AN ANALYTICAL APPROACH AND SELECTIVE STIFFENING TECHNIQUE FOR THE SOURCE REDUCTION OF NOISE AND VIBRATION IN HIGHLY LOADED MECHANICAL POWER-TRANSMISSION SYSTEMS

by

Allen C. Royal
Applied Technology Laboratory
US Army Research and Technology Laboratories (AVRADCOM)
Fort Eustis, Virginia

and

R. J. Drago and J. W. Lenski, Jr.
Boeing Vertol Company
PO Box 16858, P32-09
Philadelphia, Pennsylvania

FIFTH EUROPEAN ROTORCRAFT AND POWERED LIFT AIRCRAFT FORUM

SEPTEMBER 4 – 7TH 1979 - AMSTERDAM, THE NETHERLANDS

AN ANALYTICAL APPROACH AND SELECTIVE STIFFENING TECHNIQUE FOR THE SOURCE REDUCTION OF NOISE AND VIBRATION IN HIGHLY LOADED MECHANICAL POWER-TRANSMISSION SYSTEMS

by

Allen C. Royal
Applied Technology Laboratory
US Army Research and Technology Laboratories (AVRADCOM)
Fort Eustis, Virginia

and

R. J. Drago and J. W. Lenski, Jr.
Boeing Vertol Company
PO Box 16858, P32-09
Philadelphia, Pennsylvania

1. ABSTRACT

A finite element NASTRAN model of the complete forward rotor-transmission system for the CH-47C helicopter has been developed and applied, along with a technique for incorporating high modulus aluminum oxide fibers into a magnesium housing, to reduce transmission vibration and noise at its source. In addition to a description of the model, an analytical technique for vibration and noise prediction, and a technique for selective stiffening of a structure during casting are outlined. Also included are the dynamic response as predicted by NASTRAN, the use of strain-energy methods to optimize the housing for minimum vibration and noise, and a description of temporary design modifications which were subsequently manufactured and tested to correlate the analysis. The test data showed a 7 dB noise reduction for the analytically improved transmission housing as compared to the existing baseline housing. Test data for the fiber reinforced components showed a strain-to-load ratio approximately 3.5 times better for the reinforced specimen than that of the baseline AZ91C-T6 material. The techniques presented are not restricted to helicopters but are applicable to any power-transmission system. The transmission-housing model developed can be used further to evaluate dynamic stresses; thermal distortions; load paths, failsafety/vulnerability and composite materials.

2. INTRODUCTION

Considerable attention has been focused in recent years on the reduction of noise levels for both Military and civil helicopters as evidenced, for example, by the noise requirements outlined in specification MIL-A-8806, The Occupational Safety and Health Act (OSHA), and the Walsh-Healy Act. Helicopter noise emanates from three major sources - the rotor blades, engines and transmissions. The major portion of the perceived interior noise level, as shown in Figure 1, is due to the transmission system. The conventional means of controlling transmission noise has generally been to just add limp material blanketing and acoustical enclosures after the hardware has been built and a noise problem has been identified.

In order to define the best compromise between an optimum acoustical environment and the penalties imposed on an aircraft by the inclusion of sound-reducing treatments, methods of predicting noise levels and tools to perform trade studies (i.e., source noise reduction versus enclosures) are required. Transmission noise is a symptom of the inherent structural vibrations which generate this noise. Until recently, analytical methods have not been available to predict transmission vibration/noise levels in advance.

3. ANALYTICAL APPROACH

Transmission Noise Generation and Reduction

Gear shaft deflections and their effect on case deflections and noise production have been under investigation for several years (References 1, 2, 3, and 4). The mechanism which has been identified is that the noise is generated by the transmission housing as a result of the nonuniform transfer of torque between mating gears. The nonuniform transfer of torque is due to tooth profile and spacing tolerance, the elastic deformation of the gear teeth under load, and the deformation of the tooth backup rim, and web. This nonuniform transfer of torque produces a dynamic force at the gear mesh frequency and its multiples for a simple gear set. The apparent frequency of the planetary set is quite different since the calculation must account for planet precession. The dynamic force which excites the coupled torsional/lateral/vertical vibratory modes of the gear shaft is then applied at the tooth mesh to excite a finite element model of the internal components. The vibration produces displacements at the bearing locations, which excite the housing and cause it to vibrate at all mesh frequencies, thus radiating noise. The dynamic characteristics of the internal components may magnify this excitation. Furthermore, the dynamic characteristics of the housing may magnify its displacements and the resulting noise, as shown schematically in Figure 2.

By controlling the dynamic response of the internal components, resonances can be avoided and displacements at the bearing locations can be reduced. This is a desirable approach to vibration/noise reduction, since it also leads to increased bearing life and improved overall transmission performance and reliability. An analysis for the reduction of vibration/noise at its source has been developed which includes the reduction of dynamic excitation and response of the shafts and housing.

Description of the Model

The internal components of the CH-47C forward rotor transmission and their finite element model representation are shown in Figures 3 and 4, respectively. Using this model, the dynamic response of the shafts was determined and subsequently input into a second program to identify those shaft segments demonstrating high strain energy densities at the mesh frequencies. By modifying these segments, the dynamic response of the shaft was altered most effectively.

Dynamic Response - Internal Components

As shown in Table 1, the main exciting frequencies are very close to some of the natural frequencies. A review of the first 20 mode shapes indicated that the results are primarily coupled bending/torsion modes and that the input pinion and bevel gear/planetary systems are also coupled (as shown in Figure 4).

The damped forced response of the internal components was also calculated. Excitation at the lower planetary first harmonic is coupled vertical-lateral motion confined mainly to the pinion components.

The dynamic loads calculated for each bearing at each mesh frequency are phased (sine and consine components due to structural damping) and subsequently applied to determine the forced response of the transmission case and resulting dB output.

Description of the Model-Housing

The forward rotor transmission housing is composed of three major sections: upper cover (provides lugs for mounting the transmission and transmits the rotor system loads into the airframe), case (supports the main spiral bevel gears, lube pump and oil sump), ring gear (connects the upper cover and case, contains the planetary gear system). This natural division of the housing was adhered to for ease of modeling, as shown in Figure 5.

Dynamic Response-Housing

These housing components were analyzed separately with suitable boundary conditions applied at their interface points. It should be noted that the natural frequencies are embedded in the exciting frequencies (Table 1) which could lead to considerable amplification of noise and vibration. The results of this dynamic response were then evaluated using the strain energy program to define the optimum areas for modification (i.e., the areas of highest strain energy density).

System Detuning

Every structure possesses characteristic frequencies, called natural frequencies, at which it will vibrate when excited. Furthermore, sound is emitted and the pitch of this sound is determined by the frequency of the wave motion. Since this natural vibratory motion is periodic, it possesses a natural frequency.

If succeeding applications of the exciting force are timed to begin to act just at the instant that the wave motions are about to repeat themselves, a condition known as resonance occurs. Energy will accumulate in the structure to such an extent that the amplitude of the vibration becomes out of proportion to the exciting force producing it. In the case of a transmission housing, the exciting forces are the tooth meshing loads. If this exciting frequency or its harmonics coincides with a housing natural frequency, resonance will exist. At resonance, since the wave motions produce repeating deflections which may become large, a fatigue stress will be imposed. If this stress combined with other stresses exceeds the fatigue strength of the material, failure will occur. In practice, however, due to inherent structural damping noise generation rather than fatigue failure is the primary concern.

To optimize the transmission components for minimum vibration/noise, the eigenvectors (modes shapes) and natural frequencies are calculated and compared with the gear mesh exciting frequencies to identify each mode shape whose natural frequency is close to an exciting frequency and which it is desirable to shift. For each appropriate mode shape, the strain density (strain energy/volume) distribution throughout the structure is calculated. The structural elements with the highest strain density are the best candidates for effective modification of the natural frequency, since a minimal weight change will yield a maximum shift in natural frequency. By locally altering the housing wall to change the mass and stiffness in these areas of high strain density, the natural frequency may be shifted away from the exciting frequency.

For noise and vibration reduction it is important to minimize the displacement at the bearing locations. This may be done by relocating the bearings, reducing the dynamic tooth forces, changing the shaft stiffness and mass distribution, and changing the bearing stiffness. Thus the possibility of resonance is eliminated and the vibration and radiated noise are reduced.

In summary, therefore, the approach to reducing the noise produced by the transmission was divided into two distinct but complimentary areas. First the internal components were detuned so that their natural frequencies were moved as far as practical from the various mesh excitation frequencies. Second, the housing itself was detuned by identifying the areas of highest strain energy density and then selectively stiffening these areas to reduce the case response to the gear excitation. In addition, the mode shapes of the internal components were evaluated to insure that the excitation at the bearing supports was minimum.

Structural Modification

Based on the foregoing analysis, the response of the housing was reduced by applying a detuning ring to the stationary ring gear and by bonding two doubler plates to the case (at areas determined to be the most optimum in terms of strain density and accessibility), while that of the sun/bevel gear was reduced by pressing a sleeve onto the shaft section O.D. and into the shaft I.D., thus changing its stiffness and mass characteristics. The latter changes are illustrated in Figure 6.

Experimental Approach

Test Program

In order to determine the effects of the above noted changes an extensive experimental program was under taken. Since the three changes were developed from uncoupled models, they were tested as individual changes and in various combinations. All testing was conducted in a closed loop test rig. The transmission was surrounded by an acoustical enclosure to isolate it from the rest of the test stand. Noise data were recorded by six microphones installed in close proximity to the transmission within the enclosure, while vibration data were recorded by six accelerometers. All data were recorded on 14 channel wide band FM tape. The test transmission was run at various speeds and torque conditions to simulate aircraft operation.

Test Results

Although all combinations tested yielded significant noise reductions, the combination shown in Figure 6 provided the greatest reduction - 7 dB. The implication of a 7 dB source noise reduction is a substantial weight savings in the acoustical treatment required to comply with Table IV of noise specification MIL-A-8806 (normal cruise power condition). In the case of the CH-47C helicopter as shown in Table 2, this amounts to a savings of at least 20 percent of the total material weight required. The CH-47C helicopter has two main rotors which are driven by very similar transmissions thus, the tested aircraft weight savings was based on similar modifications of both transmissions and acoustical treatment of the entire cabin area.

4. SELECTIVE STIFFENING

Using an iterative process, the transmission components are optimized by comparing the eigenvectors (mode shapes) and natural frequencies with the gear mesh exciting frequencies to identify each mode shape whose natural frequency is close to an existing frequency and which is desirable to shift. For each appropriate mode shape, the strain density (strain energy/volume) distribution throughout the structure is calculated. The structural elements with the highest strain density have been identified as the best candidates for effective modification of the natural frequency. Studies have shown that a minimal mass or stiffness change in these areas will yield a maximum shift in the natural frequency. By locally altering the housing wall to change the mass and stiffness in these areas of high strain density the excitation of the wall can be reduced and tests have demonstrated a reduction in vibration and noise levels.

A view of the CH-47 forward transmission case showing areas of high strain density is shown in Figure 7. As can be seen in this figure, there are many isolated areas that would require changes in order to effectively reduce the overall excitation of the housing. This creates a problem in how to stiffen the housing case without a substantial increase in component weight. Previous work has been limited to varying the wall thickness of the cast magnesium housing. The analytical techniques have indicated the potential for more efficient application of the optimization methods by taking advantage of the improved properties and capability for selective reinforcement offered by composite materials. The ability to buildup composite material elements with specified property orientations allows selective reinforcement of predetermined housing regions and provides improved flexibility for optimization of the structure.

Application of Metal Matrix

The use of metal matrix materials for helicopter transmission housings has many potential benefits including increased component life and efficiency and reduced vibration/noise levels. These improvements result from the selective stiffening of identified areas and the increased overall stiffness of the housing.

The following equation relates, in matrix form, the mass and stiffness distributions in a housing to local displacements and accelerations for a specified load condition (structural damping is ignored):

$$F = M\ddot{X} + KX \tag{1}$$

The static deflections are given by

$$X = K^{-1} F \tag{2}$$

and the natural frequencies of the structure can be obtained from the roots of the characteristic equation.

$$\left[\lambda M - K \right] = 0 \tag{3}$$

where the λ's are the squares of the natural frequencies. An examination of the above equations show that increasing the housing wall stiffness will result in reduced static and dynamic displacement and shift the natural frequencies to higher values. Further, the amplitudes of the vibrations and hence the resulting sound waves are inversely proportional to the stiffness, the overall effect of increased stiffness is to reduce vibration/noise level.

A further and probably more significant benefit of metal matrix application is that it is better suited for selective stiffening than a conventional monolithic cast structure. The mechanical properties of fiber FP/magnesium metal matrix material are summarized in Table 3 and illustrate the potential availability to tailor a wide range of modulus of elasticity (stiffness) to meet various local design requirements.

To illustrate the effect of using a high modulus metal matrix as a transmission housing material, the CH-47 forward transmission housing model was used to compare load-deflection characteristics and frequency spectrum for a conventional cast magnesium and metal matrix composite material. The modulus used for magnesium was 6.5×10^6 and 22×10^6 for the metal matrix material. The model assumed that the complete housing was fabricated from the specified material.

A plot of displacement versus load for a CH-47C forward transmission housing fabricated from various materials is shown in Figure 8 and indicates that the magnitude of the housing displacements will be reduced substantially by the use of stiffer metal-matrix materials. Steel is shown in the figure as a point of reference. Since the amplitude of the vibration and resulting soundwaves is proportional to the magnitude of the displacement of the structure, the overall effect of increased stiffness would be to reduce the vibration and noise level. In addition, weight savings can be achieved through the use of metal-matrix materials to achieve a desired stiffness. If the standard magnesium housing was replaced with a steel housing of the same dimensions, a weight increase of 182 pounds would result. The use of 100% metal-matrix material to achieve a stiffness close of that of steel would result in a weight increase of less than 30 pounds. It is anticipated that only local stiffening of the housing will be required to achieve significant results, therefore, a weight increase of less than 30 pounds will be required to achieve sizable noise reductions. This weight penalty will be offset by a large reduction of weight for various acoustical treatment materials required to achieve the same results.

An additional benefit of the metal-matrix configuration is that it is much better suited for detuning of the housing than the conventional monolithic cast structure. A typical transmission frequency spectrum is presented in Figure 8. Because of the multiple forcing frequencies (bevel mesh and lower planetary) and the many natural frequencies of the structure which occur, detuning of a housing is extremely difficult. Although the natural frequencies can be shifted by modifying the wall thickness, the multitude of closely packed frequencies generally will cause the tuning process to be only minimally effective. The new frequency spectrum which results after substituting a typical metal-matrix material for monolithic magnesium is also presented in Figure 9. It is significant that the natural frequencies have been shifted toward the high end of the spectrum and that only about 40 percent of the frequencies remain in the range of interest (below approximately 5,000 Hz) as compared to the solid magnesium configuration. Those natural frequencies remaining in the range of interest also have been dispersed and are thus much more amenable to the detuning process. Using the selective stiffening capability provided by the metal-matrix design, the housing can be tuned to reduce the vibration and noise levels. Further areas of potential vibration and noise improvement for the metal-matrix structure include structural damping and acoustic transmission loss (TL). Little data is available on these aspects, and therefore they must be investigated fully.

To further investigate the feasibility of fabricating a full scale transmission housing from a metal matrix composite material, a program was conducted to demonstrate that typical housing specimens can be selectively reinforced by the use of a high modulus fiber via present state-of-the-art casting technology.

Metal Matrix Fabrication Techniques

There are many fibers such as boron, graphite and silicon carbides which have been considered as potential reinforcement fibers for a magnesium housing. Presently, little success has been achieved with these materials in the fabrication of simple test specimens. Recently a joint program has been conducted between the DuPont Pioneering Research Laboratory and the Boeing Vertol Company on the experimental fabrication of specimens using FP aluminum filaments in a magnesium matrix. This program has resulted in good quality, complex shape test specimens which have given added confidence to the successful application of this material to helicopter transmission housings.

Fiber FP is an experimental continuous aluminum oxide fiber currently under development at the DuPont Company (Reference 5). Fiber FP is essentially 100 percent polycrystalline α - aluminum at a purity of greater than 99 percent AL_2O_3, fired to a density of 98 percent of theoretical. The fiber is therefore inherently stable at elevated temperatures ($>$ 1000°C) and is compatible with a variety of metal system including aluminum and magnesium. The fiber is a continuous, multifilament yarn which offers flexibility, relative ease of handling and projected competitive cost. Some basic material properties for a 50% volume fill of unidirectional Fiber FP in a cast magnesium matrix is shown in Figure 10 and Table 3.

A direct-casting process appears to be the most practical means of producing a helicopter transmission case structure. The direct-cast process would require a fiber that is stable at the temperatures of molten magnesium. Fiber FP is stable at these temperatures and is compatible and therefore suitable for the process. A flow chart for the vacuum metal infiltration technique used to prepare FP/metal composites is shown in Figure 11. As the first step, FP yarn is made into a handleable FP tape with a fugitive organic binder in a manner similar to producing a resin-matrix composite prepreg. The fiber tapes are then laid up in the desired orientation, fiber volume loading and shape and are then inserted into a casting mold of steel or other suitable material. The fugitive organic binder is burned away and the mold is infiltrated with molten metal. This method has been successfully used to fabricate various shaped specimens which represent various cross sections of a typical transmission housing. Figure 12 shows several of the cast test specimens.

Test Evaluation

A test program was conducted to evaluate the material properties of this material. One test involved the use of an outer rim/web/inner rim test specimen mounted in a back-to-back arrangement as shown in Figure 13. This test setup simulated the conditions expected for the bearing support of the spiral bevel gear of the CH-47 forward transmission as shown in Figure 3. An offset load was applied to the test specimens to simulate an equivalent thrust and moment overload produced by the bearing reacting the bevel gear loads. The specimens were strain gaged at critical areas and dial indicators were also used to record specimen deflections under applied loads.

The reduced data from this test is presented in the form of plots (Figure 14) showing the average measured strain versus applied load. Also contained in Figure 14 are curves showing the results of a finite element analysis performed for this test configuration which indicates a good correlation between predicted and measured values. The specimens were also analyzed as if they were fabricated from AZ91C-T6 magnesium alloy which is used in conventional transmission housings.

Deflections of the loading fixture center section were also measured during the back-to-back test. These deflections would represent the displacements and rotation of the specimen's inner rim flange which would support the bearing in an actual transmission housing. Figure 15 shows the measured average vertical displacements and rotations of the fixture center section as a function of load. For comparison, data representing the displacements and rotation of a conventional AZ91C-T6 specimen is also included. Figure 15 shows that a significant reduction in system deflections can be achieved through the use of a FP/magnesium metal matrix structure versus that of a conventional AZ91C-T6 structure. Further analysis of the data indicated that a longitudinal elastic modulus of approximately 22.7 x 10^6 PSI was achieved in these test specimens.

5. CONCLUSIONS

Analytical approach to the source reduction of the noise and vibration associated with high power, high speed, lightweight transmissions has been developed, applied to a test case, and verified by extensive testing.

In addition, analytical and fabrication technology has been developed for complex metal-matrix structures made from Fiber FP - reinforced cast magnesium. The analysis and test program has indicated that metal-matrix composites can be effectively employed to selectively stiffen a helicopter transmission housing. Tests have shown that a stiffened housing will provide a better support for the dynamic components and should result in increased component life and reduced vibration/and noise. Progress to date offers considerable encouragement for the full-scale development, fabrication and evaluation of a minimum noise, maximum stiffness metal-matrix composite helicopter main transmission housing.

6. ACKNOWLEDGEMENT

The work described in this paper was performed under Contract Number DAAJ02-74-C-0040, "Helicopter Transmission Vibration/Noise Reduction Program", supported by the Applied Technology Laboratories (AVRADCOM), Fort Eustis, Virginia. It is reported in documents numbered USARTL-TR-78-2A, -2B and -2C.

7. REFERENCES

1. Sciarra, John J. Howells, Robert W., Lenski, Joseph W., Jr., Drago, Raymond J., and Schaeffer, Edward G., Helicopter Trabsmission Vibration and Noise Reduction Program, Boeing Vertol Company, Philadelphia, Pennsylvania; USARTL-TR-78-2A, Volume I, Technical Report, Applied Technology Laboratory, U. S. Army Research and Technology Laboratories (AVRADCOM), Fort Eustis, Virginia, March 1978.

2. Sciarra, John J., Howells, Robert W., Lenski, Joseph W., Jr., and Drago, Raymond J., Helicopter Transmission Vibration and Noise Reduction Program, Boeing Vertol Company, Philadelphia, Pennsylvania; USARTL-TR-78-2B, Volume II, User's Manual, Applied Technology Laboratory, U. S. Army Research and Technology Laboratories (AVRADCOM), Fort Eustis, Virginia, March 1978.

3. Lenski, Jr., Joseph W., Helicopter Transmission Vibration and Noise Reduction Program, Boeing Vertol Company, Philadelphia, Pennsylvania; USARTL-TR-78-2C, Volume III, Evaluation of Metal-Matrix Housing Specimens, Applied Technology Laboratory, U. S. Army Research and Technology Laboratories (AVRADCOM), Fort Eustis, Virginia, October 1978.

4. Howells, Robert W., and Sciarra, John J., Finite Element Analysis for Complex Structures (Helicopter Transmission Housing Structural Modeling), Boeing Vertol Company, Philadelphia, Pennsylvania; USAAMRDL-TR-77-32, Applied Technology Laboratory, U. S. Army Research and Technology Laboratories (AVRADCOM), Fort Eustis, Virginia, January 1978.

5. Chamption, A. R., Krueger, W. H., Hartman, H. S., and Dhingra, A. K., Fiber FP Reinforced Metal Matrix Composites, Second International Conference on Composite Materials, Toronto, Canada, April 1978.

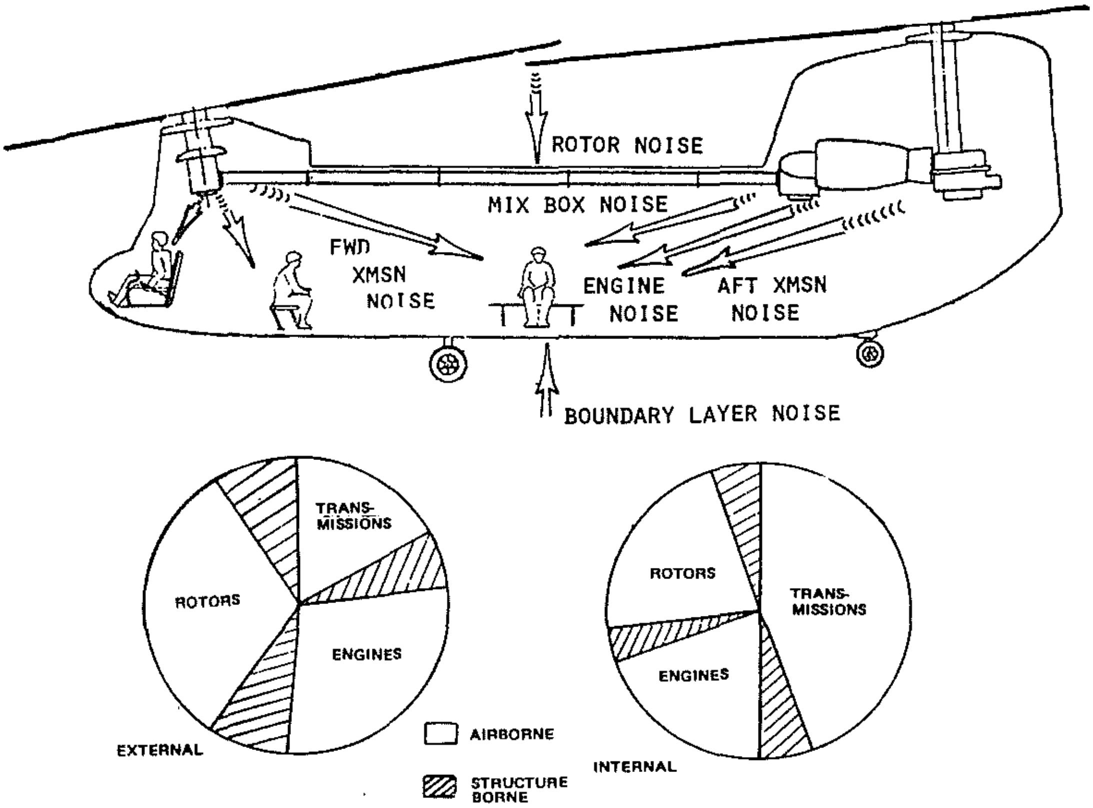

Figure 1. Sources of Helicopter Noise.

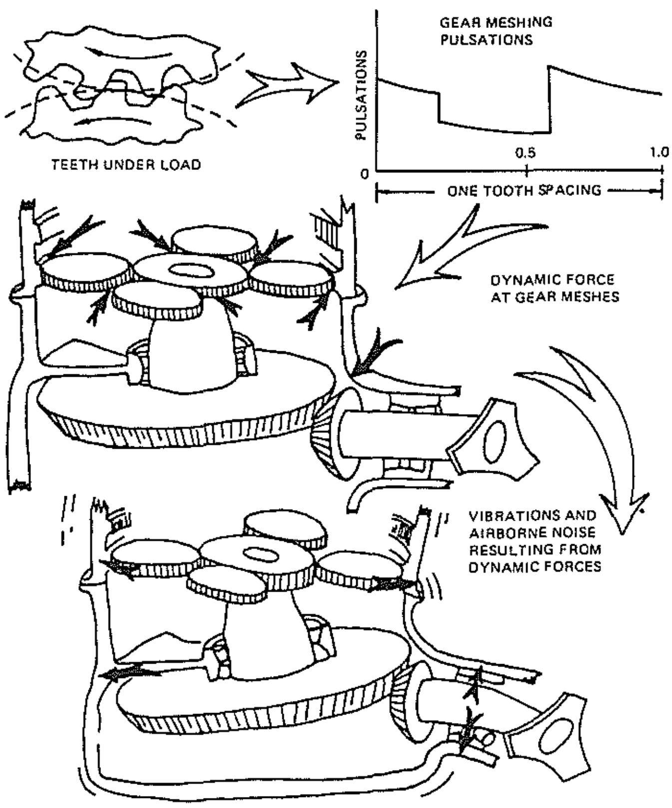

Figure 2. Sources of Transmission Noise.

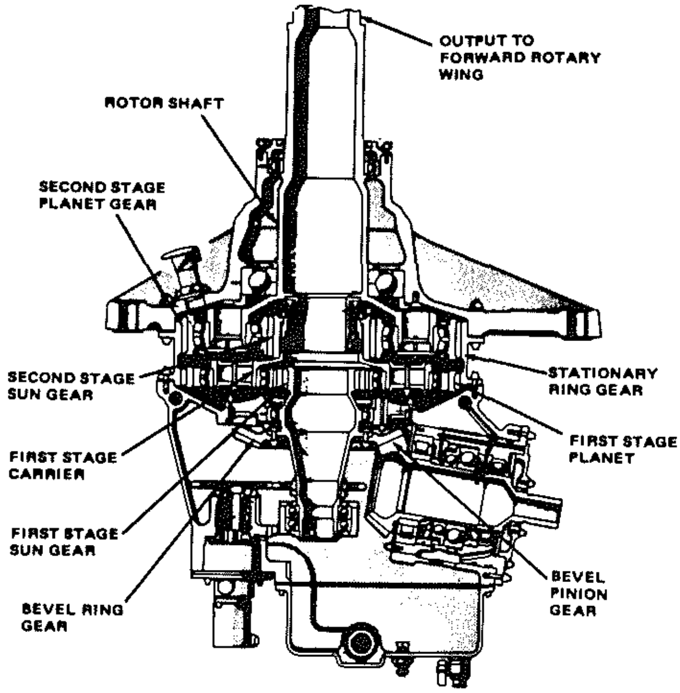

Figure 3. CH-47 Forward Rotor Transmission.

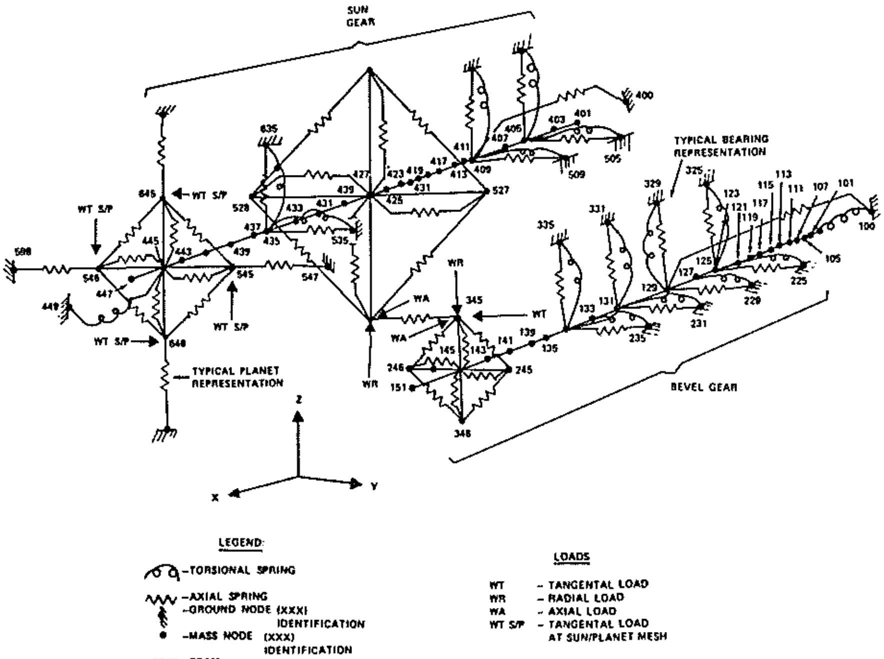

Figure 4. Finite Element Model of Transmission Internal Components.

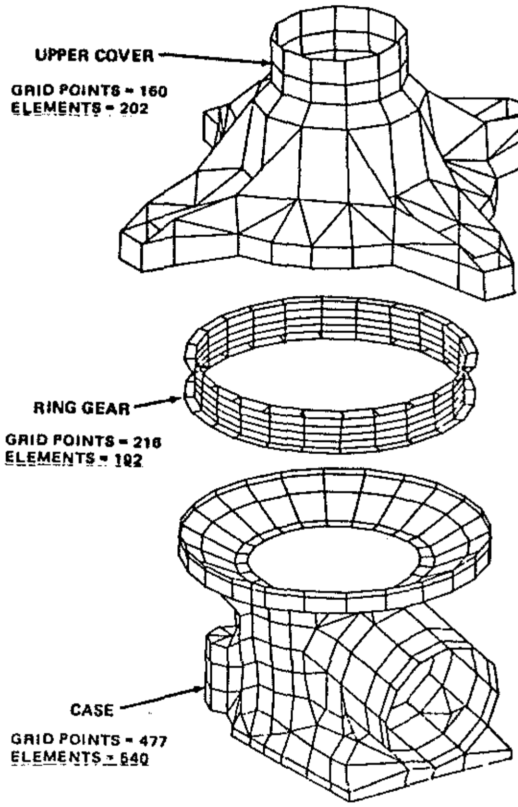

Figure 5. Transmission Housing Model.

Table 1. Spectrum of Representative Natural and Primary Exciting Frequencies for Forward Transmission

		CALCULATED NATURAL FREQUENCIES			
SOURCE	EXCITING FREQUENCIES	INTERNAL COMPONENTS	UPPER COVER	RING GEAR	CASE
Lower Planetary 1st Harmonic	1566	1526 1769	1518 1568	- 2334	1541 1603
Lower Planetary 2nd Harmonic	3132	3044 3151	3069 3133	2565 3206	3103 3181
Bevel Mesh	3606	3404 3855	3570 3653	3206 4130	3588 3664
Lower Planetary 3rd Harmonic	4698	4544 5150	4577 4775	4130 4770	4667 4735

NOTE: Only those natural frequencies adjacent to each exciting frequency are shown.

Table 2. Acoustical Treatment Weight Comparison for Baseline and Modified Transmissions

	FORWARD TRANSMISSION		AFT TRANSMISSION	
	BASELINE	MODIFIED	BASELINE	MODIFIED
1. Average Noise Level, dB	120	113	120	113
2. Noise Limit of MIL-A-8806A, Table IV, dB	86	86	86	86
3. Transmission Enclosure Attenuation Required, dB	34	27	34	27
4. Product of Surface Weight and Frequency, HZ x LB/FT^2	1000	560	1500	770
5. Surface Weight of Required Enclosure, LBS/FT^2	0.64	0.36	0.96	0.49
6. Area of Enclosure (Approximate), FT^2	110	110	155	155
7. Required Treatment Weight, LBS	70	40	149	76
8. Enclosure Weight Reduction (Baseline WT - Modified WT = Reduction)	-	30	-	73
9. Total Weight of Transmission Modifications (Gear Shaft and Magnesium Plates), LBS	-	3.3	-	3.3
10. Net Weight Reduction Per Transmission, LBS	-	26.7	-	69.7

TOTAL WEIGHT SAVINGS OF
96.4 LBS PER AIRCRAFT

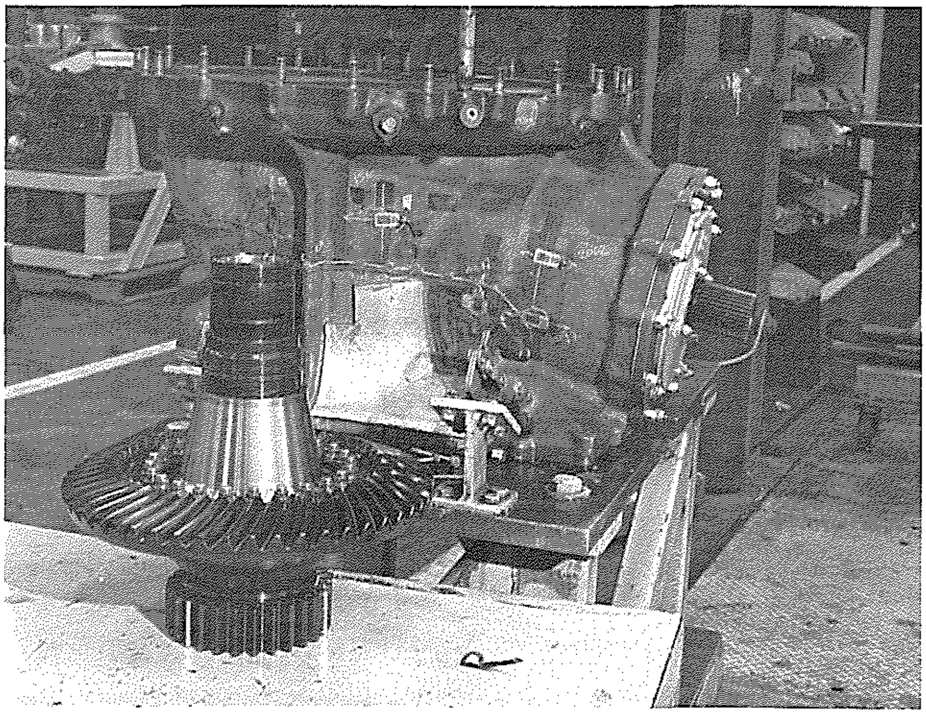

Figure 6. Installation of Detuning Sleeves on Sun/Bevel Gear and Plates on Transmission Lower Case.

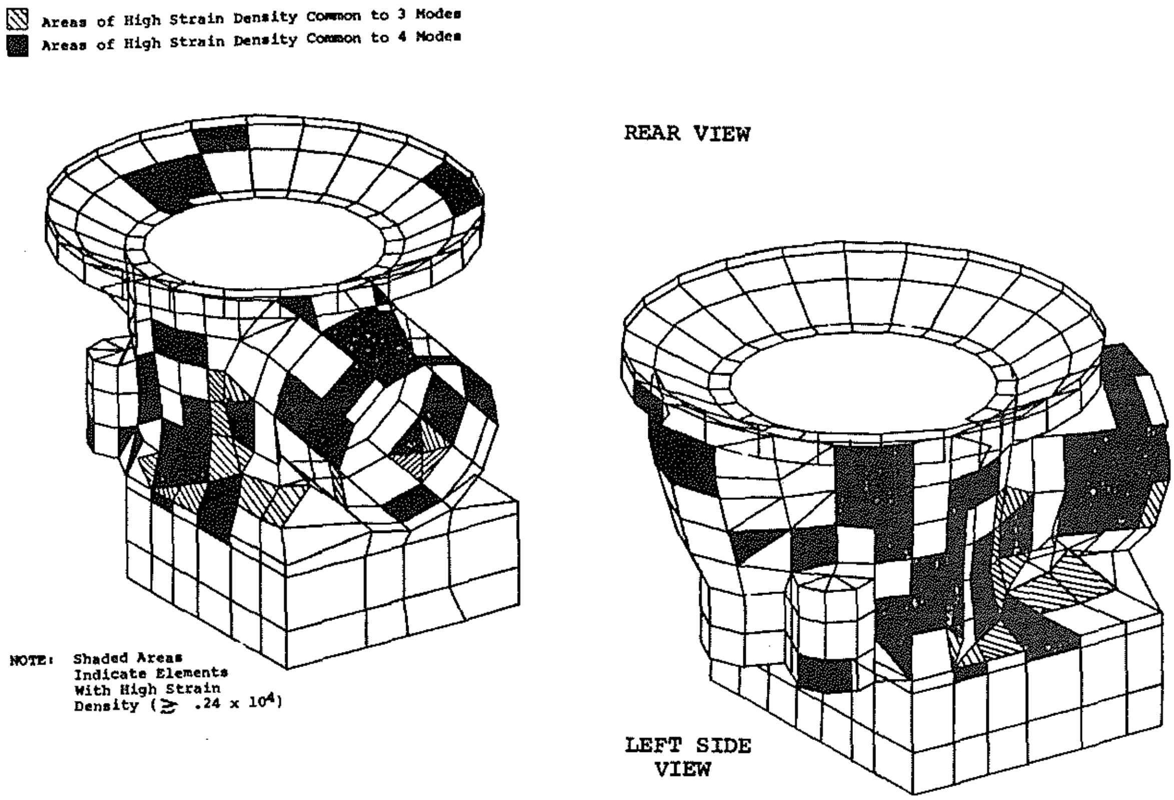

Figure 7. CH-47C Forward Rotor Transmission Case (With Sump) NASTRAN Model: Areas of High Strain Density.

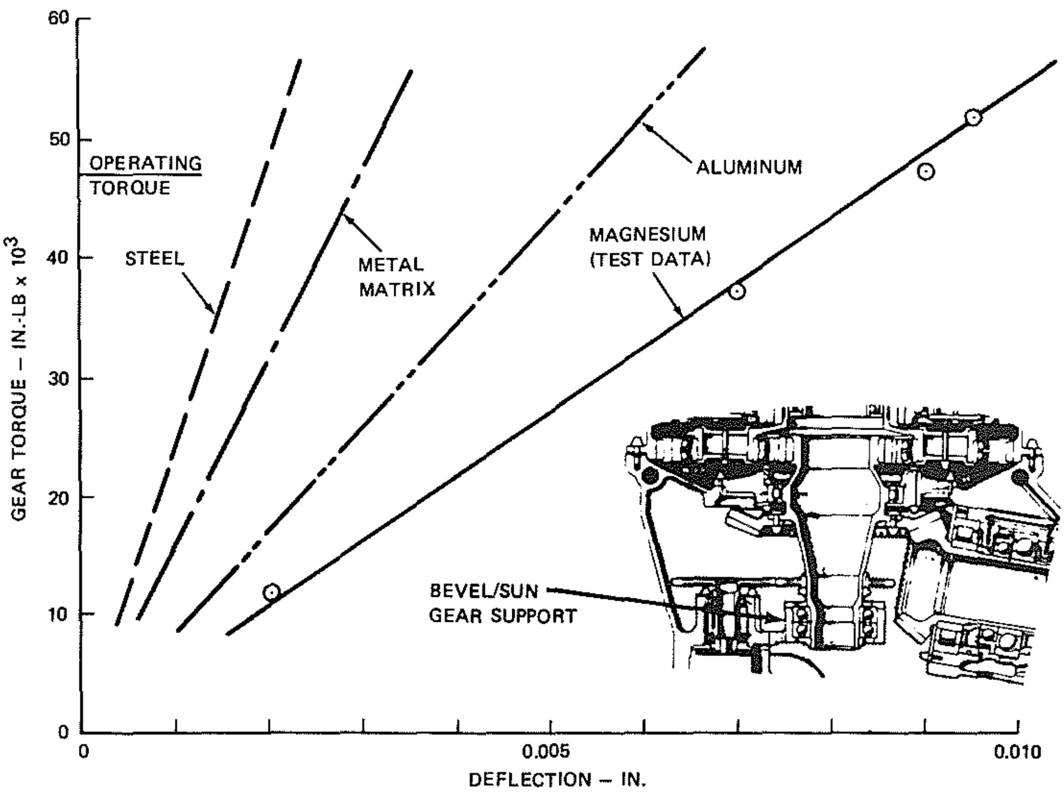

Figure 8. CH-47C Forward Transmission Dynamic Housing Deflection at Operating Speed.

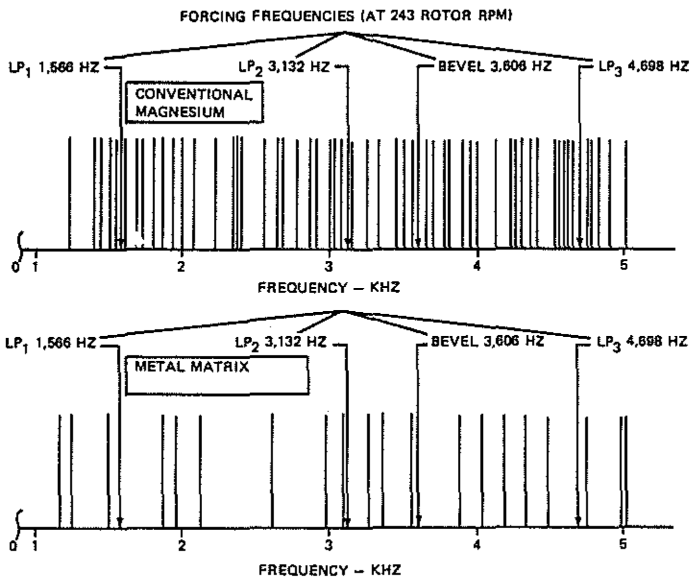

Figure 9. Typical Spectrum of Forcing Frequencies Versus Natural Frequencies.

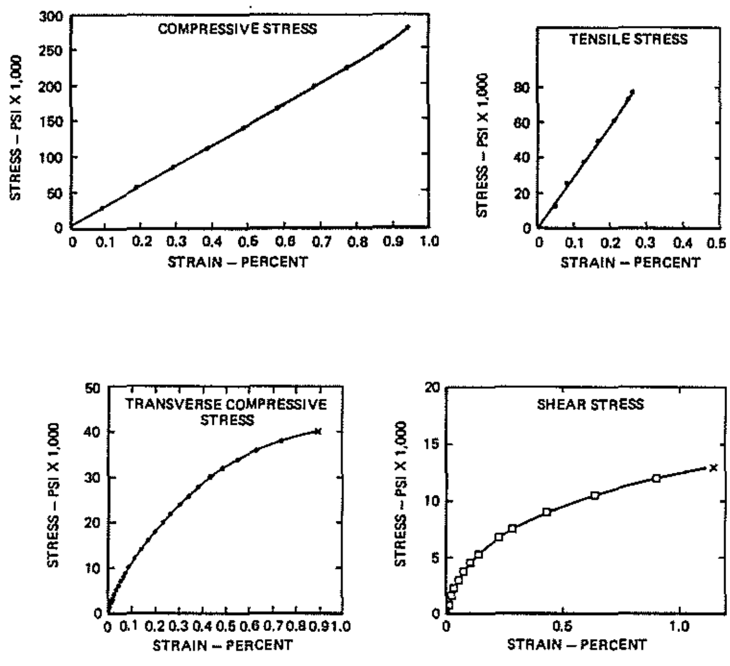

Figure 10. Properties of FP/Magnesium Composites (Unidirectional Fiber).

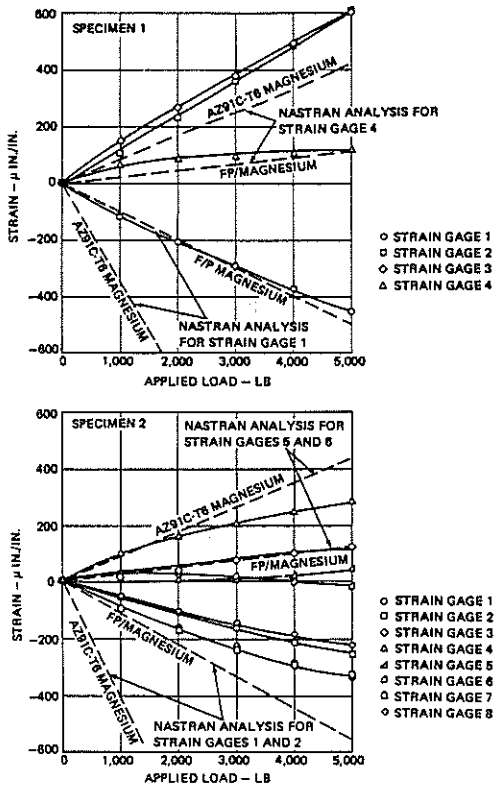

Figure 14. Strain Versus Load Data From Back-to-Back Test.

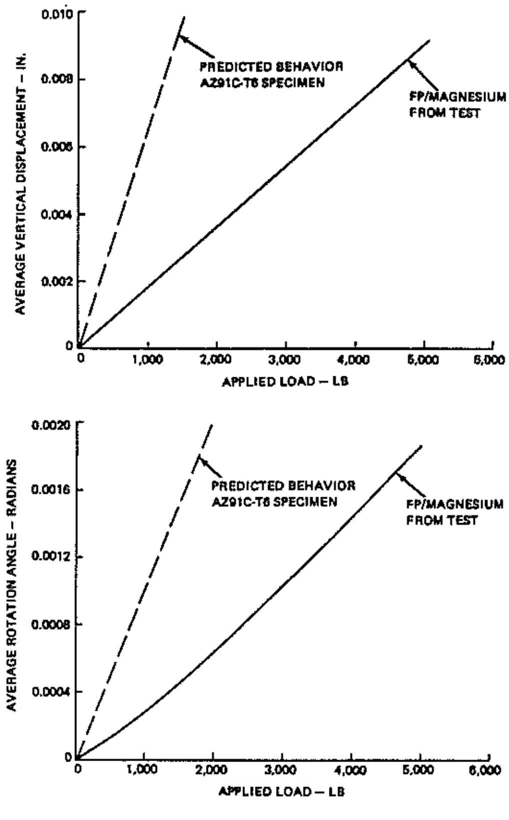

Figure 15. Vertical Displacement and Rotation of Center Section Versus Load From Back-to-Back Test.