HYGROTHERMALLY STABLE LAMINATES WITH OPTIMAL BEND-TWIST COUPLINGS

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ABSTRACT

The necessary and sufficient conditions for hygrothermal curvature stability of laminated composite plates have been derived in a prior publication and shown to be material independent. From within these conditions, various couplings are being investigated to determine any improvements over previously known optima. Extension-twist coupling has been investigated in previous work. In this work, bend-twist coupling is investigated to demonstrate the achievable level of coupling under the hygrothermal stability conditions. Results for laminates consisting of two through ten plies are presented along with results from intuitive solutions and unconstrained optimizations. Nonlinear and finite element models are used to predict the response, and testing is used to verify the models.

INTRODUCTION

Laminated composite materials can be tailored to generate coupling between deformation modes that will produce an advantageous structural response [1]. For example, use of extension-twist coupled blades on a tilt-rotor aircraft to passively adjust the blade spanwise twist distribution has resulted in measurable horsepower savings [2]. In this case, the centrifugal force of an extension-twist coupled rotor blade can change the twist distribution and, hence, the angle of attack. Since rotor blades are also subject to aerodynamic lift forces, the bending moment produced in the rotor blade can also be used to change its twist distribution if the structure exhibits bend-twist coupling.

Some couplings, such as extension-twist coupling, require asymmetric stacking sequences, which may be hygrothermally unstable [3], defined as out-of-plane deformation due to changes in temperature or moisture. Bend-twist coupling, however, is achievable with both asymmetric and symmetric stacking sequences, the latter of which guarantee hygrothermal stability. A unidirectional off-axis laminate is the most intuitive solution that will produce bend-twist coupling. In the past, symmetric laminates were used wherever possible to avoid potential hygrothermal instabilities. Further, previous attempts to create structures with coupling focused first on achieving the desired mechanical response, then addressing potential hygrothermal instabilities.

In contrast, this work makes use of the necessary and sufficient conditions for hygrothermal stability to perform a survey of the range of available couplings from within all hygrothermally stable families. Next, a constrained optimization is performed to identify laminates with maximum bend-twist coupling for two through ten plies. Nonlinear and finite element models predict the response and testing validates the models. An investigation of the sensitivity of coupled laminates to errors in fiber orientation angle is also undertaken.

HYGROTHERMAL STABILITY

The necessary and sufficient conditions for hygrothermal stability have been derived in previous work [4]; a brief overview will be provided subsequently for convenience. As given in classical lamination theory (CLT) [5] for a laminate made of specially orthotropic plies, the non-mechanical in-plane stress resultants and out-of-plane moment resultants an are related to the in-plane strains and out-of-plane curvatures as

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ \hline M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} = \left[\begin{array}{ccc|ccc} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ \hline B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{array}\right] \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \hline \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

where A_{ij}, B_{ij}, and D_{ij} are the in-plane, coupling, and bending stiffness coefficients, respectively, and $(\ )^{(T,H)}$ indicates non-mechanical quantities. Since hygrothermal stability can be defined as having the out-of-plane curvatures equal to zero for any change in temperature or moisture, this can be expressed as

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ \hline M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} = \left[\begin{array}{ccc} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \\ \hline B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{array}\right] \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{2}$$

For a laminate where all plies have the same mechanical and hygrothermal properties in their principle material directions, the non-mechanical in-plane stress resultants and out-of-plane moment resultants for an n-ply laminate are given by

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}^{(T,H)} = T_2 \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} + T_3 \sum_{k=1}^{n} \begin{Bmatrix} \cos 2\theta_k \\ -\cos 2\theta_k \\ \sin 2\theta_k \end{Bmatrix}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} = T_1 \sum_{k=1}^{n} \begin{Bmatrix} \cos 2\theta_k \\ -\cos 2\theta_k \\ \sin 2\theta_k \end{Bmatrix} (2k - n - 1) \tag{3}$$

where θ_k is the angle of the k^{th} ply, and T_1, T_2, and T_3 are solely functions of the material properties and temperature and moisture changes.

Solving Equation (2) and making use of Equation (3), it can be proven that the necessary and sufficient conditions to ensure hygrothermal stability are either [5]

$$N_{xx}^{(T,H)} = N_{yy}^{(T,H)} \text{ and } N_{xy}^{(T,H)} = M_{xx}^{(T,H)} = M_{yy}^{(T,H)} = M_{xy}^{(T,H)} = 0 \quad \text{or} \quad B_{ij} = 0. \tag{4a, b}$$

It should be noted that when the conditions are substituted into (3), all material-dependent parameters cancel, making the conditions material independent. Unlike extension-twist coupling, bend-twist coupling is achievable using symmetric stacking sequences, for which B_{ij}=0 necessarily, in addition to asymmetric stacking sequences. Therefore, either condition given in Equation (4) can be used to achieve hygrothermally stable bend-twist coupled laminates.

OPTIMIZATION STUDY

Haynes *et al* [6] and Haynes and Armanios [7] have demonstrated the use of an optimization routine to identify hygrothermally stable laminates with maximum extension-twist coupling.

Objective Function

A linear theory is needed as the objective function in order to keep computational cost low enough to be practical and allow for the enforcement of non-linear constraints. For a hygrothermally stable laminates, CLT states that the constitutive law is given by

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{16} & \beta_{11} & \beta_{12} & \beta_{16} \\ \alpha_{12} & \alpha_{22} & \alpha_{26} & \beta_{21} & \beta_{22} & \beta_{26} \\ \alpha_{16} & \alpha_{26} & \alpha_{66} & \beta_{61} & \beta_{62} & \beta_{66} \\ \beta_{11} & \beta_{21} & \beta_{61} & \delta_{11} & \delta_{12} & \delta_{16} \\ \beta_{12} & \beta_{22} & \beta_{62} & \delta_{12} & \delta_{22} & \delta_{26} \\ \beta_{16} & \beta_{26} & \beta_{66} & \delta_{16} & \delta_{26} & \delta_{66} \end{bmatrix} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} \tag{5}$$

The twist rate in a laminated composite strip, φ, due to a nominal moment, M_{xx}, alone may then be calculated as

$$\varphi = \frac{1}{2}\left(\delta_{16} M_{xx}\right). \qquad (6)$$

resulting in an objective function given by

$$g\left(\left\{\theta_k : k = 1 \ldots n\right\}\right) = -\delta_{16}^2 . \qquad (7)$$

Implementation

The sequential quadratic programming (SQP) [8] implementation in MATLAB™ was used to perform the stacking sequence optimization numerically. In all cases the objective function in (7) is minimized. First, the optimizer is run with the constrains given in (4a) enforced; next the constraints from (4b) are used. A constraint to unidirectional stacking sequences was also optimized separately. Finally, an unconstrained optimization was performed to allow for comparisons with the maximum coupling achievable from a laminate with a given number of plies. For each run, the optimizer is initialized with a stacking sequence sampled from a uniform random number generator. Several hundred optimization runs are performed starting from different random initializations to enable identification and rejection of sub-optimal local minima.

RESULTS

The results of the optimization are provided in Table 1 for laminates constrained to hygrothermal stability according to Equation (4a), Equation (4b), and a unidirectional stacking sequence. The results of the unconstrained optimization are provided in Table 2. Also included in both tables is a parameter, ζ, given by

$$\varsigma = \frac{\delta_{16}(nt)^3 E_{11}}{12} = \delta_{16} D_{11} \approx \frac{\delta_{16} M_{xx}}{\kappa_{xx}} = \frac{2\phi}{\kappa_{xx}} \qquad (8)$$

which is used to compare the level of bend-twist coupling between laminates. Note that the global optima are symmetric, meaning that the laminates with the most coupling are also hygrothermally stable by meeting Equation (4b), except for the seven- and eight-ply laminates. Nonetheless, the optimal hygrothermally stable seven- and eight-ply laminates are within a quarter of a percent of the maximum coupling achievable, given by the global optima. The intuitive unidirectional off-axis laminates produce coupling to a level less than 5% below that of the optimal stacking sequence, and laminates that meet Condition A produce at least 15% less coupling than the optima.

As can be seen from the tables, all optimal hygrothermally stable laminates have the same

Table 1. Laminates Optimized for Bend-twist Coupling with Various Constraints

# of Plies	Equation (4a) (°)	ζ	Equation (4b) (°)	ζ	Unidirectional (°)	ζ
2	N/A	-	[30.5]$_2$	8.77	[30.5]$_2$	8.77
3	N/A	-	[-31.6/$\overline{87.8}$]$_s$	8.99	[30.5]$_3$	8.77
4	[-24.3/65.7]$_s$	7.08	[32.8/-88.2]$_s$	9.16	[30.5]$_4$	8.77
5	[27.8/-76.7/$\overline{-24.4}$]$_s$	7.83	[33.3/-88.4/$\overline{33.3}$]$_s$	9.18	[30.5]$_5$	8.77
6	[-87.0/-18.0/3.2/51.1 /72.4/-38.6]	6.61	[33.5/-88.5/33.5]$_s$	9.16	[30.5]$_6$	8.77
7	[-26.6/76.7/70.5/24.4/ -21.9/-27.8/75.4]	7.46	[33.6/-88.5/33.6/$\overline{-88.5}$]$_s$	9.14	[30.5]$_7$	8.77
8	[25.6/23.0/-67.0/ -64.4]$_s$	7.17	[-32.8$_2$/88.2$_2$]$_s$	9.16	[30.5]$_8$	8.77
9	[26.7/26.1/-74.6 /-66.9/$\overline{-23.2}$]$_s$	7.79	[-33.1$_2$/88.4$_5$/-33.1$_2$]	9.18	[30.5]$_9$	8.77
10	[-28.2/-27.8/78.0 /75.0/24.3]$_s$	7.74	[-33.3$_2$/88.4$_2$/-33.3]$_s$	9.18	[30.5]$_{10}$	8.77

Table 2. Global Optimal Bend-twist Coupled Laminates with Coupling Loss due to Various Constraints

# of Plies	Global Optima (°)	ζ	Coupling Loss with Equation (4a)	Coupling Loss with Equation (4b)	Coupling Loss with Unidirectional
2	[30.5]$_2$	8.77		0.00%	0.0%
3	[-31.6/$\overline{87.8}$]$_s$	8.99		0.00%	-2.4%
4	[32.8/-88.2]$_s$	9.16	-22.7%	0.00%	-4.2%
5	[33.3/-88.4/$\overline{33.3}$]$_s$	9.18	-14.7%	0.00%	-4.4%
6	[33.5/-88.5/33.5]$_S$	9.16	-27.9%	0.00%	-4.2%
7	[33.2/-88.4/33.4/ -88.5/-88.4/33.0$_2$]	9.16	-18.6%	-0.23%	-4.3%
8	[-33.3/-33.2/88.4/88.5/ -33.5/-33.4/88.4/-33.3]	9.17	-21.9%	-0.17%	-4.4%
9	[-33.1$_2$/88.4$_5$/-33.1$_2$]	9.18	-15.1%	0.00%	-4.4%
10	[-33.3$_2$/88.4$_2$/-33.3]$_s$	9.18	-15.7%	0.00%	-4.4%

form of their stacking sequence, namely plies oriented near 30^o as the outermost plies and some plies oriented near 90^o in the middle, except the two-ply laminate, which only has plies oriented near 30^o. The plies near 30^o make the bend-twist coupling possible while the plies near 90^o reduce the bending and twisting stiffnesses. Having plies oriented near both 30^o and 90^o distributed throughout the stacking sequence make the laminate more resistant to splitting failure than a unidirectional laminate.

VALIDATION

The five- through ten-ply optimal bend-twist coupled laminates have been manufactured from a T300/976 graphite-epoxy material system with properties given in Table 3 and tested to demonstrate the expected level of coupling. Each

Table 3. Properties of T300/976 Graphite/Epoxy

Property	Value
E_{xx}	125 GPa
E_{yy}	8.45 GPa
G_{xy}	3.9 GPa
ν_{xy}	0.328
t	0.152 mm

ply was cut from a pre-impregnated roll, laid up in a flat aluminum mold, and cured in an autoclave. After curing, each laminate was cut into four specimens. The specimens had test dimensions of 3.81cm by 22.9cm (1.5” by 9.0”) and were tested by clamping one end and applying a transverse load to the other end. At each load level, the tip twist was measured and recorded. The average moment in the specimen was found by taking one half of the transverse load multiplied by the specimen test length. Details of the test procedure are provided in Reference 9. Figure 1 provides sample results from the testing of the eight-ply laminate along with nonlinear and finite element model predictions and the linear prediction according to CLT as given in Equation (6).

An analytical finite-deformation model was developed, the details of which are provided in Reference 9. A summary is provided here for reference. The assumptions are that the laminate is thin relative to its width, w, and its length, L, is much longer than its width, i.e., $h \ll w \ll L$ and the laminate undergoes pure bending with no constraint on warping. The laminate is assumed to deform into a helical shape, which was demonstrated using finite element modeling, and

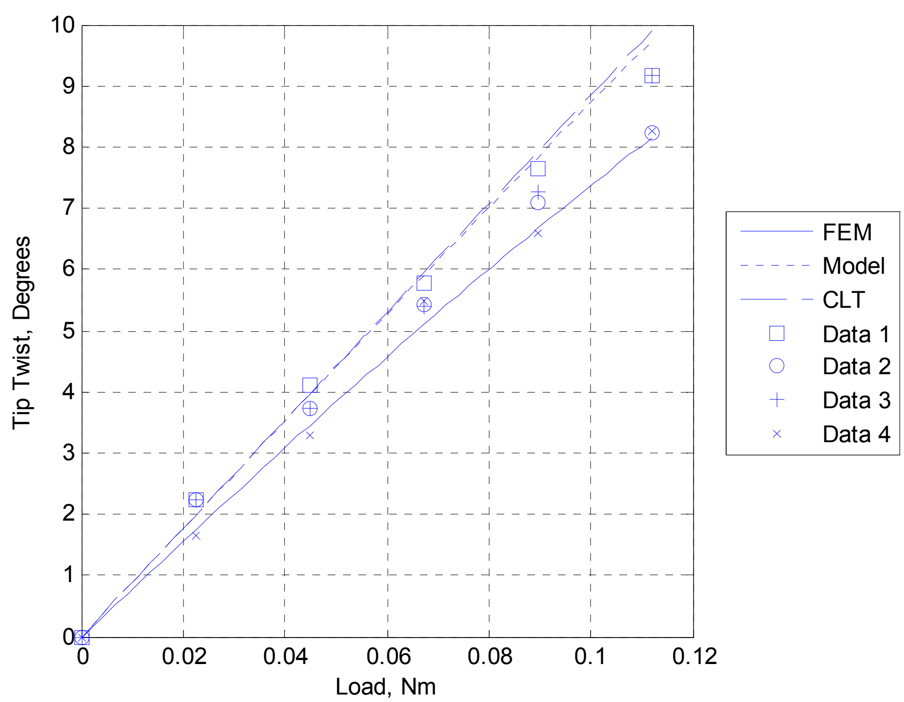

Figure 1. Test Data for Optimal Eight-ply Laminate

assumed to have a symmetric stacking sequence. Using Lagrangian strain tensors and keeping terms up to the order of magnitude of the maximum strain, a strain field was developed. Curvatures were found using the assumption that the through-the-thickness strains have a linear distribution.

Using the principle of virtual work and the definition of stress and moment resultants from CLT, a set of equilibrium equations were arrived at. Making use of the constitutive law from CLT, the strains were related to the resultants, and solved for the unknowns. The final form of the model is

$$P = b_1\phi + b_2\phi^3 \tag{9}$$

where P is the bending moment and φ is the twist rate of the strip. Coefficients b_1 and b_2 are functions of material and geometric parameters. It is interesting to note that for bend-twist coupling the largest nonlinear effect can be accounted for in a cubic term, suggesting that the response will be quasi-linear; however, for extension-twist coupling the nonlinear effects can be largely accounted for through a trapeze effect [10].

Finally, finite element modeling (FEM) was used to verify both the nonlinear model and predict test results. Very good agreement was observed between FEM and nonlinear models when identical boundary conditions are used. However, as seen in Figure 1, there is a discrepancy between the nonlinear and finite element models. The reason for this is that the FEM used in Figure 1 recreated the cantilevered boundary conditions of the test setup, which constrained warping to be zero at the clamped end, resulting in a stiffer model, whereas the nonlinear model assumes no warping constraint, and therefore, results in a more compliant model.

RESULTS

A summary of test data from the five- through ten-ply laminates is provided in Figure 2 with error bars indicating one standard deviation of the test data. Also included in Figure 2 are the nonlinear and finite element models. As the number of plies increases, the coupling tends to decrease. This can be explained by considering that as the laminates become thicker, the bending and torsional stiffnesses become greater.

A sensitivity analysis has been performed to evaluate the robustness of the optimal laminate to errors in ply angle representative of manufacturing

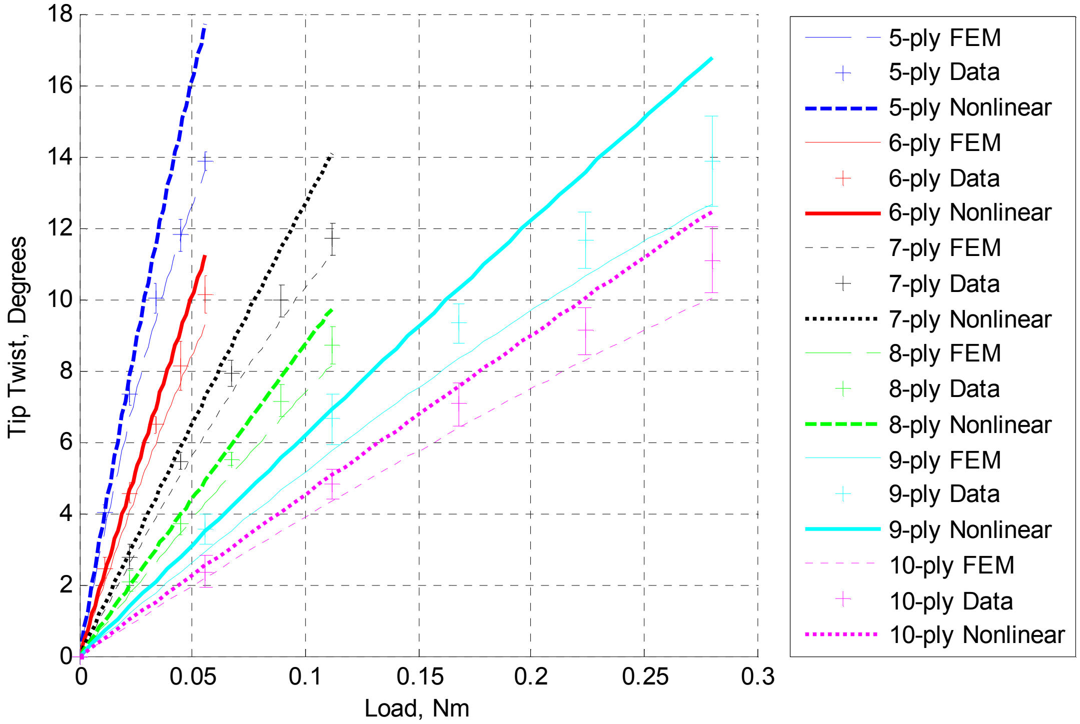

Figure 2. Summary of All Bend-twist Coupling Results

tolerances. To this end, a Monte Carlo simulation was performed to evaluate the loss of coupling due to small perturbations in ply angle over the uniform distribution $\theta_k \pm 2°$. The optimal bend-twist coupled six-ply stacking sequence was chosen, namely $[33.5/\text{-}88.5/33.5]_S$. A set of 10^6 samples was taken from the distribution, and ζ was calculated. Figure 3 gives a normalized histogram of the error from that of the optimized stacking sequence. It is expected that 0% error is the upper bound since it has been established that the stacking sequence used also produces maximum bend-twist coupling from all six-ply laminates. The lower bound of the error is around -4%.

Finally, to confirm the comparison parameter, ζ, another comparison parameter corresponding to one that exists in literature was formulated and compared alongside ζ. Rehfield *et al.* [11] established an extension-twist coupling parameter, equivalent to $\frac{B_{16}}{\sqrt{A_{11}D_{66}}}$. A corresponding comparison parameter for bend-twist coupling could be given by $\frac{|D_{16}|}{\sqrt{D_{11}D_{66}}}$. Since compliance coefficients are used in this work to define coupling, the compliance coefficients, as given in Equation (5), were also considered for use in a similar comparison parameter as $\frac{\delta_{16}}{\sqrt{\delta_{11}\delta_{66}}}$. Both

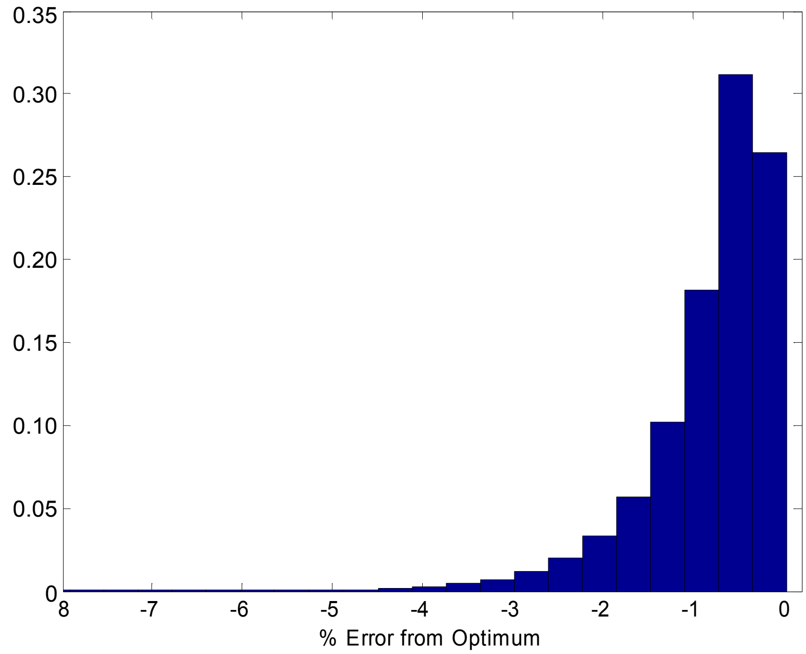

Figure 3. Distribution of Error in Coupling from Optimal Hygrothermally Stable Bend-twist Coupled Stacking Sequence; 10^6 Cases

the stiffness-based- and compliance-based-comparison parameters have been calculated for each of the hygrothermally stable optimal stacking sequences provided in Table 1. The results are provided in Table 4 and are plotted in Figure 4. Also included in Table 4 is the parameter ζ^* scaled such that $\zeta^* = \zeta/12.13$, where $12.13 = \frac{\zeta\sqrt{\delta_{11}\delta_{66}}}{\delta_{16}}$ for the six-ply laminate, which is in the middle of the laminates of interest. The result is that $\zeta^* = \frac{\delta_{16}}{\sqrt{\delta_{11}\delta_{66}}}$ for the six-ply laminate to allow for ease of comparison between ζ and $\frac{\delta_{16}}{\sqrt{\delta_{11}\delta_{66}}}$. There is less than 1.5% error between ζ^* and $\frac{\delta_{16}}{\sqrt{\delta_{11}\delta_{66}}}$, suggesting that ζ is an valid estimator of bend-twist coupling.

CONCLUSIONS

In this work, hygrothermally stable two- through ten-ply laminates have been optimized for bend-twist coupling. Optimal laminates have plies oriented in such a way as to mitigate their propensity for splitting failure. For comparison, unidirectional-constrained and unconstrained optimizations were performed. It was found that the coupling level for all hygrothermally stable laminates was within 0.25% of the maximum achievable coupling. The expected level of coupling was predicted using nonlinear and finite

Table 4. Various Comparison Parameters

# of Plies	Equation (4b) (°)	ζ^*	$\delta_{16}/\sqrt{(\delta_{11}*\delta_{66})}$	$\lvert D_{16}\rvert/\sqrt{(D_{11}*D_{66})}$
2	$[30.5]_2$	0.7177	0.7072	0.8662
3	$[-31.6/\overline{87.8}]_s$	0.7352	0.7314	0.8620
4	$[32.8/-88.2]_s$	0.7493	0.7512	0.8497
5	$[33.3/-88.4/\overline{33.3}]_s$	0.7509	0.7525	0.8360
6	$[33.5/-88.5/33.5]_s$	0.7492	0.7492	0.8263
7	$[33.6/-88.5/33.6/\overline{-88.5}]_s$	0.7478	0.7465	0.8209
8	$[-32.8_2/88.2_2]_s$	0.7493	0.7512	0.8497
9	$[-33.1_2/88.4_5/-33.1_2]$	0.7510	0.7532	0.8423
10	$[-33.3_2/88.4_2/-33.3]_s$	0.7509	0.7525	0.8360

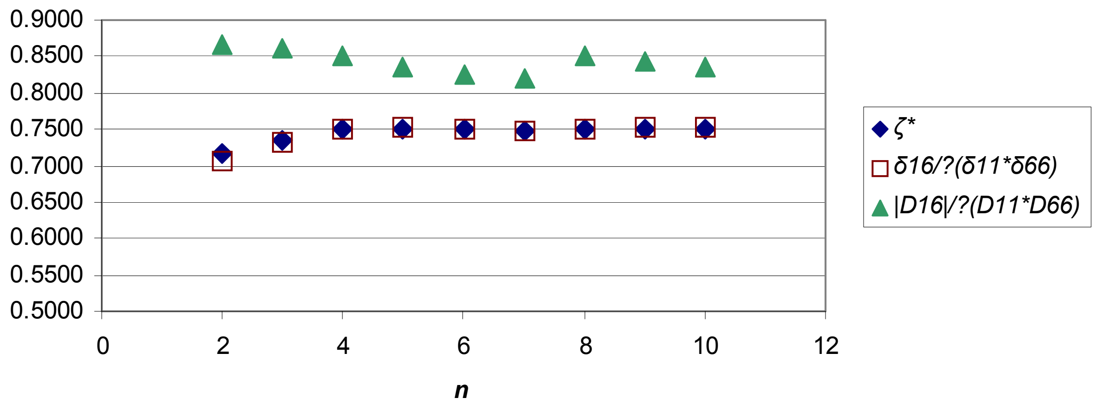

Figure 4. Various Comparison Parameters

element models and verified through manufacture and testing of the five- through ten-ply laminates. A robustness study indicated that errors in the fiber orientation angle of each ply typically seen in hand layup can cause a loss of coupling up to 4%.

REFERENCES

1. Weisshaar, T.A. 1980. "Divergence of Forward Swept Composite Wings," *Journal of Aircraft*, 17(6): 442-448.
2. Nixon, M.W. 1988. "Improvements to Tilt Rotor Performance Through Passive Blade Twist Control," NASA TM-100583.
3. Hyer, M. W. (1982). The Room-temperature Shapes of Four-layer Unsymmetric Cross-ply Laminates, *Journal of Composite Materials*., 16(4): 318–340.
4. Cross, R. J., Haynes, R. A., and Armanios, E. A. 2008. "Families of Hygrothermally Stable Asymmetric Laminated Composites," *Journal of Composite Materials*, 42(7): 697-716.
5. Jones, R. M. 1999. *Mechanics of Composite Materials*. 2nd Ed., Taylor & Francis, Philadelphia, PA, pp. 190-203.
6. Haynes, R.A., Carey, R., and Armanios, E. A. 2009. "A New Class of Hygrothermally Stable Laminates with Extension-twist Coupling," presented at the AHS 65th Annual Forum & Technology Display, May 27-29, 2009.
7. Haynes, R.A. and Armanios, E. A. 2009. "Overview of Hygrothermally Stable Laminates with Improved Extension-twist Coupling," presented at the 17th International Conference on Composite Materials, July 27-31, 2009.
8. Gill, P.E., W. Murray, and M.H. Wright. 1981. *Practical Optimization*, Academic Press, London, UK.

9. Haynes, R.A., “Hygrothermally Stable Laminated Composites with Optimal Coupling,” Ph.D. Dissertation, Georgia Institute of Technology, Atlanta, GA, 2010.
10. Armanios, E.A., Makeev, A., and Hooke, D. 1996. “Finite-Displacement Analysis of Laminated Composite Strips with Extension-Twist Coupling,” *Journal of Aerospace Engineering*, 9(3): 80-91.
11. Rehfield, Lawrence W.; Atilgan, Ali R.; and Hodges, Dewey H. 1990. “Nonclassical Behavior of Thin-Walled Composite Beams with Closed Cross Sections,” *Journal of the American Helicopter Society,* 35(2): 42 – 50.