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HELICOPTER GROUND RESONANCE MATHEMATICAL MODEL CONTAINING APERIODIC LINKS

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ABSTRACT

Such a lead-lag damper which has elastic and damping elements in series connection or a landing gear chock absorber working in consecutive order with a pneumatic tire may be considered as an example of aperiodic link in a helicopter ground resonance model. The model of the rolling pneumatic tire should also be presented as an aperiodic link.

In ground resonance studies such an aperiodic links are often replaced by equivalent models with parallel connection of elastic and damping elements in order to simplify the problem or to reduce a number of variables. However, the parameters of such a model depend on the oscillations frequency, while this particular frequency is to be defined from the system solution results. This makes a sort of uncertainty while defining the parameters mentioned.

The proposed paper gives several helicopter ground resonance examples in which the aperiodic links are presented without any simplification.

OSCILLATIONS OF THE BLADE WITH THE ELASTO-HYDRAULIC DAMPER

The blade and damper elements equilibrium equations for in-plane oscillations have the following form:

$$\begin{aligned} &\ddot{\xi} + \nu_0^2\omega^2\xi + \frac{K}{I}\dot{\xi}_0 = 0 \\ &K\dot{\xi}_0 = C(\xi - \xi_0) \end{aligned} \qquad (1)$$

where ξ - blade in-plane swing angle;
ξ_0 - damper angular deflection;
K, C - damper elements damping and elastic coefficients;
ν_0 - pendulum form oscillations frequency;
ω - rotor angular velocity.

The characteristical equation for the case described above is of the third order and for the standard case it has one aperiodic root. However there exists a possible situation when all the roots are real.

Fig. 1 shows the relation between the values of the characteristical equation roots and the rotor angular velocity for the sequence of stiffness and damping parameters. The example being considered is the Mi-26 helicopter main rotor blade.

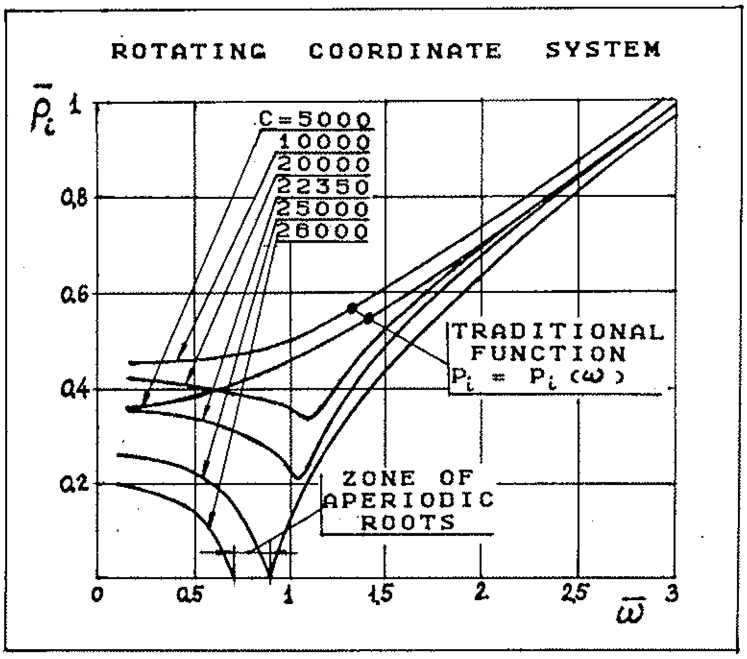

fig.1

Unlike the conventional comprehension of the blade oscillation frequency as increasing unvariedly with angular velocity rising, the case considered gives the evidence of a probable decreasing of the blade oscillation frequency with the angular velocity growth. A certain combination of C and K parameters results in the zero oscillation frequency. This may occur in some interval of rotor r.p.m. Within this interval all three roots become real and the motion is aperiodic.

The described attribute of the frequency should be accounted of while identifying the results of flight tests as well as analyzing the ground resonance safety.

GROUND RESONANCE

The multi-bladed rotor with cyclic symmetry is being considered. In this case the ground resonance equations have been derived for fixed coordinate system though a well-known variables substitution shown below:

$$\eta = \sum_{k=1}^{n} \xi_k \sin \psi_k ; \quad \varsigma = \sum_{k=1}^{n} \xi_k \cos \psi_k$$

Coleman introduced this substitution in his works. In Russia Gerebtsov and Proskurjakov applied this method. For the case being considered in the paper we have introduced the following coordinates in a similar way:

$$u = \sum_{k=1}^{n} \xi_{0_k} \sin \psi_k; \quad v = \sum_{k=1}^{n} \xi_{0_k} \cos \psi_k$$

where k - the ordinal number of the blade;
n - the number of blades.

The substitutions applied resulted in the opportunity to describe the rotor motion with the following system of equations:

$$\begin{cases} \ddot{\eta} + \dfrac{K}{I}(\dot{u} - \omega v) - \omega^2 \eta \left(1 - \nu_0^2\right) - 2\omega\dot{\varsigma} - \dfrac{n}{2}\dfrac{S}{I}\ddot{X} = 0 \\ \ddot{\varsigma} + \dfrac{K}{I}(\dot{v} + \omega u) - \omega^2 \varsigma \left(1 - \nu_0^2\right) + 2\omega\dot{\eta} - \dfrac{n}{2}\dfrac{S}{I}\ddot{Z} = 0 \\ \dot{u} - \omega v = \dfrac{C}{K}(\eta - u) \\ \dot{v} + \omega u = \dfrac{C}{K}(\varsigma - v) \end{cases} \qquad (2)$$

These equations can be supplemented with support motion equations to describe the coordinates X and Z. These motion equations are possible to be derived for a fuselage model with one or more degrees of freedom along each of X or Z directions. We shall not consider the equations in details within the frames of the paper nevertheless we should note here that the rotor - support system stability investigations are supposed to be made by the conventional manner on the basis of characteristical polynome roots analysis.

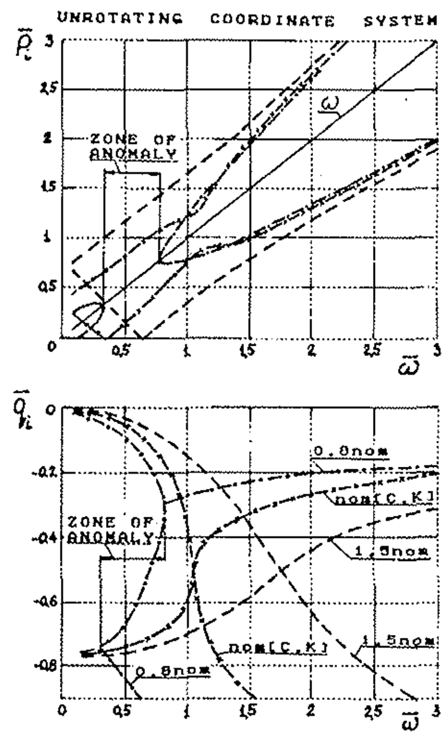

fig.2

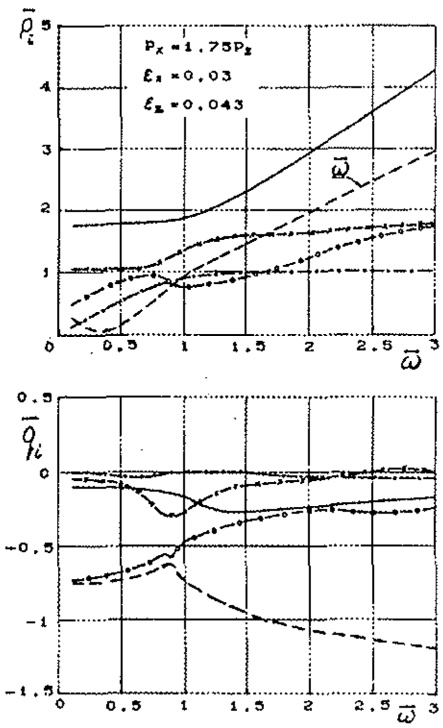

fig.3

The analysis results are shown on the fig. 2 - 3 for Mi-26 main rotor blade parameters as it was done before. Fig.2 shows the frequencies and corresponding decrements of rotor self - oscillation free from the support. As it was emphasized before the frequencies are given in an unrotating coordinate system. The relations displayed indicate the described above root characteristics shown on fig.1 for one individual blade. Fig.3 shows the characteristical polynome roots for rotor - support system with two degrees of freedom support. One can see the areas where the roots affect each other significantly. Here the impact of aperiodic motion forms on decrement values is obvious. The potential instability zones are sure depend upon this impact.

GROUND RESONANCE IN THE LANDING RUN

When a helicopter oscillates during ground run, the additional degree of freedom appears and it should be taken into account when considering the elastic properties of an undercarriage. As a deformed tyre rolls, the center of tire-to-ground contact area travels along some curvilinear trajectory. So a lateral displacement of this center depending on the wheel properties and the rolling speed occures. Wheel lateral response will depend not only on the displacement of its center, but also on the additional displacement of contact area, so-called "drift". Besides it also follows from this that the wheel dynamic response will have a phase shift relative to the given travel of wheel center in contrast to the case of non-rolling wheel oscillations (in the last case the displacement and the response both occur in the same phase if a rubber material hysteresis is not taken into account). This very peculiarity is at the bottom of aperiodic features of this link.

We can use for the analysis of this case the simplified mathematical model derived through the use of drift hypothesis from more complicated Keldysh model of rolling. Drift hypothesis assumes that there is a linear relation between angular and linear deformations of the tire in the following form:

$$\lambda = r \cdot \varphi$$

where λ - coordinate of the area of contact with supporting surface;
φ - angular deformation;
r - radius of non-deformed tire.

The lateral displacement Z_k of wheel diametral plane is defined by the following equation:

$$\dot{Z}_k = \frac{V}{r}\lambda + \dot{\lambda} \qquad (3)$$

where V - helicopter rolling speed.

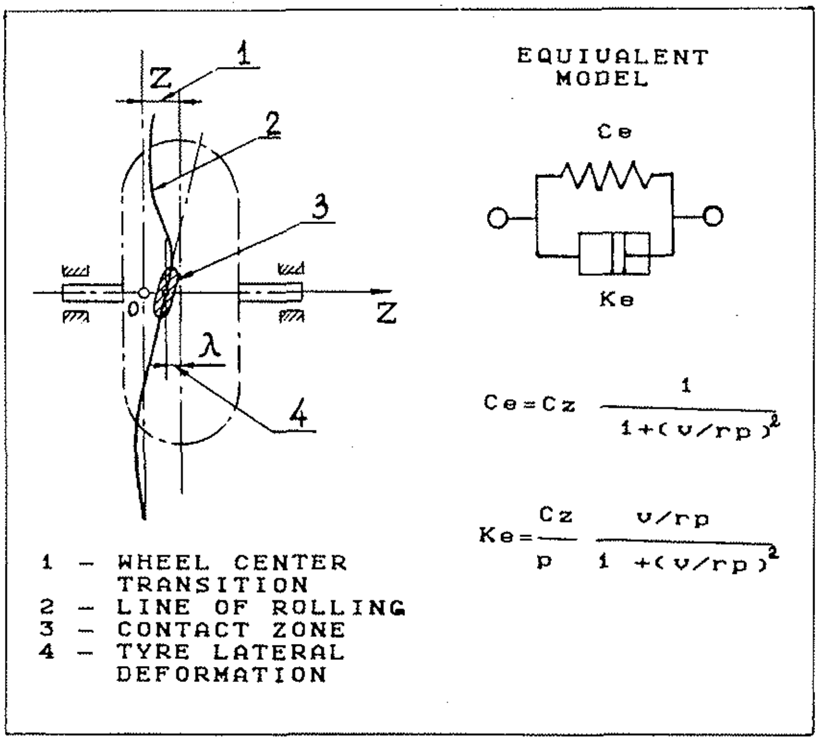

fig.4

As a result the lateral displacement Z_k of wheel diametral plane is defined by the following equation:

$$R_z = C_z \lambda \qquad (4)$$

where C_z - lateral stiffness coefficient of the tire.

Sometimes when considering the ground resonance problem the model similar to the one described is used. Such a model involves a spring and a damper connected in parallel. It's easy to define lateral stiffness coefficients values from equations (4) and (3) for a certain frequency p:

$$Ce = C_z \frac{1}{1+\left(\frac{V}{r \cdot p}\right)^2}; \quad Ke = \frac{C_z}{p} \cdot \frac{\frac{V}{r \cdot p}}{1+\left(\frac{V}{r \cdot p}\right)^2} \qquad (5)$$

Here a definite correspondence between the rolling speed and the oscillation frequency can be found in the only case when the model of the base has one degree of freedom. When the base has several degrees of freedom, some uncertainty in the model equivalent parameters selection appears and the ground resonance problem should be considered using the additional equations (4) and (3) as presented in the reference [3].

Fig.5 shows the results of ground resonance calculations for the ground run of the Mi-28 helicopter taken as an example. Helicopter fuselage has two degrees of freedom which are lateral displacement Z and roll angle. In addition to that the coordinates of contact area lateral displacement are taken into account. Alterations of oscillation frequencies and decrements with the running speed increase from zero at rest up to the maximum operational speed are shown.

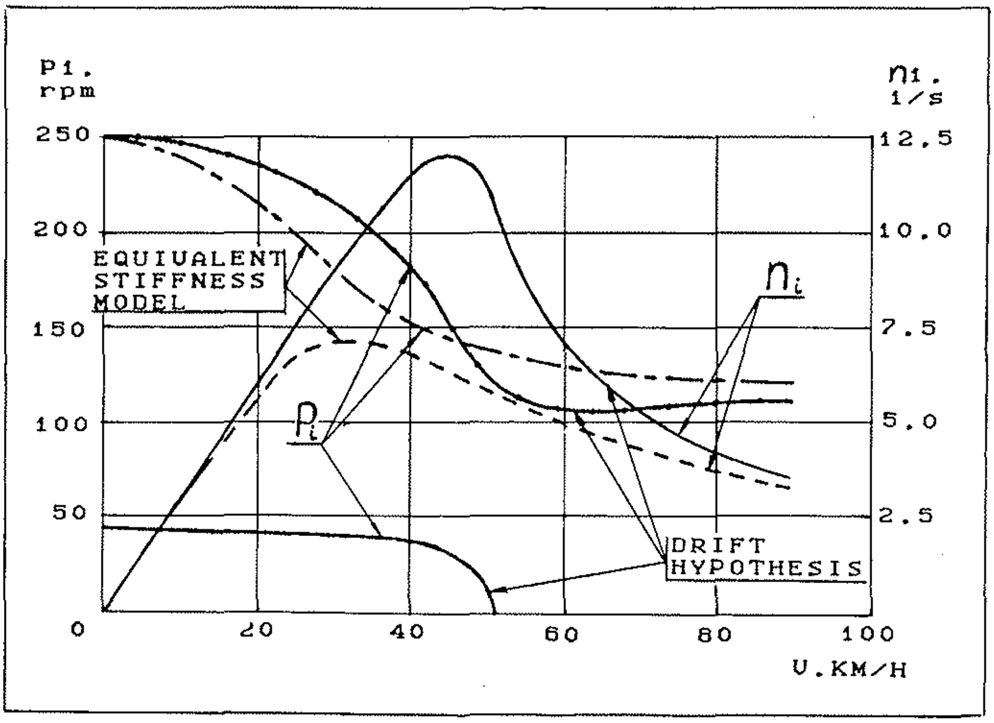

fig.5

The lower oscillation frequency p_1 corresponding mainly to the lateral displacement turns into zero quite soon and the motion becomes aperiodic. For the upper frequency p_2 corresponding mostly to the helicopter roll gradual frequency decrease appears as the running speed grows. This frequency decrease is limited by a certain value inherent to roll angular oscillations with the zero lateral stiffness of the tires. Damping of this oscillation form gradually increase up to the extreme value at the operational range of ground run speed. Such damping caused only by response phase shift during deformed tire rolling is called sometimes the kinematic damping. The affect of such a damping on general stability and ground resonance safety is quite significant, especially when the self oscillation second tone frequency becomes lower than rotor angular velocity at a certain helicopter running speed.

For the purpose of comparison results of calculations obtained for Mi-4 helicopter through the use of stiffness and damping equivalent values (as in formula (5)) are also presented. Comparison shows that for the range of medium helicopter running speed it is possible to obtain considerable accuracy improvements for the frequency and dumping values and for instability boundaries position.

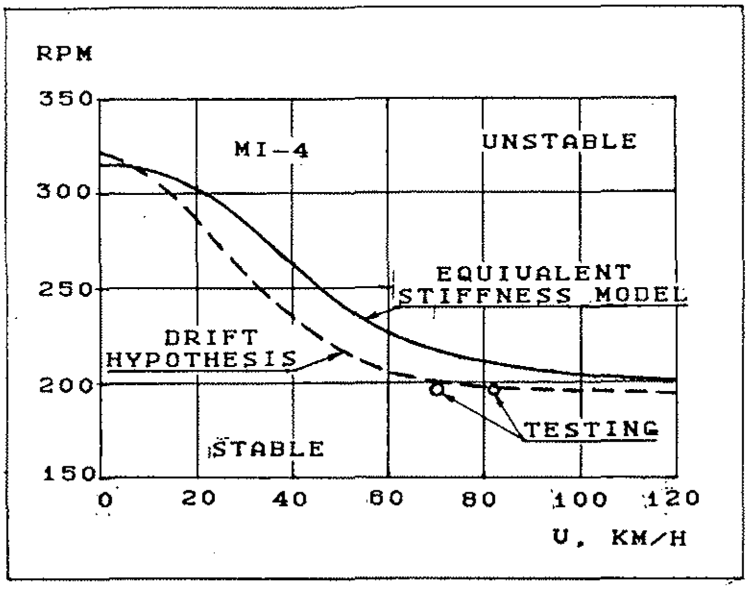

fig.6

REFERENCES:

1. Миль М.Л. и др. Вертолеты. Расчет и проектирование. Кн.2 Москва, Машиностроение - 1967
2. Келдыш М.В. Шимми переднего колеса самолета. "Труды ЦАГИ" вып.564, 1945
3. Мягков Ю.А., Годес Б.С. О земном резонансе вертолета на пробеге. Межвузовский сборник научных трудов "Прочность, надежность и долговечность авиационных конструкций" вып.3 Киев 1976