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A COMPLETE METHOD FOR COMPUTATION OF BLADE MODE CHARACTERISTICS AND RESPONSES IN FORWARD FLIGHT

by

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1– GENERAL

The dynamic behaviour of a helicopter rotor is too complex to be fully theoretically treated, at present, in spite of all advanced computers available now.

Therefore, it is out of question, in this paper, to present the complete equations of the phenomena which would exceed, anyway, the capabilities of present-day computers.

This paper deals with some of the methods used at Aerospatiale and will focus first on the three following points which were the subject of concurrent research efforts at Marignane for the last few years :

- hypotheses and form of equations
- methods of calculation
- numerical adaptation.

Obviously, the hypotheses are the most critical point ; for that reason we will insist especially on their choice, more or less restrictive, depending both upon the present state of knowledge and the permissible length of the calculations. These hypotheses lead directly to the formulation of the phenomena, formulation it is unnecessary to explain in detail any longer thanks to modular programming.

We will afterwards deal with the two important problems raised by a spinning rotor :

- determination of natural modes qualified by their eigen frequency and damping.
- behaviour in forward flight, dealt with by the azimuth method which avoids those difficulties inherent to the modal methods.

Some examples will illustrate the calculation process to be used to improve a rotor while at design stage.

2– HYPOTHESES

2.1 – Rotor break-down into its constituent parts

The blades, rotor head and pitch control components are considered **an assembly of rotating parts.**

Each part is considered as an heterogeneous beam specified in its own system of reference, which takes into account the pre-settings in position and orientation.

One of the axes of this system of reference constitutes the line of centres of cross-sections. Along this line, the geometrical, elastic, dynamic and aerodynamic characteristics of the part are given per unit length.

A part can be split into elements of finite length whose ends are the junctions of the assembly. Each junction enforces a set of punctual boundary conditions expressing the type of attachment : rigid, articulated, flexible attachment ..., with or without damping. Mobility of rotor centre constitutes a particular boundary condition .

2.2 – Twelve components state vector and blade deformation

The basic hypothesis, associated with beam-representation, is that a **cross-section remains plane and free of deformation.**

The deformation, expressed in the initial system of reference of the part, is then entirely defined through

1) the 3 components of the displacement of the center of each section and
2) the 3 **angles of rotation of the local system of reference associated to this section** (local reference system which would be super-imposed on the initial reference system of the part in the absence of deformation).

The part of the element located beyond a section applies onto the part of the element located below this section a system of forces which are equivalent at the centre of the section to **a force** and **a moment** whose components are specified in the system of reference associated to the section : that is 6 components altogether.

The total of these 12 figures constitutes the **state vector** whose evolution, at a given time, along the line of centres of the element determines the total deformation.

$$\text{Notations}\begin{cases}\text{blade deformation :} \\ \begin{bmatrix}D_1\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}d_1\end{bmatrix},\begin{bmatrix}\delta_1\end{bmatrix}\end{bmatrix}\begin{cases}\begin{bmatrix}d_1\end{bmatrix}\text{ : displacement}\\ \begin{bmatrix}\delta_1\end{bmatrix}\text{ : rotation}\end{cases} \\ \text{Force and moment} \\ \begin{bmatrix}F_1\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}T_1\end{bmatrix},\begin{bmatrix}M_1\end{bmatrix}\end{bmatrix}\begin{cases}\begin{bmatrix}T_1\end{bmatrix}\text{ : force}\\ \begin{bmatrix}M_1\end{bmatrix}\text{ : moment}\end{cases} \\ \text{State vector :} \\ \begin{bmatrix}X_1\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}D_1\end{bmatrix},\begin{bmatrix}F_1\end{bmatrix}\end{bmatrix}\quad \text{12 components row vector}\end{cases}$$

2.3 — Splitting of an element

An element is in turn divided into an **infinite sequence of slices** of infinitesimal thickness limited by cross-sections and in **relative equilibrium** with respect to parts of elements situated on either side.

Each elementary slice is itself composed of **individual fibers** linking the homologous points of the two sections limiting the slice.

The fibers and the slices are submitted to forces, which are calculated with the two following hypotheses :

- the strain of a fiber submitted to forces at its two ends is reckoned using the laws of linear elasticity
- the whole forces acting on a slice (internal forces, aerodynamic forces, weight, inertial forces) form a system equivalent to zero.

2.4 — Elastic scheme

If $[D_1]$ is the deformation at the center of the section located at abscissa x , the deformation at the centre of the section of the slice located at abscissa x + dx is $[D_1] + [D'_1]$ dx where prime (') is the spatial derivative with respect to x . The values $[D_1]$ and $[D'_1]$ entirely characterise the deformation of each fiber when one takes into account the hypothesis of cross-sections non-deformability.

The deformation of each fiber is broken down into elongation and shear and the force producing this deformation is the sum of a force proportional to elongation and a force proportional to shear.

The reduction of these forces at center of section of abscissa x gives the stress $[F_1] \equiv [T_1] , [M_1]$ expressed as a function of $[D_1]$ and $[D'_1]$:

$$[D'_1] = \Phi\left([D_1] \; , \; [F_1]\right)$$

(equation of deformation)

This differential equation for the six components of $[D_1]$ is non-linear ; it introduces the section elastic characteristics under the form of a stiffness matrix. No particular hypothesis is retained as regards section contour and its characteristic points and axes.

2.5 — Dynamic scheme

The adjacent slices submit a given slice to the following forces :

- forces reducible to ($-\vec{T_1}$, $-\vec{M_1}$) at centre C_1 of abscissa x section,
- forces reducible to ($\vec{T_1} + \vec{T'_1}$ dx , $\vec{M_1} + \vec{M'_1}$ dx) at centre C'_1 of (x + d x) abscissa section.

Moreover, the slice is submitted to the action of inertia and gravity forces, reducible in turn at C_1 to ($\vec{ti_1}$ dx , $\vec{mi_1}$ dx) and to the action of aerodynamic forces reducible at C_1 to ($\vec{ta_1}$ dx , $\vec{ma_1}$ dx).

The equilibrium of the slice obtained by reducing all the forces at C_1 can then be written :

$$\vec{T'_1}\, dx + (\vec{ti_1} + \vec{ta_1})\, dx = 0$$

$$\vec{M'_1}\, dx + \overrightarrow{C_1 C'_1} \wedge (\vec{T_1} + \vec{T'_1}\, dx) + (\vec{mi_1} + \vec{ma_1})\, dx = 0$$

In the latter equation, the term $\overrightarrow{C_1 C'_1} \wedge \vec{T'_1}$ dx, being of second order, must be neglected.

The gravity force acting upon an individual fiber is of the form $-\rho \vec{g}\, \delta S\, dx$ where ρ is the density of the fiber, $\vec{g}$ the gravity and δS the surface element associated to the fiber.

The inertia force is of the form $-\rho \vec{\gamma}\, \delta S\, dx$ where $\vec{\gamma}$ is the local acceleration associated to motion. The identification of a point of the rotor being simply expressed by means of a succession of translations and rotations from a Galilean system of reference, the expression of $\vec{\gamma}$ remains easy, provided one uses a matricial formulation.

Reduced to a force and a moment at centre C_1 of abscissa x section, the expressions of $\vec{ti_1}$ and $\vec{mi_1}$ involve $[D_1]$ and its first and second time derivatives $[\dot{D}_1]$ and $[\ddot{D}_1]$.

The aerodynamic forces which apply on the external skin of the slice have an expression depending on the aerodynamic model selected. In a first stage, we have chosen the **steady bidimensional model** using **airfoil polars.** This model enables to reduce the aerodynamic forces in C_1 under the form of a force and moment expressed as a function of $[D_1]$ and $[\dot{D}_1]$. Calculation requires assessment of the local induced velocity obtained via a **ring method** (or more simply, for forward flight calculations, via Meyer-Drees formulae).

The equilibrium equations are finally gathered to form the equation :

$$[F'_1] \equiv \Psi\left([D_1] \; , \; [\dot{D}_1] \; , \; [\ddot{D}_1] \; , \; [F_1]\right)$$

(equation of stresses)

which is a partial differential equation with 6 components. This equation is still non-linear ; it involves geometrical, inertial and aerodynamic characteristics of the section.

3– MAIN AND COMPLEMENTARY EQUATIONS

The equation of deformations, the equation of stresses and all the boundary conditions define the true state (E_1) of the rotor.

The first two equations can further be gathered as :

$$[X'_1]=G\left([X_1]\ ,\ [\dot{X}_1]\ ,\ [\ddot{X}_1]\right) \qquad (1)$$

expressing the evolution of the state vector on each element considered separately.

The boundary conditions taken together can also be written

$$C\left([Y_1]\ ,\ [\dot{Y}_1]\right)=0 \qquad (2)$$

where $[Y_1]$ would be the junction of state vectors of the ends of all elements.

The equations (1) and (2) form the **main equations** whose solution seems at first to be inextricable. We will then consider that the true state (E_1) is the addition of an approximate state (E_0) and of a complementary state (E) :

- (E_0) is described by a state vector $[X_0]$ which checks (1) without necessarily checking (2).
- (E) is the complementary state described by a complementary state vector $[X]$ whose components are defined with respect to the systems of reference associated to the sections in the (E_0) state in the same way as the components of $[X_0]$ are defined in the reference systems of the pieces before any deformation.

State (E_0) is considered close enough to (E_1) for components of state $[X]$ to be infinitely small indeed. Linearization of equation (1) (which reads $[X'_0] = G\left([X_0]\ ,\ [\dot{X}_0]\ ,\ [\ddot{X}_0]\right)$ around state (E_0), gives :

$$\Rightarrow [X'] = [X]\ \alpha_0 + [\dot{X}]\ \beta_0 + [\ddot{X}]\ \gamma_0 \qquad \text{(1bis)}$$

and linearization of equation (2), again around state (E_0), gives :

$$C\left([Y_0]\ ,[\dot{Y}_0]\right) + [Y]\ \lambda_0 + [\dot{Y}]\ \mu_0 = 0 \qquad \text{(2bis)}$$

where matrices α_0 , β_0 , γ_0 , λ_0 , μ_0 are associated to state (E_0).

The equations (1bis) and (2bis) form the **complementary equations.** They constitute two constant coefficients linear differential systems.

4– NATURAL MODES

Definition :

Natural modes are the **possible transient states considered as infinitely close** to a static state, the latter being by definition independent of time.

Previous identification of a static state

According to their definition, state vector time derivatives have to be considered null. The rotor axis is necessarily vertical and without any horizontal motion.

Equations (1) which are now $[X'_0] = G\left([X_0]\ , 0, 0\right)$ form a differential system for which the knowledge of the state vector at the origin of each element is sufficient to calculate entirely an approximate (or estimated) state (E_0).

Equations (1bis) are now reduced to $[X'] = [X].\alpha_0$ are also checked by the matrix of the partial derivatives of the local complementary state in relation to the complementary state at the origin of each element. This matrix, which is a unit matrix at the origin of any element, can then also be calculated simultaneously with (E_0).

Solution of system (2bis) which writes $C\left([Y_0]\ ,[0]\right) + [Y]\ \lambda_0 = 0$ specifies then the linear corrections to be brought to the state vectors of (E_0) at the origins in order to come closer to state (E_1).

Naturally, the method used to obtain the final steady state E_1, through the methods we have just described, is necessarily numerical.

Runge and Kutta type methods make it possible to reduce the numerical error at will. The solution process is iterative.

The steady state (E_1) being physically possible, the process is necessarily converging.

Calculation of natural modes

The ultimate steady state obtained above is an (E_0) steady state verifying now equations (1) and (2) in which time derivatives are set to zero.

The neighbouring states verify equations (1bis) and (2bis), that is :

$$[X'] = [X]\alpha_0 + [\dot{X}]\ \beta_0 + [\ddot{X}]\ \gamma_0 \qquad \text{(1bis)}$$

$$[Y]\ \lambda_0 + [\dot{Y}]\ \mu_0 = 0 \qquad \text{(2bis)}$$

which form a linear, homogeneous set of equations.

Let $[Z]$ designate the gathering of all state vectors of the successive cross-sections considered as infinitely close. Then, the above set of equations is equivalent to :

$$[\ddot{Z}]\ \Gamma_0 + [\dot{Z}]\ B_0 + [Z]\ A_0 = 0$$

which is a second order system of infinite dimension. The general solution of such a system is the following :

$$[Z] = \Sigma\ k_j\, e^{r_j t}\ [Z_j]$$

Where :

- k_j designates arbitrary constants
- r_j represents the characteristic equation roots
- $[Z_j]$ is defined within a multiplicative constant and represents the mode shape of mode j

r_j numbers are complex numbers and two by two conjugated :

$$r_j = \alpha_j \pm i\,\omega_j$$

- ω_j is the eigen pulsation of mode j
- α_j is the eigen damping of mode j .

Equations (1bis), verified by the matrix of the partial derivatives of the local complementary state as compared to the complementary state at the origin of the element, are always integrable in the complex field for a given r_j number, and we know now that one may write, within a multiplicative factor :

$$[X] = e^{r_j t}\,[X_j]$$

Therefore
$$\left\{\begin{array}{l}[\dot{X}] = r_j \cdot e^{r_j t}\,[X_j] \\ [\ddot{X}] = r_j^2 \cdot e^{r_j t}\,[X_j]\end{array}\right.$$

Equations (2bis) can be reduced to :

$$[Y_j^*]\;\lambda^*_{0j} = 0$$

where $[Y^*_j]$ is the gathering of all state vectors at the origin of the elements.

The r_j number must be selected so as to obtain :

$$\boxed{\text{Determinant } \lambda^*_{0j} = 0}$$

a relation equivalent to the characteristic equation.

$[Y^*_j]$ permits calculating effectively the mode shape.

The pair of numbers (α_j, ω_j) can always be determined by a 2-variables Newton type iterative method. To initialize the process, it is worthwhile starting with the sequence of natural uncoupled frequencies which can easily and rapidly be calculated with modern computers.

Remark 1 : Determination of modes damping also helps solve the problem inherent to linear stability of the rotor in hover.

Remark 2 : The orthogonality property of the natural modes immediately stems from their definition.

Indeed if we write : $[\dot{Z}] = [V]$

we notice that numbers r_j are the eigen values of the matrix :

$$Q(r) = \begin{bmatrix} r.1 & A_0 \\ -1 & r\,\Gamma_0 + B_0 \end{bmatrix}$$

To each eigen value corresponds an eigen vector $\begin{bmatrix}[Z_j] \\ [V_j]\end{bmatrix}$ such that :

$$\begin{bmatrix}[Z_j] & [V_j]\end{bmatrix}\; Q(r_j) = 0$$

and an eigen joint vector $\left\{\begin{matrix}\{T_j\} \\ \{U_j\}\end{matrix}\right\}$ such that :

$$Q(r_j)\left\{\begin{matrix}\{T_j\} \\ \{U_j\}\end{matrix}\right\} = 0$$

Two distinct eigen values r_j and r_k are such that :

$$[Z_j]\,\Big((r_j + r_k)\,\Gamma_0 + B_0\Big)\,\{U_k\} = 0$$

(orthogonality property)

This orthogonality relation can easily take an integral form if one recalls that $\{U_j\}$ represents the sequence of joint vectors $\{u_j\}$ verifying the joint system :

$$\{u'_j\} = -\left(r_j^2 \cdot \gamma_0 + r_j \cdot \beta_0 + \alpha_0\right)\{u_j\} \qquad \text{(1ter)}$$

The sequence of systems of equations (1ter) can be initialized on the set of elements by solving :

$$\lambda^{\bullet}_{0j}\,\{U^*_j\} = 0$$

The orthogonality property leads to the use of **modal methods** which allow to describe any possible **small** movement, resulting from an excitation of the rotor, around its steady state.

5– DYNAMIC BEHAVIOUR IN FORWARD FLIGHT (Azimuth method)

The difficulty inherent to the use of modal methods rests in the particularity intrinsic to the definition of modes which are **small movements,** whereas some parameters often present fairly large variations over one revolution in forward flight. For that reason, we will no longer mention the modal methods in this paper.

The calculation of the dynamic behaviour of a rotor in forward flight can however be solved using a simple although quite different method (the azimuth method) which, in addition, avoids preliminary modes calculation.

This method applies mainly to stabilized flights and particularly to flights in which rotor configuration can be considered **repeatable after one revolution.**

In such flights, excitation occurs at revolution frequency i.e.

$$\frac{\Omega}{2\Pi} \quad (\Omega = \text{rotor speed})$$

or at multiples of this revolution frequency (in case of multicyclic pitch).

The rotor behaviour can then be considered developable in Fourier series whose basic frequency is $\frac{\Omega}{2\Pi}$, the terms of the series being considered negligible beyond n^{th} harmonic.

When harmonics higher than n are effectively neglected, the development coefficients count shows that rotor behaviour can be best described in (2 n + 1) successive positions (or azimuths) regularly spread over the revolution.

The values of the time derivatives of any variable in each of these (2 n + 1) azimuthal positions are expressed in a simple way in terms of the values of this very variable, in the (2 n + 1) positions, as follows :

$$\dot{z}_j = \sum_{i=1}^{2n+1} z_i \cdot \lambda^{*}_{ij} ,$$

$$\ddot{z} = \sum_{i=1}^{2n+1} z_i \cdot \lambda^{**}_{ij}$$

where i and j are position numbers and λ^{*}_{ij} , λ^{**}_{ij} depend on these positions only.

The calculation of the rotor behaviour in forward flight is then analogous to that of a static distortion in hover, simultaneously determined on the (2 n + 1) positions.

Remark 1 : Accuracy only depends on the number n of harmonics retained. Selection of n number depends on the capacity of the computer.

Remark 2 : This method can be used to determine sub-harmonics components if one considers that the rotor configuration is repeatable after several revolutions.

6– DESCRIPTION OF FIGURES

Figures illustrating modes calculation (Fig. 1 to 4)

Fig. 1 shows a 4-blade AS 332 tail rotor.

Each section includes :

- a blade
- a Triflex arm
- a sleeve
- a pitch rod controlled by a spider plate.

The Triflex arm connects the blade to the rotor centre.

The sleeve attached to the blade supports the pitch control, fairs the arm and bears against the center section of the hub.

The complexity of the configuration was taken into account in the mathematical modeling, (modeling is limited to a single blade and its attachment).

Figures 2 and 3 indicate the calculated ranges of varia-

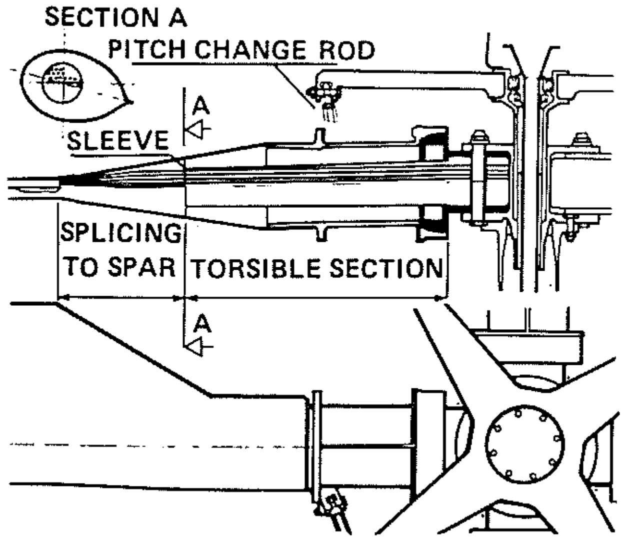

Fig. 1 : AS 332 TRIFLEX TAIL ROTOR

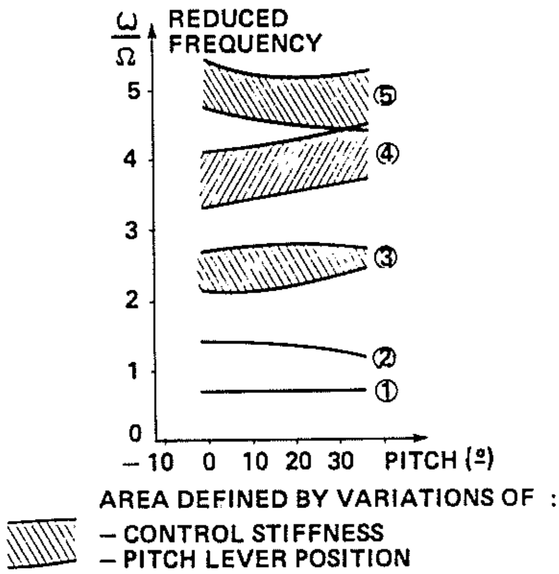

Fig. 2 : AS 332 TRIFLEX TAIL ROTOR

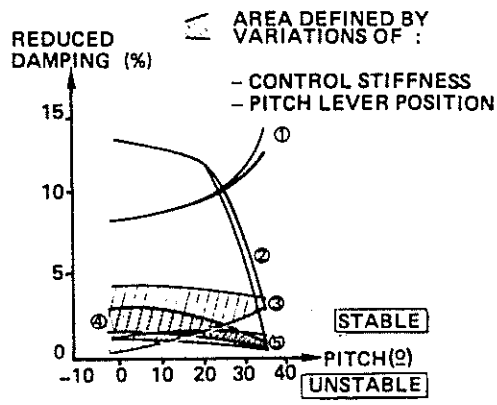

Fig. 3 : AS 332 TRIFLEX TAIL ROTOR

tion determined for natural frequency and damping, while adjusting :

- Rigidity of spider plate (Flexibility of pitch control)
- Rigidity of rest
- Length of pitch lever.

These two figures show what it is possible to achieve, in a parametric study , in order to adjust the natural frequencies and, simultaneously, to improve the rotor stability.

Figure 4 describes the influence of the sleeve on the natural damping of a few modes for three cases :

▲ : The sleeve, considered as ideal, is infinitely rigid and weightless

● : The sleeve is infinitely rigid but has its effective weight

+ : The sleeve has its effective rigidity and weight.

This figure shows the importance of a most complete mathematical modeling.

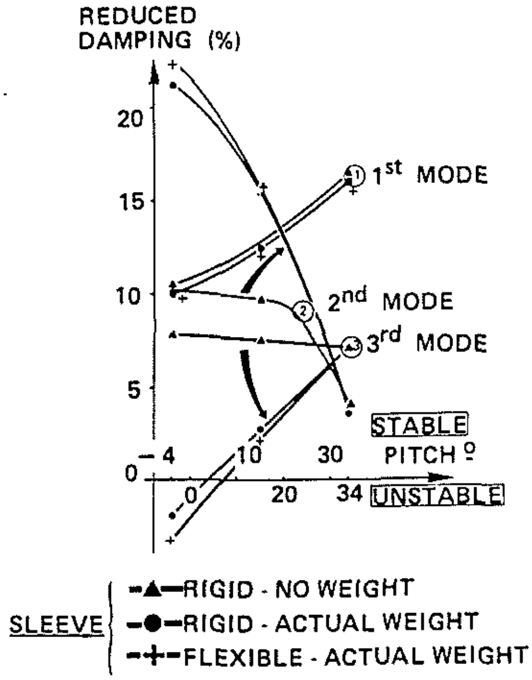

Fig. 4 : AS 332 TRIFLEX TAIL ROTOR

Figures illustrating a calculation of the behaviour of a rotor in forward flight (Fig. 5 to 7)

These figures illustrate a study undertaken to design a new 349-2 A/C blade.

- Figure 5 first indicates, for the 349-Z version of the blade, the vertical acceleration (divided by g) **calculated** at the pilot (γ_Z , RH) and the copilot (γ_Z , LH) stations as a function of speed (solid lines). These results, obtained on a 349-2 A/C equipped with 349-Z blades, were determined in two steps :
  1) by applying azimuths method to a succession of flight configurations : 3 P loads and moments are deduced at the center of the rotor in a non rotating reference system.
  2) by subsequently applying transfer functions (between the center of the rotor and the pilot and copilot stations) determined from laboratory measurements on the suspended aircraft structure.

  The accuracy of the calculations is evidenced on figure 5 where dots represent direct in-flight measurements of the accelerations on the same aircraft fitted with the same blades.
- A parametric study has then been undertaken in order to design a new blade for the 349-2 A/C with reduced levels of accelerations in the cabin. As above the calculations proceeded in two steps : azimuth method and use of measured transfer function.

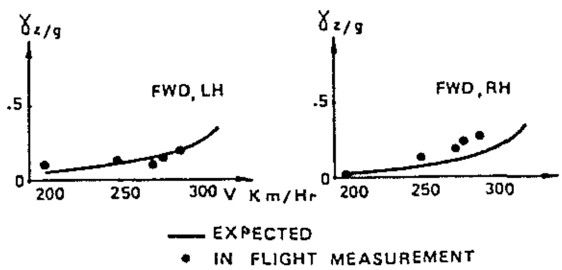

Fig. 5 : 3P VIBRATION LEVEL

Initial data are :

Rotor diameter : 10.5 m

Airfoil : OA 209

Chord : 0.35 m over the feathered section

Rotational speed Ω : 40 rd/s

Twist : -1.4^{o}/m over the feathered section

3-blade NAT hub

Flapping stiffness over current section :
$E\,I_B = 9{,}000$ N.m^2

Lead-lag stiffness over current section :
$E\,I_T = 400{,}000$ N.m^2

Torsional stiffness over current section :
$G\,J = 10{,}000$ N.m^2

Mass per unit length of current section :
$m' = 5{,}5$ kg/m

The influence of each parameter was tested separately and in groups (Figure 6 a, b, c, d). Such results allowed us to define the characteristics of a final version (see 10 **Fig. 6 (d)**) whose levels of accelerations in the cabin seem acceptable.

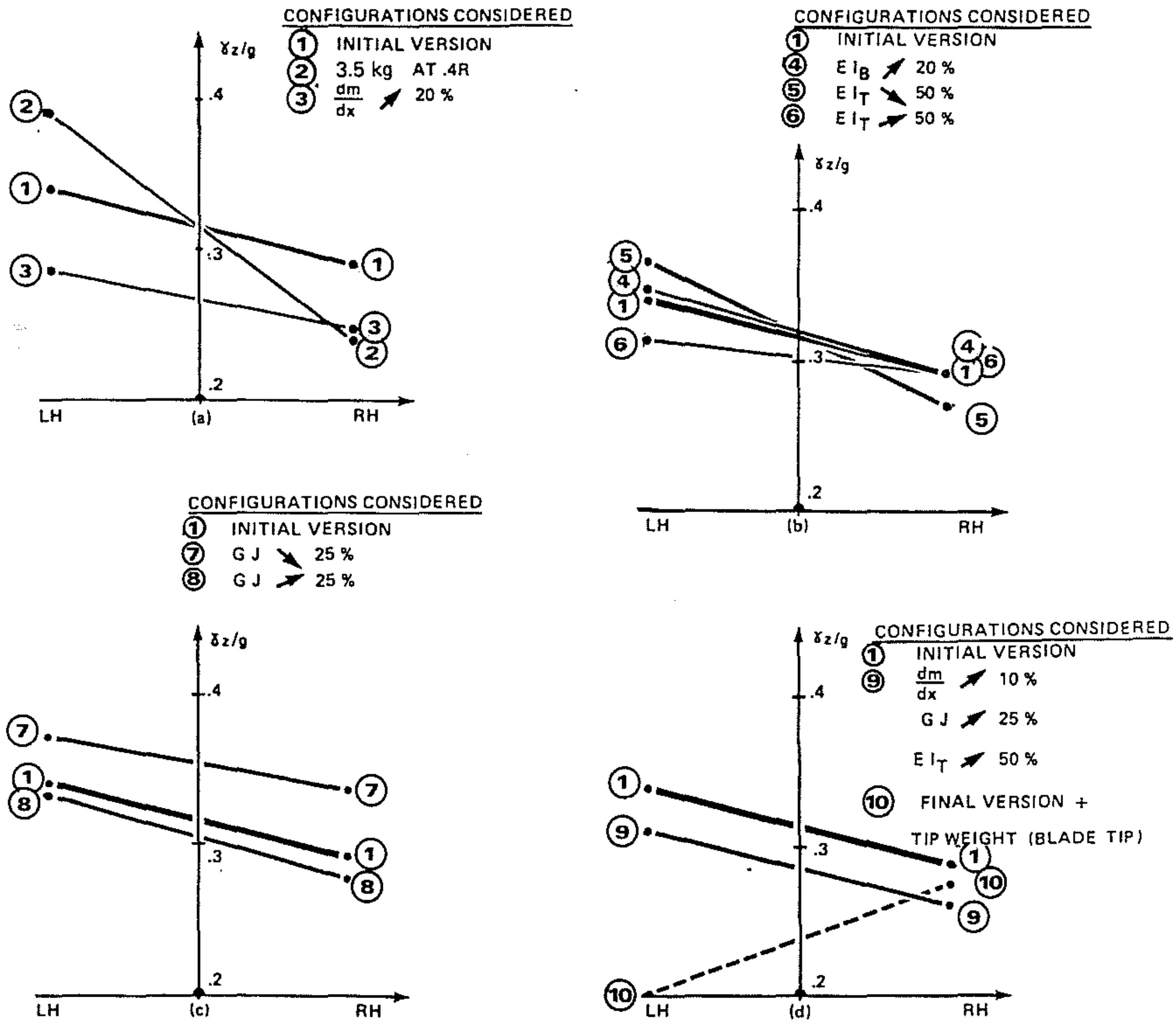

Fig. 6 : PARAMETRIC STUDY OF VIBRATION LEVELS IN CABIN (3 Ω)
(SA 349-2 ; V = 320 km/h)

Remark : The segments connecting RH and LH points have no physical significance : they help connect homologous points only.

Figure 7 outlines an other type of results that can be obtained by the azimuths method.

This figure shows the vertical displacements at the various points of the blade over one revolution with respect to a plane perpendicular to the shaft. This figure demonstrates in particular that blades considered flexible no longer really describe a cone.

Similar figures could be drawn up in a similar manner for other terms of the state vector.

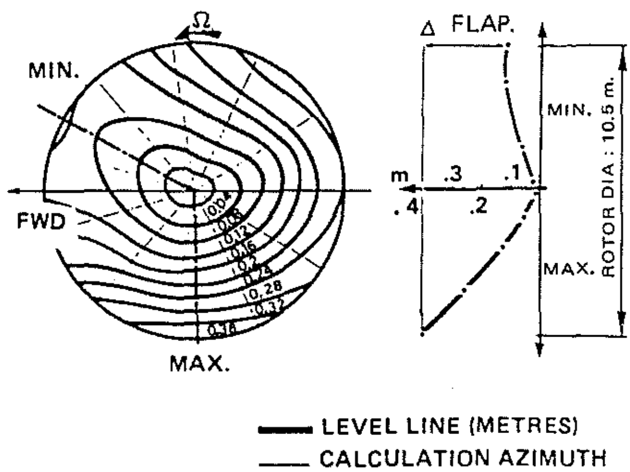

Fig. 7 : MOVEMENT OF A BLADE 349-2 ; V = 220 km/hr

7– CONCLUSIONS

It would have been difficult in only a few pages to give more than an outline of such a large subject.

What needs to be remembered is that the calculation methods we have described constitute an irreplaceable tool for parametric studies and help the rotor designer to avoid wrong choices.

These calculation methods are widely used at Aerospatiale : we have given a few results only in this paper in order to illustrate the possibilities of such methods. Many other results are described in paper No 74 by Blachère and D'Ambra.

The accuracy of the mathematical methods used to solve the linear and non linear set of equations is not questionable. Only hypotheses concerning the rotor representation still need further verification :

- The hypothesis concerning non warping of the cross sections is questionable. It was however adopted to facilitate calculation methods setting.

  In the future it is envisaged to discard this rather restrictive hypothesis.

- the stationary bidimensional aerodynamic model also is an intermediate step and we intend to introduce an improved model.

  Computer's advances will naturally be of help in the effective implementation of these projects.

In any case, the methods we presently have at our disposal should be commonly used to design a rotor, thanks to the efforts devoted to calculations time reduction.

Experience will show whether a rotor thus designed proves easy to develop.