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ABSTRACT

At present, a simplified approach to the prediction of rotor blade airloads is urged to be developed in the engineering application.

In this paper, firstly, relations of first two harmonic induced velocities to the lower and same-order harmonic circulations are obtained from the generalized classical vortex theory of the rotor. Then, based on the blade element theory, a closed form of equations for circulation is established and, by taking the flapping condition into account, simplified formulae for predicting rotor blade airloads are set up. In particular, expressions of flapping coefficients are derived, including the effect of variable induced velocity distribution but in terms of blade parameters and flight parameters only.

Finally, a calculation of a typical example is made and comparisions of airloads with those from the more accurate numerical solution are shown that the present method is fairly suitable for aerodynamic analysis and preliminary design of helicopters.

NOTATION

Ω —— rotational speed of the rotor

R —— radius of the rotor

v —— induced velocity

$\bar{v}=v/\Omega R$ —— nondimensional

Γ —— circulation

$\bar{\Gamma}=\Gamma/\Omega R^2$ —— nondimensional

(r, θ) —— polar coordinates in the disk plane

(ρ, ψ) —— dummy polar coordinates

$\bar{r}=r/R$ —— nondimensional

$\bar{\rho}=\rho/R$ —— nondimensional

ρ_H —— air density

k —— number of blades

V_0 —— forward velocity of the rotor

$\bar{V}_0=V_0/\Omega R$ —— nondimensional

V_1 —— resultant velocity of the air—stream, constant over the disk plane

$\bar{V}_1=V_1/\Omega R$ —— nondimensional

α_0 —— angle of attack of the rotor with respect to V_0

α_1 —— angle of attack of the rotor with respect to V_1

b —— blade chord

$\bar{b}=b/R$ —— nondimensional

c_y —— blade section lift coefficient

a_∞ —— two dimensional lift curve slope

T_1 —— thrust of one blade

$T=kT_1$ —— thrust of the rotor

ϑ —— blade section pitch angle

U —— relative velocity of blade section

$\bar{U}=U/\Omega R$ —— nondimensional

ϑ_0 —— blade pitch at the root

$\Delta\vartheta$ —— blade twist

ϑ_1 —— cosine term of blade feathering

ϑ_2 —— sine term of blade feathering

U_x —— velocity component in the disk plane

U_y —— velocity component normal to the disk plane

$\mu=V_0\cos\alpha_0/\Omega R$ —— advance ratio

$\lambda_0=V_0\sin\alpha_0/\Omega R$ —— inflow ratio

K —— factor of coupling between flapping and feathering

β_e —— flapping angle with origin at pin

β —— flapping angle with origin at center

a_0 —— coning angle

a_n —— cosine term of blade flapping

b_n —— sine term of blade flapping

e —— flapping pin offset

$\bar{e}=e/R$ —— nondimensional

m_1 —— blade mass

$\bar{m}_1=m_1/\rho_H R^3$ —— nondimensional

$C_T=T\Big/\frac{1}{2}\rho_H\pi R^2\Omega^2R^2$ —— thrust coefficient

J_e —— inertia moment of one blade about the pin

S_e —— mass moment of one blade about the pin

$(M_A)_e$ —— thrust moment of one blade about the pin

$(M_G)_e$ —— gravity moment of one blade about the pin

g —— gravity acceleration

$\varkappa$ —— root and tip losses factor after integration

1 INTRODUCTION

The prediction of rotor blade airloads in flapping plane is one of the fundamental problems in helicopter aerodynamics and dynamics. This is because not only helicopter flight performance, handling quality, but also rotor fatigue life, aeroelastic instability all depend on the understanding of the rotor blade airloads, particularly of the blade thrust loads.

Since 1960s, various investigators have done much work in the area and got great success. In 1973, AGARD organized a specialists meeting on "Helicopter Rotor Loads Predicion Methods" in Italy (ref. 1). It was a survey of the situation of the analytical methods used by different airframe manufacturers. However, as stated by some reviews and later in many papers (ref. 2, 3), the improvement in recent years is not so significant even with the high speed, large scale digital computer owing to the complexity of the rotor behavior. Rather, it is required to have a simplified method for predicting rotor blade airloads available to the engineer and the designer at a working level.

In this paper, firstly, relations of first two harmonic induced velocities to the lower and same-order harmonic circulations are obtained from the generalized classical vortex theory of the rotor. Then, based on the blade element theory, a closed form of equations for circulation is established. And finally, by taking the flapping condition into account, simplified formulae for calculating rotor blade airloads are set up.

2 INDUCED VELOCITIES

According to the generalized vortex theory of rigid wake of the rotor (ref. 4), the axial induced velocity at any point ($\bar{r}$, θ) on the rotor disk is a function of the bound vortex circulation $\bar{\Gamma}(\bar{\rho}, \psi)$:

$$\bar{v}=\bar{v}(\bar{\Gamma}) \qquad (2\text{—}1)$$

If the circulation $\bar{\Gamma}$ is expanded into Fourier series:

$$\overline{\Gamma}=\overline{\Gamma}_0(\bar{\rho})+\sum_{m=1}[\overline{\Gamma}_{mc}(\bar{\rho})\cos m\theta+\overline{\Gamma}_{ms}(\bar{\rho})\sin m\theta] \tag{2—2}$$

the induced velocity $\bar{v}$ could be written into Fourier series also:

$$\bar{v}=\bar{v}_0(\bar{r})+\sum_{n=1}[\bar{v}_{nc}(\bar{r})\cos n\psi+\bar{v}_{ns}(\bar{r})\sin n\psi] \tag{2—3}$$

Here, every harmonic component of $\bar{v}$, in general, is induced by all harmonic components of circulation. In this work, as a simplification, only the lower and same-order harmonics of circulation to the induced velocity are taken into account by considering the major contribution of vortices, i.e.

$$\left.\begin{aligned}
&\bar{v}_0=\bar{v}^0_0\\
&\bar{v}_{1c}=\bar{v}^0_{1c}+\bar{v}^{1c}_{1c}\\
&\bar{v}_{1s}=\bar{v}^0_{1s}+\bar{v}^{1s}_{1s}\\
&\bar{v}_{2c}=\bar{v}^0_{2c}+\bar{v}^{1c}_{2c}+\bar{v}^{1s}_{2c}+\bar{v}^{2c}_{2c}\\
&\bar{v}_{2s}=\bar{v}^0_{2s}+\bar{v}^{1c}_{2s}+\bar{v}^{1s}_{2s}+\bar{v}^{2s}_{2s}\\
&\cdots\cdots\cdots\cdots
\end{aligned}\right\} \tag{2—4}$$

where the superscripts denote the harmonic orders of circulation.

Based on Wang's vortex theory (ref. 5) and limited to second harmonics, we find (induced velocity is positive as downward):

$$\bar{v}_0=-\frac{k}{4\pi\overline{V}_1}(-\overline{\Gamma}_0)$$

$$\begin{aligned}
\bar{v}_{1c}=-\frac{k}{4\pi\overline{V}_1}\Big\{(c+c)\Big[&\int_0^{\bar{r}}\frac{d\overline{\Gamma}_0}{d\bar{\rho}}\,\frac{\bar{\rho}^2}{\bar{r}^2}\,\frac{1}{2}\cdot F\Big(\frac{3}{2},\ \frac{1}{2},\ 2,\frac{\bar{\rho}^2}{\bar{r}^2}\Big)\,d\bar{\rho}+\\
&+\int_{\bar{r}}^1\frac{d\overline{\Gamma}_0}{d\bar{\rho}}\,\frac{\bar{r}}{\bar{\rho}}\,\frac{1}{2}\cdot F\Big(\frac{3}{2},\ \frac{1}{2},\ 2,\frac{\bar{r}^2}{\bar{\rho}^2}\Big)d\bar{\rho}\Big]+\\
&+(1-c^2)(-\overline{\Gamma}_{1c})\Big\}
\end{aligned}$$

$$\bar{v}_{1s}=-\frac{k}{4\pi\overline{V}_1}\Big\{(c+c)\Big(\frac{-\overline{V}_1}{\bar{r}}\Big)\overline{\Gamma}_0+(1+c^2)\ (-\overline{\Gamma}_{1s})\Big\}$$

$$\begin{aligned}
\bar{v}_{2c}=-\frac{k}{4\pi\overline{V}_1}\Big\{&(c^2+c^2)\int_0^{\bar{r}}\frac{d\overline{\Gamma}_0}{d\bar{\rho}}\,\frac{\bar{\rho}^2}{\bar{r}^2}d\bar{\rho}+\\
&+(c^3-c)\Big[\int_0^{\bar{r}}\frac{-d\overline{\Gamma}_{1c}}{d\bar{\rho}}\,\frac{\bar{\rho}^3}{\bar{r}^3}\,\frac{1}{2}\cdot F\Big(\frac{3}{2},\ \frac{1}{2},\ 2,\frac{\bar{\rho}^2}{\bar{r}^2}\Big)d\bar{\rho}-
\end{aligned}$$

$$-\int_{\bar{r}}^{1}\frac{d\overline{\Gamma}_{1c}}{d\bar{\rho}}\,\frac{1}{2}\cdot F\left(\frac{3}{2},\ \frac{1}{2},\ 2,\frac{\bar{r}^2}{\bar{\rho}^2}\right)d\bar{\rho}\ \Big]+$$

$$+\ (c^3+c)\left(\frac{\overline{V}_1}{\bar{r}}\right)\int_0^{\bar{r}}\frac{d\overline{\Gamma}_{1s}}{d\bar{\rho}}\ \frac{\bar{\rho}}{\bar{r}}d\bar{\rho}+$$

$$+(1+c^4)\,(-\overline{\Gamma}_{2c})\Big\}$$

$$\bar{v}_{2s}=-\frac{k}{4\pi\overline{V}_1}\Big\{(c^2+c^2)\ \left(\frac{-\overline{V}_1}{\bar{r}}\right)\Big[\int_0^{\bar{r}}\frac{d\overline{\Gamma}_0}{d\bar{\rho}}\cdot F\left(\frac{3}{2},\ \frac{-1}{2},\ 1,\frac{\bar{\rho}^2}{\bar{r}^2}\right)d\bar{\rho}-$$

$$-\int_r^1\frac{d\overline{\Gamma}_0}{d\bar{\rho}}\ \frac{\bar{r}^3}{\bar{\rho}^3}\ \frac{1}{8}\cdot F\left(\frac{3}{2},\ \frac{3}{2},\ 3,\frac{\bar{r}^2}{\bar{\rho}^2}\right)d\bar{\rho}\Big]+$$

$$+\ (c^3-c)\left(\frac{\overline{V}_1}{\bar{r}}\right)\int_0^{\bar{r}}\frac{d\overline{\Gamma}_{1c}}{d\bar{\rho}}\ \frac{\bar{\rho}}{\bar{r}}d\bar{\rho}+$$

$$+\ (c^3+c)\Big[\int_0^{\bar{r}}\frac{d\overline{\Gamma}_{1s}}{d\bar{\rho}}\ \frac{\bar{\rho}^3}{\bar{r}^3}\ \frac{1}{2}\cdot F\left(\frac{3}{2},\ \frac{1}{2},\ 2,\frac{\bar{\rho}^2}{\bar{r}^2}\right)d\bar{\rho}+$$

$$+\int_{\bar{r}}^1\frac{d\overline{\Gamma}_{1s}}{d\bar{\rho}}\frac{1}{2}\cdot F\left(\frac{3}{2},\ \frac{1}{2},\ 2,\frac{\bar{r}^2}{\bar{\rho}^2}\right)d\bar{\rho}\Big]+$$

$$+(1-c^4)\,(-\overline{\Gamma}_{2s})\Big\}\qquad(2\text{—}5)$$

where the hypergeometric functions are defined as follows:

$$F(a,\ b,\ d,\ z)=1+\sum_{k=1}\frac{(a)_k\cdot(b)_k}{(d)_k\cdot k!}z^k$$

in which

$$|z|<1,$$

$$(a)_k=a(a+1)(a+2)\cdots(a+k-1)$$

$$d\neq-1,\ -2,\ \cdots\cdots$$

and the symbol c is

$$c=\frac{1-|\sin\alpha_1|}{\cos\alpha_1}=\frac{\cos\alpha_1}{1+|\sin\alpha_1|}\leqslant 1$$

The expressions of induced velocity harmonics above are the key to the settlement of our problem. The hypergeometric functions involved, according to reference 8, might be cut down to the first several terms in the series.

3 EQUATIONS OF CIRCULATION

Based on the blade element theory (ref. 6) and the famous Joukowsky formula, the rotor blade airloads can be expressed as

$$\frac{dT_1}{dr}=\rho_H U\Gamma=\frac{1}{2}\rho_H U^2 bc_y \tag{3—1}$$

here, the conventional assumptions are adopted:

$$U\approx U_x$$

$$c_y\approx a_\infty\left(\vartheta_*-\frac{U_y}{U_x}\right)$$

Thus, the circulation relation (nondimensional) is

$$\bar{\Gamma}=\frac{\bar{b}}{2}a_\infty(\bar{U}_x\vartheta_*-\bar{U}_y) \tag{3—2}$$

For articulated rotor, the blades are considered as rigid with hinge offset e and coupling factor between flapping and feathering K, and for hingless rotor, the blades could be considered as deflected to first elastic mode and treated as an equivalent model. Therefore,

$$\vartheta_*=\vartheta_0+\bar{r}\Delta\vartheta+\vartheta_1\cos\psi+\vartheta_2\sin\psi-K\beta_e$$

$$\bar{U}_x=\bar{r}+\mu\sin\psi$$

$$\bar{U}_y=-\lambda_0+\bar{v}+\mu\cos\psi\cdot\beta_e+(\bar{r}-\bar{e})\frac{d\beta_e}{d\psi} \tag{3—3}$$

and

$$\beta_e=\frac{\beta}{1-\bar{e}}=\frac{1}{1-\bar{e}}\left[a_0-\sum_{n=1}(a_n\cos n\psi+b_n\sin n\psi)\right] \tag{3—4}$$

If introduce following notations

$$\left.\begin{aligned}
\lambda_0^*&=\lambda_0+\vartheta_2^*\mu\\
a_1^*&=a_1-\vartheta_2^*\\
b_1^*&=b_1+\vartheta_1^*\\
\vartheta_0^*&=\vartheta_0-K\frac{a_0}{1-\bar{e}}\\
\vartheta_1^*&=\vartheta_1+K\frac{a_1}{1-\bar{e}}\\
\vartheta_2^*&=\vartheta_2+K\frac{b_1}{1-\bar{e}}
\end{aligned}\right\} \tag{3—5}$$

and neglect smaller terms which contained $\frac{\bar{e}}{1-\bar{e}}$ and the flapping coefficients higher than third order a_3, $b_3\cdots$, we have

$$\bar{\Gamma}_0=\frac{a_\infty \bar{b}}{2}\left[\vartheta_0^*\bar{r}+\Delta\vartheta\bar{r}^2-\bar{v}_0+\lambda_0^*+\frac{1}{2}\mu a_1^*\right]$$

$$\bar{\Gamma}_{1c}=\frac{a_\infty \bar{b}}{2}\left[-\bar{v}_{1c}-\mu a_0+b_1^*\bar{r}+\frac{1}{2}\mu a_2+\frac{1}{2}\mu K b_2\right]$$

$$\bar{\Gamma}_{1s}=\frac{a_\infty \bar{b}}{2}\left[\vartheta_0^*\mu+\Delta\vartheta\mu\bar{r}-\bar{v}_{1s}-a_1^*\bar{r}-\frac{1}{2}\mu K a_2+\frac{1}{2}\mu b_2\right]$$

$$\bar{\Gamma}_{2c}=\frac{a_\infty \bar{b}}{2}\left[-\bar{v}_{2c}+\frac{1}{2}\mu a_1^*+K a_2\bar{r}+2b_2\bar{r}\right]$$

$$\bar{\Gamma}_{2s}=\frac{a_\infty \bar{b}}{2}\left[-\bar{v}_{2s}+\frac{1}{2}\mu b_1^*-2a_2\bar{r}+K b_2\bar{r}\right] \qquad (3\text{—}6)$$

It should be noted that, under the premise of the simplified treatment (i.e. only the lower and same-order harmonics of circulation to the induced velocity are taken into account), the relationship between circulation and the induced velocity is obtained as a closed form.

From (2—5) and (3—6), the harmonic components of the induced velocity might be written as follows

$$\bar{v}_0=\frac{A_0^0}{1+A_0^0}\left[\vartheta_0^*\bar{r}+\Delta\vartheta\bar{r}^2+\lambda_0^*+\frac{1}{2}\mu a_1^*\right]$$

$$\bar{v}_{1c}=\frac{A_{1\,c}^{1\,c}}{1+A_{1\,c}^{1\,c}}\left[-\mu a_0+b_1^*\bar{r}+\frac{1}{2}\mu a_2+\frac{1}{2}\mu K b_2\right]+\frac{1}{1+A_{1\,c}^{1\,c}}\Delta\bar{v}_{1c}$$

$$\Delta\bar{v}_{1c}=\frac{A_{1c}^{0}}{1+A_0^0}\Big[\left(\lambda_0^*+\frac{1}{2}\mu a_1^*\right)\left(\frac{1}{2}\bar{r}+\frac{1}{3}\bar{r}^3\right)+$$

$$+\vartheta_0^*\left(-\frac{8}{9}\bar{r}+\frac{10}{7}\bar{r}^2+\frac{1}{10}\bar{r}^3\right.+$$

$$\left.+\Delta\vartheta\left(-\frac{1}{2}\bar{r}-\frac{1}{7}\bar{r}^2+\frac{8}{7}\bar{r}^3\right)\right]$$

$$\bar{v}_{1s}=\frac{A_{1\,s}^{1\,s}}{1+A_{1\,s}^{1\,s}}\left[\vartheta_0^*\mu+\Delta\vartheta\mu\bar{r}-a_1^*\bar{r}-\frac{1}{2}\mu K a_2+\frac{1}{2}\mu b_2\right]+\frac{1}{1+A_{1\,s}^{1\,s}}\Delta\bar{v}_{1s}$$

$$\Delta\bar{v}_{1s}=\frac{A_{1s}^{0}}{1+A_0^0}\left[\vartheta_0^*+\Delta\vartheta\bar{r}+\frac{1}{\bar{r}}\left(\lambda_0^*+\frac{1}{2}\mu a_1^*\right)\right]$$

$$\bar{r}_{2c}=\frac{A_{2\,c}^{2\,c}}{1+A_{2\,c}^{2\,c}}\left[\frac{1}{2}\mu a_1^*+K a_2\bar{r}+2b_2\bar{r}\right]+\frac{1}{1+A_{2\,c}^{2\,c}}\Delta\bar{v}_{2c}$$

$$\Delta\bar{v}_{2c}=\frac{A_{2c}^{0}}{1+A_0^0}\left[-\frac{1}{3}\vartheta_0^*\bar{r}-\frac{1}{2}\Delta\vartheta\bar{r}^2\right]+$$

$$+\frac{A^{1c}_{2c}}{1+A^{1c}_{1c}}\left\{\frac{A^0_{1c}}{1+A^0_0}\left[\left(\lambda^*_0+\frac{1}{2}\mu\,a^*_1\right)\left(-\frac{1}{15}\bar{r}-\frac{1}{10}\bar{r}^3-\frac{7}{20}\bar{r}^5\right)+\right.\right.$$

$$+\vartheta^*_0\left(\frac{3}{10}\bar{r}-\frac{2}{9}\bar{r}^3-\frac{1}{3}\bar{r}^5\right)+\varDelta\vartheta\left(\frac{2}{15}\bar{r}+\frac{16}{39}\bar{r}^3-\frac{11}{16}\bar{r}^5\right)\Big]+$$

$$\left.+\mu\,a_0\left(-\frac{1}{2}-\frac{1}{3}\bar{r}^2\right)+b^*_1\left(\frac{1}{16}\bar{r}+\frac{3}{8}\bar{r}^2+\frac{1}{4}\bar{r}^4\right)\right\}+$$

$$+\frac{A^{1s}_{2c}}{1+A^{1s}_{1s}}\overline{V}_1\left[-\frac{1}{2}\left(1-\frac{A^0_{1s}/\cos\alpha_1}{1+A^0_0}\right)\mu\cdot\varDelta\vartheta+\frac{1}{2}a^*_1-\right.$$

$$\left.-\frac{A^0_{1s}/\cos\alpha_1}{1+A^0_0}\,\mu\left(\lambda^*_0+\frac{1}{2}\mu\,a^*_1\right)\frac{1}{\bar{r}^2}\,(1+\ln\bar{r})\right]$$

$$\bar{v}_{2s}=\frac{A^{2s}_{2s}}{1+A^{2s}_{2s}}\left[\frac{1}{2}\mu\,b^*_1-2a_2\,\bar{r}+Kb_2\,\bar{r}\right]+\frac{1}{1+A^{2s}_{2s}}\varDelta\bar{v}_{2s}$$

$$\varDelta\bar{v}_{2s}=\frac{A^0_{2s}}{1+A^0_0}\overline{V}_1\left[\left(\lambda^*_0+\frac{1}{2}\mu\,a^*_1\right)\left(\frac{1}{\bar{r}}+\frac{1}{8}\bar{r}^2+\frac{1}{5}\bar{r}^4\right)+\right.$$

$$\left.+\vartheta^*_0\left(\frac{4}{7}+\frac{5}{32}\bar{r}^2+\frac{2}{7}\bar{r}^4\right)+\varDelta\vartheta\left(\frac{1}{8}\bar{r}+\frac{3}{8}\bar{r}^2+\frac{1}{3}\bar{r}^4\right)\right]+$$

$$+\frac{A^{1c}_{2s}}{1+A^{1c}_{1c}}\overline{V}_1\left\{\frac{A^0_{1c}}{1+A^0_0}\left[\left(\lambda^*_0+\frac{1}{2}\mu\,a^*_1\right)\left(-\frac{1}{4}-\frac{1}{9}\bar{r}^2-\frac{1}{5}\bar{r}^4\right)+\right.\right.$$

$$+\vartheta^*_0\left(-\frac{4}{21}-\frac{2}{9}\bar{r}^2-\frac{9}{50}\bar{r}^4-\frac{1}{4}\ln\bar{r}\right)$$

$$\left.\left.+\varDelta\vartheta\left(\frac{1}{4}-\frac{1}{10}\bar{r}-\frac{2}{5}\bar{r}^2-\frac{2}{7}\bar{r}^4\right)\right]+\frac{1}{2}b^*_1\right\}+$$

$$+\frac{A^{1s}_{2s}}{1+A^{1s}_{1s}}\left[\left(\lambda^*_0+\frac{1}{2}\mu\,a^*_1\right)\frac{A^0_{1s}/\cos\alpha_1}{1+A^0_0}\,\mu\left(-\frac{37}{40}\frac{1}{\bar{r}}-\frac{1}{15}\bar{r}^2-\frac{3}{10}\bar{r}^4\right)+\right.$$

$$+\vartheta^*_0\left(1-\frac{A^0_{1s}/\cos\alpha_1}{1+A^0_0}\right)\mu\left(\frac{1}{2}+\frac{1}{3}\bar{r}^2\right)+$$

$$+\varDelta\vartheta\left(1-\frac{A^0_{1s}/\cos\alpha_1}{1+A^0_0}\right)\mu\left(\frac{1}{16}\bar{r}+\frac{3}{8}\bar{r}^2+\frac{1}{4}\bar{r}^2\right)+$$

$$\left.+a^*_1\left(-\frac{1}{16}\bar{r}-\frac{3}{8}\bar{r}^2-\frac{1}{4}\bar{r}^4\right)\right]\qquad(3\text{—}7)$$

where A^0_0, A^0_{1c}, A^{1c}_{1c}……are defined as follows

$$A^0_0=\frac{a_\infty\bar{b}}{2}\,\frac{k}{4\pi\overline{V}_1}$$

$$A^{1c}_{1c}=A^0_0\,\frac{2\sin\alpha_1}{1+\sin\alpha_1},\qquad A^{1s}_{1s}=A^0_0\,\frac{2}{1+\sin\alpha_1}$$

$$A^{2c}_{2c}=A^0_0\,\frac{2+2\sin^2\alpha_1}{(1+\sin\alpha_1)^2},\qquad A^{2s}_{2s}=A^0_0\,\frac{4\sin\alpha_1}{(1+\sin\alpha_1)^2}$$

$$A_{1c}^{0}=A_{0}^{0}\frac{2\cos\alpha_1}{1+\sin\alpha_1}, \qquad A_{1s}^{0}=A_{1c}^{0}$$

$$A_{2c}^{0}=A_{0}^{0}\frac{2-2\sin\alpha_1}{1+\sin\alpha_1}, \qquad A_{2s}^{0}=A_{2c}^{0}$$

$$A_{2c}^{1c}=A_{0}^{0}\frac{2\cos\alpha_1\sin\alpha_1}{(1+\sin\alpha_1)^2}, \qquad A_{2s}^{1c}=A_{2c}^{1c}$$

$$A_{2c}^{1s}=A_{0}^{0}\frac{2\cos\alpha_1}{(1+\sin\alpha_1)^2}, \qquad A_{2s}^{1s}=A_{2c}^{1s}$$

Here must be mentioned that, in doing the integration for the induced velocity, the lower limit of the integrals should be changed to $\bar{r}_0$ instead of o, where $\bar{r}_0$ is the nondimensional radial distance at blade root cutout, if the infinite occures.

Since circulation $\bar{\Gamma}$ is expanded into Fourier series, the blade airload could be also written as Fourier series:

$$\frac{dC_{T1}}{d\bar{r}}=\frac{2}{\pi}\bar{U}\,\bar{\Gamma}=\left(\frac{dC_{T1}}{d\bar{r}}\right)_0+\sum_{m=1}\left[\left(\frac{dC_{T1}}{d\bar{r}}\right)_{mc}\cos m\psi+\left(\frac{dC_{T1}}{d\bar{r}}\right)_{ms}\sin m\psi\right] \tag{3—8}$$

From expressions (2—2), (3—1) and (3—3), we have

$$\left(\frac{dC_{T1}}{d\bar{r}}\right)_0=\frac{2}{\pi}\left(\bar{\Gamma}_0\bar{r}+\frac{1}{2}\mu\bar{\Gamma}_{1s}\right)$$

$$\left(\frac{dC_{T1}}{d\bar{r}}\right)_{1c}=\frac{2}{\pi}\left(\bar{\Gamma}_{1c}\bar{r}+\frac{1}{2}\mu\bar{\Gamma}_{2s}\right)$$

$$\left(\frac{dC_{T1}}{d\bar{r}}\right)_{1s}=\frac{2}{\pi}\left(\bar{\Gamma}_{1s}\bar{r}+\mu\bar{\Gamma}_0-\frac{1}{2}\mu\bar{\Gamma}_{2c}\right)$$

$$\left(\frac{dC_{T1}}{d\bar{r}}\right)_{2c}=\frac{2}{\pi}\left(\bar{\Gamma}_{2c}\bar{r}-\frac{1}{2}\mu\bar{\Gamma}_{1s}+\frac{1}{2}\mu\bar{\Gamma}_{3s}\right)$$

$$\left(\frac{dC_{T1}}{d\bar{r}}\right)_{2s}=\frac{2}{\pi}\left(\bar{\Gamma}_{2s}\bar{r}+\frac{1}{2}\mu\bar{\Gamma}_{1c}-\frac{1}{2}\mu\bar{\Gamma}_{3c}\right) \tag{3—9}$$

Then, the relations between harmonics of the blade airloads and harmonics of circulation are established.

As for the thrust coefficient of the whole rotor, it is easily given as

$$C_T=k\int_{\bar{r}_0}^{\bar{r}_1}\left(\frac{dC_{T1}}{d\bar{r}}\right)_0 d\bar{r}=\kappa k\int_0^1\left(\frac{dC_{T1}}{d\bar{r}}\right)_0 d\bar{r} \tag{3—10}$$

where $\bar{r}_0$ is the nondimensional radial distance at blade root cutout and

$\overline{r}_1$ is the tip loss factor, if it is desired to be considered.

4 FLAPPING CONDITION

Since there are flapping coefficients in the expressions of circulation and the induced velocuty, it is necessary to study the flapping motion.

For articulated rotor, the flapping motion of one blade, according to reference 6, is given as

$$\frac{d^2\beta_e}{dt^2}J_e+\beta_e\Omega^2(J_e+eS_e)=(M_A)_e-(M_G)_e \qquad (4\text{—}1)$$

where

$$J_e=\int_e^R(r-e)^2dm_1$$

——inertia moment of one blade about the flapping pin

$$S_e=\int_e^R(r-e)dm_1$$

——mass moment of one blade about the flapping pin

$$(M_A)_e=\int_{r_0}^{r_1}(r-e)dT_1$$

——thrust moment of one blade about the flapping pin

$$(M_G)_e=gS_e$$

——gravity moment of one blade about the flapping pin

or in nondimensional form:

$$\frac{d^2\beta_e}{d\psi^2}\overline{J}_e+\beta_e\,\overline{\nu}^2\,\overline{J}_e=(\overline{M}_A)_e-(\overline{M}_G)_e$$

And it can be written as

$$\frac{d^2\beta}{d\psi^2}\overline{J}+\beta\overline{\nu}^2\,\overline{J}=\overline{M}_A-\overline{V}_G \qquad (4\text{—}2)$$

where

$$\overline{J}_e=J_e/m_1R^2, \qquad \overline{J}=\overline{J}_e/(1-\overline{e})^2$$

$$\overline{\nu}^2=1+\frac{\overline{e}\overline{S}_e}{\overline{J}_e}=1+\frac{\overline{e}}{1-\overline{e}}\,\frac{\overline{S}}{\overline{J}}$$

$$\overline{S}_e=\overline{S}_e/m_1R, \qquad \overline{S}=\overline{S}_e/(1-\overline{e})$$

$$(\overline{M}_A)_e=(M_A)_e/m_1\Omega^2R^2, \qquad \overline{M}_A=(\overline{M}_A)_e/(1-\overline{e})$$

$$(\overline{M}_G)_e=(M_G)_e/m_1\Omega^2R^2, \qquad \overline{M}_G=\overline{g}\overline{S}$$

$$\overline{g}=g/\Omega^2R$$

If we express $\overline{M}_A$ into a Fourier series

$$\overline{M}_A=(\overline{M}_A)_0+\sum_{n=1}[(\overline{M}_A)_{nc}\cos n\psi+(\overline{M}_A)_{ns}\sin n\psi] \qquad (4\text{—}3)$$

then, we find

$$a_0\,\overline{\nu}^2\,\overline{J}=(\overline{M}_A)_0-\overline{g}\,\overline{S}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$a_n(n^2-\overline{\nu}^2)\overline{J}=(\overline{M}_A)_{nc}$$

$$b_n(n^2-\overline{\nu}^2)\overline{J}=(\overline{M}_A)_{ns} \qquad (4\text{—}4)$$

and

$$(\overline{M}_A)_0=\frac{\rho_H R^3}{m_1}\int_{\overline{r}_0}^{\overline{r}_1}\frac{\overline{r}-\overline{e}}{1-\overline{e}}\,\frac{\pi}{2}\left(\frac{dC_{T1}}{d\overline{r}}\right)_0 d\overline{r}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$(\overline{M}_A)_{nc}=\frac{\rho_H R^3}{m_1}\int_{\overline{r}_0}^{\overline{r}_1}\frac{\overline{r}-\overline{e}}{1-\overline{e}}\,\frac{\pi}{2}\left(\frac{dC_{T1}}{d\overline{r}}\right)_{nc} d\overline{r}$$

$$(\overline{M}_A)_{ns}=\frac{\rho_H R^3}{m_1}\int_{\overline{r}_0}^{\overline{r}_1}\frac{\overline{r}-\overline{e}}{1-\overline{e}}\,\frac{\pi}{2}\left(\frac{dC_{T1}}{d\overline{r}}\right)_{ns} d\overline{r} \qquad (4\text{—}5)$$

thus, in order to calculate flapping coefficients a_0, $\cdots\cdots a_{nc}$, a_{ns}, we must solve $(\overline{M}_A)_0$, $\cdots\cdots(\overline{M}_A)_{nc}$, $(\overline{M}_A)_{ns}$ before. These are long integrals. In a simple case, put $\overline{b}$=constant, $\overline{e}=0$, $\overline{\nu}^2=1$, and denote

$$\gamma=\frac{\rho_H R^3}{m_1}\,\frac{a_\infty \overline{b}}{2}\Big/\overline{J}$$

we get

$$a_0=\kappa\gamma\Big[\vartheta_0^*\Big(\frac{1}{4}+\frac{1}{4}\mu^2\Big)+\Delta\vartheta\Big(\frac{1}{5}+\frac{1}{6}\mu^2\Big)+\frac{1}{3}\lambda_0^*-$$

$$-\int_0^1\overline{v}_0\overline{r}^2\,d\overline{r}-\int_0^1\frac{1}{2}\mu\overline{v}_{1s}\,\overline{r}\,d\overline{r}\Big]-\frac{\overline{g}\overline{S}}{\overline{J}}$$

$$a_1^*=\Big[\frac{2}{3}\vartheta_0^*\,\mu+\frac{1}{2}\Delta\vartheta\,\mu+\frac{1}{2}\lambda_0^*\,\mu-\int_0^1\overline{v}_{1s}\,\overline{r}^2\,d\overline{r}-\int_0^1\mu\,\overline{v}_0\,\overline{r}\,d\overline{r}+$$

$$+\int_0^1 \frac{1}{2}\mu\,\bar{v}_{2c}\,\bar{r}\,d\bar{r}\Big]\Big/\Big(\frac{1}{4}-\frac{1}{8}\mu^2\Big)$$

$$b_1^* = \Big[\int_0^1 \bar{v}_{1c}\,\bar{r}^2\,d\bar{r} + \int_0^1 \frac{1}{2}\mu\,\bar{v}_{2s}\,\bar{r}\,d\bar{r} + \frac{1}{3}a_0\,\mu\Big]\Big/\Big(\frac{1}{4}+\frac{1}{8}\mu^2\Big)$$

$$a_2 = (q_1p_1+q_2p_2)/(q_1^2+q_2^2)$$

$$b_2 = (q_1p_2-q_2p_1)/(q_1^2+q_2^2) \qquad (4\text{—}6)$$

where

$$p_1 = -\frac{1}{4}\vartheta_0^*\mu^2 - \frac{1}{6}\Delta\vartheta\,\mu^2 + \int_0^1 \frac{1}{2}\mu\,\bar{v}_{1s}\,\bar{r}\,d\bar{r} - \int_0^1 \bar{v}_{2c}\,\bar{r}^2\,d\bar{r} + \frac{1}{3}\mu\,a_1^*$$

$$p_2 = -\int_0^1 \frac{1}{2}\mu\,\bar{v}_{1c}\,\bar{r}\,d\bar{r} - \int_0^1 \bar{v}_{2s}\,\bar{r}^2\,d\bar{r} - \frac{1}{4}\mu^2 a_0 + \frac{1}{3}\mu\,b_1^*$$

$$q_1 = \frac{3}{\kappa\gamma} - K\Big(\frac{1}{4}+\frac{1}{8}\mu^2\Big)$$

$$q_2 = \frac{1}{2} - \frac{1}{8}\mu^2$$

Here, when we determine a_n, b_n, only a_{n-2}, b_{n-2}, a_{n-1}, b_{n-1}, are taken into account, but a_{n+1}, b_{n+1}, a_{n+2}, b_{n+2} are not, as we noticed that the magnitude of higher order harmonics of flapping coefficients is smaller than lower ones.

And also we have

$$C_T = \kappa\frac{k}{\pi}a_\infty\bar{b}\Big[\vartheta_0^*\Big(\frac{1}{3}+\frac{1}{2}\mu^2\Big) + \Delta\vartheta\Big(\frac{1}{4}+\frac{1}{4}\mu^2\Big) +$$

$$+\frac{1}{2}\lambda_0^* - \int_0^1 \bar{v}_0\,\bar{r}\,d\bar{r} - \int_0^1 \frac{1}{2}\mu\,\bar{v}_{1s}\,d\bar{r}\Big] \qquad (4\text{—}7)$$

In the formulae of flapping coefficients a_0, a_1^*, b_1^*, ……and thrust coefficient C_T, by contrast to classical formulae, there are additional terms of induced velocity integrals. Using equations (3—7), we can integrate them out and further obtain:

$$C_T = \kappa\frac{k}{\pi}a_\infty\bar{b}\Big[\vartheta_0^*\Big(\frac{1}{3(1+A_0^0)} + \frac{1}{2(1+A_1^1{}_s)}\mu^2 - \frac{A_{1s}^0/\cos\alpha_1}{2(1+A_0^0)(1+A_1^1{}_s)}\mu^2\Big) +$$

$$+\Delta\vartheta\Big(\frac{1}{4(1+A_0^0)} + \frac{1}{4(1+A_1^1{}_s)}\mu^2 - \frac{A_{1s}^0/\cos\alpha_1}{4(1+A_0^0)(1+A_1^1{}_s)}\mu^2\Big) +$$

$$+\lambda_0^*\Big(\frac{1}{2(1+A_0^0)} - \frac{A_{1s}^0/\cos\alpha_1}{(1+A_0^0)(1+A_1^1{}_s)}\mu^2\Big)\Big] \qquad (4\text{—}8)$$

$$a_0=\kappa\gamma\Big[\vartheta_0^*\Big(\frac{1}{4(1+A_0^0)}+\frac{1}{4(1+A_{1s}^1)}\mu^2-\frac{A_{1s}^0/\cos\alpha_1}{4(1+A_0^0)(1+A_{1s}^1)}\mu^2\Big)+$$

$$+\varDelta\vartheta\Big(\frac{1}{5(1+A_0^0)}+\frac{1}{6(1+A_{1s}^1)}\mu^2-\frac{A_{1s}^0/\cos\alpha_1}{6(1+A_0^0)(1+A_{1s}^1)}\mu^2\Big)+$$

$$+\lambda_0^*\Big(\frac{1}{3(1+A_0^0)}-\frac{A_{1s}^0/\cos\alpha_1}{2(1+A_0^0)(1+A_{1s}^1)}\mu^2\Big)\Big]-\frac{\bar{g}S}{J} \qquad (4\text{—}9)$$

$$a_1^*=\Big[\vartheta_0^*\mu\Big(\frac{1}{3(1+A_0^0)}+\frac{1}{3(1+A_{1s}^1)}-\frac{A_{1s}^0/\cos\alpha_1}{3(1+A_0^0)(1+A_{1s}^1)}\Big)+$$

$$+\varDelta\vartheta\mu\Big(\frac{1}{4(1+A_0^0)}+\frac{1}{4(1+A_{1s}^1)}-\frac{A_{1s}^0/\cos\alpha_1}{4(1+A_0^0)(1+A_{1s}^1)}\Big)+$$

$$+\lambda_0^*\mu\Big(\frac{1}{2(1+A_0^0)}-\frac{A_{1s}^0/\cos\alpha_1}{2(1+A_0^0)(1+A_{1s}^1)}\Big)\Big]\Big/$$

$$\Big/\Big[\frac{1}{4(1+A_{1s}^1)}+\frac{A_{1s}^0/\cos\alpha_1}{4(1+A_0^0)(1+A_{1s}^1)}\mu^2-\frac{1}{4(1+A_0^0)}\mu^2+\frac{1}{8}\mu^2\Big]$$

$$(4\text{—}10)$$

$$b_1^*=\Big[\frac{1}{3(1+A_{1c}^1)}\mu a_0+\frac{A_{1c}^0}{(1+A_0^0)(1+A_{1c}^1)}\,(0.08\vartheta_0^*+0.04\varDelta\vartheta+$$

$$+0.2\Big(\lambda_0^*+\frac{1}{2}\mu a_1^*\Big)\Big]\Big/\Big[\frac{1}{4(1+A_{1c}^1)}+\frac{\mu}{8}\Big] \qquad (4\text{—}11)$$

in which, $\bar{v}_{2c}$ and $\bar{v}_{2s}$ are neglected.

It can be seen from equations (4—8), (4—9) and (4—11) that, under the same flight condition (λ_0^*, μ, ϑ_0^*), the values of C_T and a_0, in which the variable induced velocity distribution is taken into consideration, are smaller than that of considering constant induced velocity distribution, while the value of b_1^* is much larger. As already discussed in references 7 and 9, the longitudinal induced velocity distribution has a pronounced influence on the sine flapping coefficient b_1^* and it is very important to the lateral control. In this paper, we first bring up the analytical expressions for C_T and flapping coefficients with the effect of variable induced velocity distribution but in terms of blade parameters and flight parameters only.

Furthermore, we have

$$a_2=(q_{22}p_{11}+q_{12}p_{22})/(q_{11}q_{22}+q_{12}q_{21})$$

$$b_2=(q_{11}p_{22}-q_{21}p_{11})/(q_{11}q_{22}+q_{12}q_{21}) \tag{4—12}$$

where

$$p_{11}=\vartheta_0^*\left[\frac{A_{2c}^{0}}{12(1+A_0^0)(1+A_{2c}^{2c})}-\frac{1}{4(1+A_{1s}^{1s})}\mu^2\left(1-\frac{A_{1s}^{0}/\cos\alpha_1}{1+A_0^0}\right)\right]+$$

$$+\Delta\vartheta\left[\frac{A_{2c}^{0}}{10(1+A_0^0)(1+A_{2c}^{2c})}-\frac{1}{6(1+A_{1s}^{1s})}\mu^2\left(1-\frac{A_{1s}^{0}/\cos\alpha_1}{1+A_0^0}\right)\right]+$$

$$+\frac{1}{6}\mu a_1^*\left[\frac{1}{1+A_{1s}^{1s}}+\frac{1}{1+A_{2c}^{2c}}\right]+\frac{16A_{2c}^{1c}}{125(1+A_{1c}^{1c})(1+A_{2c}^{2c})}b_1^*,$$

$$p_{22}=\vartheta_0^*\mu\left[-\frac{3A_{1c}^{0}}{79(1+A_0^0)(1+A_{1c}^{1c})}-\frac{5A_{2s}^{0}/\cos\alpha_1}{19(1+A_0^0)(1+A_{2s}^{2s})}-\right.$$

$$\left.-\frac{20A_{2s}^{1s}}{87(1+A_{1s}^{1s})(1+A_{2s}^{2s})}\left(1-\frac{A_{1s}^{0}/\cos\alpha_1}{1+A_0^0}\right)\right]+$$

$$+\Delta\vartheta\mu\left[-\frac{A_{1c}^{0}}{77(1+A_0^0)(1+A_{1c}^{1c})}-\frac{2A_{2s}^{0}/\cos\alpha_1}{13(1+A_0^0)(1+A_{2s}^{2s})}-\right.$$

$$\left.-\frac{16A_{2s}^{1s}}{125(1+A_{1s}^{1s})(1+A_{2s}^{2s})}\left(1-\frac{A_{1s}^{0}/\cos\alpha_1}{1+A_0^0}\right)\right]-$$

$$-\frac{1}{4(1+A_{1c}^{1c})}\mu^2 a_0+\frac{16A_{2s}^{1s}}{125(1+A_{1s}^{1s})(1+A_{2s}^{2s})}a_1^*+$$

$$+\frac{1}{6}\mu b_1^*\left(\frac{1}{1+A_{1c}^{1c}}+\frac{1}{1+A_{2s}^{2s}}\right),$$

$$q_{11}=\frac{3}{\kappa\gamma}-\left[\frac{1}{4(1+A_{2c}^{2c})}+\frac{1}{8(1+A_{1s}^{1s})}\mu^2\right]K,$$

$$q_{12}=\frac{1}{2(1+A_{2c}^{2c})}-\frac{1}{8(1+A_{1s}^{1s})}\mu^2,$$

$$q_{22}=\frac{3}{\kappa\gamma}-\left[\frac{1}{4(1+A_{2s}^{2s})}+\frac{1}{8(1+A_{1c}^{1c})}\mu^2\right]K,$$

$$q_{21}=\frac{1}{2(1+A_{2s}^{2s})}-\frac{1}{8(1+A_{1c}^{1c})}\mu^2,$$

in p_{11} and p_{22}, some smaller terms are neglected.

5 BLADE AIRLOADS

Substituting the expressions of the induced velocity harmonics (3—7) into the equations of the circulation harmonics (3—6), we obtain the latter in a matrix form:

$$\begin{pmatrix} \bar{\Gamma}_0 \\ \bar{\Gamma}_{1c} \\ \bar{\Gamma}_{1s} \\ \bar{\Gamma}_{2c} \\ \bar{\Gamma}_{2s} \end{pmatrix} = \frac{a_\infty \bar{b}}{2} [Q] \cdot \begin{pmatrix} \vartheta_0^* \\ \Delta\vartheta \\ \lambda_0^* \\ a_0 \\ a_1^* \\ b_1^* \\ a_2 \\ b_2 \end{pmatrix} \tag{5—1}$$

where all elements in matrix [Q] are given in Appendix 1 .

Next, substituting the expressions of the circulation harmonics (5—1) into the equations of the blade airloads (3—9), we obtain finally:

$$\begin{pmatrix} \left(\frac{dC_{T1}}{d\bar{r}}\right)_0 \\ \left(\frac{dC_{T1}}{d\bar{r}}\right)_{1c} \\ \left(\frac{dC_{T1}}{d\bar{r}}\right)_{1s} \\ \left(\frac{dC_{T1}}{d\bar{r}}\right)_{2c} \\ \left(\frac{dC_{T1}}{d\bar{r}}\right)_{2s} \end{pmatrix} = \frac{a_\infty \bar{b}}{\pi} [P] \cdot \begin{pmatrix} \vartheta_0^* \\ \Delta\vartheta \\ \lambda_0^* \\ a_0 \\ a_1^* \\ b_1^* \\ a_2 \\ b_2 \end{pmatrix} \tag{5—2}$$

where matrix [P] is:

$$[P] = \begin{pmatrix} \bar{r} & 0 & \frac{\mu}{2} & 0 & 0 \\ 0 & \bar{r} & 0 & 0 & \frac{\mu}{2} \\ \mu & 0 & \bar{r} & -\frac{\mu}{2} & 0 \\ 0 & 0 & -\frac{\mu}{2} & \bar{r} & 0 \\ 0 & \frac{\mu}{2} & 0 & 0 & \bar{r} \end{pmatrix} \cdot [Q]$$

6 AN EXAMPLE

In illustration of the present method, we take the rotor blades of Y—2 Helicopter as an example and compute the flapping coefficients for $\mu=0.05$, 0.075, 0.10, 0.125, 0.15, 0.20, 0.24 and the thrust loads for $\mu=0.20$ with a calculator. The initial data are given as follows:

$R=5\ m$	$\bar{b}=0.0486$
$\Delta\vartheta=-0.1396$ rad.	$\bar{e}=0.014$
$K=0.3$	$k=3$
$m_1=2.755$ kg-sec$^2/m$	$\Omega=37.48$ rad/sec
$\rho_H=0.108$ kg-sec$^2/m^4$	$a_\infty=5.73$

and the flight parameters are taken from trim calculation. For instance, at $\mu=0.20$, we find:

$\bar{V}_1=0.2053$	$\cos\alpha_1=0.9741$
$\vartheta_0^*=0.2409$	$\lambda_0^*=-0.02494$

Then, according to the formulae of calculating the flapping coefficients (4—9), (4—10), (4—11), (4—12) and (4—13), the results of a_0, a_1^*, b_1^*, a_2, and b_2 versus advance ratio μ are obtained and plotted in Figures 1, 2, 3, 4 and 5 respectively.

In those figures, the results of the flapping coefficients for constant induced velocity distribution are also plotted in comparision. It can be seen that, as stated before, the curve of a_0 for variable induced velocity distribution is lower than that of a_0 for constant distribution. The curves of a_1^* for two distributions are nearly the same. However, the curves of b_1^* for two distributions are quite different. The former is larger than the latter, particularly, there is a peak at low speeds. This phenomenon was observed in many tests (ref. 9). The curves of a_2 and of $(-b_2)$ are slightly similar to that of b_1^*, but the magnitude of a_2 and of $(-b_2)$ are one-tenth smaller than that of b_1^*. And it is resonable in fact that we might neglect the higher order flapping coefficients when calculating the lower ones.

Finally, from formulae of the induced velocity harmonics (3—7) and from formulae of the blade airloads harmonics (5—2), the values of v_0, v_{1c}, v_{1s}, v_{2c}, v_{2s} and of $\left(\frac{dT_1}{dr}\right)_0$, $\left(\frac{dT_1}{dr}\right)_{1c}$, $\left(\frac{dT_1}{dr}\right)_{1s}$, $\left(\frac{dT_1}{dr}\right)_{2c}$, $\left(\frac{dT_1}{dr}\right)_{2s}$ are calculated along radius for $\mu=0.20$. And the results are shown in Figs. 6—10 and in Figs. 11—15. In order to verify the accuracy of the simplified method, the results of the blade airloads harmonics from the numerical integration method (ref. 10) are also plotted in Figs. 11—15. It can be seen that the curves of the airloads from different methods are in good coincidence. Besides, in Fig. 16, the curves of the blade airloads along azimuth for different radial distances are plotted for illustration. The tendency of these curves are very similar to those, which were found in reference 1.

7 CONCLUSIONS

The major conclusions obtained form the present study can be summarized below.

(1) Based on the generalized classical rotor vortex theory and the blade element theory, a closed form of relations between the induced velocity and circulation is established.

(2) It might be the first time to set up the analytical expressions of flapping coefficients and blade airloads, including the effect of variable induced velocity distribution but in terms of blade parameters and flight parameters only.

(3) The method developed here for predicting rotor blade airloads is simplified for calculation and it is believed to be suitable for engineering application.

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APPENDIX I

The elements in matrix [Q] of the equation (5—1) are given as follows:

$$Q_{11}=\frac{1}{1+A_0^0}\bar{r},$$

$$Q_{12}=\frac{1}{1+A_0^0}\bar{r}^2,$$

$$Q_{13}=\frac{1}{1+A_0^0},$$

$$Q_{14}=0,$$

$$Q_{15}=\frac{1}{2(1+A_0^0)}\mu,$$

$$Q_{16}=Q_{17}=Q_{18}=0,$$

$$Q_{21}=\frac{A_{1c}^0}{(1+A_0^0)(1+A_{1c}^{1c})}\left(\frac{8}{9}\bar{r}-\frac{10}{7}\bar{r}^2-\frac{1}{10}\bar{r}^3\right),$$

$$Q_{22}=\frac{A_{1c}^0}{(1+A_0^0)(1+A_{1c}^{1c})}\left(\frac{1}{2}\bar{r}+\frac{1}{7}\bar{r}^2-\frac{8}{7}\bar{r}^3\right),$$

$$Q_{23}=\frac{A_{1c}^0}{(1+A_0^0)(1+A_{1c}^{1c})}\left(-\frac{1}{2}\bar{r}-\frac{1}{3}\bar{r}^3\right),$$

$$Q_{24}=\frac{1}{1+A_{1c}^{1c}}(-\mu),$$

$$Q_{25}=\frac{A_{1c}^0}{(1+A_0^0)(1+A_{1c}^{1c})}\left(-\frac{1}{4}\bar{r}-\frac{1}{6}\bar{r}^3\right),$$

$$Q_{26}=\frac{1}{1+A_{1c}^{1c}}\bar{r},$$

$$Q_{27}=\frac{1}{2(1+A_{1c}^{1c})}\mu,$$

$$Q_{28}=\frac{1}{2(1+A_{1c}^{1c})}\mu K,$$

$$Q_{31}=\frac{1}{1+A_{1s}^{1s}}\left(1-\frac{A_{1s}^0/\cos\alpha_1}{1+A_0^0}\right)\mu,$$

$$Q_{32}=\frac{1}{1+A^{1s}_{1s}}\left(1-\frac{A^{0}_{1s}{}'/\cos\alpha}{1+A^{0}_{0}}\right)\mu\bar{r},$$

$$Q_{33}=\frac{1}{1+A^{1s}_{1s}}\left(-\frac{A^{0}_{1s}{}'/\cos\alpha_1}{1+A^{0}_{0}}\right)\mu\frac{1}{\bar{r}},$$

$$Q_{34}=0,$$

$$Q_{35}=\frac{1}{1+A^{1s}_{1s}}\left(-\frac{A^{0}_{1s}/\cos\alpha_1}{1+A^{0}_{0}}\frac{1}{2}\mu^2\frac{1}{\bar{r}}-\bar{r}\right),$$

$$Q_{36}=0,$$

$$Q_{37}=\frac{1}{1+A^{1s}_{1s}}\left(-\frac{1}{2}\mu K\right),$$

$$Q_{38}=\frac{1}{1+A^{1s}_{1s}}\frac{1}{2}\mu,$$

$$Q_{41}=\frac{1}{1+A^{2c}_{2c}}\left[\frac{A^{0}_{2c}}{3(1+A^{0}_{0})}\bar{r}+\frac{A^{1}_{0c}A^{1c}_{2c}}{(1+A^{0}_{0})(1+A^{1c}_{1c})}\left(-\frac{3}{10}\bar{r}+\right.\right.$$

$$\left.\left.+\frac{2}{9}\bar{r}^3+\frac{1}{3}\bar{r}^5\right)\right],$$

$$Q_{42}=\frac{1}{1+A^{2c}_{2c}}\left[\frac{A^{0}_{2c}}{2(1+A^{0}_{0})}\bar{r}^2+\frac{A^{0}_{1c}A^{1c}_{2c}}{(1+A^{0}_{0})(1+A^{1c}_{1c})}\left(-\frac{2}{15}\bar{r}-\right.\right.$$

$$\left.\left.-\frac{16}{39}\bar{r}^3+\frac{11}{16}\bar{r}^5\right)+\frac{A^{1s}_{2c}/\cos\alpha_1}{2(1+A^{1s}_{1s})}\left(1-\frac{A^{0}_{1s}/\cos\alpha_1}{1+A^{0}_{0}}\right)\mu^2\right],$$

$$Q_{43}=\frac{1}{1+A^{2c}_{2c}}\left[-\frac{A^{0}_{1c}A^{1c}_{2c}}{(1+A^{0}_{0})(1+A^{1c}_{1c})}\left(\frac{1}{15}\bar{r}+\frac{1}{10}\bar{r}^3+\frac{7}{20}\bar{r}^5\right)+\right.$$

$$\left.+\frac{A^{0}_{1c}A^{1s}_{2c}/\cos^2\alpha_1}{(1+A^{0}_{0})(1+A^{1s}_{1s})}\mu^2\frac{1}{\bar{r}^2}(1+\ln\bar{r})\right],$$

$$Q_{44}=-\frac{A^{1c}_{2c}}{(1+A^{1c}_{1c})(1+A^{2c}_{2c})}\mu\left(\frac{1}{2}+\frac{1}{3}\bar{r}^2\right),$$

$$Q_{45}=\frac{1}{1+A^{2c}_{2c}}\mu\left[\frac{1}{2}+\frac{A^{0}_{1c}A^{1c}_{2c}}{(1+A^{0}_{0})(1+A^{1c}_{1c})}\left(\frac{1}{30}\bar{r}+\right.\right.$$

$$\left.+\frac{1}{20}\bar{r}^3+\frac{7}{40}\bar{r}^5\right)+\frac{A^{1s}_{2c}/\cos\alpha_1}{1+A^{1s}_{1s}}\left(-\frac{1}{2}+\right.$$

$$\left.\left.+\frac{A^{0}_{1s}/\cos\alpha_1}{1+A^{0}_{0}}\frac{1}{2}\mu^2\frac{1}{\bar{r}^2}(1+\ln\bar{r})\right)\right],$$

$$Q_{46}=\frac{A^{1c}_{2c}}{(1+A^{1c}_{1c})(1+A^{2c}_{2c})}\left(-\frac{1}{16}\bar{r}-\frac{3}{8}\bar{r}^2-\frac{1}{4}\bar{r}^4\right),$$

$$Q_{47}=\frac{1}{1+A^{2c}_{2c}}K\bar{r},$$

$$Q_{48}=\frac{1}{1+A_{2\,c}^{2\,c}}2\bar{r},$$

$$Q_{51}=\frac{1}{1+A_{2\,s}^{2\,s}}\Big[\frac{A_{2s}^{0}/\cos\alpha_1}{1+A_0^0}\mu\Big(-\frac{4}{7}-\frac{5}{32}\bar{r}^2-\frac{2}{7}\bar{r}^4\Big)+$$

$$+\frac{A_{1c}^{0}A_{2\,s}^{1\,c}/\cos\alpha_1}{(1+A_0^0)(1+A_{1\,c}^{1\,c})}\mu\Big(\frac{4}{21}+\frac{2}{9}\bar{r}^2+\frac{9}{50}\bar{r}^4+\frac{1}{4}\ln r\Big)+$$

$$+\frac{A_{2\,s}^{1\,s}}{1+A_{1\,s}^{1\,s}}\Big(1-\frac{A_{1s}^{0}/\cos\alpha_1}{1+A_0^0}\Big)\mu\Big(-\frac{1}{2}-\frac{1}{3}\bar{r}^2\Big)\Big],$$

$$Q_{52}=\frac{1}{1+A_{2\,s}^{2\,s}}\Big[\frac{A_{2s}^{0}/\cos\alpha_1}{1+A_0^0}\mu\Big(-\frac{1}{8}\bar{r}-\frac{3}{8}\bar{r}^2-\frac{1}{3}\bar{r}^4\Big)+$$

$$+\frac{A_{1c}^{0}A_{2\,s}^{1\,c}/\cos\alpha_1}{(1+A_0^0)(1+A_{1\,c}^{1\,c})}\mu\Big(-\frac{1}{4}+\frac{1}{10}\bar{r}+\frac{2}{5}\bar{r}^2+\frac{2}{7}\bar{r}^4\Big)+$$

$$+\frac{A_{2\,s}^{1\,s}}{1+A_{1\,s}^{1\,s}}\Big(1-\frac{A_{1s}^{0}/\cos\alpha_1}{1+A_0^0}\Big)\mu\Big(-\frac{1}{16}\bar{r}-\frac{3}{8}\bar{r}^2-\frac{1}{4}\bar{r}^4\Big)\Big],$$

$$Q_{53}=\frac{1}{1+A_{2\,s}^{2\,s}}\Big[\frac{A_{2s}^{0}/\cos\alpha_1}{1+A_0^0}\mu\Big(-\frac{1}{\bar{r}}-\frac{1}{8}\bar{r}^2-\frac{1}{5}\bar{r}^4\Big)+$$

$$+\frac{A_{1c}^{0}A_{2\,s}^{1\,c}/\cos\alpha_1}{(1+A_0^0)(1+A_{1\,c}^{1\,c})}\mu\Big(\frac{1}{4}+\frac{1}{9}\bar{r}^2+\frac{1}{5}\bar{r}^4\Big)+$$

$$+\frac{A_{2\,s}^{1\,s}}{1+A_{1\,s}^{1\,s}}\,\frac{A_{1s}^{0}/\cos\alpha_1}{1+A_0^0}\mu\Big(\frac{37}{40}\cdot\frac{1}{\bar{r}}+\frac{1}{15}\bar{r}^2+\frac{3}{10}\bar{r}^4\Big)\Big],$$

$$Q_{54}=0,$$

$$Q_{55}=\frac{1}{1+A_{2\,s}^{2\,s}}\Big[\frac{A_{2s}^{0}/\cos\alpha_1}{1+A_0^0}\mu\Big(-\frac{1}{2\bar{r}}-\frac{1}{16}\bar{r}^2-\frac{1}{10}\bar{r}^4\Big)+$$

$$+\frac{A_{1c}^{0}A_{2\,s}^{1\,c}/\cos\alpha_1}{(1+A_0^0)(1+A_{1\,c}^{1\,c})}\mu\Big(\frac{1}{8}+\frac{1}{18}\bar{r}^2+\frac{1}{10}\bar{r}^4\Big)+$$

$$+\frac{A_{1s}^{0}A_{2\,s}^{1\,s}/\cos\alpha_1}{(1+A_0^0)(1+A_{1\,s}^{1\,s})}\mu\Big(\frac{37}{80}\cdot\frac{1}{\bar{r}}+\frac{1}{30}\bar{r}^2+\frac{3}{20}\bar{r}^4\Big)+$$

$$+\frac{A_{2\,s}^{1\,s}}{1+A_{1\,s}^{1\,s}}\Big(\frac{1}{16}\bar{r}+\frac{3}{8}\bar{r}^2+\frac{1}{4}\bar{r}^4\Big)\Big],$$

$$Q_{56}=\frac{1}{1+A_{2\,s}^{2\,s}}\,\frac{1}{2}\mu\Big(1-\frac{A_{2\,s}^{1\,c}/\cos\alpha_1}{1+A_{1\,c}^{1\,c}}\Big),$$

$$Q_{57}=\frac{1}{1+A_{2\,s}^{2\,s}}(-2\bar{r}),$$

$$Q_{58}=\frac{1}{1+A_{2\,s}^{2\,s}}K\bar{r}\ .$$

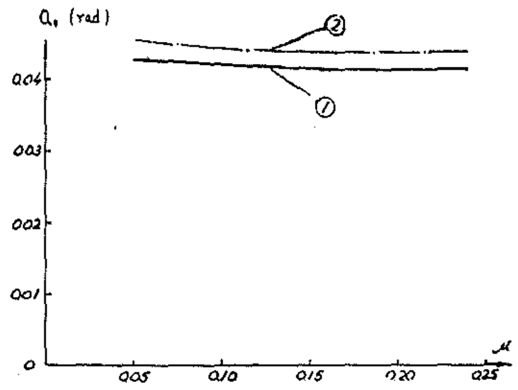

Fig.1

① present study

② classical formula

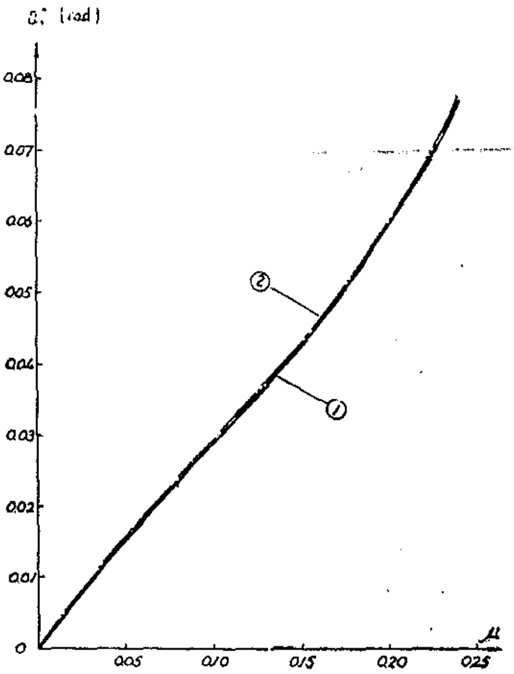

Fig.2

① present study

② classical formula

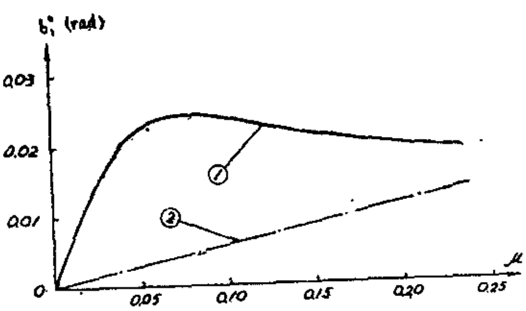

Fig.3

① present study

② classical formula

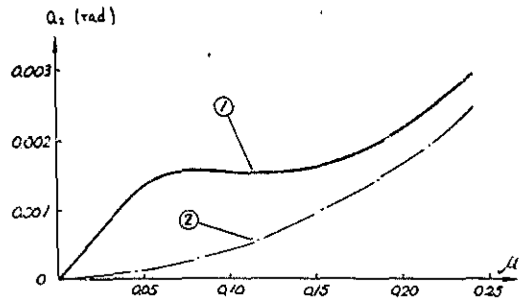

Fig.4

① present study

② classical formula

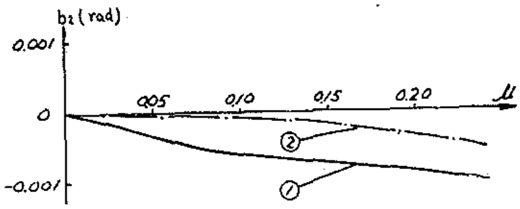

Fig.5

① present study

② classical formula

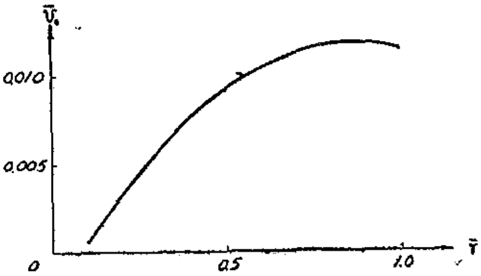

Fig.6

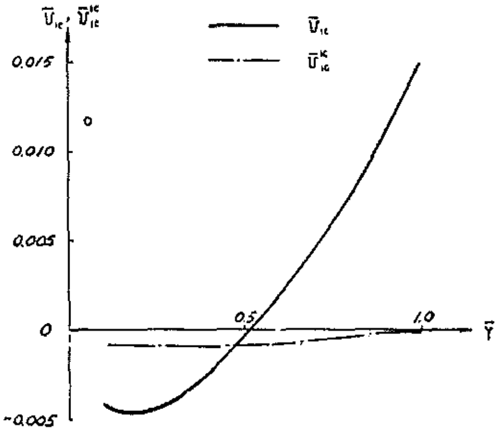

Fig.7

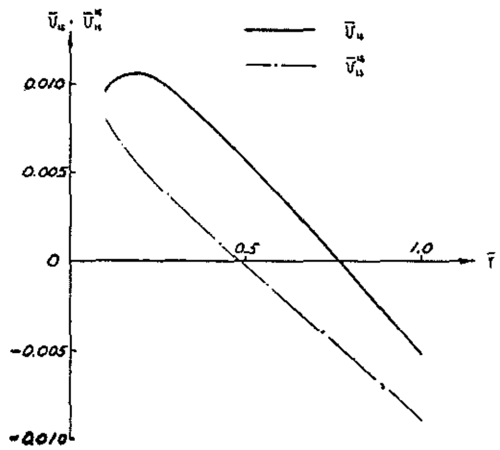

Fig.8

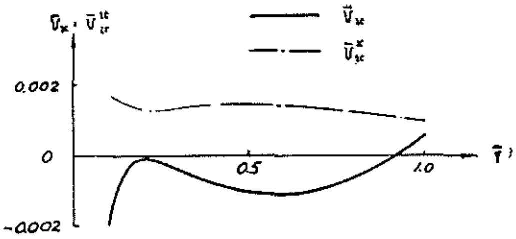

Fig.9

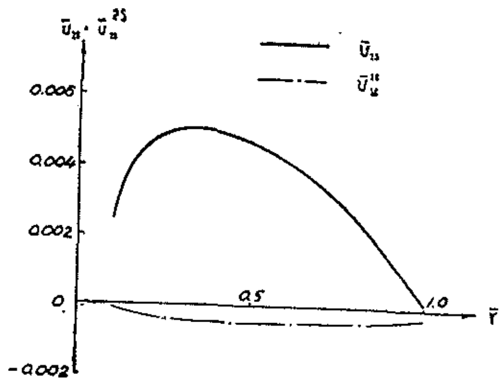

Fig.10

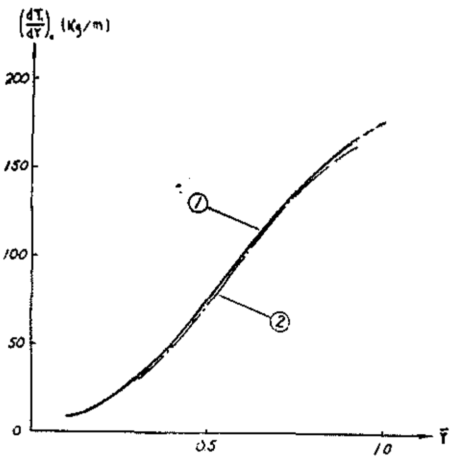

Fig.11

① present study

② numerical solution

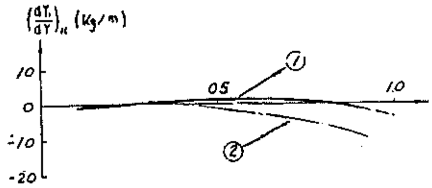

Fig. 12
① present study
② numerical solution

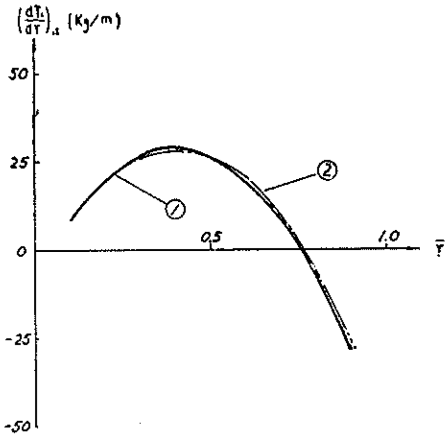

Fig. 13
① present study
② numerical solution

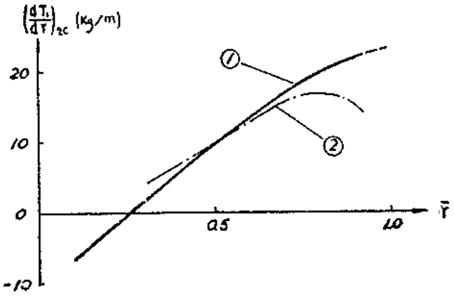

Fig. 14
① present study
② numerical solution

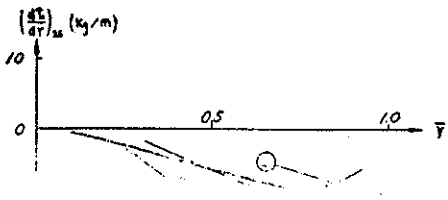

Fig. 15
① present study
② numerical solution

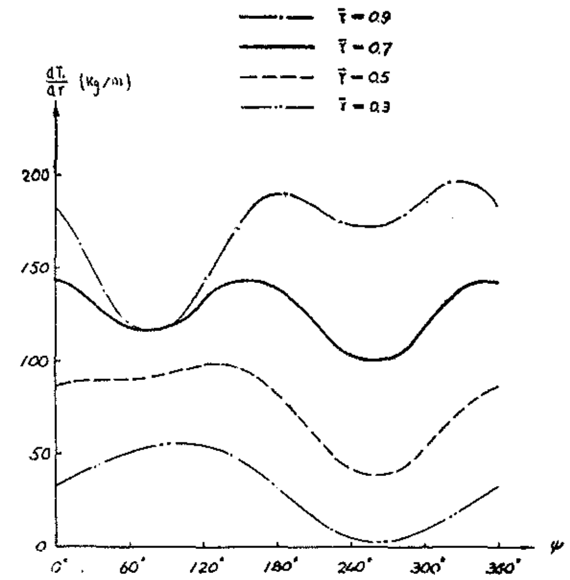
r̄ = 0.9
r̄ = 0.7
r̄ = 0.5
r̄ = 0.3
dT/dr (kg/m)
200
150
100
50
0
0°
60°
120°
180°
240°
300°
360°
ψ

Fig.16