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COMPLEX MODES IN GROUND RESONANCE STABILITY ANALYSIS

Colombo Attilio

AGUSTA S.p.A - Aeromechanics Dept.
Via G. Agusta 520, 21017 C. Costa di Samarate, Italy

Abstract

The typical approach used to evaluate ground resonance stability characteristics of the helicopter is based on the use of a modal representation of the fuselage, derived either experimentally or by means of some mathematical model. When this model includes also the effects of damping, that can arise, for example, from the inclusion of the landing-gear oleo strut, the resulting modes are, generally speaking, complex. Complex modes are also assumed by most algorithms implemented in the modal analysis packages that calculate fuselage modal characteristics from experimentally measured FRFs.

In this work, a method to deal with complex modes in a coupled rotor-fuselage stability analysis is presented. For simplicity, only the motion in the rotor plane has been considered (blade lead-lag and hub in-plane translations). This technique overcomes the limitations of presently used codes which accepts only real (or normal) modes, and gives the user a more straightforward interface with the output of the fuselage modal analysis codes (both analytical and experimental). Furthermore, the use of complex modes, which contain also the information on the phase delay spatial distribution, allows to obtain a more representative fuselage model. The stability of the resulting coupled rotor-fuselage system can be analysed both with Floquet theory or using multiblade coordinates, as appropriate depending on the specific case considered.

1 Introduction

Ground resonance analysis of the helicopter involves the evaluation of the stability characteristics of a set of equations that describe the dynamics of the coupled rotor-fuselage system. The degrees of freedom that are considered in the classical analysis [1,2,3] are the translations of the rotor hub centre in the rotor disk plane and the lead-lag motion of the blades, that can be assumed almost as a rigid motion around the hinge for articulated and soft-in plane rotors.

Even within this basic frame, the analytical derivation of the equations of motions that describe the dynamic behaviour of the fuselage supported by its undercarriage, taking into account the various geometrical and structural parameters of the configuration being analysed, can be a laborious task. Therefore, a commonly used approach is based on the use of some general purpose structural modelling tool to get the dynamic characteristics of the fuselage in terms of its eigenmodes. This is also the same kind of information that can be derived from experimental dynamic tests of the airframe on the ground, which in any case represent an important step in the development phase of the helicopter, since allows to obtain more realistic data and also to explore the possible non-linear effects which may be present [4].

Once the modes of the airframe have been obtained, either analytically or experimentally, they can be used as input data for the codes (both in-house developed [5] and CAMRAD/JA [6]) which perform the stability calculation taking into account the rotor dynamics. The assumption which is implied by both codes is that these modes are "real" (or "normal") modes, *i.e.* their eigenvectors are purely real. In this case, the motion of different points of the structure, for each mode considered, is either in-phase or out-of-phase with respect to each other.

This is not the most general case, however. In fact, "real" modes analytically exist only when the system dynamic matrices obey to some specific conditions, which may not represent the more general case, especially when the damping matrix is included in the analysis. But this is, indeed, the situation which has to be considered when the final objective is the stability analysis of the complete coupled rotor-fuselage system since the knowledge of the damping behaviour of the airframe is a fundamental issue to be addressed. Complex eigenvalue analysis is needed to evaluate the modal damping coefficients of the fuselage, which result from the inclusion in the airframe model of the damping properties of the structure, particularly with respect to the landing gear characteristics.

In the complex modes case, eigenvectors turn out to be complex too, with phase delays between the various degrees of freedom of the model which can assume, in principle, any value in the range from 0 to 2π. Nodes, *i.e.* points of the structure with null motion are replaced by the so called “galloping nodes”, and the motion of each individual point of the model, instead of being an oscillation along a line, is represented by a more or less pronounced elliptical trajectory.

The situation is similar also on the experimental side, since the software packages used to extract the eigenmodes from the measured frequency response functions (FRFs) generally adopt a complex modes modal formulation.

Therefore, using complex modes directly as input data for the stability analysis codes, without having to introduce some approximate real mode representation of the dynamics of the airframe, could be a useful possibility from the user point of view.

The procedure to include complex airframe modes in the coupled rotor-fuselage ground resonance stability analysis will be presented considering a rotor model having only rigid lead-lag motion DOF. This simplified assumption is used to retain only the basic behaviour of the system dynamics, but the method can be applied also to more general rotor models.

2 Review of real vs. complex modes

Complex eigenmodes analysis is a well know technique [7,8,9,10] and for the purposes of this work only a brief review of the theory and of the main results will be presented.

Let us consider a second order dynamic system that can described by the following set of N linear, constant coefficients, differential equations:

$$M\ddot{x} + C\dot{x} + Kx = 0 \qquad (1)$$

where x is the vector of the N degrees of freedom and M,C,K are the [NxN] simmetric mass, damping and stiffness matrices of the system. The structure of these matrices is, generally speaking, full, *i.e.* they can contain various coupling terms between different DOF.

Looking for solutions of the form $x(t) = x_o e^{j\omega t}$ to the undamped form of the system:

$$M\ddot{x} + Kx = 0 \qquad (2)$$

leads to the following eigenvalue problem:

$$\left[M\omega^2 + K\right]x_o = 0 \qquad (3)$$

from which the eigenvalues ω and corresponding eigenvectors x_o of the system can be found. Considering the modal matrix Φ of the eigenvectors, it can be shown that this matrix has the following orthonormalization properties with respect to the system mass and stiffness matrices:

$$\Phi^T M\Phi = M_m \quad \text{and} \quad \Phi^T K\Phi = K_m \qquad (4)$$

where M_m and K_m are the so-called modal mass and modal stiffness diagonal matrices.

Coming back to the original system (1), it is possible to see that in the case of proportional damping, *i.e.* when the damping matrix can be expressed as a linear combination of the mass and stiffness matrix:

$$C = \alpha M + \beta K$$

the eigenvector matrix Φ can be used to diagonalize also the damping matrix

$$\Phi^T C\Phi = \alpha M_m + \beta K_m = C_m \qquad (5)$$

where the modal damping matrix C_m is again a diagonal matrix. The condition of proportional damping is not very common since it implies a very peculiar variation of modal damping with the modal frequency [7], characterised by a minimum damping ratio of $\sqrt{\alpha\beta}$ occurring at $\omega_{\min} = \sqrt{\alpha / \beta}$ rad/sec and greater damping values for lower and higher frequencies, which is not exhibited by typical actual structures.

For the most general form of system (1) the procedure to extract complex eigenvalues/eigenvectors starts with the addition of the following set of N identities:

$$M\dot{x} - M\dot{x} = 0 \qquad (6)$$

In this way, the original second order system of N equations can be transformed to the following set of 2N first order equations:

$$\begin{bmatrix} 0 & M \\ M & C \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix} \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (7)$$

Introducing the state vector (of dimension 2N):

$$\{y\} = \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix}$$

the system can be written compactly as:

$$[A]\dot{y} + [B]y = 0 \qquad (8)$$

where $[A] = \begin{bmatrix} 0 & M \\ M & C \end{bmatrix}$; $[B] = \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}$

In the presence of the damping matrix, solutions of system (8) will be of the form:

$$y(t) = y_o e^{\lambda t} \quad \text{with} \quad \lambda = \lambda_R + i\lambda_I \qquad (9)$$

where now the exponent λ has also a real part which accounts for the time decay of the solutions.
Substituting (9) into system (8) gives rise to the following eigenvalue problem:

$$\begin{bmatrix} -M^{-1}C & -M^{-1}K \\ U & 0 \end{bmatrix} \{y_o\} = \lambda \{y_o\} \qquad (10)$$

The solutions, in terms of both eigenvalues and eigenvectors, are now, in general, complex. The eigenvector matrix Ψ which now has dimension [2Nx2N] can be shown to have the following orthonormalization properties with respect to the dynamic state matrices A and B:

$$\Psi^T A \Psi = A_m \quad \text{and} \quad \Psi^T B \Psi = B_m \qquad (11)$$

A_m and B_m are diagonal complex matrices (the so-called modal_A and modal_B matrices), and can be shown to satisfy the following relation: $B_m = -\Lambda A_m$ where Λ is the diagonal matrix of the eigenvalues.
When the system dynamic matrices M,C,K are real, it follows that eigenvalues and corresponding eigenvectors appear either as complex conjugate pairs, or as purely real (distinct or not).
This is no more true when one consider the case of structural damping included as complex terms of the stiffness matrix. However, the use of structural damping in the form of complex stiffness should be carefully evaluated when analysing the system stability since it goes beyond the limits of applicability of the mathematical model which is confined to the analysis of steady-state harmonic oscillations [9] and could lead to unstable eigenvalues of the airframe itself.

3 Stability analysis using real modes

Let us consider the classical equations of motion of an articulated rotor with N_b rigid lagging blades coupled, at the rotor hub centre, to a dynamic support (fuselage) [1,2,3] (see fig. 1).

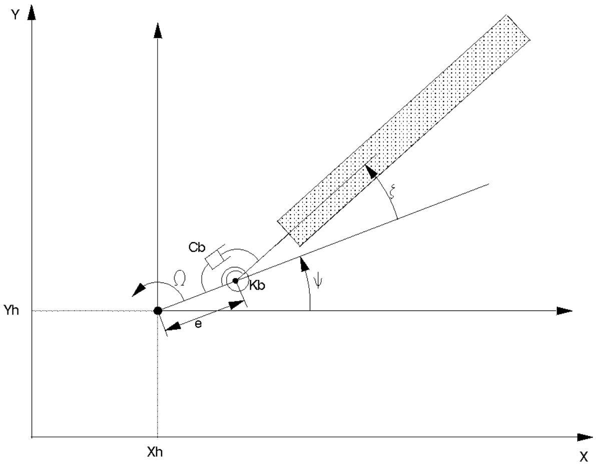


Figure 1: Rotor model schematic and DOF.

Under the assumption of small blade motion ζ«1, these equations can be written, for convenience, in a compact matrix form as follows:

$$\begin{bmatrix} I_R & -S_S^T \\ -S_S & M_H + M_R \end{bmatrix} \begin{Bmatrix} \ddot{\zeta} \\ \ddot{x}_H \end{Bmatrix} + \begin{bmatrix} C_R & 0 \\ -2\Omega S_C & C_H \end{bmatrix} \begin{Bmatrix} \dot{\zeta} \\ \dot{x}_H \end{Bmatrix} + \begin{bmatrix} K_R & 0 \\ \Omega^2 S_S & K_H \end{bmatrix} \begin{Bmatrix} \zeta \\ x_H \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (12)$$

where I_R, C_R and K_R are diagonal matrices representing the rotor dynamics and having the following expression of the diagonal terms:

$$I_R(j,j) = I_j; \quad C_R(j,j) = C_j; \quad K_R(j,j) = K_j + e\Omega^2 S_j$$

S_j, I_j, and C_j, K_j are, respectively, the j[th] blade first and second mass moment of inertia with respect to the hinge, and the angular damping and stiffness coefficient, *e* is the hinge offset and Ω the rotor angular speed. M_H, C_H and K_H are the structural matrices of the airframe with dimensions $[N_H x N_H]$, and assuming to have ordered the degrees of freedom in such a way that the x and y translation of the hub centre appears as first and second term of the vector x_H.

The other matrices which appear in the equations are:

$$M_R(i,j) = \begin{cases} \sum_{k=1}^{N_b} M_k = m_R & for \quad i = j = 1,2 \\ 0 & for \quad i \neq j \end{cases}$$

$$S_S(i,j) = \begin{cases} +S_j \sin(\psi_j) & for \quad i = 1 \\ -S_j \cos(\psi_j) & for \quad i = 2 \end{cases}$$

$$S_C(i,j) = \begin{cases} +S_j \cos(\psi_j) & for \quad i = 1 \\ +S_j \sin(\psi_j) & for \quad i = 2 \end{cases}$$

where $\psi_j(t) = \Omega t + 2\pi(j-1)/N_b$

and M_k is the mass of the k^{th} blade.

These matrices contain also some other zero rows (for i≥3). However, since these terms will have only the effect of removing the explicit dependency from the airframe DOF different from the x and y hub translations, for simplicity of notation, they have not been indicated.

Knowing the normal modes of the support, it is possible to expand the motion of the hub centre as a linear superposition of the modal contributions:

$$x_H = [\Phi]\{\eta\}$$

where the modal matrix [Φ] contains the components of the support eigenvectors corresponding to the x and y in-plane degrees of freedom at the hub:

$$[\Phi] = \begin{bmatrix} \varphi_{1x} & \varphi_{2x} & \ldots & \varphi_{Mx} \\ \varphi_{1y} & \varphi_{2y} & \ldots & \varphi_{My} \end{bmatrix} \qquad (13)$$

for the M modes considered.

Now it is possible to introduce the following global transformation matrix:

$$\begin{Bmatrix} \zeta \\ x_H \end{Bmatrix} = \begin{bmatrix} U & 0 \\ 0 & \Phi \end{bmatrix} \begin{Bmatrix} \zeta \\ \eta \end{Bmatrix} = [Q] \begin{Bmatrix} \zeta \\ \eta \end{Bmatrix} \qquad (14)$$

Applying this transformation, and premultiplying by Q^T the set of equations (12), leads to the following system of 2N equations:

$$\begin{bmatrix} I_R & -S_S^T\Phi \\ -\Phi^T S_S & \Phi^T M_H^* \Phi \end{bmatrix} \begin{Bmatrix} \ddot{\zeta} \\ \ddot{\eta} \end{Bmatrix} + \begin{bmatrix} C_R & 0 \\ -2\Omega\Phi^T S_C & \Phi^T C_H \Phi \end{bmatrix} \begin{Bmatrix} \dot{\zeta} \\ \dot{\eta} \end{Bmatrix} + \begin{bmatrix} K_R & 0 \\ \Omega^2 \Phi^T S_S & \Phi^T K_H \Phi \end{bmatrix} \begin{Bmatrix} \zeta \\ \eta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (15)$$

where $M_H^* = M_H + M_R$ represents the mass matrix of the airframe with the addition of the (concentrated) rotor mass m_R at the hub centre.

These equations now contain only the modal characteristics of the airframe, expressed by the modal matrix [Φ] of the eigenvectors at the rotor hub centre, and the diagonal, real valued, modal mass M_m^*, modal damping C_m and modal stiffness K_m which replace the products $\Phi^T \cdot \lfloor M_H^*, C_H, K_H \rfloor \cdot \Phi$ in virtue of the orthonormalisation properties of the real modes.

Due to the definition of the augmented mass matrix of the airframe M_H^*, there are two options to account for the rotor mass contribution to the airframe modal mass. The first assume that the rotor mass m_R is included as a concentrated mass in the airframe model (or test item). The second considers the airframe without the rotor mass, and adds the rotor inertial forces weighted by the airframe mode shapes:

$$M_m^* = \Phi^T M_H^* \Phi = \Phi^T M_H \Phi + m_R \Phi^T \Phi$$

System (15) is still periodic, with the periodicity included in the S_S and S_C coupling matrices. It can be analysed for stability using Floquet theory [12]. In the case of an isotropic rotor it can be converted, using multiblade coordinates [11,13], to a set of constant coefficients equations whose (complex) eigenvalues will give frequencies and damping ratios of the coupled rotor-fuselage system.

4 Stability with complex airframe modes

We have seen the form of the transformation that can be applied in the case of real fuselage modes, and we also know that for the complex modes case, the orthonormalization properties of the eigenvector matrix is valid only for the first order dynamic matrices A and B. The basic idea of the procedure presented here to include airframe complex modes is based on a transformation similar to the one used for the normal modes, in the sense that it will produce a new set of equations which will contain only modal parameters and modal variables for the support dynamics, but extended to the general case of complex modes, plus rotor-related physical degrees of freedom and characteristics. The difference with the previously seen method lies in the fact that the transformation, instead of being applied to the original second order form of the system, will be applied to the support partition of the first order form of the coupled rotor-fuselage system that can be obtained adding to equations (15) the following identities:

$$\begin{bmatrix} M_H^* & 0 \\ 0 & I_R \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\zeta} \end{Bmatrix} - \begin{bmatrix} M_H^* & 0 \\ 0 & I_R \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\zeta} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (16)$$

The two systems (15) and (16), written reordering the equations in order to partition the support from the rotor degrees of freedom, can be expressed in the following way:

$$\begin{bmatrix} 0 & M_H^* & 0 & 0 \\ M_H^* & C_H & -S_S & -2\Omega S_C \\ 0 & 0 & 0 & I_R \\ -S_S^T & 0 & I_R & C_R \end{bmatrix} \begin{Bmatrix} \ddot{x}_H \\ \dot{x}_H \\ \ddot{\zeta} \\ \dot{\zeta} \end{Bmatrix} +$$
$$\begin{bmatrix} -M_H^* & 0 & 0 & 0 \\ 0 & K_H & 0 & \Omega^2 S_S \\ 0 & 0 & -I_R & 0 \\ 0 & 0 & 0 & K_R \end{bmatrix} \begin{Bmatrix} \dot{x}_H \\ x_H \\ \dot{\zeta} \\ \zeta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (17)$$

It is convenient to rewrite this set of first order equations in a more compact form as:

$$\lfloor A_y \rfloor \{\dot{y}\} + \lfloor B_y \rfloor \{y\} = 0 \qquad (18)$$

having introduced the new state vector:

$$\{y\} = \begin{Bmatrix} \dot{x}_H \\ x_H \\ \dot{\zeta} \\ \zeta \end{Bmatrix}$$

and partitioning the dynamic matrices A_y and B_y in the rotor and support (hub) components:

$$[A_y] = \begin{bmatrix} A_{HH} & A_{HR} \\ A_{RH} & A_{RR} \end{bmatrix}$$
$$[B_y] = \begin{bmatrix} B_{HH} & B_{HR} \\ B_{RH} & B_{RR} \end{bmatrix}$$

This system of first order equations has twice the dimensions of the original system, and is still periodic in time due to the presence of the S_S and S_C sub-matrices. Again, when the rotor is isotropic, *i.e.* when all the blades have the same characteristics, it is possible to use the multiblade coordinates transformation to eliminate this periodicity and obtain a system of constant coefficients equations to analyse for stability. It is convenient to introduce the multiblade transformation at this point of the procedure, along with the complex modal variable change for the airframe dynamics. In this way all the computations will be condensed in a single global transformation operation acting simultaneously, even if separately and in a different way, on the whole system equations and variables.

The multiblade transformation, for an isotropic rotor with 3 or more blades, is defined as follows:

$$\{\zeta\} = [Q_{MB}]\{b\} \qquad (19)$$

where b is the vector of the N_b multiblade rotor DOF, and:

$$[Q_{MB}] = \begin{bmatrix} 1 & \sin\psi_1 & \cos\psi_1 & \dots & -1 \\ 1 & \sin\psi_2 & \cos\psi_2 & \dots & +1 \\ \dots & & & \dots & \dots \\ 1 & \sin\psi_{N_b} & \cos\psi_{N_b} & \dots & +1 \end{bmatrix}$$

(the last column of the matrix is present only when the number of blades is even).

The general form of the transformation that will be applied to the state vector y of system (18) can be written as:

$$\{y\} = [T]\{q\} \quad \text{with} \quad [T] = \begin{bmatrix} \Psi & 0 \\ 0 & Q_R \end{bmatrix} \qquad (20)$$

where Ψ is the (2Mx2M) complex modal matrix of the support at the hub centre. Q_R is the rotor transformation matrix which, depending on the particular case being analysed, can be:

- isotropic rotor: $Q_R = Q_{MB}$
- nonisotropic rotor: $Q_R = U$

To apply the transformation defined by eqs. (20) to the system (18), it is necessary to take into account the possible explicit dependency of the T matrix from the time variable t (which is due to the rotor transform matrix Q_R in the multiblade coordinates case). Therefore, the derivative of the state vector will be:

$$\dot{y} = T\dot{q} + \dot{T}q = \begin{bmatrix} \Psi & 0 \\ 0 & Q_R \end{bmatrix} \dot{q} + \begin{bmatrix} 0 & 0 \\ 0 & \dot{Q}_R \end{bmatrix} q \qquad (21)$$

With the change of variable defined by eqs. (20) and (21), and premultiplying system (14) by T^T we obtain:

$$\left(T^T A_y T\right)\dot{q} + (T^T A_y \dot{T} + T^T B_y T)q = 0$$

or, shortly:

$$\lfloor A_q \rfloor \{\dot{q}\} + \lfloor B_q \rfloor \{q\} = 0 \tag{22}$$

where the new dynamic state matrices A_q and B_q have the following partitioned expressions:

$$\lfloor A_q \rfloor = \begin{bmatrix} \Psi^T A_{HH} \Psi & \Psi^T A_{HR} Q_R \\ Q_R^T A_{RH} \Psi & Q_R^T A_{RR} Q \end{bmatrix}$$

$$\lfloor B_q \rfloor = \begin{bmatrix} \Psi^T B_{HH} \Psi & \Psi^T (B_{HR} Q_R + A_{HR} \dot{Q}_R) \\ 0 & Q_R^T (B_{RR} Q_R + A_{RR} \dot{Q}_R) \end{bmatrix}$$

Now it is possible to see that using the orthonormalisation properties of the airframe complex eigenvector matrix, the terms $\Psi^T A_{HH}\ \Psi$ and $\Psi^T B_{HH}\ \Psi$ will be replaced by the diagonal modal_A and modal_B matrices which represent the analogous of the modal mass, stiffness and damping terms of the "real" mode case. The other elements of the A_q and B_q matrices depend from the rotor characteristics, the rotor-fuselage coupling terms and the rotor transformation matrix which can be evaluated from the rotor characteristics and from the complex support eigenvector matrix.

From the form of the final system (22) matrices it is possible to see that although the method has been applied here to a rotor having only the lead-lag rigid DOF, it can be extended to more general rotor models following the partitioning scheme that has been adopted and changing appropriately the expression of the rotor-related matrix Q_R and of the coupling matrices A_{HR}, A_{RH} and B_{HR}, B_{RH}.

5 Analysis of a 2 DOF fuselage model

The procedure described above will be applied to a rotor support having two planar DOF (x and y translations in the rotor disk plane). This simple case allows to check the stability results obtained with the complex airframe mode approach, versus the results of a direct stability analysis of the system equations expressed in physical coordinates.

The airframe dynamics at the rotor hub centre can be described by the following equations:

$$\begin{bmatrix} m_x & 0 \\ 0 & m_y \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} c_x & c_{xy} \\ c_{xy} & c_y \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} +$$

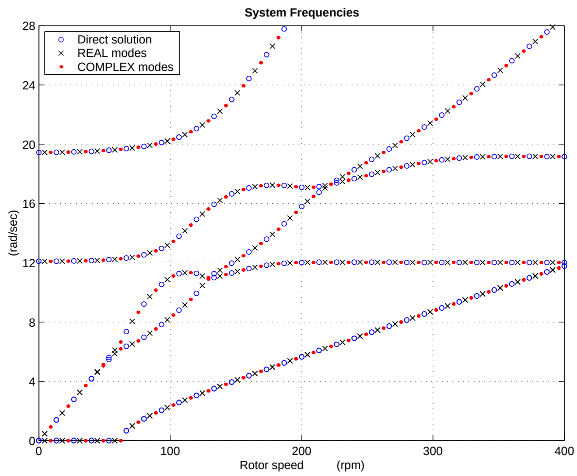
$$\begin{bmatrix} k_x & k_{xy} \\ k_{xy} & k_y \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{23}$$

A first run has been made using for the system parameters the values listed in table 1 (taken from ref. [2] considering the nonisotropic hub case).

Support characteristics				Rotor data			
m_x =	550	slugs		N_b =	4	
m_y =	225	slugs		M_b =	6.5	slugs
k_x =	85000	lb/ft		S_b =	65.0	slug.ft
k_y =	85000	lb/ft		I_b =	800.0	slug.ft^2
k_{xy} =	0	lb/ft		e =	1.0	ft
c_x =	3500	lb.sec/ft		K_b =	0.0	ft.lb/rad
c_y =	1750	lb.sec/ft		C_b =	3000.0	ft.lb.sec/rad
c_{xy} =	0	lb.sec/ft				

Table 1: Values of the parameters for the nonisotropic uncoupled (real modes) hub case

In this case the support equations are uncoupled, since the structural coupling terms k_{xy} and c_{xy} have been neglected. The support matrices are therefore diagonal and the resulting modes real.



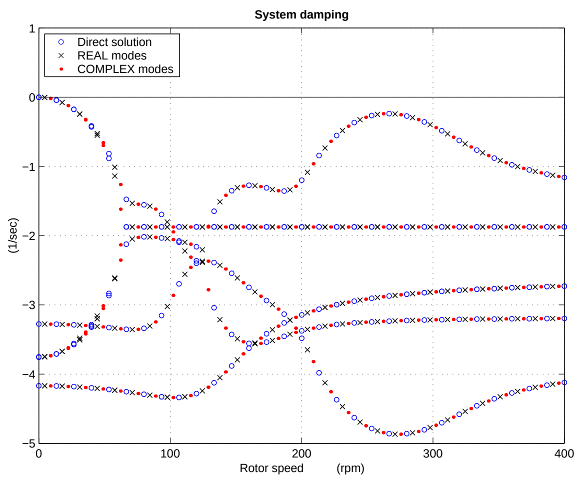


Figure 2: Results of the analysis for the uncoupled nonisotropic hub case.

Figure 2 shows the results of the stability calculation performed using three different approaches:

– direct analysis with physical DOF;
– analysis with real airframe modes;
– analysis with complex support modes.

These graphs can be compared with fig. 5 of ref. [2] to see that the three methods give in this case the same results in terms of system frequencies and dampings. The difference with ref. [2] lies in the fact that the present analysis consider all the rotor modes, including also, in this particular case, the collective and reactionless lead-lag modes.

Working with this simple 2 DOF support model it is possible to control the "degree" of complexity of the hub mode shapes simply changing the value of the off-diagonal coupling term c_{xy} of the support damping matrix. This allows to make a comparison with the stability results that would be obtained using approximate real modes for the support dynamics in place of the actual complex modes calculated from the support equations.

To perform this comparison it is necessary to specify in which way complex modes can be converted to real modes. Two different possible methods have been considered.

The first – "*undamped system modes*" - takes the real eigenvectors at the hub centre from the undamped form of the airframe dynamics (this can be done performing for example, a NASTRAN SOL 103 analysis) without changes, and modifies the eigenvalues to account for the appropriate modal damping by assigning the values obtained from the complex analysis of the airframe to the eigenvalues corresponding to the same or most similar eigenvectors.

The second – "*amplitude normalisation*" - considers the complete set of complex eigenvectors and eigenvalues (obtained either analytically or experimentally) and transform the complex mode shapes to real format keeping only the modulus of the eigenvectors, with a proper handling of the signs (this can be done, for example, looking at the residues). This operation correspond to force an alignment of the eigenvector phases along a given reference angle.

The first method can be applied only to analytical models since it requires the knowledge of the mass and stiffness matrices to calculate the undamped system modes. The second one is more linked to the experimental modal analysis package, which offer the possibility of exporting the extracted modes using either complex or real format in a universal file.

Both these methods are approximate, since in any case the phase delay relations between different DOF will be altered by the conversion of the eigenvectors to a real format with respect to the actual distribution contained in the original complex mode shapes.

Therefore, it is possible that some differences in the final results can be found as a result of the "normalisation" procedure. These differences should be less evident when the airframe modes are less complex which corresponds to a support dynamics such that the hub in-plane modal trajectories are closer to a line or to a very thin ellipse. On the other side, for a more complex mode, the centre of the hub will show an elliptical modal path with lower eccentricity (*i.e.* a "fatter" ellipse), and in this case the conversion of the eigenvector to the real format will produce a greater alteration of the eigenvector.

The effect of the degree of complexity of the airframe modes has been analysed considering the values of the simplified 2 DOF model parameters listed in the following table 2.

Support characteristics			Rotor data		
m_x =	400	kg	N_b =	4	
m_y =	400	kg	M_b =	50.0	kg
k_x =	3e+5	N/m	S_b =	70.0	kg.m
k_y =	1e+5	N/m	I_b =	230.0	kg.m2
k_{xy} =	0	N/m	e =	0.25	m
c_x =	600	N.sec/m	K_b =	0.0	N.m/rad
c_y =	400	N.sec/m	C_b =	3000.0	N.m.sec/rad

Table 2: Values of the parameters

The rotor considered in this example is a four bladed rotor, with a first lead-lag frequency ν_{lag}=0.28 */rev*, which is in the typical range of articulated rotors.

Two different values of the out-of-diagonal term c_{xy} of the damping matrix have been considered:

a) c_{xy} = -1000 N.sec/m;
b) c_{xy} = -3000 N.sec/m.

These values are not necessarily representative of a realistic case, being used in this example only to modify the complexity of the support modes. However, the results of the stability analysis are not affected by these assumptions.

The complex support modes corresponding to the two values of the coupling parameter c_{xy} are listed in the following table 3.

		Case a): c_{xy}=-1000		Case b): c_{xy}=-3000	
		Mode 1	Mode 2	Mode 1	Mode 2
Eigenvalue	λ (1/sec)	-0.7452 +27.203i	-0.5048 +15.904i	-0.6979 +25.668i	-0.5521 +16.855i
Frequency	ω_d (Hz)	4.3294	2.5312	4.0852	2.6825
Damping ratio	ξ (%)	2.7384	3.1724	2.7178	3.2741
Modal_a	$a=2\lambda_I i$	54.406i	31.808i	51.336i	33.709i
Modal_b	$b=-\lambda a$	1480.0 +40.544i	505.87 +16.056i	1317.7 +35.825i	568.18 +18.612i
Complex eigenvector	x	0.0502 +8.9e-6i	-6.5e-5 +0.004i	0.0522 +0.0001i	-0.0003 +0.0151i
	y	2.5e-6 -0.0069i	0.0505 +1.9e-7i	0.0001 -0.0246i	0.0557 +3.4e-5i

Table 3: Calculated complex modes of the support.

Figures 3a) and 3b) show the hub centre modal trajectories associated to the calculated eigenvectors. It is possible to see that the more complex mode shapes associated to the higher value of c_{xy} produce more circular modal paths.

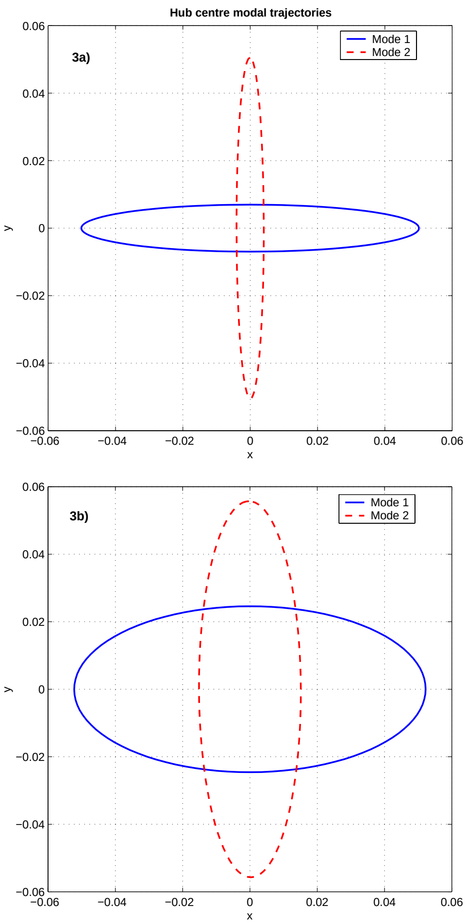


Figure 3: Hub centre modal trajectories for the two values of c_{xy} considered.

It should be noticed that the degree of complexity of the modes is not necessarily linked to high modal damping values, since in both cases analysed the damping ratios are in the order of a few percent. The more circular paths are associated to the condition where the x and y components of the eigenvector have phase delay closer to 90° with amplitudes of the same order of magnitude.

Converting these modes to real (or normal) mode format using the two methods described above leads to the eigenvectors listed in the following tables 4a and 4b (eigenvalues remain the same listed in previous table 3), normalising to unity modal mass:

Table 4a): Undamped system modes		Case a): c_{xy}=-1000		Case b): c_{xy}=-3000	
		Mode 1	Mode 2	Mode 1	Mode 2
Real eigenvector	x	0.05	0	0.05	0
	y	0	0.05	0	0.05

Table 4b): Amplitude normalised modes		Case a): c_{xy}=-1000		Case b): c_{xy}=-3000	
		Mode 1	Mode 2	Mode 1	Mode 2
Real eigenvector	x	-0.0502	0.0040	-0.0522	0.0151
	y	-0.0070	-0.0505	-0.0246	-0.0557

Eigenvectors listed in tab. 4a, obtained from the undamped form of the support equations, are not affected by the value of the c_{xy} term, since they are evaluated without considering the damping matrix.

The stability analysis of the coupled rotor-support system has been performed using both these normal modes and the complex modes listed in tab. 3, in order to compare the results obtained with the two "*realisation*" procedures.

Figure 4 shows a comparison of the results obtained for the first case considered (c_{xy}=-1000), which exhibits moderately complex support modes. For better legibility of the figure, only the envelope of the maximum real parts of the final rotor-support system eigenvalues has been plotted.

In this case the results obtained with the various methods are almost the same, even if the curves corresponding to the normal support modes show a very small, but still visible, difference with respect to the reference analysis. The application of the complex airframe modes analysis reproduces exactly the results of the direct solution of the system equations expressed in physical coordinates.

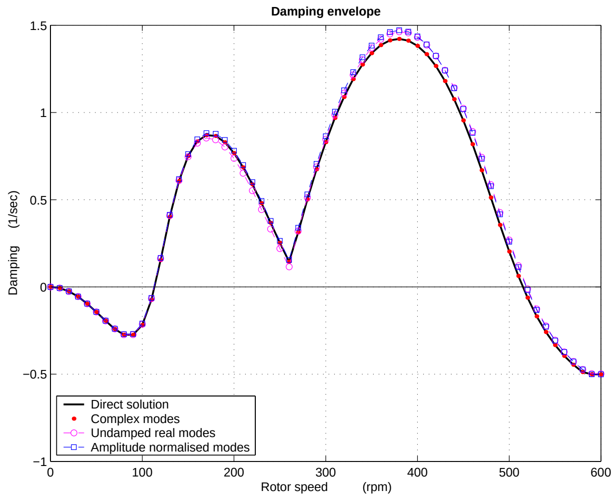


Figure 4: Envelope of the maximum real parts of system eigenvalues for the case c_{xy}=-1000.

The results for the second case, with c_{xy}=-3000, are shown in figure 5 for the damping envelope, and figure 6 for the coupled rotor-support system frequencies.
In this case, the increased degree of complexity of the support mode shapes leads to greater approximation in the support modes converted to real format with respect to the original complex mode shapes. This approximation, along with the associated less accurate representation of the support damping behaviour, produces now to greater differences in the final stability curves that can be seen in fig. 5. Looking at this figure it is possible to observe that complex support modes analysis is able to reproduce the stability results of the direct solution also in this case.

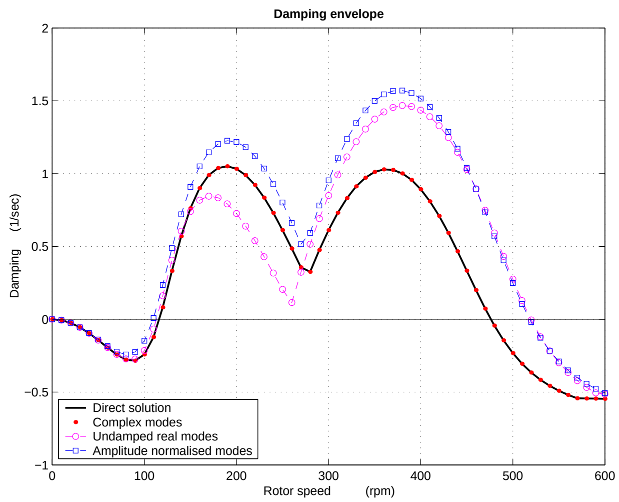


Figure 5: Envelope of the maximum real parts of system eigenvalues for the case c_{xy}=-3000.

The differences between complex and real support modal transformation are related to the different capability of the two methods of reproducing the dynamic behaviour of the support, particularly for the effects induced by the damping matrix, being the rotor model the same for both methods. These differences appear clearly also from the rotor-fuselage system frequencies trend towards zero rotor angular speed plotted in fig. 6, where the rotor induced effects on the system eigenvalues is lower. In fact it is possible to see that at low angular rotor speed (in the limit, with the rotor stopped) the frequencies evaluated with real support modes differ from the reference, direct solution, analysis results.

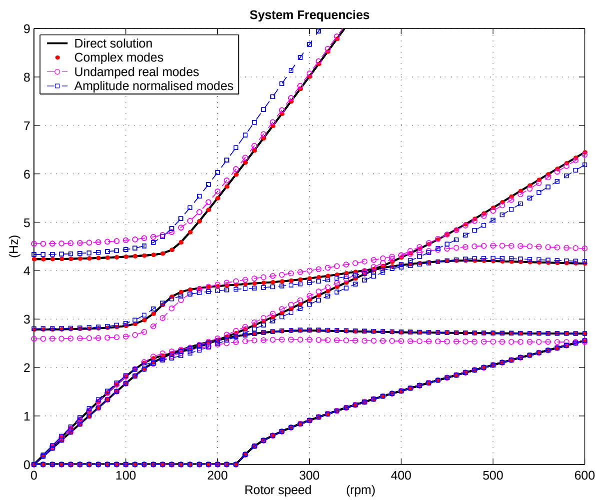


Figure 6: System frequencies (imaginary part of eigenvalues) for the case c_{xy}=-3000.

6 Conclusions

A method to include complex airframe modes into the stability analysis of the coupled rotor-fuselage system, applied here to the study of the ground resonance characteristics, has been presented. The use of this modal method produces results which are consistent with the direct solution of the system equations of motion expressed in physical coordinates, regardless of the particular structure of the support dynamic matrices.
This method can be viewed as an extension of the real modal representation of the fuselage dynamics and allows to use in a more straightforward way the complex support modal parameters obtained either analytically or experimentally.

The stability results obtained converting complex mode shapes to real mode shapes, with the two different methods described, are close to the exact solution when the degree of complexity of the modes being considered is low. The applicability of real support modes will depend on the particular behaviour of the actual fuselage configuration being analysed. The present complex mode method can therefore be used to evaluate the effects induced by the use of approximate real mode shapes, comparing the stability curves obtained using the two approaches.

7 References

1. Coleman, R.P. and Feingold, A.M. "*Theory of Self_Excited Mechanical Oscillations of Helicopter Rotors with Hinged Blades*", NACA Report 1352, 1958;
2. Hammond, C.E. "*An Application of Floquet Theory to Prediction of Mechanical Instability", AHS/NASA Specialists'* meeting on Rotorcraft Dynamics, February 13-15, 1974;
3. Johnson, W. "*Helicopter Theory*", Princeton University Press, Princeton, N.J., 1980;
4. Luti, L. and Speziali, S. "*Detection and Identification of the Nonlinearities Present on a Helicopter from a Frequency Response Function Set*", 19th European Rotorcraft Forum, September 14-16, 1993, paper nr. 7;
5. Orlandi, Ceriotti, Russo, Marchese, Maino, Nagaraja "*Analytical and Experimental Results of the Ground Resonance Phenomenon for A129*", 10th European Rotorcraft Forum, August 28-31, 1984;
6. CAMRAD JA *Users and Theory Manual*;
7. Lang, G.F. "*Demistifying Complex Modes*", Sound and Vibration, January 1989;
8. Duncan, W.J. "*Elementary Matrices*", MacMillan Company, New York, 1946;
9. Thomson, W.T. *"Theory of Vibration with Applications*", Prentice-Hall, New Jersey, 1972;
10. LMS Cada-X *Modal Analysis Manual*;
11. Hohenemser, K.H. and Yin, S.K. *"Some Applications of the Method of Multiblade Coordinates*", AHS Journal, Vol. 17, No. 3, July 1972, pp. 3-12;
12. Peters, D.A. and Hohenemser, K.H. *"Application of the Floquet Transition Matrix to Problems of Lifting Rotor Stability*", AHS Journal, Vol. 16 No. 2, April 1971, pp. 25-33;
13. Dugundji, J. And Wendell, J.H. "*Some Analysis Methods for Rotating Systems with Periodic Coefficients*", AIAA Journal, Vol. 21, No. 6, June 1983, pp. 890-897.