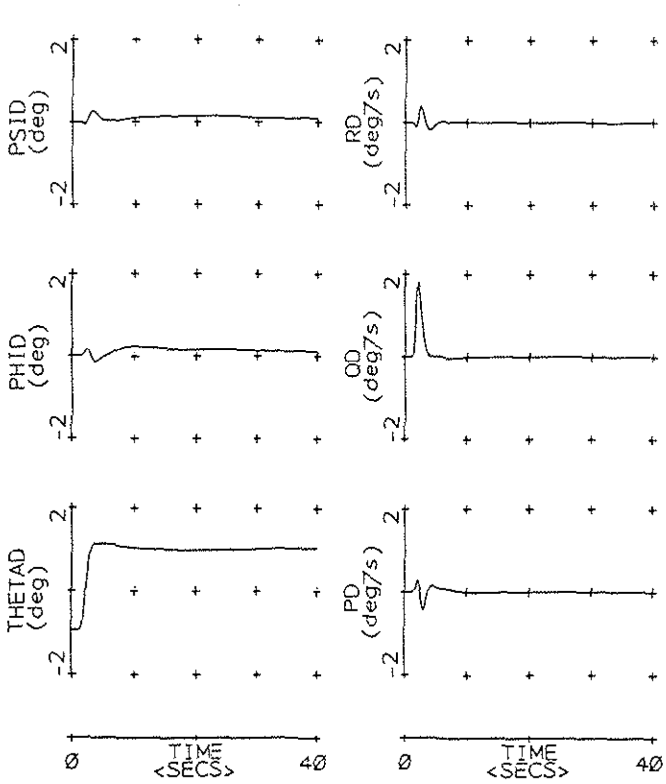


Fig. 7-1. Case 1: 80 kts, step in pitch

Case 2 : A two degree step in roll at 80 knots forward velocity (Fig. 7-2) shows good control in all three body axes and all three angular rates (PD, QD, and RD). The lateral translational velocity V shows a slight sideslip due to the new roll angle.

The time responses to step inputs at 40 and 146 knots forward velocity were also performed, and although the performance was inferior to that at 80 knots, control was generally good in position and very good in rate.

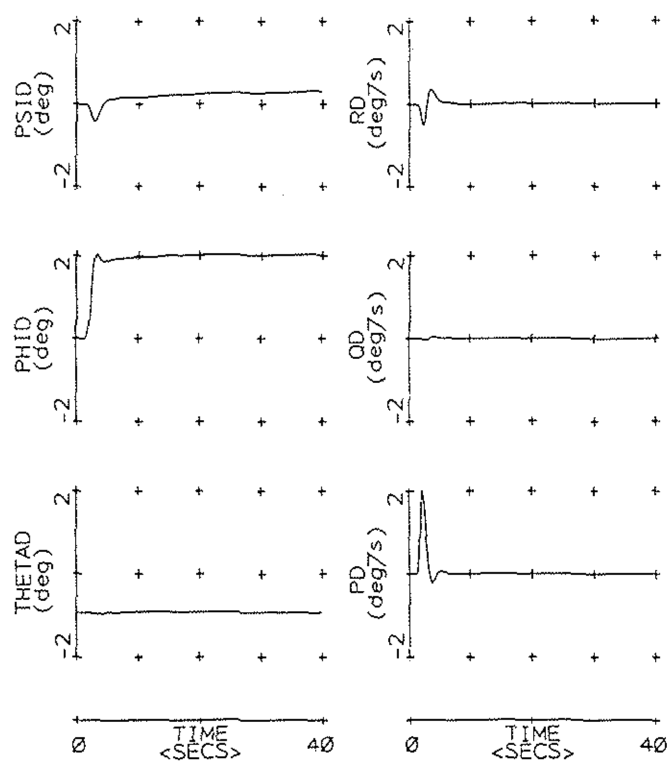


Fig. 7-2. Case 2: 80 kts, step in roll

Optimal Controller

Case 3: The time-response of the helicopter controlled by the full state dynamic controller designed using CSDS has been tested at first by entering a square wave ("pulse") input from the cyclic stick, both in the lateral and in fore/aft axes. The square wave started at $t = 1$ second, had an amplitude of the 10% of the control authority and a duration of one second positive plus one second negative. After a five-seconds transient, the control system keeps the helicoper within the desired bounds about the initial trimmed equilibrium position.

Case 4: A roll command of $\varphi = 2$ degrees has been entered as a reference to the integrator as described above. The helicopter response has been verified along a 40-second time-response, in three different forward flight trim conditions of, respectively, 80 knots, 146 knots and 40 knots. The reference is a $+2$ step, smoothed with a sine-type transient. It is evident how the response is quite satisfactory, with limited overshoots and fast transients in all the evaluated flight conditions. Since ψ has no integral control, the response of this state to the step command is not led to zero, but shows a slow transient

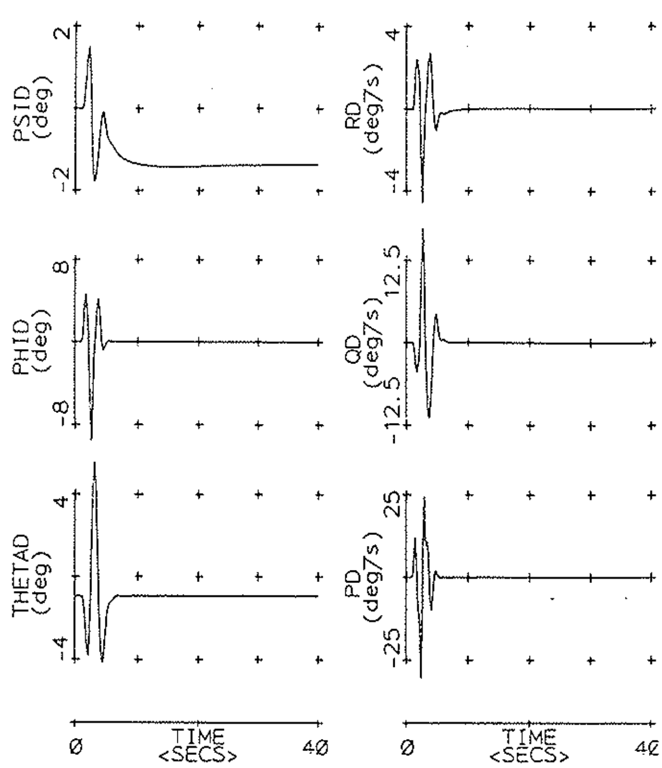


Fig.7-3 Case 3: 80 kts, Pulse Disurbance, Optimal controller

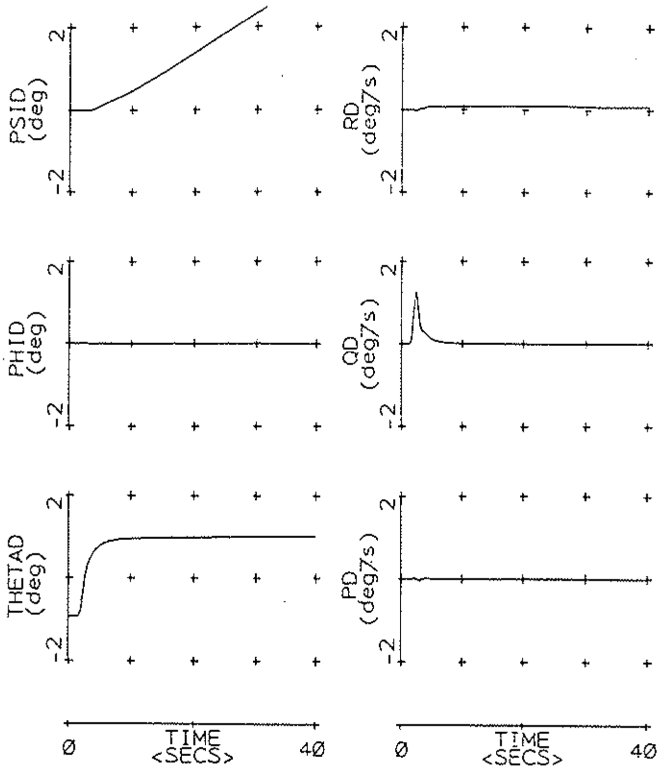


Fig.7-5. Case 5: 80 kts, step in pitch, Optimal Controller

to a new equilibrium position. The ψ response is mainly driven by the control acting on the yaw rate r (RD_).

Case 5: A pitch command of $\theta = 2$ degrees has been entered as a refernce to the integrator. The helicopter response has been verified along a 40-seconds time-response, in three different forward flight trim conditions of, respectively, 80 knots, 146 knots and 40 knots. The reference is a $+2$ step, smoothed with a sine-type transient. It is evident how the response in θ and φ is again fast with limited overshoots in all the evaluated flight conditions, and the yaw rate response is maintained within tight bounds, also if it converges slowly to the required zero value.

Cases 6 & 7: The responses of the system with a traditionally-designed AFCS to step commands in pitch and roll are also included for comparison.

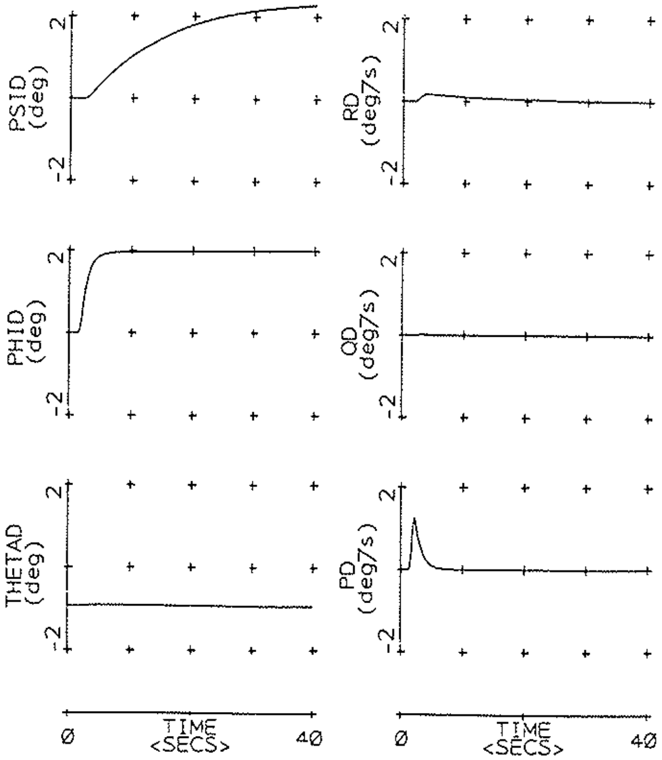


Fig.7-4 Case 4: 80 kts, step in roll, Optimal controller

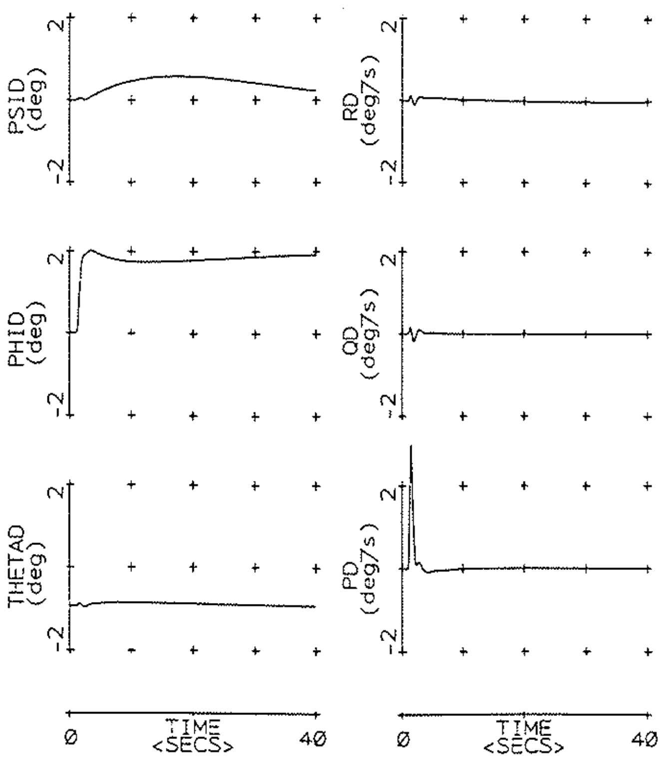


Fig.7-6. Case 6: 80 kts, step in roll, basic ASE

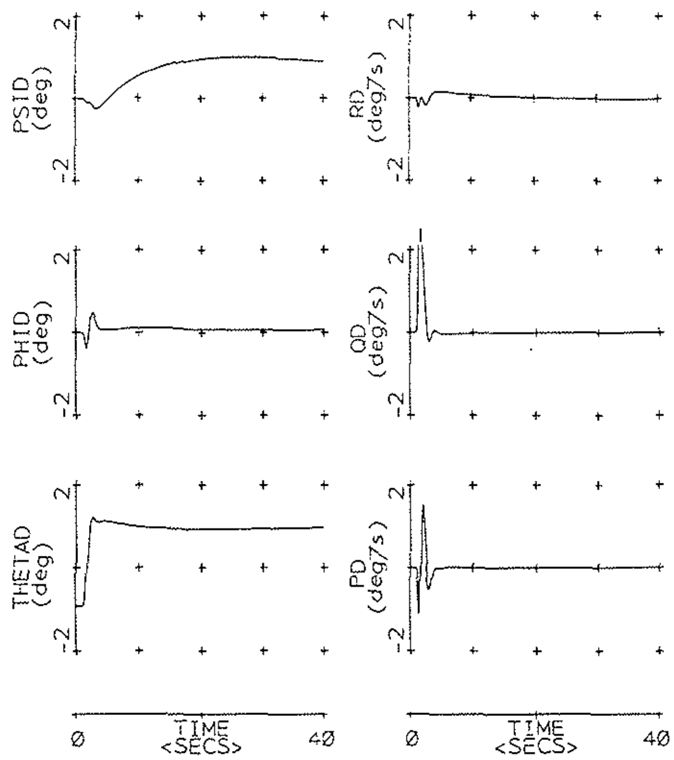


Fig.7-7. Case 7, 80 kts, step in roll, basic ASE

8. CONCLUDING REMARKS

The objectives of the work described in this paper, as stated in the introduction, were to investigate the applicability of modern multivariable techniques in the design of advanced flight control systems and to evaluate the potentialities of these methods by comparing their performances with more conventional AFCS designs. Of the two approaches that were analyzed, i.e., frequency domain and time domain, the former was expected to show a higher potential than we could in fact experiment. Due to the great complexity of the rotorcraft dynamics, both the Characteristic Loci method and Nyquist Array method showed an inherent difficulty in obtaining the design target.

In spite of the fact that, all the transmission zeros had negative real part, and therefore no intrinsic difficulty was forseen in the design, many iterations were instead required to obtain acceptable results with the DNA method; moreover, the C.L. method was abandoned after a few iterations because of the limited available development time. The state space, i.e., time domain approach, on the other hand resulted in a much smoother design sequence, and in the end gave better results in terms of development time and control system performance, as can be easily seen from the simulation results presented in the previous section.

Having made this remark, we can state, however that both design methods produced substantial improvement over the traditional AFCS design as can be observed by comparison with the time responses of the same helicopter model with a typical state of the art flight control system (see Figures 7-6 and 7-7).

It is the authors' belief that, taken to their limits, all multivariable design methods can deliver comparable results, and that the differences arise entirely in the detailed execution of the design.

In evaluating the apparent superiority of the LQG design one should in fact consider that, while the output regulation target was the same as the DNA design (i.e., ACAH response with regulated θ, φ and r), the number of measured variables was much higher, as it included the entire set of angles, rates and velocities. A detailed analysis then would be necessary to evaluate, for instance, the behavior in case of sensor failures. This was not performed in the present study, although it is expected that a different controller would be necessary for each different failure type.

From this point of view, the DNA method yields an inherently superior design, as the system de-coupling and the single loop design guarantees that in case of sensor failure, stability and performance will be retained for the non-failed loops.

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