

PHYSICS-BASED DETAILED DESIGN OF A DUCTED-FAN DRIVEN ROTORCRAFT

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Abstract

A physics-based detailed design approach is suggested for an accurate estimation of a ducted-fan driven rotorcraft. First, classification of the major components is conducted, and the layout of the inner components in both fuselage and the main wing is attempted. Then the equipment and components will be considered as point masses and arranged to locate the center of gravity appropriately. To evaluate the structural integrity, the pertinent aerodynamic loads are applied. The optimization procedure will be established and performed. Finally, the present approach will be capable of estimating a brand-new compound rotorcraft accurately.

1. INTRODUCTION

Unlike fixed-wing aircraft, conventional rotorcrafts have unique characteristics such as vertical take-off and hover. Due to these characteristics, rotorcrafts have been utilized in the civilian and military areas. However, compressibility on its advancing side and stall on the retreating side usually occur during high-speed forward flight. Due to these aerodynamic features, its forward speed will be limited to approximately 150 kts[1].

By such limitation, various compound rotorcrafts have been suggested by the U.S Army Joint Multi-Role Technology Demonstrator [2] and NASA Heavy Lift Rotorcraft System investigation [3]. Through those projects, interest in compound rotorcraft has been increased. However, complex aerodynamic interactions such as the blockage effect and the wake interaction enforced it to be difficult to design the compound rotorcraft [4].

In addition, unlike the conventional aircraft, the counter-rotating blade or tilt-wing mechanism will bring the structural complexity. For such reason, it is necessary to establish an appropriate preliminary design suitable for a compound rotorcraft.

As a result of those factors, several preliminary design programs for the compound rotorcraft were developed. Johnson [5] analyzed the characteristics of the compound rotorcraft and developed the NASA Design and Analysis of RotorCraft (NDARC) software. He designed and investigated a variety of compound rotorcrafts including the lift offset coaxial, tiltrotor, and ducted fan [6]. Lee et al [7] designed and analyzed three types of compound rotorcrafts such as the compound coaxial, tip-jet, and fan-in-fuselage. The weight was estimated by the trend formulas. However, the more the new configuration differs from the aerodynamic and structural characteristics of the existing aircraft, the more the accuracy of the trend formula will be degraded.

In particular, the compound rotorcraft based on the lift fan featured unique characteristics [4]. Unlike an open rotor, a lift fan is installed in the duct which increases the lift by 50% ideally. During transition flight, high drag is generated in the duct due to the aerodynamic imbalance. But when the duct is closed, it is capable of operating as a fixed-wing. In addition, as the fan was installed in the wing or fuselage, the structural design would become more complicated than it was in the conventional rotorcraft [8,9]. Such complexity will degrade the accuracy of the trend formula.

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For that, the physics-based approach using three-dimensional finite element (FE) has been suggested by several researchers [10,11]. Schwinn et al. [10] estimated the rotorcraft fuselage weight. They used a three-dimensional FE methodology to obtain the accurate weight of a fuselage. And they performed a structural sizing loop to reduce the weight of a fuselage. Wunderlich et al.[11] also used FE approach to estimate the weight of the main wing. Those results showed that the three-dimensional approach based on the finite element was capable of obtaining an accurate weight of the fuselage. However, such approach has several limitations. First, as the complexity of the aircraft is increased, more detailed input will be needed to obtain the credibility of estimation[10]. Such approach needed a considerable amount of computational time. And, it will be a challenge to estimate the weight automatically[10]. Thus, a further design framework will be required to overcome such limitations.

The primary purpose of this paper is to propose a physics-based detailed design approach for an accurate estimation of a ducted fan driven rotorcraft. The internal component in both fuselage and main wing will be designed by considering a ducted fan. For the physics-based design approach, parallel computation will be attempted. Because this paper will focus on the establishment of the physics-based detailed design approach, the result of the trend formula and detailed aerodynamic analysis will not be performed. Instead, the existing results [7, 12] will be adopted.

2. PRESENT STRUCTURAL DESIGN PROCEDURE

As mentioned in Introduction, this paper focuses on the physics-based detailed design procedure for the ducted-fan compound rotorcraft. First, the result of the trend formula and aerodynamic analysis will be introduced briefly. Then, the present approach will be suggested.

2.1. Trend formula result

Since the present physics-based detailed design approach is capable of handling complex configurations, a ducted-fan compound rotorcraft is selected as a design task. Lee et al. [7] estimated the weight and performance of a ducted-fan compound rotorcraft.

The common mission segments were used and detailed information is included in Ref. 7. The baseline configuration had two ducted fans which

are located in the fuselage. The number of fan blades are six and that for the propeller is five. The detailed configuration is shown in Fig. 1.

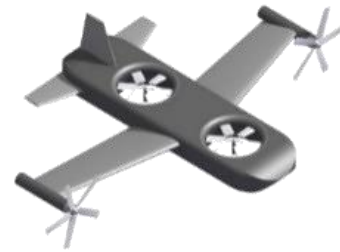


Figure 1. Baseline configuration of the present ducted fan driven rotorcraft [7]

2.2. Aerodynamic analysis result

The baseline configuration of the present ducted fan driven rotorcraft has unique aerodynamic features. It is divided into two parts such as the interaction between the duct and fuselage, and the blockage effect due to the pusher rotor. Due to such unique aerodynamic features, it is not straightforward to utilize a low fidelity aerodynamic approach. To identify the aerodynamic phenomenon of the baseline configuration, a high-fidelity aerodynamic analysis will be needed.

The aerodynamics of the present configuration was investigated by Lee et al.[12] by using computational fluid dynamics(CFD). They used CFD to obtain accurate aerodynamic features of the present configuration. And they constructed the relevant aerodynamics table. Thus, such results will be adopted to design the internal component. As they did not perform the trim analysis, a simplified trim analysis will be attempted based on that result.

2.3. Physics-based detailed design approach

In this section, the overall procedure of the present physics-based design will be listed as follows.

- 1) A baseline configuration is divided into the following four components: the fan, propeller, fuselage, and wing. The fan and propeller were designed by Im et al[13]. The weight of the fan and propeller will be considered as the point mass.

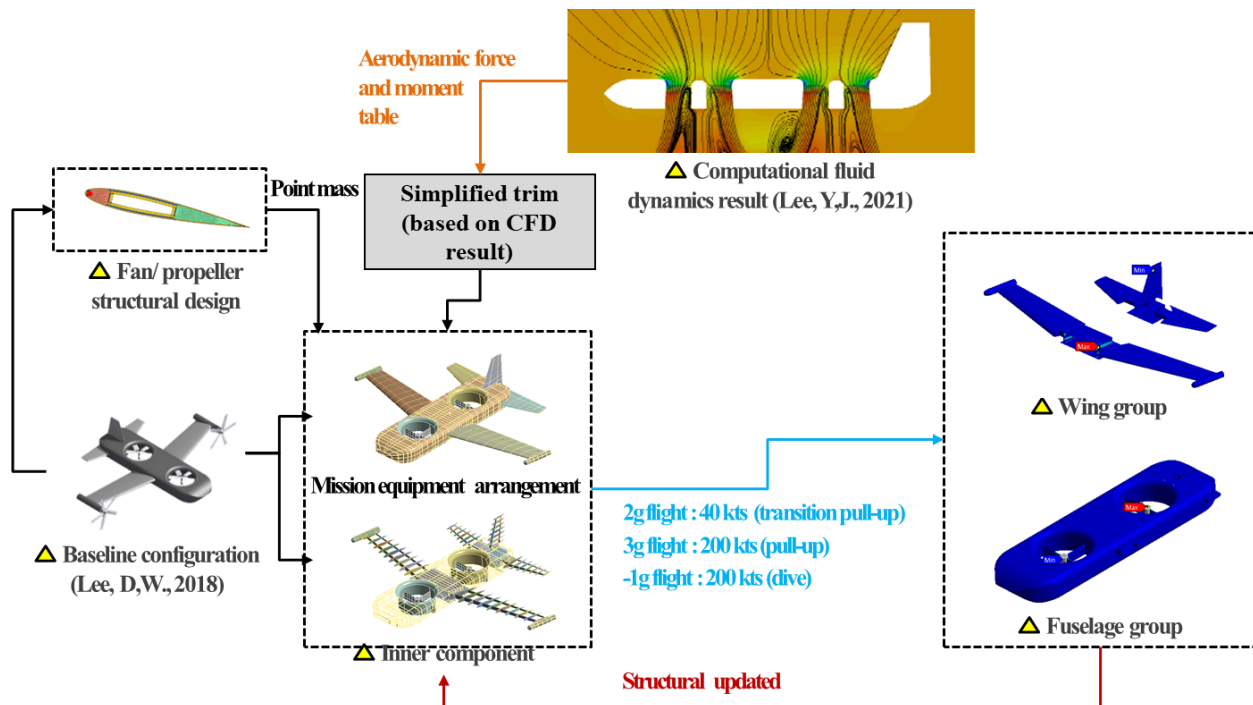


Figure 2. Present physics-based detailed design approach

- 2) After the structural design of the fan and propeller blade is completed, the arrangement of the components will be performed. First, the location of the center of gravity (C.G.) has to be selected appropriately. For the fixed-wing aircraft, the mean aerodynamic chord will be the appropriate location. However, there exist no detailed criteria for C.G. location of the compound rotorcraft. In this paper, the moment generated by the length between the ducted fan and C.G. will be taken into account. Thus, the desired C.G. will be located in the middle of the fans and it will be located near the position of 25% chordwise location of the main wing. Then, the groups such as the propulsion, equipment, and payload will be arranged in the appropriate location.
- 3) To perform the structural analysis, the physics-based aerodynamic load will be needed. However, the baseline configuration with a duct fan reveals complex aerodynamics. In order to analyze this configuration, computational fluid dynamics (CFD) will be selected. As this paper is focused on the physics-based detailed design, the CFD results [12] will be borrowed. Lee et al[12]. analyzed and established the aerodynamic table about the baseline configuration. Trim analysis was not performed on the aerodynamic results established by Lee.
- 4) Due to that, a simplified trim analysis will be executed. The three assumptions will be defined to perform a simplified trim analysis. The details are listed as follows: First, the lift of the main wing will be affected by the variation of the propeller collective angle. Second, the locations of the control surface are only arranged at the horizontal and vertical tail wings. The last one is that the control surface is an all-moving type. The following three flight conditions will be used to perform the trim analysis: transition, pull-up, and dive. The flight speed for the transition will be 40 kts, and other flight speed conditions will be 200 kts. By the present simplified trim analysis, the aerodynamic loads such as 3g pull-up, -1g dive, and 2g transition will be obtained.
- 5) Using those aerodynamic loads, the structural analysis will be performed. To evaluate the structural integrity, the safety margin will be used. The criteria of safety margin will be selected as 25%. Such criteria are based on the guide of unmanned aircraft proposed by U.S. Military Standard Joint Service Specification Guide [14]. The isotropic material properties will be checked by the equivalent stress and the composite material will be evaluated by the Tsai-Wu criterion. The relevant procedure is illustrated in Fig. 2.

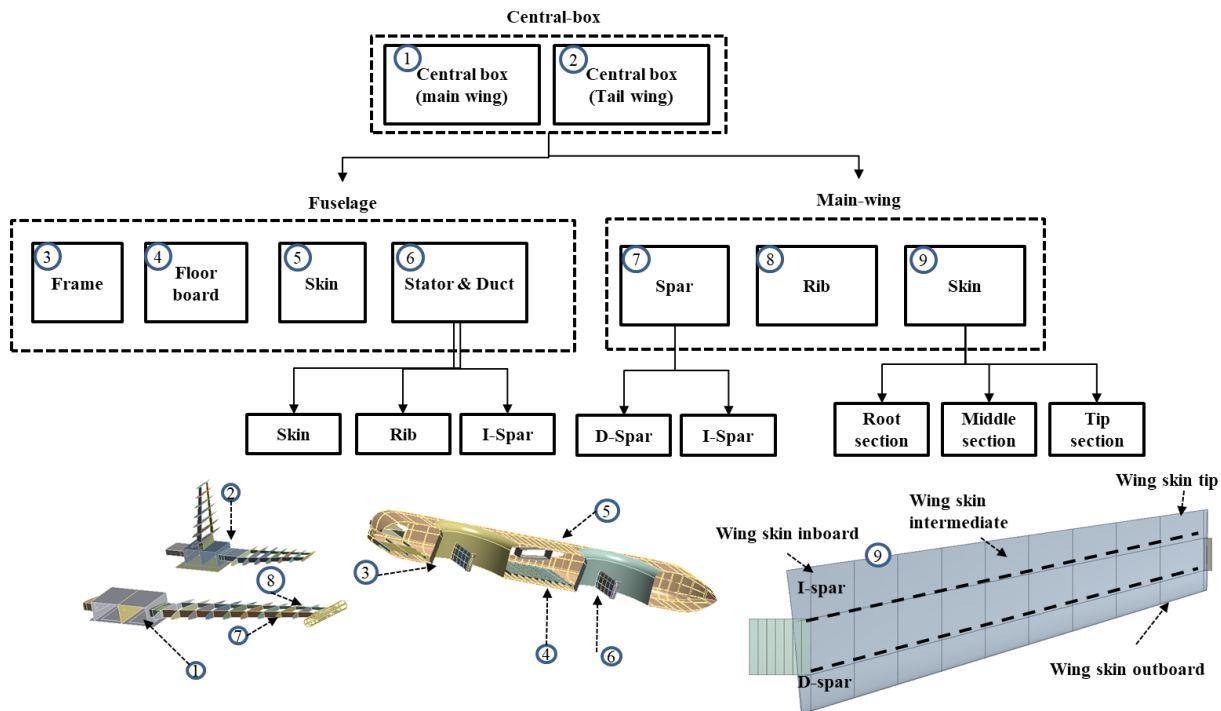


Figure 3. List of the detailed components

2.4. Detailed structural design

This section will describe the classification of the major components. The central box is composed of two kinds. The first one is connected with the main wing and the other one is connected to the tail wing. It is attached to the wing rib and skin. The fuselage will be divided into the following three parts: frame, floor board, and skin. The frame of the fuselage will be composed of the frame, duct, stator, floorboard, and skin. The configuration of the longitudinal stiffener and lateral rib and stiffener will be based on the previous result [15-16]. The rib arrangement will be evaluated by dividing it into 5 parts with regard to the span direction. The floorboard will be supported the central box and heavy-weight components such as the engine. In addition, the duct and stator will be constructed. Such configuration is referred for the result of FANRAFT™[17-18].

The skin of the fuselage will be made of composite material. The main wing will have the following three components: spar, rib, and skin. The spar will consist of D and I shape ones. The rib will be located uniformly from the wing root to the tip. The skin will be composed of the following four areas: inboard, intermediate, outboard, and the entire area. The detailed description of the major

components is illustrated in Fig.3.

3. OPTIMIZATION PROCESS

The framework of the optimization for the physics-based detailed design was established by the authors [15-16]. Based on that framework, the present optimization process will be attempted. The overall process of the optimization will be introduced briefly. Then, the specification such as objective function, design variable, and constraint will be explained.

3.1. Overall process of optimization

For optimization, the detailed three-dimensional configuration will be selected as the initial variables. The upper and lower limits of the design variables will be selected within a certain range which satisfy the minimum structural integrity. As the automatic procedure for the physics-based approach is a challenge [10], the breakpoint will be added for preventing the unexpected exit from the optimization. The two breakpoints are set in the present procedure. To alleviate the computational time, parallel computation will be performed. It will need an amount of time to estimate the gradient of the function. Due to that, parallel computation will be used to reduce the time of such gradient procedure. The details of the optimization process are included in Refs. 15-16.

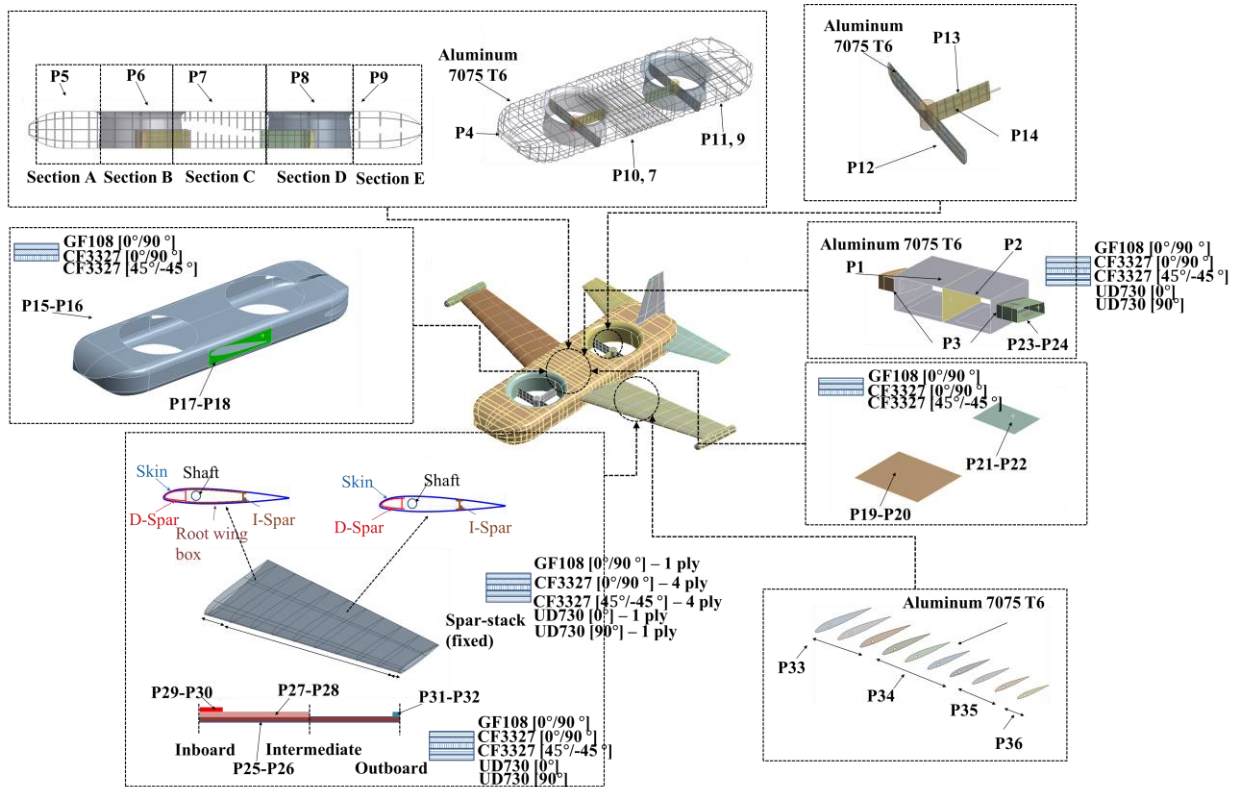


Figure 4. Design variables selected and used in the present design

Table 1. Specification of the present optimization

Objective function	Symbol (Fig. 4)	Design variable	Symbol (Fig. 4)	Design variable
Min (gross weight)	P1-P3	Center box thickness	P19-P22	floor board (No. of plies)
Constraint	P4	Thickness of fuselage frame(inner)	P23-P24	Central box connect (No. of plies)
Safety margin > 30 %	P5-P9	Thickness of fuselage rib	P25-P26	Entire wing skin (No. of plies)
1) Central wing box	P10-P11	Lower floor part depth	P27-P28	Intermediate wing skin (No. of plies)
2) Fuselage frame stator/floor board/skin	P12-P14	Duct support (stator skin, airfoil thickness)	P29-P30	Inboard wing skin (No. of plies)
3) Wing skin/spar/rib	P15-P16	Base fuselage skin (No. of plies)	P31-P32	Outboard wing skin (No. of plies)
	P17-P18	Reinforced fuselage skin (No. of plies)	P33-P36	Main wing rib thickness

3.2. Detail specification of optimization

Before optimization, a problem definition will be needed. First, the definition of the objective function will be performed. For the purpose of the present paper, the take-off gross weight (TOGW) must be minimized. However, the present optimization only handles the major structural weight such as the main wing, duct, and fuselage. Due to that, the propulsion system and fuel weight will not be optimized in this paper. The information of internal components such as engine, and transmission, will be borrowed from the result of the trend formula. To obtain a reliable result, the constraints will be applied. Each component such as the central box, fuselage frame, and wing will be evaluated by the safety margin. The criterion of the safety margin is based on the U.S. Military Standard Joint Service Specification Guide [14].

The safety margin should be larger than 0.25.

It is important to select an appropriate number of the variables for optimization. The computational time will be shorter when the number of design variables is fewer. But the amount of computational time will be significant as a number if the design variable is increased. Due to that, a proper number of the design variables is selected and it is shown in Fig 4. 36 design variables will be used. And certain components will fix the thickness or location: 1) The fuselage and main wing rib locations will be kept as constant. 2) The stack sequence of the spar will be kept as constant.

4. RESULTS OF PRESENT APPROACH

4.1. Load factors

To evaluate the structural integrity of aircraft, the critical load needs to be considered. Federal

Aviation Administration (FAA) and European Union Aviation Safety Agency (EASA) suggested the regulation of a flight envelope such as V-n diagram[19]. Such V-n diagram is defined for a fixed-wing aircraft. It should define the stall speed of an aircraft. However, a compound rotorcraft generates a sufficient lift before its stall speed. Due to that, V-n diagram needs to be modified. Based on the aerodynamic table and simplified trim, the flight envelope will be defined and three sets of load factors will be selected. In a transition flight, 2g pull-up will be selected as the relevant load factor. The lift of the rotorcraft will be generated by both fan and wing. And a 3g pull-up and -1g dive are selected at 200 kts. Such aerodynamic load is provided by the aerodynamic table and present simplified trim. The aerodynamic load will be applied to the propeller, nacell, fan, duct, main wing, tail wing, and fuselage. V-n diagram which provides the present load factor is shown in Fig. 5. V_T is the transition flight speed and V_s is the stall speed of the present configuration. V_c and V_D are the cruise speed and maximum flight speed, respectively.

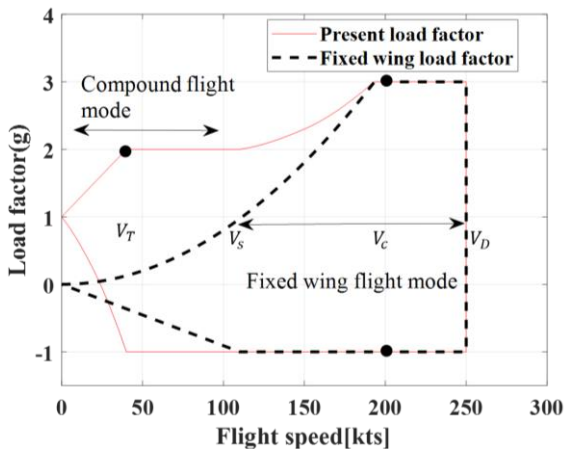


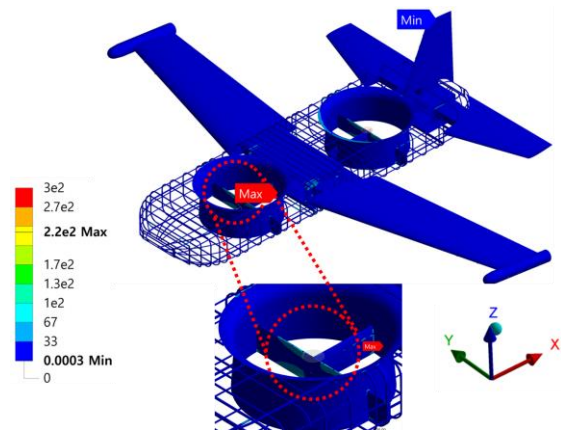
Figure 5. Present load factor (V-n diagram)

4.2. Structural analysis

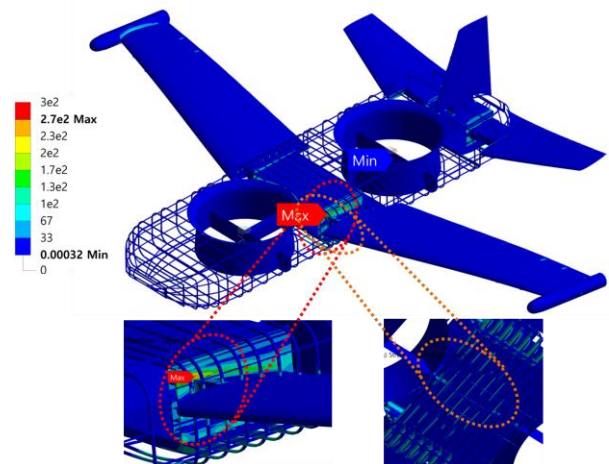
By using those load factors, the structural integrity will be evaluated. To analyze the present configuration, the material properties will be selected. The central box, fuselage frame, duct, rib, stator, and nacelle frame will be composed with Al-7075-T6. The skin of the fuselage, wing, spar, and floorboard will be composed of composite materials such as CF108, CF3327, and UD730.

The present structural analysis result is illustrated in Fig 6. For the 2g transition flight, the aerodynamic load of fan will be transferred to the transmission and stator. Due to that, it is found that the stress is concentrated in the stator. Such result

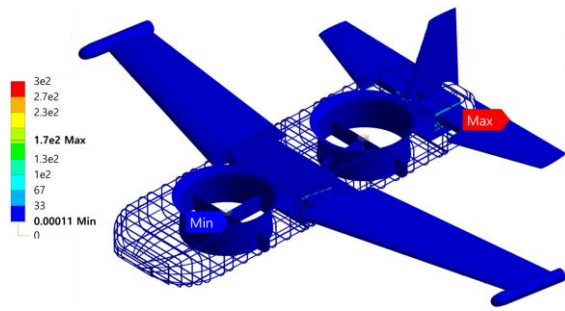
is shown in Fig. 6 (a). Unlike the transition flight case, the present rotorcraft will stop the fan and be operated in fixed-wing mode at cruise flight. 3g pull-up flight will be performed at 200 kts and such result is shown in Fig. 6 (b). The result of 3g pull-up flight shows that the stress is concentrated at the main wing root and central box. In addition, the concentration of the stress is observed at the lower fuselage frame. That trend occurs due to the heavy-weight components such as the engines, fuel, and the central box. As inertial loads are influenced by such components, the middle location of the fuselage will show the concentration of stress. In Fig. 6(c), the result of -1g flight is shown and the stress is also concentrated at the wing root and central box. However, the maximum stress level shows significant discrepancy. By the comparison on the safety margin results, the lowest safety margin is found at the 3g load factor. Through these analyses, inertial load will need to be considered for evaluating the integrity of the compound rotorcraft. By these results, the detailed three-dimensional configuration is obtained and constructed.



(a). 2g transition at 40 kts: von-Mises stresses



(b). 3g flight at 200 kts: von-Mises stresses



(c). -1g flight at 200 kts: von-Mises stresses

Figure 6. Result of the present structural analysis

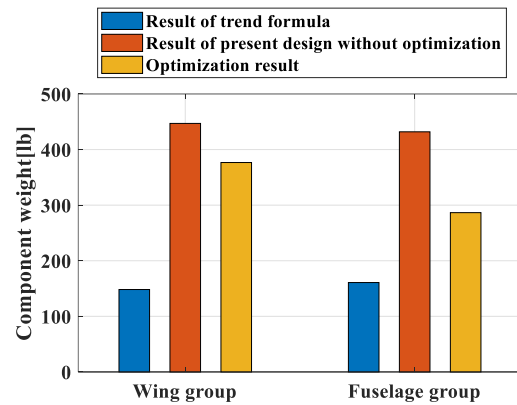
4.3. Result of the present optimization

Based on the structural analysis, the detailed three-dimensional configuration is constructed. As such configuration is not optimized yet, the optimization procedure will be performed. TOGW and component weight will be compared and its details are listed as follows.

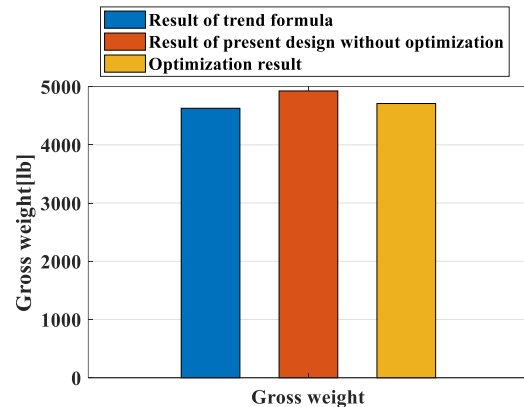
- 1) The weight of three configurations such as the trend formula, detailed three-dimensional configuration without optimization and optimization result will be compared.
- 2) MIL-STD-1374 [20] will be used for statement of the component weight.
- 3) The structural design for the fan and propeller is not included in this optimization process. Such components are designed before the execution of the present framework and those point mass only considered.
- 4) Due to that, the fuselage and main wing will be optimized and evaluated by the structural integrity.

In Fig. 7 (a), the weight comparison is performed and its details are illustrated. The result of the detailed three-dimensional configuration without optimization shows the increasing trend against the trend formula. The weight of the wing and fuselage group is significantly increased. There are several reasons and those are listed as follows. First, the additional components such as floor board, central box, and stator are considered in the detailed three-dimensional configuration. It will not be straightforward to estimate such components using trend formula approach. Another reason is that the unique feature of ducted-driven rotorcraft will increase the weight. As mentioned in the load factor section, the present baseline configuration has unique load factors. Unlike the conventional fixed-wing aircraft, such result does not need to consider the stall speed. As the present baseline

configuration was affected by that feature, the wing area will be smaller than conventional fixed-wing aircraft. Due to that fact, the wing loading will increase significantly when the load factor increases. For those reasons, the weight of both fuselage and wing group will be increased than that by the trend formula.



(a) Comparison of the major structural weight



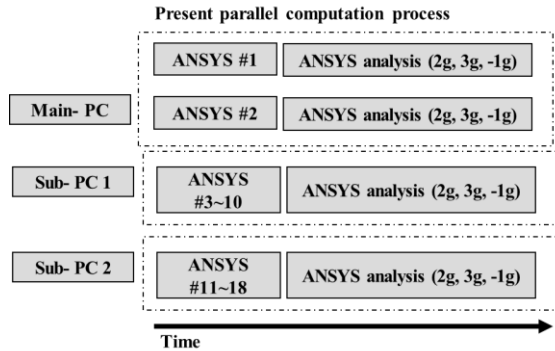
(b) Comparison of the gross weight

Figure 7. Weight comparison result

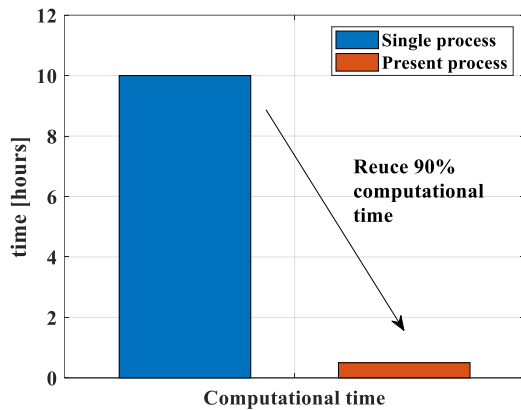
The optimization results are also shown in Figure 7(a). The optimization result of both wing and fuselage group is larger than that by the trend formula. It is not straightforward to minimize weight while satisfying the structural integrity. However, the weight of the wing group obtained through optimization is reduced by 13% compared with that before optimization. For the fuselage group, 33% is reduced by optimization. Based on that result, the optimization process is well established.

In addition, the comparison of the gross weight is shown in Fig. 7 (b). The detailed three-dimensional configuration without optimization is only increased by 6.4% than that by the trend formula. And the optimization weight is reduced by 4.58% than that by the result obtained before optimization. In addition, the optimization result is larger than that by the trend formula by only 2%. Unlike the

increment rate of the major structural weight, the variation of gross weight is small. The reason is that the design and optimization process only consider the major structural components such as the fuselage and wing group.



(a) Process of the present parallel computation



(b) Computational time comparison between the single CPU and parallel process

Figure 8. Computational method and the resulting time

To obtain the optimization result faster, a reduction of computational time will be necessary. For that, the present paper implements the parallel computational approach in optimization process. Such process is illustrated in Fig. 8 (a). As shown in Fig. 8 (a), three individual PC will be used to compute the gradient of each design variable. The main PC uses two cores and eight cores are used by each sub PC respectively. Based on such procedure, total 18 cores are updated simultaneously. As summarized in Table 1, the number of design variables for this study is 40. It is straightforward to obtain the gradient in each step using present parallel computation process. Single CPU approach needs the 40 iterations to obtain the gradient in each step. And using single CPU approach, approximately 10 hours will be required. However, the present parallel approach requires

only 30 min to obtain the gradient result. The computational time is reduced by about 90%. Based on this result, the present physics-based FE approach is an appropriate one to the design procedure to estimate the accurate weight of the new rotorcraft configuration.

5. CONCLUSION AND FUTURE WORKS

In this study, a physics-based detailed design approach is proposed for an accurate estimation of a ducted-fan driven rotorcraft. For that, the following procedure is performed. First, classification of the major components in both fuselage and main wing are conducted and designed. Then, the load factors for the present configuration are selected. Using that, the structural analysis is performed to check the safety margin. Since such result is not optimal solution, the optimization process is performed. The major contributions of this paper are listed as follows.

- 1) To estimate the accurate weight of the new concept of rotorcraft, the physics-based detailed three-dimensional approach is attempted. The procedures for detailed three-dimensional configuration are established and its details are listed as follows: classification of structural configuration, structural analysis, and optimization.
- 2) To evaluate the structural integrity of the compound rotorcraft, an appropriate load factor is suggested. The compound rotorcraft generates sufficient lift before stall speed. Due to that, the conventional v-n diagram is not appropriate. To compensate for this limitation, the load factor is constructed and applied to the compound mode and fixed-wing mode, respectively.
- 3) The optimization process is established and performed. The weight of the wing and fuselage group obtained by optimization is reduced by 13% and 33% respectively than that before optimization. And the weakness of the physics-based FE approach which needs an amount of computational time to obtain the result is compensated. The parallel approach shows that the computational time is reduced by 90% to obtain the gradient in each step.

In the future, the design variable will become more specific to reduce the weight of each component. Through those efforts, the accuracy of optimization will increase and be reliable.

6. ACKNOWLEDGMENTS

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