TIME DOMAIN IDENTIFICATION METHOD WITH IMPROVED ROBUSTNESS FOR ROTORCRAFT FLIGHT DYNAMICS MODELING

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Abstract
A time domain identification method based on set-membership theory for rotorcraft flight dynamics modeling is developed in this paper. In order to solve the difficulties in applying standard set-membership identification algorithm to identify the flight dynamics model of a rotorcraft due to its inherent complexity, an improved Optimal Bounding Ellipsoid (OBE) method is established. The basic OBE algorithm for multi-input multi-output system is derived firstly, and then an indirect OBE algorithm for state space model is established. The concept of generalized noise is introduced and the noise bound optimization is implemented. Finally, the established identification method is used to identify the flight dynamics models of a helicopter and a quad-rotor aircraft separately. The results show that the method developed in this paper is able to identify the flight dynamics model of a rotorcraft with high accuracy as well as robustness.

1. INTRODUCTION

Identification technology has become a very powerful tool in rotorcraft flight dynamics modeling. It can provide models with higher confidence than theoretical modeling technique. The principle of system identification is to obtain a model or a set of models from experimental data based on the parameter estimation theory. The accuracy of an identified model is determined by many factors such as the quality of the experiment data, selection of proper model structure and suitable identification strategy. The first attempt to apply system identification technique to the rotorcraft flight dynamics modeling was developed more than 50 years ago [1]. A series of research works in this area have been carried out recently. Most of these works were concentrated on linear flight dynamics model identification. For many applications such as handing quality assessment and flight control system design, a linear flight dynamics model is good enough. For linear flight dynamics model identification of a rotorcraft, the unknown parameters to be identified are aerodynamic stability derivatives and control derivatives which determine the static and dynamic characteristics of the rotorcraft near certain trim condition. Since the rotorcraft is heavily coupled, at least 6 degrees of freedom (DOFs) model are required which results in large numbers of parameters to be identified. The identification of rotorcraft flight dynamics model is a difficult problem, especially for high order models. There are different techniques both in time domain and frequency domain that solve the problems of identifying rotorcraft flight dynamics model well. These methods are called conventional identification methods in this paper. The conventional identification methods [2-9] are mostly based on the least square and the maximum likelihood estimation. These estimators usually require the information of mean value and covariance of the noise or error vectors in the model, but it is difficult and sometimes even impossible to obtain such information during the flight test. In order to solve this problem, the conventional methods either assume ideal random noise or no noise. This is absolutely not true in the real flight test. Therefore the accuracy and robustness of identification are reduced.

In recent years, there are some research works that concentrate on applying new methods to aircraft (including rotorcraft) flight dynamics model identification such as artificial neural network modeling [10] and subspace identification [11]. The new methods are quite different from the conventional ones, and these methods have many benefits that overcome some of the difficulties in the conventional methods. The neural network modeling does not need a conventional state space model structure, so the conventional identification procedure is avoided. The subspace identification does not require the detailed noise

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information, and it has already been shown that this kind of method will increase the identification accuracy when the test data contains high level of noise. However, the newer methods have deficiencies too. The parameters in neural network models lacks of physical meaning, so it is not convenient to analyze the model. Some research [11] demonstrates the disadvantages of using subspace identification alone when applying it to identify rotorcraft flight dynamics model, but improves when combined with conventional methods. System identification methods have had significant developments during the past years, as new identification theory becomes more and more mature. These include one powerful tool - the set-membership identification.

The theory of set-membership identification was first proposed by Fogel [12] in 1979 and further developed in [13-22]. The set-membership identification theory is based on the Unknown-But-Bounded (UBB) noise assumption, where the only knowledge required about the noise is bound. No ideal noise assumption is needed. Even the data contains colored noise, and this will not affect the identification result. Therefore, this kind of identification method is especially beneficial for the rotorcraft flight dynamics modeling since the rotorcraft flight test data is usually contaminated considerably by measurement noise.

There are plenty of papers that summarize the set-membership identification method, but most of these papers are concentrated on theoretical algorithm development. Research on application of set-membership identification to complex systems such as aircraft flight dynamics modeling are very few, and it has been shown that it is not easy to use this method to decouple fixed wing aircraft identification [16]. So it will be difficult to apply this method to rotorcraft flight dynamics modeling. The aim of this paper is to develop a comprehensive method based on set-membership identification that can be applied to rotorcraft flight dynamics modeling. The standard set-membership identification methods can only be applied to those models which have explicit relationships between model outputs and parameters to be identified. The rotorcraft flight dynamics model is represented as state space differential equation that does not meet the requirement of set-membership identification. In order to solve this problem, the first part of this paper develops an indirect identification algorithm. Then a comprehensive set-membership identification method with two steps is established for the identification of rotorcraft flight dynamics model. The first step is transformed the rotorcraft flight dynamics model into linear algebraic equation form to satisfy the requirement of set-membership identification. An Optimal Boundary Ellipsoid (OBE) algorithm is used to identify the model directly. Secondly, indirect identification methods are applied to the state space equation form flight dynamics model to refine the results by setting previous identified parameters as initial values. Moreover, in both identification steps, the concept of generalized noise is introduced, and the noise bound is determined by an optimization algorithm. Finally, the method is applied to identify the flight dynamics models of a light helicopter and a quad-rotor aircraft in hover condition.

2. SET-MEMBERSHIP IDENTIFICATION METHOD

Set-membership identification is used to identify the following affine-in-parameters model which has the form:

(1) $$\boldsymbol{y} = \boldsymbol{\theta}^{\mathrm{T}} \bullet \boldsymbol{x} + \boldsymbol{\varepsilon}$$

Where $\boldsymbol{y}$ is the measured $m \times 1$ vector, $\boldsymbol{x}$ is the measured $n \times 1$ vector, $\boldsymbol{\theta}$ is the $n \times m$ matrix of parameters to be identified, $\boldsymbol{\varepsilon}$ is the $m \times 1$ bounded noise vector. The bounded noise vector can be described as:

(2) $$|\boldsymbol{\varepsilon}| \le \boldsymbol{\sigma}$$

Based on Eq. (1) and inequality (2), two hyperplanes at each sample time t can be obtained:

$$\boldsymbol{H}_t^+ = \left\{ \boldsymbol{\theta} \,\middle|\, \boldsymbol{y}_t = \boldsymbol{\theta}^{\mathrm{T}} \bullet \boldsymbol{x}_t + \boldsymbol{\sigma} \right\}$$
$$\boldsymbol{H}_t^- = \left\{ \boldsymbol{\theta} \,\middle|\, \boldsymbol{y}_t = \boldsymbol{\theta}^{\mathrm{T}} \bullet \boldsymbol{x}_t - \boldsymbol{\sigma} \right\}$$

The true values of parameters must be laid between these hyperplanes. The intersection of these hyperplanes over time forms a sequence of polytopes. The task is to use a mathematic function to describe these polytopes which is the feasible solution set (FSS) of parameters to be identified. It is usually not possible to give accurate description of these polytopes, and approximate representation is always used to solve this problem. The most frequently used method is the optimal bound ellipsoid (OBE) method, which attempts, at each t, to use a bound ellipsoid to tightly bound the exact polytopes in some sense. The principle of OBE method can be illustrated geometrically as shown in Fig. 1.

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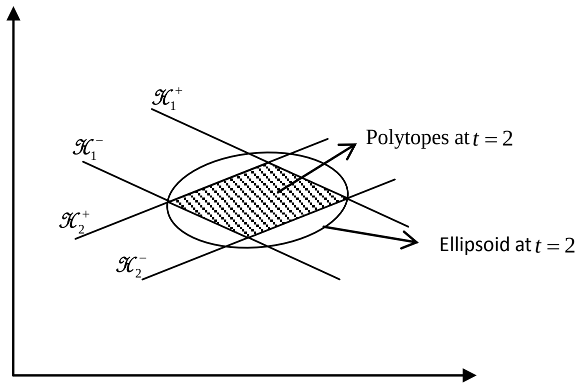

Fig. 1 Geometric Illustration of OBE Method

The bound ellipsoid at each sample time t can be described as:

(3) $\Theta_t = \left\{ \boldsymbol{\theta} \middle| \left(\boldsymbol{\theta}-\boldsymbol{\theta}_t^c\right)^{\mathrm{T}} \bullet \boldsymbol{P}_t^{-1} / \kappa_t \bullet \left(\boldsymbol{\theta}-\boldsymbol{\theta}_t^c\right) \le 1 \right\}$

Where $\boldsymbol{\theta}_t^c$ is the center of bound ellipsoid at time t, $\boldsymbol{P}_t$ is the covariance matrix at time t and κ_t is a weighting coefficient at time t.

2.1. OBE Algorithm for Multi Input Multi Output System

Ref. 17 gives a standard OBE algorithm for a Multi Input Single Output (MISO) system. However, the helicopter is a Multi Input Multi Output (MIMO) system. An extended OBE algorithm for MIMO system can be derived as follows.

First, the FSS of Eq. (1) can be obtained easily by substituting Inequality (2) into Eq. (1):

(4) $\Theta = \left\{ \boldsymbol{\theta} : \sum_{i=1}^{t} \lambda_i \left\| \boldsymbol{y}_i - \boldsymbol{\theta}^T \bullet \boldsymbol{x}_i \right\|_2^2 \le \sum_{i=1}^{t} \lambda_i \bullet tr(\boldsymbol{\sigma}_i \boldsymbol{\sigma}_i^T) \right\}$

Where λ_t is a weighting coefficient which can be used to define the shape of the ellipsoid, $\|\bullet\|_2$ is the vector norm operator, and tr(·) is the trace of a matrix.

The left side of above solution set is expanded and after some mathematical manipulation, a standard bound ellipsoid equation can be obtained as shown in Eq. (5).

(5) $\Theta_t = \left\{ \boldsymbol{\theta} \middle| tr\left[\left(\boldsymbol{\theta}-\boldsymbol{\theta}_t^c\right)^T \bullet \boldsymbol{P}_t^{-1} / \kappa_t \bullet \left(\boldsymbol{\theta}-\boldsymbol{\theta}_t^c\right) \right] \le 1 \right\}$

Where,

(6) $\boldsymbol{\theta}_t^c = \boldsymbol{P}_t \sum_{i=1}^{t} \lambda_i \boldsymbol{x}_i \boldsymbol{y}_i^T$

(7) $\boldsymbol{P}_t^{-1} = \sum_{i=1}^{t} \lambda_i \boldsymbol{x}_i \boldsymbol{x}_i^T$

(8) $\kappa_t = \sum_{i=1}^{t} \lambda_i \bullet tr(\boldsymbol{\sigma}_i \boldsymbol{\sigma}_i^T) - \sum_{i=1}^{t} \lambda_i \bullet tr(\boldsymbol{y}_i \boldsymbol{y}_i^T) + tr(\boldsymbol{\theta}_t^{cT} \bullet \boldsymbol{P}_t^{-1} \bullet \boldsymbol{\theta}_t^c)$

It is more convenient to rewrite Eq. (6), Eq. (7), and Eq. (8) as iterative form shown below for programming:

(9) $$\begin{cases} \boldsymbol{\theta}_t^c = \boldsymbol{\theta}_{t-1}^c + \lambda_t \boldsymbol{P}_t \boldsymbol{x}_t \boldsymbol{e}_t^T \\ \boldsymbol{P}_t = \boldsymbol{P}_{t-1} - \dfrac{\lambda_t \boldsymbol{P}_{t-1} \boldsymbol{x}_t \boldsymbol{x}_t^T \boldsymbol{P}_{t-1}}{1+\lambda_t \boldsymbol{x}_t^T \boldsymbol{P}_{t-1} \boldsymbol{x}_t} \\ \kappa_t = \kappa_{t-1} + \lambda_t tr(\boldsymbol{\sigma}_t \boldsymbol{\sigma}_t^T) - \dfrac{\lambda_t \bullet tr(\boldsymbol{e}_t \boldsymbol{e}_t^T)}{1+\lambda_t \boldsymbol{x}_t^T \boldsymbol{P}_{t-1} \boldsymbol{x}_t} \end{cases}$$

Where, $\boldsymbol{e}_t = \boldsymbol{y}_t - \boldsymbol{\theta}_{t-1}^{cT} \bullet \boldsymbol{x}_t$ is the model prediction error at time t.

Finally, the bound ellipsoid is optimized by solving the weighting coefficient λ_t. An optimized ellipsoid must be the smallest figure that bound the polytopes well. There are different criteria for optimization, and one commonly used criterion is the volume of the ellipsoid. Ref. 17 gives the definition of ellipsoid volume and for our case it can be defined as Eq. (10). Where, det(·) is the determinant of a matrix.

(10) $\mu_{vol} = \det(\boldsymbol{P}_t \bullet \kappa_t)$

Substituting Eq. (9) into Eq. (10) and minimizing the volume of the ellipsoid by letting $\partial \mu_{vol} / \partial \lambda_t = 0$, one can obtain the optimized weighting coefficient λ_t as the largest positive root of the following 2nd order algebraic equation if such exists. If there is no positive root, the data at time t did not provide useful information and no updating of the parameter vector θ_t^c is required. When this situation occurs, it indicates the two hyperplanes at this data point do not intersect with the current bound ellipsoid.

(11) $a_2 \lambda_t^2 + a_1 \lambda_t + a_0 = 0$

Where,

(12) $a_2 = (n-1) \bullet tr(\boldsymbol{\sigma}_t \boldsymbol{\sigma}_t^T) \bullet (\boldsymbol{x}_t^T \boldsymbol{P}_{t-1} \boldsymbol{x}_t)^2$

(13) $a_1 = \left[(2n-1) \bullet tr(\sigma_t \sigma_t^T) + tr(\boldsymbol{e}_t \boldsymbol{e}_t^T) - \kappa_{t-1} x_t^T \boldsymbol{P}_{t-1} x_t \right] \bullet x_t^T \boldsymbol{P}_{t-1} x_t$

(14) $a_0 = n\left[tr(\boldsymbol{\sigma}_t \boldsymbol{\sigma}_t^T) - tr(\boldsymbol{e}_t \boldsymbol{e}_t^T) \right] - \kappa_{t-1} \boldsymbol{x}_t^T \boldsymbol{P}_{t-1} \boldsymbol{x}_t$

Combines above equations, The OBE algorithm for MIMO system now can be obtained as shown in Fig. 2.

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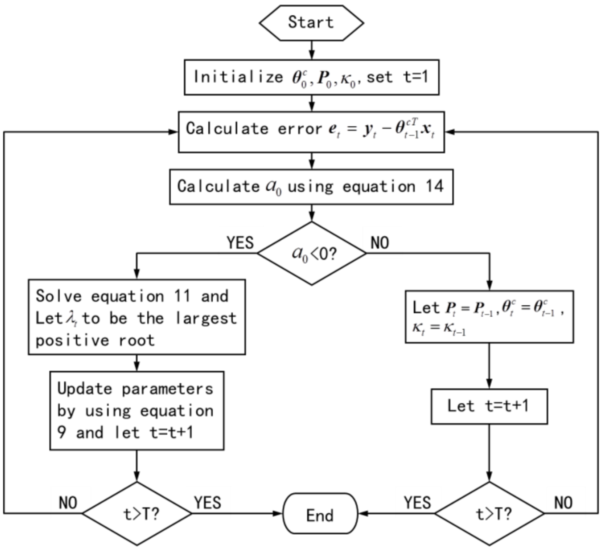

Fig. 2 OBE Algorithm for MIMO System

2.2. Indirect Identification Algorithm for State Space Model

The linear flight dynamics model of a rotorcraft is represented as the state space equation below:

(15) $\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{BU}$

Where, ***X*** is the state vector, ***U*** is the control input vector with equivalent time delays, ***A*** and ***B*** are the stability matrix and control matrix respectively, which contain the parameters to be identified.

It is difficult to find the explicit relationship between the state vector and the parameters to be identified from Eq. (15), so it is impossible to use the OBE algorithm directly to identify such model. In order to solve this problem, an indirect identification method is developed. First, add an observation equation, shown as Eq. (16).

(16) $\boldsymbol{Y} = \mathbf{C}\boldsymbol{X} + \boldsymbol{\varepsilon}$

Where, ***Y*** is the observation vector, ***C*** is observation matrix, ***X*** is state vector in Eq. (15) and $\boldsymbol{\varepsilon}$ is the bounded noise vector.

Since ***X*** is the function of all parameters to be identified, assume the parameter vector $\boldsymbol{\theta}$, and let $\boldsymbol{\theta}^c$ be the estimation of $\boldsymbol{\theta}$. Then at $\boldsymbol{\theta}^c$, ***X*** can be expanded as a first order Taylor series as below:

(17) $\boldsymbol{X}(\boldsymbol{\theta}) = \boldsymbol{X}(\boldsymbol{\theta}^c) + (\boldsymbol{\theta} - \boldsymbol{\theta}^c)^T \frac{\partial \boldsymbol{X}}{\partial \boldsymbol{\theta}}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^c}$

Then, Eq. (16) can be rewritten as Eq. (18)

(18) $\boldsymbol{Y} = \boldsymbol{CX}(\boldsymbol{\theta}^c) + \boldsymbol{C}(\boldsymbol{\theta} - \boldsymbol{\theta}^c)^T \frac{\partial \boldsymbol{X}}{\partial \boldsymbol{\theta}}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^c} + \boldsymbol{\varepsilon}$

Let $\tilde{\boldsymbol{Y}} = \boldsymbol{Y} - \boldsymbol{CX}(\boldsymbol{\theta}^c)$, $\tilde{\boldsymbol{X}} = \boldsymbol{C}\frac{\partial \boldsymbol{X}}{\partial \boldsymbol{\theta}}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^c}$, and $\Delta\boldsymbol{\theta} = \boldsymbol{\theta} - \boldsymbol{\theta}^c$. Then an indirect identification model can be obtained as Eq. (19).

(19) $\tilde{\boldsymbol{Y}} = \Delta\boldsymbol{\theta}^T \bullet \tilde{\boldsymbol{X}} + \boldsymbol{\varepsilon}$

Eq. (19) has the same structure as Eq. (1). So, one can use the above MIMO system OBE algorithm to identify $\Delta\boldsymbol{\theta}$ in this model. However, in order to do this, we still need to calculate $\tilde{\boldsymbol{X}}$ in Eq. (19), and this can be done by solving the parameter sensitivity equation as shown in Eq. (20).

(20) $\frac{d}{dt}\left(\frac{\partial \boldsymbol{X}}{\partial \boldsymbol{\theta}}\right) = \boldsymbol{A}\frac{\partial \boldsymbol{X}}{\partial \boldsymbol{\theta}} + \frac{\partial \boldsymbol{A}}{\partial \boldsymbol{\theta}}\boldsymbol{X} + \frac{\partial \boldsymbol{B}}{\partial \boldsymbol{\theta}}\boldsymbol{U}$

Based on above analysis, the identification of Eq. (15) can be done indirectly by using following algorithm:

1) Set initial value of $\Delta\boldsymbol{\theta}_0$, $\boldsymbol{P}_0$, κ_0 and $\boldsymbol{\theta}_t^c$, let $t=1$;

2) Solve Eq. (15) using Gill algorithm to get $\boldsymbol{X}(\boldsymbol{\theta}_t^c)$, then $\tilde{\boldsymbol{Y}}$ is obtained;

3) Solve Eq. (20) to get $\frac{\partial \boldsymbol{X}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_t^c}$, then $\tilde{\boldsymbol{X}}$ is obtained;

4) Use MIMO system OBE algorithm described in Fig. 5 to identify Eq. (19), then the estimation of $\Delta\boldsymbol{\theta}$ is obtained;

5) Let $\boldsymbol{\theta}_{t+1}^c = \boldsymbol{\theta}_t^c + \Delta\boldsymbol{\theta}$ and t=t+1, go to 2)

3. APPLICATIONS TO HELICOPTERS AND QUAD-ROTOR AIRCRAFT IN HOVER

The rotorcraft flight dynamics model contains large numbers of parameters to be identified and these parameters have different sensitivity to model responses, so it is not easy to identify these parameters. In order to solve this problem, a two-step identification method is developed.

In the first step, let all accelerations be the model output. Put all other model responses as well as control inputs into the model input vector. Then a simple parameterized model can be obtained as Eq. (21).

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(21) $\boldsymbol{y}_m = \boldsymbol{\theta}^T \bullet \boldsymbol{x} + \boldsymbol{\varepsilon}$

Since the rotorcraft is unstable in hover condition, the pilot will need make compensations during the flight test in order to keep the rotorcraft stable. This indicates that in the test data, the compensation controls and correlative responses are not negligible. The data used in the identification should contain measurements from all control channels. This brings two problems. First, the responses data in main excitation channel have a large signal to noise ratio while usually have a low signal to noise ratio in other channels. This will decrease the identification accuracy of relevant parameters in these channels. Second, a fully coupled model is required in the identification which means a large amount of parameters need to be identified. Moreover, these parameters have large differences in sensitivity to the model responses which mean it is difficult to identify all these parameters at the same time.

In order to solve these problems, conventional identification methods usually introduce a model structure identification procedure before final parameter estimation. The purpose of model structure identification is to eliminate all insensitive parameters that are difficult to identify. So after model structure identification, an optimized model structure can be obtained and it is not difficult to identify the remaining parameters with high accuracy. It is also very convenient to use the model structure identification technique in set-membership identification method, but this procedure is quite time consuming especially in time domain. Considering the noise bound determines the size of the ellipsoid, and the size of the bound ellipsoid determines the size of FSS, which affects the accuracy of point estimation of parameters. Based on this, a more efficient technique is developed in this paper to solve the problems mentioned above.

First, the generalized noise is defined in this paper. The generalized noise consists of physical noise and non-physical noise. The physical noise is the measurement noise in the test data, while the non-physical noise contains several individual components. Typical non-physical noise includes the random measurement errors of on board sensors, the model approximation error, and the errors due to numerical calculations etc.

Second, the noise bound has large influences on identification accuracy in OBE algorithm. Therefore, the value of noise bound should be set properly. In previous research, the noise bound was determined upon engineering experiences. In this paper, an optimization algorithm is established to improve the noise bound.

To this end, a cost function for noise bound optimization is defined as Eq. (22)

(22) $$J(\sigma_i) = \sum_{t=t_0}^{T} \left((\boldsymbol{e}_t^i)^T \boldsymbol{e}_t^i + \frac{1}{S_t^i} \right) \quad i = 1, 2, \cdots, m$$

Where,

$\boldsymbol{e}_t^i = \boldsymbol{y}_t^i - \boldsymbol{\theta}^{cT} \bullet \boldsymbol{x}_t^i$, is the model prediction error.

$\boldsymbol{S}_t^i = \frac{1}{n} \sum_{j=1}^{n} \frac{\partial \boldsymbol{x}_t^i / \partial \boldsymbol{\theta}_j}{\boldsymbol{\theta}_j}, \quad i = 1, 2, \cdots m$, is the parameter sensitivity function.

Minimize cost function *J* in Eq. (22) will find the noise bound for minimum model prediction error and maximum parameter sensitivity.

3.1. Identification results of helicopters

Flight test data of a light helicopter is used for model identification in hover condition. The flight test was conducted twice by using longitudinal excitation input and lateral excitation input separately. Standard data processing procedures include low pass filtering, wild value elimination, data compatibility examination and data reconstruction etc. are carried out before identification.

The optimized noise bound is determined based on preceding method, and the results can be found in Table 1. The bound values in the table are relative to the maximum variations of each state variable.

Table 1 optimized noise bound of the test helicopter

Generalized noise	Longitudinal excitation	Lateral excitation
ε_u	1.5%	5.3%
ε_v	6.1%	0.9%
ε_w	19.5%	14.6%
ε_p	5.8%	1.2%
ε_q	0.9%	6.1%
ε_r	12.2%	8.5%
ε_ϕ	6.1%	1.1%
ε_θ	0.8%	6.5%

The flight dynamics model identification results can be found in Table 2.

Table 2 Identification results for the test helicopter

DER*	Value	STD**	DER	Value	STD	DER	Value	STD
X_u	-0.0321	0.00014	Z_u	0.1241	0.002	M_u	0.0010	0.00014
X_v	-0.0329	0.00011	Z_v	-0.0519	0.0016	M_v	0.0811	0.00013
X_w	0.0147	0.00049	Z_w	-0.4833	0.0055	M_w	-0.0152	0.00067
X_p	1.1137	0.3094	Z_p	0.0854	3.6143	M_p	-0.7744	0.1637
X_q	1.2900	0.1296	Z_q	0.8446	1.475	M_q	-2.9862	0.0978
Xr	-0.3737	0.0299	Z_r	2.4377	0.3882	M_r	0.6008	0.0386
$X_{\delta lon}$	-0.0543	0.00048	$Z_{\delta lon}$	-0.0453	0.0572	$M_{\delta lon}$	0.0593	0.0038
$X_{\delta lat}$	0.0032	0.00031	$Z_{\delta lat}$	0.0029	0.0338	$M_{\delta lat}$	-0.0161	0.0015
$X_{\delta col}$	-0.0069	0.00010	$Z_{\delta col}$	-0.0151	0.0149	$M_{\delta col}$	0.0267	0.0011
$X_{\delta ped}$	0.0264	0.00056	$Z_{\delta ped}$	-0.2042	0.0642	$M_{\delta ped}$	-0.0237	0.0057
Y_u	0.0201	0.00027	L_u	-0.0949	0.00055	N_u	0.0127	0.003
Y_v	-0.0901	0.00017	L_v	-0.1029	0.0006	N_v	-0.0066	0.025
Y_w	-0.0139	0.00068	L_w	0.0848	0.003	N_w	-0.0253	0.061
Y_p	-1.2841	0.7061	L_p	-4.9555	0.4914	N_p	0.6156	0.9211
Y_q	0.1184	0.2475	L_q	1.8998	0.3860	N_q	0.9480	0.3116
Y_r	-0.1942	0.0439	L_r	0.4913	0.1678	N_r	-0.1239	0.419
$Y_{\delta lon}$	-0.0136	0.0011	$L_{\delta lon}$	-0.0094	0.0013	$N_{\delta lon}$	-0.0336	0.0015
$Y_{\delta lat}$	0.0028	0.00076	$L_{\delta lat}$	0.0773	0.0041	$N_{\delta lat}$	0.0092	0.0011
$Y_{\delta col}$	-0.0053	0.00017	$L_{\delta col}$	-0.0285	0.0041	$N_{\delta col}$	0.0082	0.0027
$Y_{\delta ped}$	0.0199	0.0009	$L_{\delta ped}$	-0.0300	0.0014	$N_{\delta ped}$	0.0094	0.0091
τ_{long}	0.1	-	τ_{lat}	0.06	-	τ_{col}	0.04	-
τ_{ped}	0.04	-						

* DER = Derivative
** STD = Standard Deviation

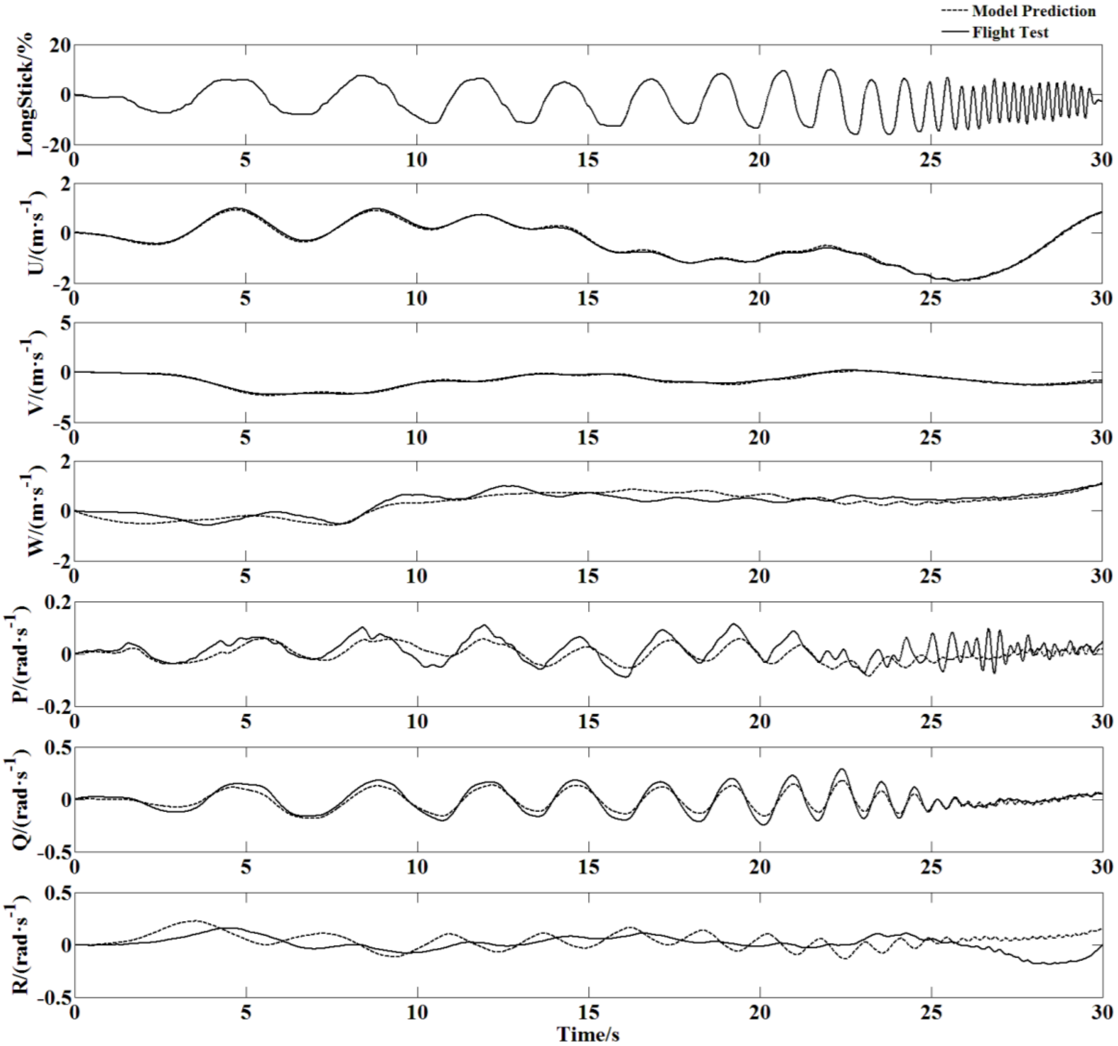

Fig. 3 Comparison of responses between model prediction and flight test in longitudinal excitation

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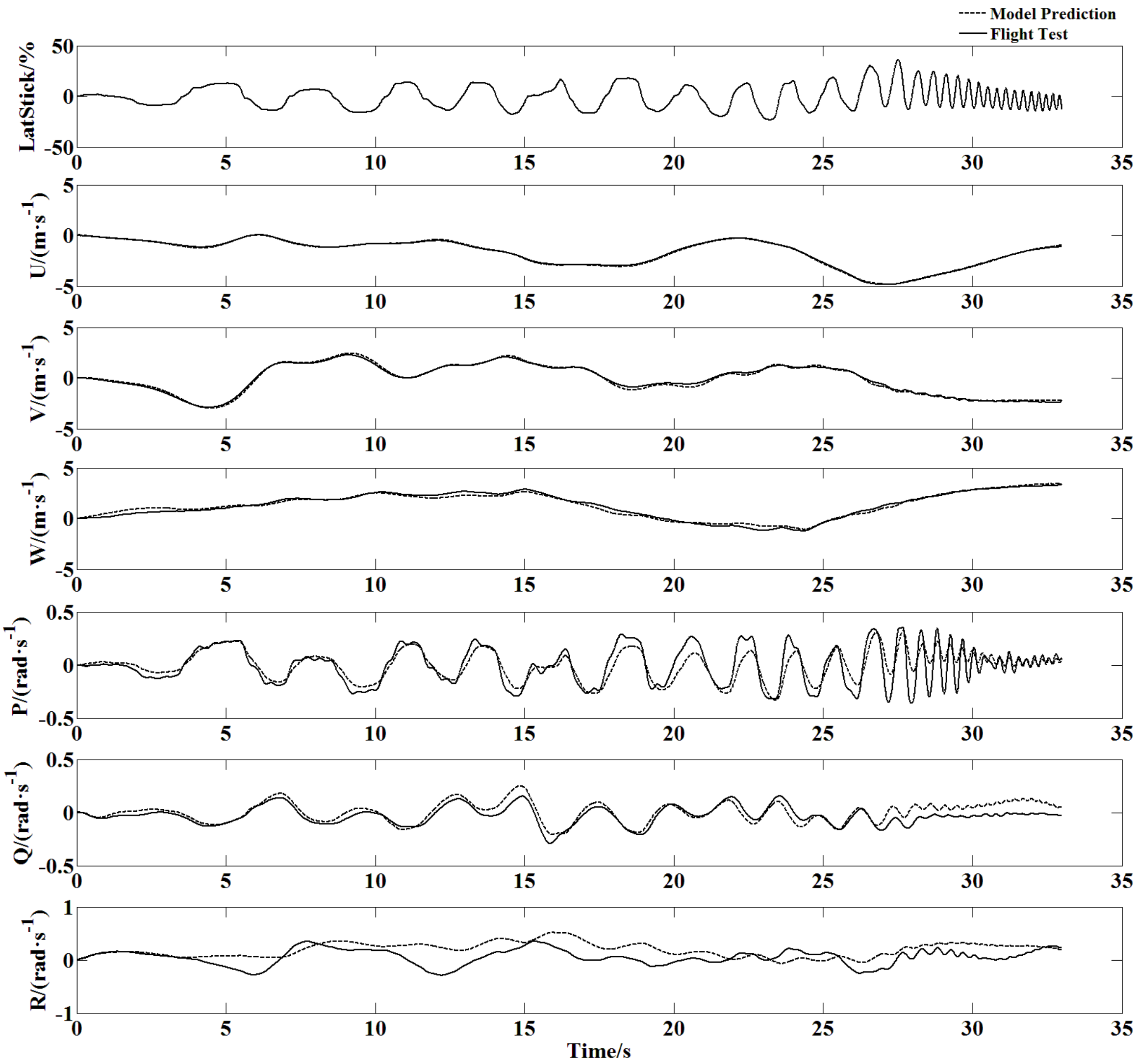

Fig. 4 Comparison of responses between model prediction and flight test in lateral excitation

It can be found in Table 2, the identification accuracy is very good for most of the parameters. It also should be noticed that, although in vertical channel, the test data are contaminated heavily by measurement noise (this can also be proven in Table 1, the noise bound in vertical channel is much larger than others), the vertical derivatives still have good estimation results. This indicates the robustness to noise of the established identification method is very good too.

The identified model is verified by additional test data which are not used in identification progress. The results are shown in Fig. 3 and Fig. 4. Still, good agreement between the model prediction and flight test data can be found.

3.2. Identification results of a quad-rotor aircraft

The developed method is also used to identify the flight dynamics model of a model-scaled quad-rotor aircraft as shown in Fig. 5. The takeoff weight of the quad-rotor aircraft is about 3 kg. The flight test of this aircraft was also conducted in hover condition as shown in Fig. 6.

The generalized noise bound for the quad-rotor aircraft can be found in Table 3. It is obviously that the average noise bound of the quad-rotor aircraft is less than the helicopters.

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Fig. 5 The quad-rotor aircraft for identification

Fig. 6 Flight test of the quad-rotor aircraft

Table 3 optimized noise bound of the test quad-rotor aircraft

Generalized noise	Longitudinal excitation	Lateral excitation
ε_u	0.6%	2.3%
ε_v	3.1%	0.7%
ε_w	6.5%	7.2%
ε_p	2.3%	0.8%
ε_q	0.7%	1.9%
ε_r	7.2%	6.1%
ε_ϕ	2.5%	0.9%
ε_θ	0.6%	1.8%

The identified flight dynamics model of the quad-rotor aircraft can be found in Eq. (23). The verification of the identified model is also performed. The result is shown in Fig. 7. It is apparently that the identification accuracy is still satisfactory.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -0.8798 & 0 & 0 & 0 & 0 & 0 & 0 & -9.8 & 0 \\ 0 & -0.8660 & 0 & 0 & 0 & 0 & 9.8 & 0 & 0 \\ 0 & 0 & -1.2994 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2672 & 0 & -3.5110 & 0 & 0 & 0 & 0 & 0 \\ 0.2679 & 0 & 0 & 0 & -3.4467 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.4987 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \\ \psi \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.02367 & 0 \\ 0 & -0.02016 & 0 & 0 \\ 0.3628 & 0 & 0 & 0 \\ 0 & 0.11068 & 0 & 0 \\ 0 & 0 & 0.11711 & 0 \\ 0 & 0 & 0 & 0.03947 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{col} \\ \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \end{bmatrix} \quad (23)$$

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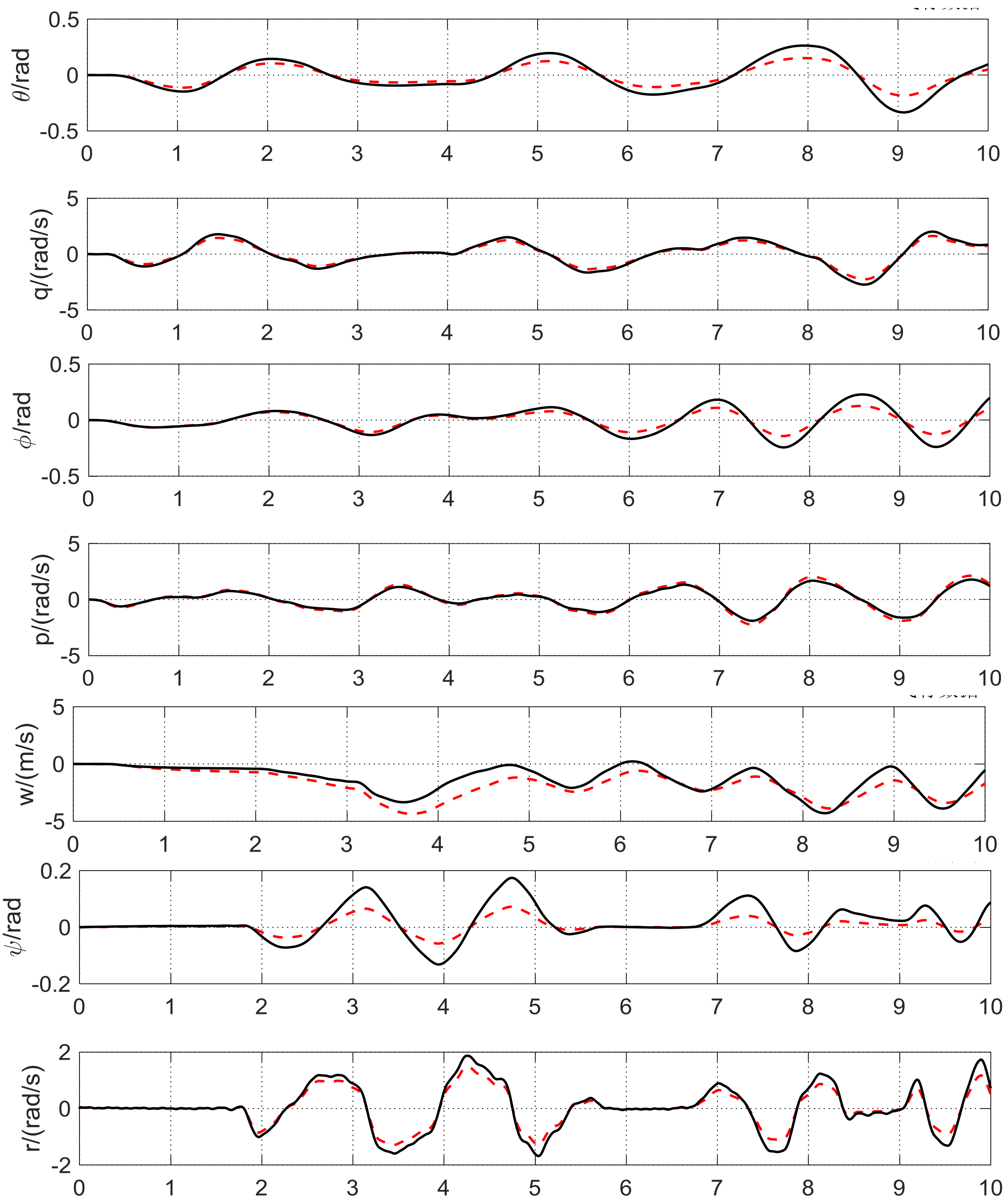

Fig. 7 Model verification of the quad-rotor aircraft

4. CONCLUSIONS

A method based on set-membership theory for rotorcraft flight dynamics model identification is developed. The new method does not require detailed information of noise and random error, but only their bounds. It is more suitable for identifying rotorcraft flight dynamics model since the test data usually contains high level measurement noise. The main work done in this paper can be summarized as follows:

- A standard OBE algorithm for MIMO system which can be used to identify linear algebraic formed models is derived.
- An indirect identification algorithm is established for systems that represented in state space equation form that cannot be identified using standard OBE algorithm directly.
- The generalized noise concept is introduced and an optimization algorithm is developed for optimal noise bound determination

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Applications of the identification method to helicopters and quad-rotor aircraft indicate the developed method in this paper has high identification accuracy as well as robustness.

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