PARAMETRIC IDENTIFICATION OF A MODEL FOR THE DYNAMICS OF HELICOPTER MOTION USING OPTIMAL CONTROL THEORY METHODS

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ABSTRACT

This paper is the result of a series of experiments to identify parameters of a model for an object (a system) using methods based on the theory for acheiving optimal control of a system (or object).

The advantages of solving this dual-problem in flight dynamics, by the use of one and the same method are demonstrated. The dual problem being that of model identification and control optimization.

An example is given. Namely, model identification for coaxial helicopter motion during a manoeuvre in a vertical plane, using one of the optimal control theory methods.

I. INTRODUCTION

Experience in the field of control systems design has clearly shown that the development of any control system is reduced to solving two interconnected tasks: task of creating an object model and of developing a control algorithm corresponding to the given model and the given purpose of the functioning system. For a given system (or object) parametric identification may be carried out at the same time as system control optimization.

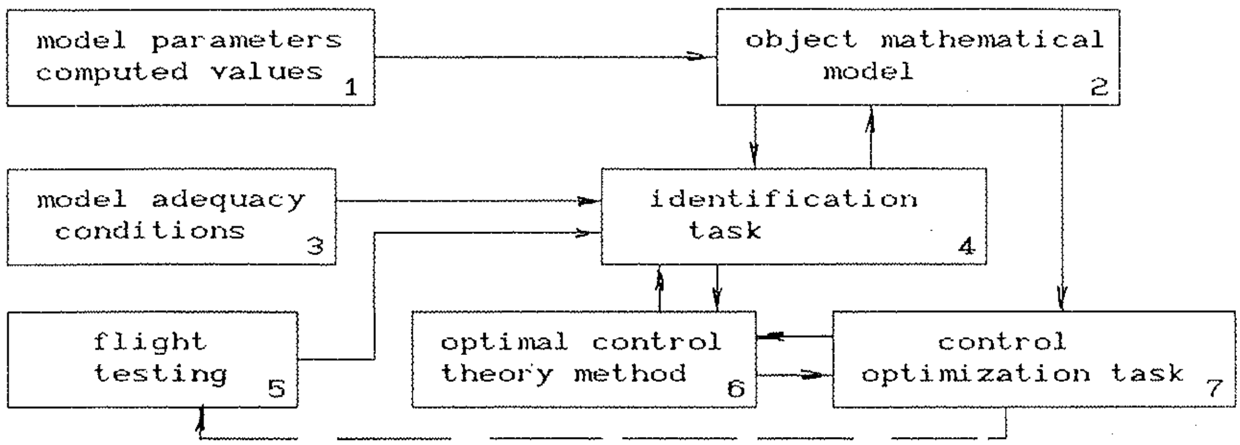
Though a lot of work has been done in the field of the control system (or object) model identification [1...4] in this case helicopters [1] by the progress of the identification theory and practice new tasks appear and get solved. One of such problem is that of an integreted approach to the solving of the flight dynamics two-sided task i.e. model identification and control optimization.

The investigations carried out proved the expediency of applying the same methods to identification tasks as to control optimization ones. Optimal control theory approximation methods are considered to be promising as they may successfully be applied to both linear and non-linear models. One of such methods will be examined below namely the method of successive improvements in control [5,6] based on V.F.Krotov theory valid in general conditions adequate for optimality.

The use of this improvement method permitted the development of the method for model parametric identification by means of a run of experiments. This method is applicable to linear and non-linear models. Non-lineality is a feature of helicopter dynamics models.

The second distinctive feature is the use of a run of experiments permitting a decrease in the effect of random factors.

The third distinctive feature is the continuity of both flight dynamics task parts (fig. 1) which in practice reduce the volume of calculations when solving the task as a whole.



1...6 identification task components
2, 6, 7 optimization task components

Fig.1 Block diagram defining the steps of the flight dynamics tasks solving.

II PROBLEM IDEFINITION

It is assumed that a run of experiments was made regarding the object in measuring control vector U (t), and part of the object state vector time variables:

$$y(t) = \{ X_1(t),\dots X_l(t)\dots X_m(t) \} \quad \dots \quad (1)$$

$$X_l(t) = F(t, X_l, U_l, A) \qquad l=1\dots n \quad \dots \quad (2)$$

X(t)- N dimensional representation phase vector;
U(t)- N order control vector;
Y(t)- R : $R \le N$ order measured parameters vector;
A - S dimensional representation corrected parameters vector;
M - number of experiments (number of modes at testing).

The problem consists of determing vector A so as to minimize the functional which consists of the squared difference integral values of the measured (X_T) and calculated (X_C) phase vector values corresponding to solutions of (2) equation system:

$$I = \int_{t_H}^{t_k} \sum_{l=1}^{M} \sum_{j=1}^{R} D_j (X_{l_{j_T}} - X_{l_{j_C}})^2 \, dt \, , \quad \dots \quad (3)$$

Dj is the weight coefficient of the phase state j component.

Further, the functional minimum is found in a similar way to the solvution of the control optimization tasks i.e. a conjugated system (vector-function) is determined by solving an auxiliary equations system

$$\frac{\partial \psi_{lj}}{\partial t} = - \frac{\partial H_l}{\partial X_j} = - \frac{\partial}{\partial X_j} (\sum_{j=1}^{N} \psi_j F_{lj}) \qquad l = 1\dots M \quad \dots (4)$$

The initial conditions for integrating the auxiliary equations system (4) for the identification task are zero:

$$\psi_{j_l}(t_k) = 0$$

Correction of the unknown parameters vector A is similar to the control correction when solving the control optimization task and is done by iterations:

$$Aj_i = Aj_{i-1} + \delta Aj \qquad \ldots . (5)$$

j= 1.....S, i- number of iteration;
A correction may be done both at each step (parameter A varies in time):

$$\delta A_j(t) = K_j \sum_{l=1}^{M} \frac{\partial H_l}{\partial A_j} \qquad \ldots . (6)$$

and integrally:

$$\delta A_j = K_j \sum_{l=1}^{M} \int_{t_H}^{t_k} \frac{\partial H_l}{\partial A_j} dt \qquad \ldots . (7)$$

2.1 Peculiarities of the problem solution

The problem of vector A definition is solved when one of the following conditions is acheived:

(a) - convergence of the calculated functional value with a predetermined degree of accuracy:

$$\varepsilon_I = \left| \frac{I_i - I_{i-1}}{I_i} \right| \le \varepsilon_{I_m}$$

(b) - convergence of the component of the parameters of vector A to the nearest approximation:

$$\varepsilon_{\alpha j} = \left| \frac{A_{j_i} - A_{j_{i-1}}}{A_{j_{i-1}}} \right| \le \varepsilon_{\alpha j_m}$$

(c) - integral convergence of the parameter of vector A to the norm:

$$\varepsilon_\alpha = \left| \frac{A_i - A_{i-1}}{A_{i-1}} \right| \le \varepsilon_{\alpha_m} \qquad A_i = \sqrt{\sum_{j=1}^{s} A^2_{ji}}$$

2.2 Weight coefficients

The necessity of specifying weight coefficients Dj (3) in the parametric identification problem is dictated by the physical meaning of the problem to be solved, since the helicopter motion parameters change within different limits. Thus, the change of flight speed is measured in the order of one to tens m/s, the change of rotor angular speed is measured in the order of 0.1 to 0.5 rad/s, the change of a maneuvering helicopter altitude - of up to a radian etc.

Since the total functional consists of several components (3) it is neccessary to specify the weight of each component according to the engeneering evaluation of this or that parameter, which is important in the mode examined. So, for example, if for practical reasons, the following deviation values are taken as equal to: 1 m/s for flight speed; 0.05 rad for pitch angle; 0.1 rad/s for main rotor angular speed, then corresponding values of weight coefficients must be selected to conform to the following ratios: D_1: D_2: D_3 = 1: 500 : 100 . Since the sensitivity of the problem change in one or other of the components of the vector state, depends on the values of weight coefficients, it is quite evident that engeneering experience will be necessary to evaluate correctly vector D.

2.3 The method application algorithm

As the components of the vector determined are not one-dimensional, two types of improvement algorithms may be applied:

a) The method of successive elimination of each of the components of the vector, that is the one component of vector A is determined, A_1 for example, (the rest A_j are not changed i.e. in formulae (6) and (7) the corresponding coefficients are taken equal to zero: $K_j = 0$, $j \neq 1$) and the calculation is done by convergence in ε_I or in ε_α. Then the obtained value is recorded and the next parameter is determined etc., so at each step the correction is calculated only for one parameter. After the determination of the vector A last component the calculation returns to the first component A with new A_j ($j \neq 1$) values and so it goes on up to the total convergence in vector A. In fig. 2 the change of the functional and the corrected parameters are presented according to the iterations in the process of minimization, upon the given diagram;

b) The method of parallel determination of components with a simultaneous correction of the corrected parameter vector components. As the computing practice demonstrates, this method is reliable enough and works faster than the first one. They are compared in fig. 2.

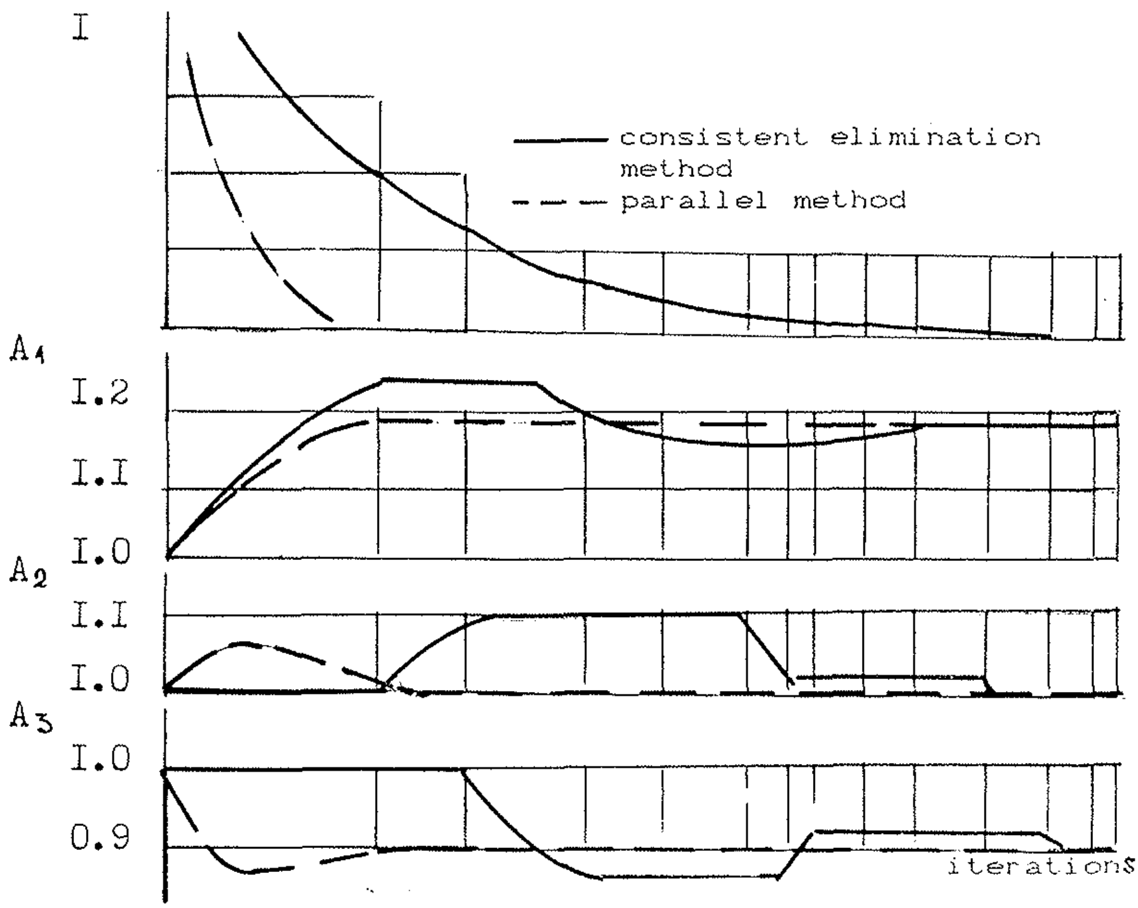


Fig. 2 Iterational change of functional I and correcting parameters A_1, A_2, A_3

III IDENTIFICATION OF A COAXIAL HELICOPTER MOTION MODEL IN A MANOEUVRE IN A VERTICAL PLANE

The analysis of the flight test results of a helicopter performing manoeuvres in the vertical plane (steep climbs, dives) proves that though the integral parameters like overload are rather well represented by the model, but the flight test results demonstrate some difference as regards the flight speed, pitch angle and main rotor speed values.

The problem demands determination of the vector of constant coefficients, correcting the calculated values of the coaxial helicopter motion equations coefficients, for the vertical manoeuvres.

3.1 Motion equations

A coaxial helicopter motion model developed by the Kamov Helicopters Scientific & Technology Company, is used as a motion model. Due to the aerodynamic symmetry of the coaxial helicopter, the side motion may be disregarded, which considerably simplifies the system.

The equations are written in the right side coordinate system related to the main rotor shaft.

Accepted designations:

- V_x, V_y – flight speed components in respect to the main rotor shaft related axes;
- ω – main rotor angular speed;
- ϑ – pitch angle;
- $\varphi_з$ – main rotor setting angle;
- ω_z – angular speed of rotation around OZ axis;
- $\bar{X}$, $\bar{Y}$ – components of forces acting upon the helicopter and referred to the helicopter mass;
- $\bar{M}_z$ – moment in respect to OZ axis referred to the helicopter inertia longitudinal moment;
- $\bar{M}_к$ – main rotor shaft torque referred to the main rotor inertia polar moment;
- $\bar{M}_{к\,дв}$ – engine moment on the main rotor shaft referred to main rotor inertia polar moment;
- δ_z – main rotor resultant force deviation angle at longitudinal control;
- φ – main rotor collective pitch angle.

$$\frac{dV_x}{dt} = \bar{X}(V_x) - \bar{X}(V_{x_o}) + V_y\omega_z - g[\sin(\vartheta-\varphi_з) - \sin(\vartheta_o-\varphi_з)+$$

$$+ \bar{X}^{\delta z}\Delta\delta z + \bar{X}^{\varphi}\Delta\varphi \;;$$

$$\frac{dV_y}{dt} = [\bar{Y}(V_x)-\bar{Y}(V_{x_o})+V_y\omega_z-g\cos(\vartheta-\varphi_з)+\bar{Y}^{\delta z}\Delta\delta z+\bar{Y}^{\varphi}\Delta\varphi](\frac{\omega_o+\Delta\omega}{\omega_o})^2-$$

$$- V_x\omega_z-g\cos(\vartheta-\varphi_з)$$

$$\frac{d\omega_z}{dt} = \Delta\bar{M}_z(V_x)-\Delta\bar{M}_z(V_{x_o})+\bar{M}^{Vy}\Delta Vy+\bar{M}^{\omega z}\Delta\omega_z+\bar{M}_z^{\delta z}\Delta\delta z+\bar{M}_z^{\varphi}\Delta\varphi$$

$$\frac{d\vartheta}{dt} = \omega_z$$

$$\frac{d\omega}{dt} = \Delta\bar{M}_{K_{ДВ}} - [\bar{M}_K(V_x) - \Delta\bar{M}_K(V_{x_0}) + \bar{M}_K^{Vy}\Delta Vy + \bar{M}_K^{\omega z}\Delta\omega_z + \bar{M}_K^{\delta z}\Delta\delta z + \bar{M}_K^{\varphi}\Delta\varphi + \bar{M}_K(V_{x_0})](\frac{\omega_0+\Delta\omega}{\omega_0})^2 + \bar{M}_K(V_x)$$

Engine operation equation considering the operation of the main rotor speed governer:

$$\frac{d\Delta\bar{M}_{КДВ}}{dt} = \frac{1}{T_{ДВ}}(-\Delta\bar{M}к_{ДВ} - K_\omega\Delta\omega)$$

The model implies some non-linear dependences, i.e. :

- dependence on the pitch angle of the force of gravity components projected on to the axes;
- dependence of the main rotor thrust and torque upon the main rotor speed (main rotor angular speed);
- dependence of the forces and moments upon the flight speed.

It is necessary to calculate the values of the coefficients A_1 , A_2 , A_3 , A_4, A_5 :

$$\bar{M}_Z^{Vy} = A_1(\bar{M}_Z^{Vy})_T \qquad \bar{M}_Z^{\omega z} = A_2(\bar{M}_Z^{\omega z})_T \qquad \bar{M}_Z^{\delta z} = A_3(\bar{M}_Z^{\delta z})_T$$

$$\bar{M}_K^{Vy} = A_4(\bar{M}_K^{Vy})_T \qquad \bar{M}_K^{\delta z} = A_5(\bar{M}_K^{\delta z})_T$$

so as to reduce the difference between the model and flight tests.

3.2 Specification of basic standard dependences

The following motion parameters obtained from flight testing are accepted as the most valid (ref. fig. 3) : flight speed (V_x), pitch angle (ϑ), main rotor revolutions (n). Except for these standard dependences, the change of control corresponding to each of the modes (X_B) is also taken from the flight test results. In the example presented, those modes are selected for identification, where only the longitudinal control changes (the value of the collective pitch angle is not changed in the process of the mode realization) i.e. the control vector is onedimensional.

The number of modes accepted as basic standard according to the flight test results, is equal to 5.

So, in the equation system describing helicopter motion in the vertical plane, orders of control vectors and corrected coefficients etc. following are defined:

Nu = 1 - control vector order;
N = 7 - initial equations system order;
S = 5 - corrected parameters vector order;
R = 3 - changed parameters vector order;
M = 5 - number of experiments.

Based on the flight test results (fig.3) the basic standard dependences are defined for calculation - change in rotor resultant force deviation angle, change in flight speed, change in pitch angle and rotor change in angular speed.

The dependences obtained for five calculated modes are presented at fig.4, where time is equal to zero at the beginning of the longitudinal control change in each mode.

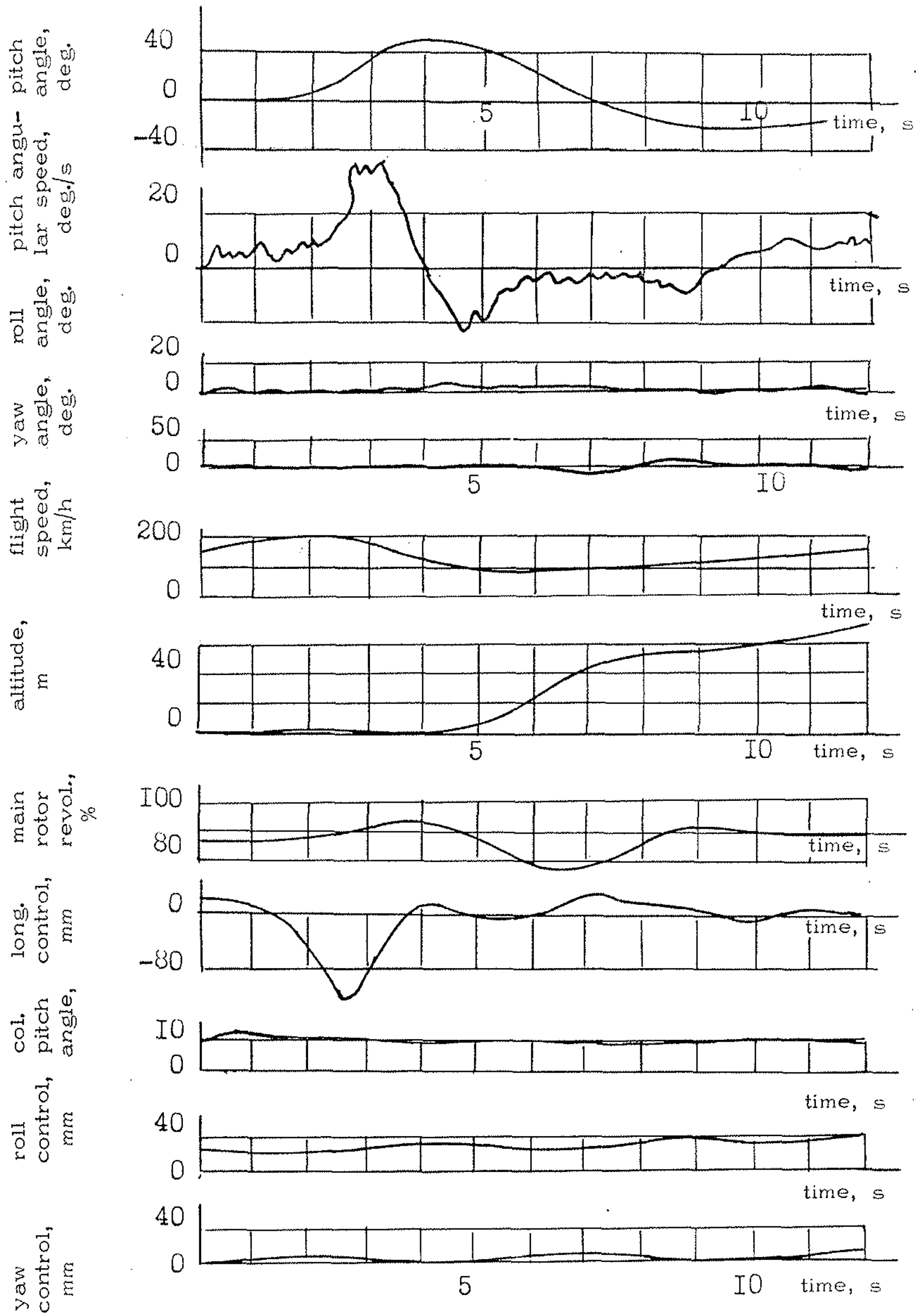


Fig.3 Change of the helicopter motion parameters in the process of performing steep climb manoeuvre

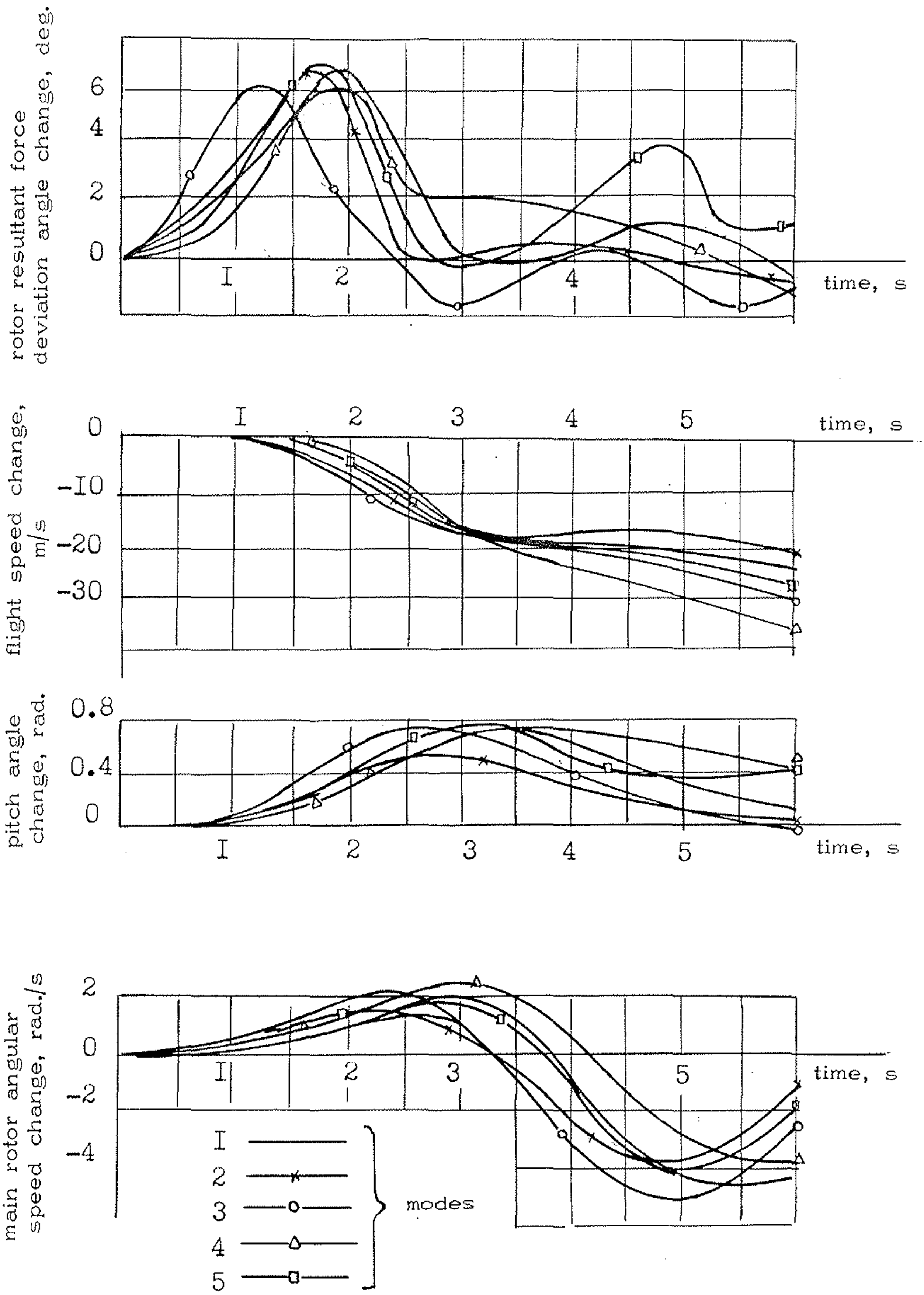


Fig. 4 Change of the longitudinal control (δ_z), pitch angle (ϑ), speed (V_x) and rotor angular speed (ω) in the process of performing steep climb manoeuvres

The minimized functional is written in form of:

$$I = \int_{t_o}^{t_k} \sum_{l=1}^{5} \sum_{j=1}^{3} D_j (X_{l_{j_T}} - X_{l_{j_C}})^2 dt = \Delta I_1 + \Delta I_2 + \Delta I_3$$

$$\Delta I_1 = D_1 \int_{t_o}^{t_k} \sum_{l=1}^{5} (\Delta V_{x_{l_T}} - \Delta V_{x_{l_C}})^2 dt , \quad \Delta I_2 = D_2 \int_{t_o}^{t_k} \sum_{l=1}^{5} (\Delta \vartheta_{l_T} - \Delta \vartheta_{l_C})^2 dt ,$$

$$\Delta I_3 = D_3 \int_{t_o}^{t_k} \sum_{l=1}^{5} (\Delta \omega_{l_T} - \Delta \omega_{l_C})^2 dt$$

where D_j - weight coefficient in the functional component: D_1 - for the flight speed, D_2 - for the pitch angle, D_3 - for the rotor angular speed.

The weight coefficients D_1 , D_2 and D_3 values relationship for the condition of the quantitative value comparability in the functional components of deviations from preliminarary determined i.e. flight speed ΔV_x in 1 m/s, pitch angle $\Delta\vartheta$ in 0.05 rad. and rotor angular speed $\Delta\omega$ in 0.1 rad/s : D_1=1, D_2= 500, D_3= 100.

To illustrate the above, the relationship of the functional components ΔI_1 , ΔI_2 , ΔI_3 at selected weight coefficient values in the process of the functional minimization when solving the problem starting with different initial values A_o = 0.5 and A_o = 2, is presented at fig. 5.

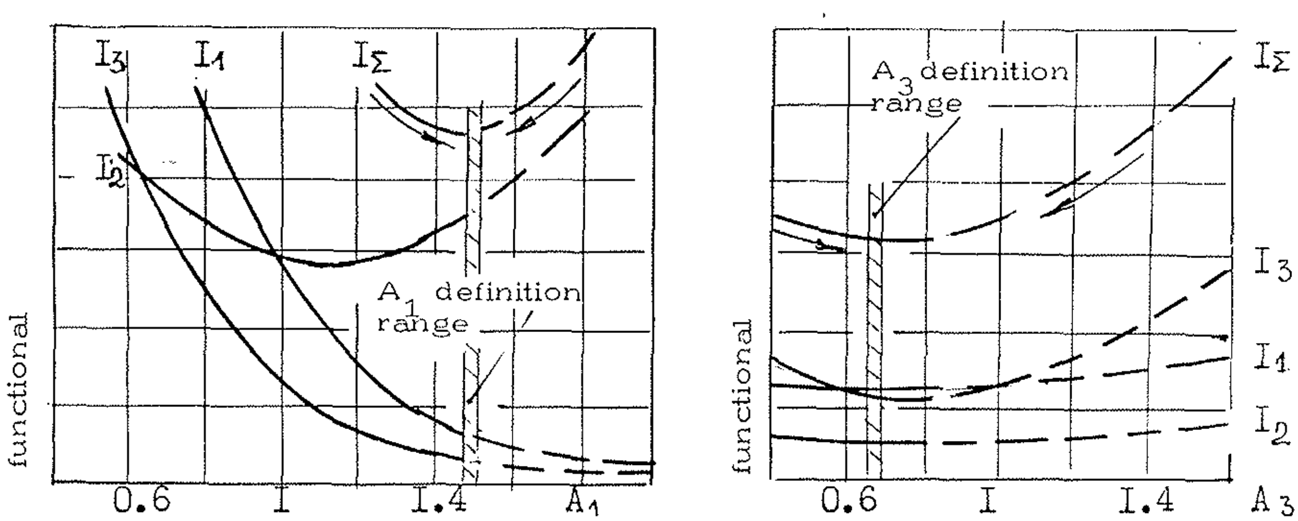


Fig.5 Functional and its components change dependences at various initial values of corrected parameters (—— Ao=0.5, --- Ao=2)

3.3 Identification results

The results of the calculations on the corrected parameters vector determination are presented at fig.6 in the form of comparing the helicopter motion parameters calculated change at one of the modes and the flight test results. At the same fig.6 the change of the motion parameters defined upon the initial (noncorrected) helicopter motion model are marked for the purpose of comparison.

To illustrate the importantce of performing the run of expiriments to solve the identification task, the scatter of corrected coefficients values at identification is shown for one mode only and the result for all the five modes is given. The identification deviation at each of the modes separately is from 10 to 20 % for various coefficients.

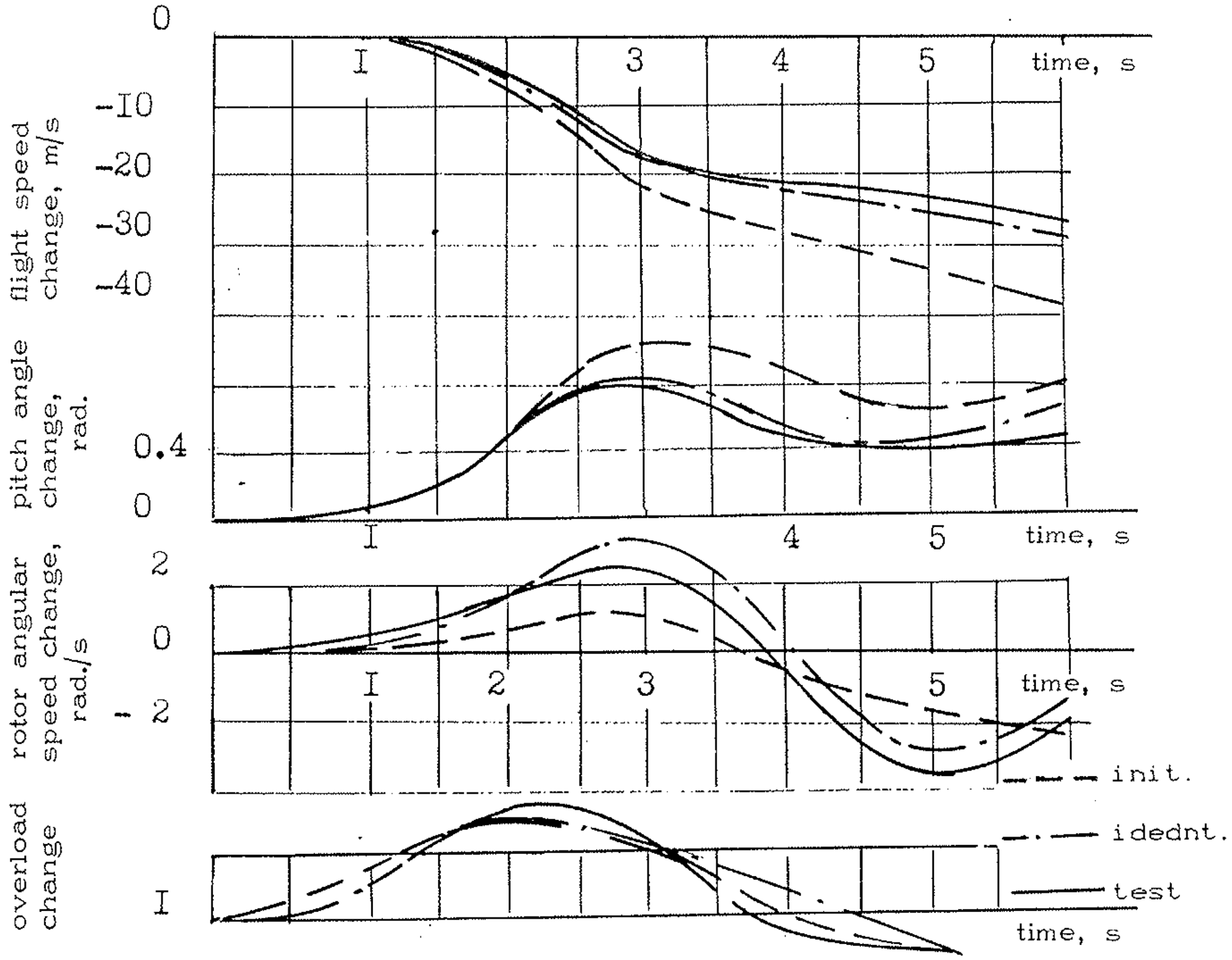


Fig.6 Comparison of motion parameters obtained through the model (initial and identified) and flight test results

However it should be kept in mind that the initial modes were selected carefully enough and did not show any explicite violations of the maneuver performance conditions (approximately equal maximal longitudinal control level and its change rate, constant collective pitch etc.) which explains for a considerably small scatter of results.

table 1

modes	corrected parameters			
	A_1	A_2	A_3	A_4
1. Identification for each mode separately				
mode 1	1.205	0.978	0.868	1.107
mode 2	1.555	0.887	0.909	0.991
mode 3	1.017	1.043	0.957	1.016
mode 4	1.249	1.192	0.996	1.358
mode 5	1.340	0.906	0.858	1.219
2. Identification for a run of five modes	1.216	1.00	0.900	1.155
3. Maximal deviation values at identification for each mode separately (as a % of the identification result for the run)	16	11.3	10.6	14.6

IV CONCLUSION

The use of one and the same mathematical apparatus for the solution of both general flight dynamics tasks i.e. for the object motion model identification and the control optimization considerably reduces the volume of computations in solving problems because of the possibility of a consequential program application for solving the both parts of the problem the model description in particular.

From the practical point of view an important feature of the proposed approach is also the possibility of identifying a model not only by means of one experiment but by a run of experiments which increases the degree of the model adequacy due to the averaging influence on random parameters.

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