THE OPTIMUM ANTI-TANK HELICOPTER

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Introduction

In the anti-tank battle the helicopter features as a force multiplier able, because of its own characteristics and those of its weapon system to engage armour at very long numerical odds. (Armour means an armoured unit having protection against air attack).

When looking to the next generation of anti-tank helicopters discussion naturally centres on improvements to the features of the helicopter and its armament and target acquisition systems. Certainly there are plenty of these in prospect but experience shows that they will also increase the cost of anti-tank helicopters and their operations.

Since, in peacetime, it is reasonable to assume that at best there is a fixed sum available for procurement, the number of helicopters which can be bought will decrease as the cost of an individual unit rises.

But it is well established that numbers, probably more than quality, have a decisive influence on the outcome of a battle. Moreover having sufficient numbers makes for flexibility, which is necessary when there have to be several widely spread units.

Thus, in pressing for increased effectiveness of the helicopter and its weapon system, the risk is run of diminishing the overall effectiveness of an anti-tank helicopter force and a study to establish the balance between quality and quantity is necessary.

A previous, superficial, investigation (Ref. 1) based on Lanchester's (Ref. 2) model of a battle suggests that if there is a fixed sum available then investment in quality will only increase overall effectiveness if the quality increases at a rate faster than the square of this cost. This is a very general conclusion, not directly connected with the helicopter-tank battle and there is a need for a deeper, more particular study in which some attempt is made to represent cost as well as the helicopter which fights a battle against tanks with missiles.

The aim is not to simulate a helicopter-tank engagement as such; models adequate to this purpose already exist. Nor is it the intention to directly compare one helicopter or armament system with another. Rather the aim is to study the consequences of investment in either quality or quantity, assuming that the two sides have the general characteristics appropriate to those of the helicopter-tank battle.

A mathematical model based on these ideas is developed in the next section.

Battle Model

This must represent an engagement with one side (the helicopter force) firing discrete missiles.

Since the aim is to take a total view rather than to investigate differences, a model which follows the fortunes of individual tanks and helicopters is unnecessary; numerical models do not provide easy insight. Lanchester's simple model will therefore be adopted; it is very much a global approach but it leads to straightforward analytical results which are most suited to investigating trends. The relative quality of the two sides is then represented by one parameter, the relative quantity by a second but the outcome of the battle depends only upon a single parameter which combines quality and quantity.

It is assumed that there are two sides - helicopters H and tanks T.

Lanchester's (Ref. 3) basic assumption is that the rate of loss of one side is proportional to the instantaneous size of the forces of the other i.e.

$$dT/dt = -aH \qquad dH/dt = -bT \tag{1}$$

The coefficients a and b are effective rates of fire averaged over the duration of the war, t is the time which has elapsed since the start of the war.

Eliminating t from (1) gives

$$eH^2 - T^2 = eH_o^2 - T_o^2 \tag{2}$$

$$e = (a/b)^{\frac{1}{2}} \tag{3}$$

where H_o, T_o are the initial numbers of helicopters and tanks. a/b is the relative rate of effective fire. e is of fundamental significance but is very hard to quantify since it is a function of the relative skills of the force commanders in choosing terrain which permits their own side but not the other, to fire; of the relative skills of the crews in making use of cover and their weapons and of the relative technical qualities of the weapons which determine the range, time of day or night (or state of the weather) at which the battle is fought. The wide speed range of the helicopter is a direct contributor to e as will be any factors which make it less vulnerable or less detectable. e is in fact a measure of the overall scenario; topological, technological, professional: it can be estimated through a combination of experience, trials and war games. In the helicopter-tank battle $e > 5$.

(2) describes the relative strengths of the two sides as the battle proceeds. The right-hand side is positive if the Lanchester coefficient L = eH_o/T_o is greater than unity.

Then when $T = 0, H>0$ and the helicopters win. Conversely if L<1 the tanks win. L is the single parameter in the Lanchester model; e measures the overall relative quality and H_o/T_o, the initial odds, measures the relative quantity.

L thus measures the degree of military superiority of the helicopters over the tanks. The fact that relative quality appears as a square root whilst relative quantity appears as a direct ratio is the feature of Lanchester's theory. Doubling the numbers of helicopters doubles the Lanchester coefficient and hence the military effectiveness. Doubling the relative rate of effective fire, which by change of tactics can be done for nothing or through changes in technology which cost the earth, increases L by only 40%.

This rolling of so many factors of such significance into a single number is simultaneously one of the weaknesses and strength of the Lanchester theory. Weakness because of the scope for doubt and argument, particularly over the effects of change or the relative significance of one factor or another at critical times. The strength is that there are so many factors, all unknown, uncertain, unquantifiable and of unknown weight, that if they were not reduced to a single parameter, nothing of meaning could be simply deduced.

The gross averaging and the uncompromising implication that victory means the total removal of the enemy forces from the field make the Lanchester model most suited to describing not so much an engagement as a long, ultimately decisive battle or campaign in which many influences, random or otherwise, are brought to bear. It is therefore helpful as a tool for planning, which must accommodate many varied and mostly unpredictable factors.

In planning for a campaign it is necessary to provide sufficient resource to see the fighting through and some definition of the finish has to be adopted. It is unlikely that, in any one engagement, every tank present will have to be destroyed before the force is effectively defeated but just what degree of loss i.e. T_o-T, will cause the tanks to call a halt? This problem is a matter for debate and separate investigation in any war game; any definition requires further assumption. Therefore, for simplicity, the end of the helicopter-tank battle will be taken to occur when no tanks remain; certainly this should give an upper bound to the cost.

The helicopters fire sufficient missiles to destroy the tanks at a rate a. Since some of the missiles fired will not destroy their targets the actual rate of firing is a/p where p is the average probability of a missile being effective. If m_o is the number of missiles originally provided for each helicopter and these are just sufficient for the duration then the campaign will come to an end after a time

$$t = am_o/p \tag{4}$$

Now the solution to (1), (2), can, as may be directly verified, be written in the form

$$T = T_o \cosh kt - eH_o \sinh kt$$
$$H = H_o \cosh kt - (T_o/e) \sinh kt \tag{5}$$

Here $k = (ab)^{\frac{1}{2}}$ and t is the time for which the battle has progressed.

At the finish $T=0$ and $t = (ab)^{-\frac{1}{2}} \tanh^{-1}(1/L)$ so that the total number of missiles which must be made available for each helicopter is

$$m_o = (e/p) \tanh^{-1}(1/L) \tag{6}$$

It will be appreciated that m_o is not the total number of missiles which must be provided for each helicopter. Others will be needed in training, to keep the repair and maintenance pipe lines filled and to make up for losses and delays in supply due to other enemy action. Also it does not follow that each helicopter will necessarily carry or be designed to carry m_o missiles; this total can be provided by re-arming at the end of each of several sorties.

When all the tanks are destroyed the number of helicopters remaining is $H_E = H_o(1-1/L^2)^{\frac{1}{2}}$. As L increases away from unity the percentage of helicopters remaining at first rises very rapidly but beyond $L \sim 1.3$ further improvement with increasing L is slow. (see Fig. 1).

One measure of the effectiveness of the helicopter force is the ratio of tanks destroyed to helicopters lost. i.e. $T_o/H_o - H_E$. Alternatively this exchange ratio X is given by

$$X = e/L - (L^2 - 1)^{\frac{1}{2}} \tag{7}$$

Helicopter Model

The characteristic of the helicopter which makes it such a formidable opponent to the tank is its wide speed range. It is able to hover, or to fly slowly in any direction and so to observe or fire from cover at long range whilst remaining undetected. It can also be flown at high speed and with great precision at very low altitudes so that it is able to use ground when moving rapidly between fire positions.

With very few exceptions these qualities are common to all helicopters. Of course, an engine, rotors and a control system are needed but as yet these do not vary much in any important respect between helicopters of comparable size. The underlying engineering principles are well understood and such central factors as the structure weight and the aerodynamic performance can be estimated fairly simply and with good accuracy. The ability to hover for a long period, for example, is only a function of the fuel used and the weight of the structure needed to make that manoeuvre possible. In an engineering and hence ultimately in a cost, sense all that matters is the efficiency of the design, constructional and operational process.

The ability to find and destroy targets at long range is more a function of the missile and target acquisition systems than of the helicopter which is essentially a vehicle making their exploitation possible.

Thus the qualities of the helicopter which make it so useful are more appropriately absorbed into a description of the battle, i.e. into the parameter e, rather than directly into a model of the helicopter. The helicopter alone can be seen as a carrier, at a cost, of the missile and target acquisition system. The factor connecting the helicopter with cost is its weight, which in turn depends upon the weight of the armament system and of the structural and energy conversion efficiences which can be achieved.

The gross weight W of a helicopter is given by

$$W = E + F + P \tag{8}$$

where $E = rW$ is the weight ready to fly, F is the fuel weight and P is the payload.

The fuel weight is $F = shWd$ where s is the specific fuel consumption, h is the power required per unit of weight and d is the endurance.

Substituting for F and E in (8)

$$W = P/1 - r - shd \tag{9}$$

The quantities r, s, h can be estimated and an endurance can be specified on the basis of operational experience. The quality 1-r in the disposable load fraction and is a measure of the structural efficiency of the design. The products (hs) in the overall efficiency of energy conversion.

If $G \equiv 1/1\text{-}r\text{-}shd$ then $W = GP$. G is characteristic of all helicopters but its precise value depends upon the endurance. A typical value is $G = 10/3$.

The payload is taken to be made up of two parts

(i) the weight of that equipment which is necessary to convert the helicopter from a flying machine into an anti-tank helicopter. The weight is denoted by P_o; the essential constituents of P_o are the weight of:-

crew	defensive measures (structural, armament, electronic)
armour	sensors
fixed fittings for missiles	communication and navigation equipment

(ii) the weight of the missiles. If the number of missiles in a full load is f then

$$P = P_o + wf \tag{10}$$

where w is the weight of a single missile

and

$$W = G(P_o + wf) \tag{11}$$

If the cost of a helicopter per unit of empty weight is c and the total cost is C then

$$C = crW = \alpha(P_o + wf) \ ; \ \alpha = crG \tag{12}$$

If in total m_o missiles have to be placed at the disposal of each helicopter and operational studies show that on average this will require n sorties then $f = m_o/n$ and

$$C = \alpha(P_o + wm_o/n) \tag{13}$$

The gross weight of the helicopter is then known as a function of m_o which is given by (6).

(13) is a sufficient cost model of the helicopter for our purposes.

Cost Model

This brings together the helicopter and battle models and provides an overall framework for judgment.

The underlying assumption is that the war or campaign will be of relatively short duration, fought with what is initially to hand. This resource must be accumulated in peace time, first through a Research and Development phase in which the weapons and systems thought to be the most suitable are selected and developed. This is followed by a Procurement phase during which the systems are procured in quantity and supplied to the forces who train with them for war.

The amount of money spent on Research and Development is essentially independent of that spent on any subsequent procurement. It is therefore a function of the firepower of individual rather than of the total of, fighting units. That which is spent on Training will greatly influence the effective firepower of the whole and of individual units. Money for these two vital activities is really a continuing, indirect spend.

On the other hand the costs incurred in the accumulation of materiel are a direct preparation for war and are those which concern us here; they are proportional to the size if the forces involved and also to the supplies, particularly ammunition, which they will require.

This is a general statement but when made particular to the helicopter - tank battle it can be put in the form

$$D = A H_o + B m_o H_o \tag{14}$$

D is the total direct cost of this battle, A is the cost of an equipped helicopter and Bm_o is the cost of the missiles which must be at the disposal of each.

If E is the cost of equipping a helicopter then

$$A = E + C = E + \alpha(P_o + w m_o / n) \tag{15}$$

hence

$$D = (\alpha P_o + E) H_o + (B + \alpha w/n)\, m_o H_o$$

$$= Q H_o + M m H_o\, e \tanh^{-1}(1/L) \tag{16}$$

$Q = \alpha P_o + E$ is the cost of the equipment plus that share of the cost of the helicopter which is necessary to carry the equipment. $M = (B + \alpha W/n)p$ is the cost of an (effective) missile together with that share of the cost of the helicopter which is necessary to allow it to carry the missile. M and Q are the costs of bringing an effective missile and its supporting equipment to the battle. In this way the costs of the helicopter are absorbed into the costs of equipment and missile, so underlining its role as a carrier.

The direct cost of destroying one tank in D/T_o where

$$\frac{D}{T_o} = \frac{QH_o}{T_o} + M L \tanh^{-1}(1/L) \tag{17}$$

or, in non-dimensional terms

$$D/MT_o = R = (Q/Me)\, L + L \tanh^{-1}(1/L) \tag{18}$$

Since M is to all intents and purposes the cost of a missile, R is the ratio of the actual cost to the minimum cost of destroying each of T_o tanks. Q/Me is a measure of both the cost - Q - and the contribution - through its share in e - of a helicopter. It is a measure of the cost - effectiveness of the anti-tank force taking into account relative military skills and the costs and performance of the missiles and equipment. The smaller Q/Me the more cost-effective is the whole.

In Fig. 1 R is plotted as a function of L for the cases $N = Q/Me = 3, 10$. There are clearly marked minimum values of R.

Differentiating (18) $w.r.t. L$ (which since e is fixed is the same as differentiating $w.r.t.$ H_o/T_o)

$$\partial R/\partial L = N + tanh^{-1}(1/L) - L/L^2 - 1 \tag{19}$$

Thus R is a minimum when

$$L/(L^2-1) - tanh^{-1}(1/L) = N = Q/Me \tag{20}$$

or

$$QH_o/MT_o = L^2/(L^2-1) - L\,tanh^{-1}(1/L) \tag{21}$$

(21) is the size of the helicopter fleet needed to destroy T_o tanks for a minimum direct cost.

The corresponding value of R (=R_m) is

$$R_m = D_m/MT_o = L^2/L^2 - 1 \tag{22}$$

The variation of R_m with L is also shown in Fig. 1. At each point on this curve N is given by (20). N decreases rapidly with L and the minimum cost decreases rapidly with increasing cost-effectiveness. But to achieve this minimum the degree of military superiority must increase more and more rapidly. This feature seems certain to impose a practical limit on what can be achieved in the way of cost-reduction through increase in military superiority and effectiveness.

The number of helicopters remaining when all the tanks are destroyed is $H_E=H_o(1-1/L^2)^{\frac{1}{2}}$. If the force is of the size and constitution for minimum cost then

$$H_E/H_o = 1/R_m^{\frac{1}{2}} \tag{23}$$

Although Fig. 1 shows R for $1<L<2$ the practical range of L is much smaller than this - for the very good reason that the aim of the enemy is to ensure that $L<1$. Therefore the variation of R with N is much greater than its variation with L and the minimum R_m is a good guide to the scale of cost at a particular value of N.

Discussion

Perhaps the most important result of this limited and elementary analysis is that there is a degree of military superiority which makes the cost of the helicopter - tank battle a minimum. The degree of military superiority is a function of the cost-effectiveness and the associated minimum cost R_m is given by (22). This last is an economic extension of Lanchester's theory for it shows that in order to win it not only necessary for $L>1$ but also that a minimum sum of money should be made available. The size of that sum depends upon the cost-effectiveness of the total system, which is a military-industrial matter.

The first step is to establish the present position on this military and economic scale. Information is of course extremely difficult to come by because in the arms trade there is usually no correlation between price and cost. But reasonable guesses are possible. For example Q/pM is 0(100) and $h \sim 0.2$; $p \sim 0.4 - 0.5$. Therefore QH_o/MT_o is 0(10) which for minimum cost requires $L \sim 1.05$ so that $e \sim 5$ and the exchange ratio is 7:1. These figures are in general agreement with the average of trials and experience; hence it may be assumed that present operations are in the region of minimum cost.

For $L \sim 1.05$, $R_m \sim 10$ so that the direct cost of destroying a tank by this means is 10 times the cost of a missile - which is about equally spaced between the cost of a tank and the cost of employing a single private soldier to rest the missile against the tank before igniting the fuse. Given the flexibility and mobility of the helicopter this is a good indication that whilst the anti-tank helicopter costs *money*, it is not *expensive*.

But there is great scope for improvement for $N \sim 10$ also, which is a very low cost-effectiveness and over 70% of the helicopter force is lost in destroying all the tanks. At $N = 10$ the minimum cost is sharply defined so that departures from the optimum *will* be expensive. Also $L = 1.05$ is in a region of steeply rising costs and rate of helicopter loss so that there is great military and financial sensitivity to the inevitable fluctuations in e.

There is in prospect a whole range of improvements which should - unless the tanks are provided with appropriate counter measures - allow a considerable increase in the rate of effective helicopter fire - although there is likely to be a corresponding increase in the cost of missiles and equipment as a consequence. But (19) shows that if Q and M rise at the same rate then an increase in e will produce a directly proportionate gain in cost effectiveness, so increasing L and decreasing R_m. The actual direct cost will then increase or decrease according as the change in missile cost is greater or less than the decrese in R_m. Thus if L is increased from 1.05 to 1.12 whilst doubling the costs of missiles and equipment R_m falls to 5 and the total direct cost is sensibly unchanged. The loss sustained by the helicopter fleet is reduced to 55%.

The cost effectiveness must then improve by a factor of 3 which requires e to treble and the effective rate of helicopter fire to improve by an order of magnitude. The number of missiles which must be provided for each helicopter is almost doubled.

The optimum helicopter then has a much increased all up weight. Precise figures are unimportant but a present-day anti-tank helicopter with 1000 kg. of equipment, a load of 6-8 missiles and two hours or so endurance would have a gross weight of 4,000 kg.

If in order to get the necessary improvements in range and lethality the missile weight is doubled and the weight of the equipment is increased by only 20%, the minimum cost anti-tank helicopter has a gross weight of 6,000 kg. The missile load assuming three sorties, is increased to 14.

Since there is no increase in overall cost, the money to pay for the more expensive equipment and missiles comes from a reduction in the number of helicopters - the fleet is reduced in size by more than 60%. Although the equipment costs are assumed to be doubled the proportion, of the total sum, which is expended on equipped helicopters falls from 90% to 67%.

Now it may be that a reduction numbers on this scale is not a viable practical proposition. But if other operational factors dictate that more such helicopters should be provided then the cost of destroying a tank will rise in proportion to the increase in fleet size.

In conclusion it is worth drawing attention to the contrast between the part apparently played by the helicopter in this operation and the part it plays in the mathematical model.

The helicopter as such virtually disappears from the military-economic equation. This is because the helicopter is only a carrier used to get the weapons system to the right place. There are other ways of delivering missiles so that the true value of the helicopter can only appear out of a larger scenario in which some sort of competition takes place. Of greater concern is the fact that theory completely discounts the possibility of any financial advantage arising from an increase in the number of helicopters. All financial improvements come as a consequence of a higher relative rate of fire; if this is got from more expensive missiles and equipment the number of helicopters must be reduced. If the cost of a missile remains fixed and improvements in effective rate of fire are sought in more expensive equipment (which includes the helicopter cost) then the cost-effectiveness will only improve if the relative rate of effective fire increases faster than the square of the cost.

Thus from the purely industrial point of view it must be a matter of priority to devise ways and means of increasing the direct contribution of the helicopter to the effective rate of fire. Ideally this should be done at lower aircraft cost for there would then be a second improvement to the cost-effectiveness. But even if the helicopter itself costs more there could very well be an overall cost reduction since improvements in cost-effectiveness have a marked effect on total cost.

To achieve improvements directly through the helicopter will require radical thought but surely it is time for that anyway.

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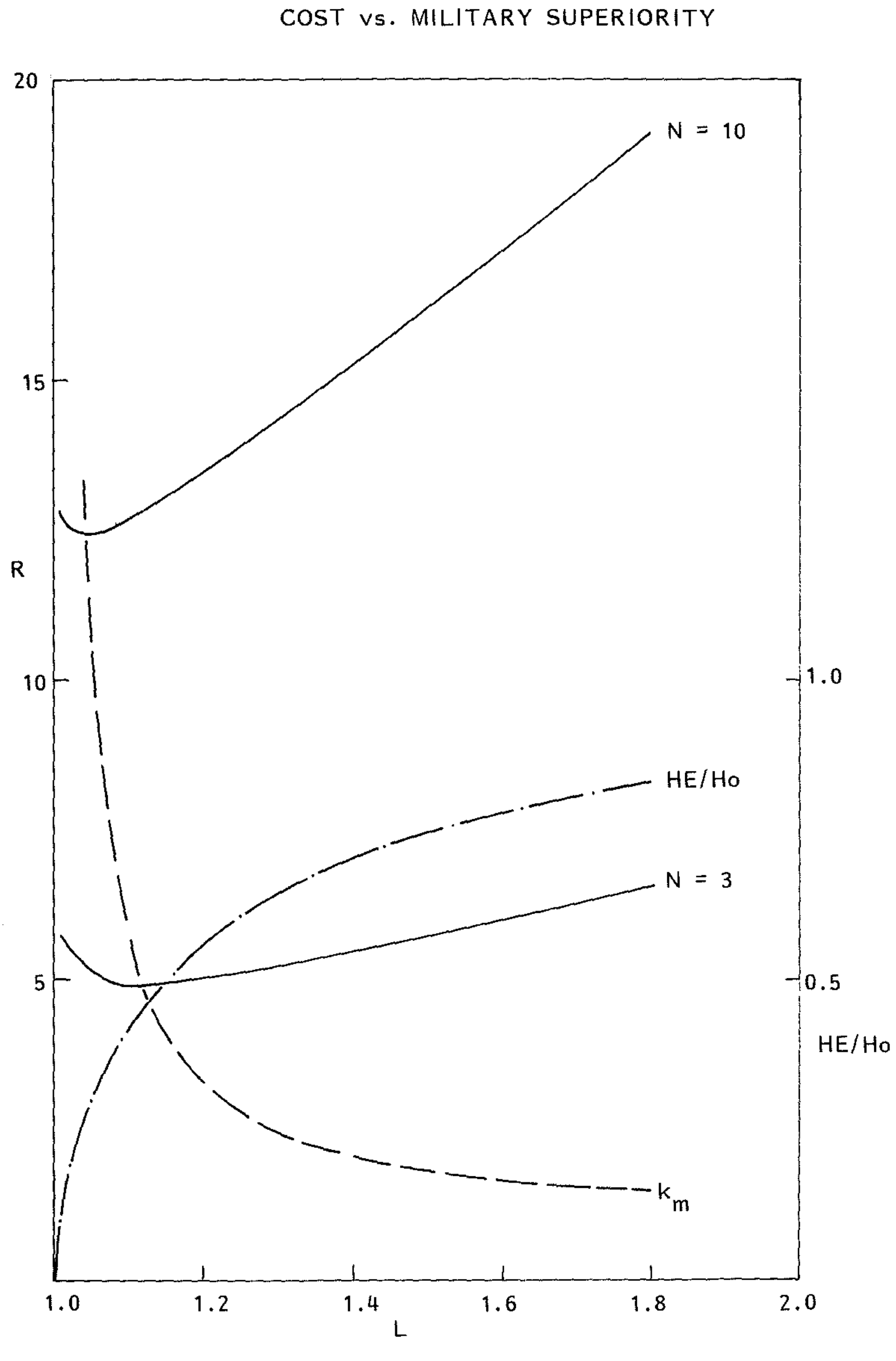
COST vs. MILITARY SUPERIORITY
20
15
10
5
R
N = 10
HE/Ho
N = 3
k_m
1.0
0.5
HE/Ho
1.0
1.2
1.4
1.6
1.8
2.0
L