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APPLICATION OF THE LOCAL MOMENTUM THEORY TO THE AERODYNAMIC CHARACTERISTICS OF TANDEM ROTOR IN YAWED FLIGHT

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SUMMARY

The local momentum theory[1], which was developed by two of authors of present paper to obtain the dynamic behaviour as well as the aerodynamic characteristics of the single rotor helicopter, has been well applied to know the aerodynamic interference between two rotors such as tandem rotor in yawed flight. Since the theory is based on the instantaneous momentum balance with the blade elemental lift at a local station of the rotor rotational plane the induced velocity generated by one rotor over another rotor can be easily calculated and thus the aerodynamic efficiency due to the interference between two rotors is simply obtained without laborious computation sometimes appeared in that of vortex theory.

The exemplified calculations have been performed for the performance of two tandem rotor helicopters, CH-46 and CH-47, in yawed flight and compared with the results obtained by other theories and the flight test data. The comparison has shown good agreement over the wide range of yawing angle in both directions and confirmed that the right yaw between 10 tO 20 degrees brings about 5 percent power reduction in the exemplified helicopters.

1. INTRODUCTION

The induced power required for flight of helicopter is dependent primarily on rotor disc loading in hovering, but on span loading of the rotor disc in forward flight. Tandem rotor helicopters, having smaller span than that of single rotor and side-by-side rotor helicopters for a given disc area, require more induced power when the speed reaches a substantial fraction of the downwash velocity. This is of importance at low-to-moderate advance ratios because of critical-engine-out performance.

It has been believed that flying a tandem rotor helicopter in yawed condition offers substantial reductions in necessary power because the span of the rotor system in yawed condition is thereby increased. This is considered to be resulted from the fact that in yawed flight blades of rear rotor pass through either the reduced downwash or even upwash flow caused by front rotor in some area of rotor operating disc.

The exact distribution of the induced velocities around a rotor (or rotors) may be obtained by integrating the induced velocities generated by an elemental vorticity of the rotor (or rotors) over the rotor disc and wake. It is, however, difficult to perform the calculation on the aerodynamic characteristics of two rotors because mutual interference among wake vortices results wake distortion and change of vorticities. Very limited computations were performed in a rigid wake system[2,3].

By considering the aerodynamic balance between the fluid momentum and the force acting on a blade element at a local station on the rotor tip path plane, the induced velocity and the airloading at that station can be determined easily and precisely. The computational method may, therefore, be called "Local Momentum Theory"[1], and can be applied to any kind of multi-rotor system without laborious computation and difficulty of convergence sometimes experienced in the usual vortex theories.

2. LIFTING LINE APPROXIMATION

In forward flight of helicopter the flow in the vicinity of a rotor resembles that of an equivalent wing which may simply be represented by a horseshoe vortex system. Cheeseman developed a simple and yet very useful theory for the performance calculation of tandem rotor by combining a lifting line approximation to translational lift of the front rotor and a stream tube model for propeller lift[4]. The theory can be extended to obtain the degree of performance improvement of the rear rotor in yawed flight of tandem rotor helicopter[2].

Let us divide a lift of the rotor, L, into two fractions, the propeller lift and the translational lift such as

$$L = 2m_0 v + L(1-v/v_0) \tag{1}$$

where v_0 and v are the induced velocities in hovering and forward flights respectively and m_0 is the mass flow being equivalent to the hovering state or

$$m_0 = \rho S v_0 = \rho(\pi R^2) v_0 . \tag{2}$$

Then, the circulation of bound vortex of the front rotor represented by a lifting line is

$$\begin{aligned} \Gamma &= L(1-v/v_0)/2\rho RV \\ &\cong (\pi/2)R^2\Omega C_T(1-v/v_0)/(V/R\Omega), \end{aligned} \tag{3}$$

where C_T is the thrust coefficient of the front rotor or $C_T = L/\rho S(R\Omega)^2$ and the rotors are assumed to have same dimensions and operating conditions except rotational direction. By referring to Fig. 1, a mean value of the induced velocity at the rear rotor center due to the horseshoe vortex system of the front rotor will be calculated in the similar way given by Ref. 4. A further downwash increment due to the propeller slip stream will be additional as the fraction of the induced velocity within the overlap region.

The induced velocity affecting the one rotor in consideration will, thus, be obtained as the sum of those generated by the propeller slip stream and the vortex system of both rotors. The parallel and normal components with respect to the tip-path plane of the rotor, $\delta\mu$ and $\delta\lambda$, are given as follows:

$$\delta\mu_{\substack{F\\R}} = [C_T\{1-\sqrt{C_T/2(\mu^2+\lambda^2)}\}\{(f_x\sin\chi+f_z\cos\chi)/16\mu\}\cos\alpha]_{\substack{R\\F}}$$

$$\delta\lambda_{\substack{F\\R}} = [C_T\{1-\sqrt{C_T/2(\mu^2+\lambda^2)}\}\{(f_x\cos\chi-f_z\sin\chi)/16\mu\}\cos\alpha]_{\substack{R\\F}} \tag{4}$$

$$+[\{C_T/\pi\sqrt{\mu^2+\lambda^2}\}\{\cos^{-1}(d/2R)-(d/2R)\sqrt{1-(d/2R)^2}\}]_{\substack{R\\F}}$$

where the subscripts F and R are those related to the front and rear rotor respectively, and where the second term of the right hand side of $\delta\lambda$ is the induced velocity due to the direct slip stream. The remaining terms are given by

$$\mu_{\substack{F\\R}} = \mu\cos\alpha_{\substack{F\\R}}+\delta\mu_{\substack{F\\R}}$$

$$\lambda_{\substack{F\\R}} = [-\mu\tan\alpha+C_T/2\sqrt{\mu^2+\lambda^2}+\delta\lambda]_{\substack{F\\R}} \tag{5}$$

$$\alpha_{\substack{F\\R}} = \alpha+\Delta\chi_{\substack{F\\R}}$$

$$\chi_{\substack{F\\R}} = \chi\mp\Delta\chi = \tan^{-1}(\mu/\lambda)_{\substack{F\\R}}\mp\Delta\chi \tag{6}$$

$$\Delta\chi = \Delta\chi_F-\Delta\chi_R$$

$$f_{x_{\substack{F\\R}}} = [(F+G-2)(z_0/d)/\{1-(y_0/d)^2\}]_{\substack{F\\R}}$$

$$f_{z_{\substack{F\\R}}} = [ln\{(1-x_0/d)^2/(G-x_0/d)(F-x_0/d)\} \tag{7*}$$

$$-(F+G-2)(x_0/d)/\{1-(y_0/d)^2\}]_{\substack{F\\R}}$$

$$F_{\substack{F\\R}} = [\{1+4(R/d)^2+4(R/d)(y_0/d)\}^{1/2}]_{\substack{F\\R}}$$

$$G_{\substack{F\\R}} = [\{1+4(R/d)^2-4(R/d)(y_0/d)\}^{1/2}]_{\substack{F\\R}} \tag{8}$$

$$(x_0/d)_{\substack{F\\R}} = \pm(s/d)\cos\Psi\sin\chi_{\substack{F\\R}}/\cos\alpha_{\substack{F\\R}}$$

$$(y_0/d)_{\substack{F\\R}} = \mp(s/d)_{\substack{F\\R}}\sin\Psi \tag{9}$$

$$(z_0/d)_{\substack{F\\R}} = \pm(s/d)_{\substack{F\\R}}\cos\Psi\cos\chi_{\substack{F\\R}}/\cos\alpha_{\substack{F\\R}}$$

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* When the yawing angle is $\Psi=\pm\pi/2$, the effect of the bound vortex on the front rotor should be dropped out in these expressions so that

$$f_x = 0,\ f_z = ln(1/FG) \tag{7'}$$

and if additionally $R/d\leq0.5$ the above terms due to the vortex system will be discarded.

$$(s/d)_{\substack{F\\R}} = [1+\cos^2\Psi\tan^2\alpha]^{-1}_{\substack{F\\R}} \tag{10}$$

$$(g/d)_{\substack{F\\R}} = [-(s/d)\tan\alpha\cos\Psi]_{\substack{F\\R}}$$

$$(v/R\Omega)_{\substack{F\\R}} = [C_T/2\sqrt{\mu^2+\lambda^2}]_{\substack{F\\R}}. \tag{11}$$

Then the thrust and torque increments of the each rotor are respectively given by

$$(\Delta C_T/C_T)_{\substack{F\\R}} = [\{\mu\theta\delta\mu-(1/2)\delta\lambda\}/\{(1/3+\mu^2/2)\theta-\lambda/2\}]_{\substack{F\\R}} \tag{12}$$

$$(\Delta C_Q/C_Q)_{\substack{F\\R}} = [\{(1/4)C_{d_0}\mu\delta\mu+(a/6)\theta\delta\lambda\}/\{(1/8)C_{d_0}(1+\mu^2)+(a/6)\theta\lambda\}]_{\substack{F\\R}}.$$

It can be seen from equations (12) that the front rotor brings some thrust decrement and little torque increment.

3. LOCAL MOMENTUM THEORY IN MULTIROTORS

Let us suppose that two rotors are travelling in a horizontal plane as shown in Fig. 2. A part of or all span of blade will be affected by either downwash or upwash generated by own and other blades in both rotors. In the local momentum theory a rotor blade is considered to be comprised of n wings. The each wing has an elliptical circulation distribution, which may be called elliptical wing, as shown in Fig. 3. A blade element partitioned by the subsequent elliptical wings is assumed to proceed intermittently in the horizontal plane within a small time interval. As shown in Fig. 4, the blade element occupied a station (l',m') on the mesh of rotor plane. It had a forward speed of $V_{i\ j-1\ k}$ and a normal component of the velocity which consisted of a uniform inflow due to the inclination of the rotor plane, V_N, and the sum of the induced velocities due to the blade element itself, $v_{i\ j-1\ k}$, and one due to the preceding blade elements having passed over the point by that time if any before, $v^{j-1}_{l'm'}$. At two other points (l,m) and (l'',m''), which will be just reached by the blade element at time t=j and t=j+1 respectively, there may also exist the induced velocity having been created by the preceding blade elements. For example, a trace of the induced velocity at a station (l,m) having been passed over by an i-th spanwise element of k-th blade at (j+1)th azimuthal or time sequence can be given by multiplying an attenuation coefficient, C^j_{lm}, into the sum of the induced velocities, $(v^j_{lm}+v_{ijk})$. The coefficient, C^j_{lm}, may be given by a ratio of the induced velocities at the time just before and after the blade passing, v/v_0, at the time sequence, t=j+1. The induced velocities v and v_0 are calculated by using the vortex theory[5] for two semi-infinite and skewed vortex cylinder, the respective upper end of which flows with the resultant velocity at the center of rotor and suddenly comes back to the rotor operational plane when the next blade hits that station. Actually, in the present study, the resultant velocity used only for determining the attenuation coefficient is simply obtained by assuming a uniform induced velocity and equal sharing of the thrust for both rotors.

That is to say, the induced velocities at the front and rear rotors are given by $v_{0,F} = v_0$ and $v_{0,R} = 3v_0$ respectively, where v_0 is the mean induced velocity generated by the wake vortex of the front rotor alone, and where $3v_0$ is resulted from the sum of fully developed downwash due to the front rotor, $2v_0$, and the downwash at the rotor plane due to the rear rotor itself, v_0.

Then the airloading of a blade element (i, j, k) and the induced velocity at the station (l,m) on the mesh of rotor plane are connected by

$$
\begin{aligned}
l_i &= \int_{x_i}^{x_{i+1}} (1/2)\rho V_i^2 c_i a(\theta_i - \phi_i)\,dx/(x_{i+1}-x_i) \\
&= \int_{x_i}^{x_{i+1}} \sum_{\nu=1}^{i} \{4L_\nu/\pi R(1-x_\nu)\}[\{R\Omega x + V\sin(\psi_{k,0} + \sum_{\lambda=1}^{j}\Delta\psi_\lambda)\}/ \\
&\qquad V_{i,c}]\sqrt{1-\{(2x-1-x_i)/(1-x_i)\}^2}\,dx/(x_{i+1}-x_i)
\end{aligned}
\tag{13}
$$

where a is the lift slope and

$$
\begin{aligned}
V_i &= V\sin(\psi_{k,0} + \sum_{\lambda=1}^{j}\Delta\psi_\lambda) + R\Omega(x_i + x_{i+1})/2 \\
c_i &= c(x=(x_i+x_{i+1})/2), \quad \theta_i = \theta(x=(x_i+x_{i+1})/2) \\
\phi_i &= (V_N + v_{lm}^j + v_{ijk})/V_i \\
V_N &= V\sin i \\
L_i &= 2\rho S_i V_{i,c}(v_{ijk} - v_{i-1\,jk}) \\
v_{lm}^i &= v_{lm,F}^j + v_{lm,R}^j = C_{lm}^{j-1}(v_{lm,F}^{j-1} + v_{lm,R}^{j-1} + \sum_{i=1}^{n}\sum_{k=1}^{b} v_{i\,j-1\,k}\cdot\delta_{lm})
\end{aligned}
\tag{14*}
$$

and where $\psi_{k,0}$ and $\Delta\psi$ are the initial azimuth angle of the k-th blade and azimuthal step respectively. Thus, equation (13) can determine the induced velocity at any local station on the rotor rotational plane as well as the airloading of the blade at any spanwise and azimuthal position.

4. YAWED FLIGHT OF A COMPLETE HELICOPTER

Any trimmed flight condition for a given set of collective and cyclic pitch inputs can be determined by solving the following equations for a specified helicopter:

$$
\begin{aligned}
&X_B - X_F\cos i_F - X_R\cos i_R + Z_F\sin i_F + Z_R\sin i_R - W\sin\Theta\cos\Phi = 0 \\
&Y_B + Y_F - Y_R + W\sin\Phi = 0 \\
&Z_B - Z_F\cos i_F - Z_R\cos i_R - X_F\sin i_F - X_R\sin i_R + W\cos\Theta\cos\Phi = 0
\end{aligned}
\tag{15}
$$

---

* δ_{lm} should be one if any blade element hits the station (l, m) at $t=j-1$ and otherwise zero.

$$
\begin{aligned}
&L_B-L_F\cos i_F+N_F\sin i_F+L_R\cos i_R-N_R\sin i_R = 0\\
&M_B+M_F+M_R+(X_F\cos i_F-Z_F\sin i_F)h_F+(X_R\cos i_R-Z_R\sin i_R)h_R\\
&\quad+(X_F\sin i_F+Z_F\cos i_F)l_F-(X_R\sin i_R+Z_R\cos i_R)l_R = 0\\
&N_B-N_F\cos i_F-L_F\sin i_F+N_R\cos i_R+L_R\sin i_R+l_FY_F+l_RY_R = 0
\end{aligned}
\tag{16}
$$

where, by referring to Fig. 5,

X_B, Y_B, Z_B and L_B, M_B, N_B : Aerodynamic forces and moments acting on the fuselage at center of gravity

X_F, Y_F, Z_F and L_F, M_F, N_F : Rotor forces and moments acting on the front hub

X_R, Y_R, Z_R and L_R, M_R, N_R : Rotor forces and moments acting on the rear hub

W : Gross weight of helicopter

Ψ, Θ, Φ : Eulerian angles of the body attitude

i_F, i_R : Inclined angles of shaft of front and rear rotors respectively

(l_F, h_F), (l_R, h_R) : Hub positions of front and rear rotors respectively

5. EXEMPLIFIED CALCULATIONS AND FLIGT TESTS

Shown in Fig. 6 and 7 are the induced velocity distribution around a model of tandem rotor operating respectively in hovering and forward flight conditions. In comparison of the present theory having assumed a constant attenuation coefficient, there are also shown results of a simple and effective momentum theory developed by Stepniewski[6] and of the concise vortex theory derived by Cheeseman[4].

Although the present theory gives strong spanwise variation of the induced velocities even in hovering flight, their mean value coincides well with the momentum theory. The spanwise variation has been resulted from the preserved effects of mutual interference of the flow in the overlapped region.

Since the present theory can give the instantaneous change of the induced velocity, its fluctuating behaviour corresponding to the blade position during one revolution can be observed in the forward flight. In Fig. 7 there are shown the maximum and minimum values of the induced velocities at the respective station in one rotor revolution and their mean values which are also consistent with the results from the simple momentum theory.

Fig. 8 shows variations of the thrust and torque coefficients for the change of yawing angle of the tandem rotor model flying at the advance ratio of μ=0.15 without any cyclic pitch input. In this calculation the attenuation coefficients have been considered variable as a function of the spanwise position and azimuth angle. It can be seen that as the yawing angle increases the thrust also increases and the torque decreases in the rear rotor. It is very much interesting to find that the right yaw (Ψ>0) brings slightly better performance improvement than the left yaw (Ψ<0). This fact has been confirmed by the flight tests too[7]. The improvement must be derived by taking the overlapped area into the blade retreating side and as the result, by compensating mutually the defficiency in the blade retreating side.

Actual performance improvements due to the right yaw have been obtained for two exemplified helicopters, CH-46 and CH-47. The drag increment of the fuselage obtained by the wind tunnel tests is shown in Fig. 9 for the both helicopters,in which a better characteristic can be observed for CH-47 because of a slightly more flattened feature of the fuselage cross section than that of the CH-46. Other necessary aerodynamic characteristics required for determining the trim conditions are given in Table 1.

The necessary horse power versus advance ratio for the both helicopters have been calculated from the present modified Cheeseman's theory and shown in Fig. 10 for various yaw angle. It can be concluded that since an optimal power line connecting the minimum powers of the respective helicopter is almost vertical in the drawing the flight speed should be kept constant through the yawed flight in a critical condition.

Shown in Fig. 11 are comparisons of the theoretical results and the flight test data in the power increment due to the right yaw. The flight test data fall a range between two theoretical results. It can, thus, be expected that the power reduction of about 5 percent will be achieved by taking the right yaw between 10 to 20 degrees in the exemplified helicopters.

In these calculations by FACOM 230-75 computer, the present local momentum theory required about four minutes to get the solutions in convergent state.

6. CONCLUSION

The local momentum theory has been applied to investigate the aerodynamic characteristics of the tandem rotor helicopter in yawed flight. The results have been compared with those obtained by other methods such as the modified Cheeseman's vortex theory and the Stepniewski's momentum theory and with the flight test data. The comparison has shown good agreement over the wide range of yawing angle in both directions and confirmed that the right yaw between 10 to 20 degrees brings about 5 percent power reduction in the exemplified helicopters.

7. ACKNOWLEDGEMENTS

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Table I. Aerodynamic characteristics of fuselage (Body axes coordinate)

$$(C_X, C_Y, C_Z) = (X_B, Y_B, Z_B)/(1/2)\rho V^2(2\pi R^2)$$

$$(C_l, C_m, C_n) = (L_B, M_B, N_B)/(1/2)\rho V^2(2\pi R^2)R$$

(a) C_X ($\times 10^{-3}$)

	CH-46			CH-47		
$\Psi \diagdown \Theta$	0.4°	5.3°	10.2°	-20°	0°	20°
-20°	-7.07	-6.79	-6.19	-4.85	-7.02	-5.22
0°	-6.97	-6.55	-5.69	-6.54	-7.43	-6.72
20°	-7.27	-6.98	-5.95	-2.78	-4.51	-2.53

(b) C_Y ($\times 10^{-2}$)

$\Psi \diagdown \Theta$	0.4°	5.3°	10.2°	-20°	0°	20°
-20°	-2.69	-2.69	-2.61	-3.71	0.035	3.54
0°	0.	0.	0.	-2.63	0.	2.79
20°	2.94	3.06	3.18	-2.39	0.124	2.41

(c) C_Z ($\times 10^{-3}$)

$\Psi \diagdown \Theta$	0.4°	5.3°	10.2°	-20°	0°	20°
-20°	0.	-3.67	-6.12	22.6	23.8	22.1
0°	-2.45	-6.36	-12.2	-6.19	2.3	-6.19
20°	0.	-3.67	-8.32	-32.9	-15.2	-32.7

(d) C_l ($\times 10^{-3}$)

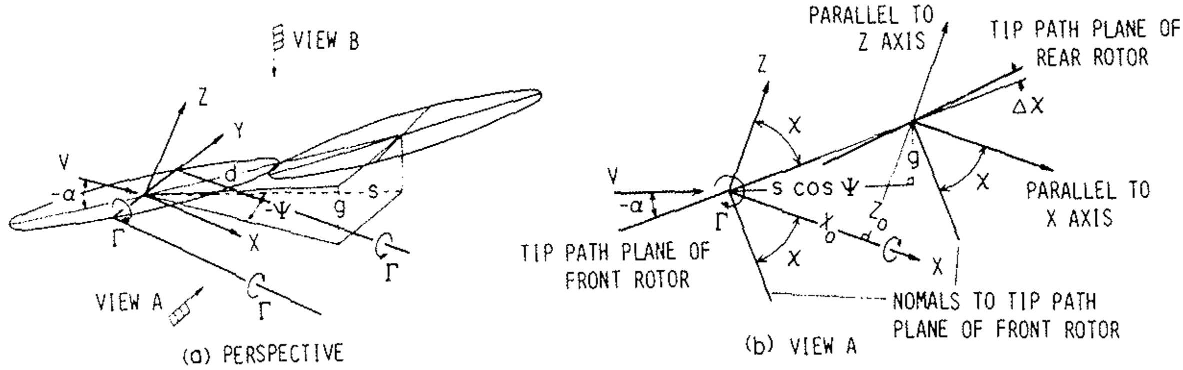
	CH-46			CH-47		
$\Psi \diagdown \Theta$	0.4°	5.3°	10.2°	-20°	0°	20°
-20°	-1.37	-0.72	0.115	-0.501	-0.059	0.315
0°	0.264	-0.024	-0.058	-0.577	-0.012	0.354
20°	1.50	0.845	0.269	-0.42	-0.289	-0.224

(e) C_m ($\times 10^{-3}$)

$\Psi \diagdown \Theta$	0.4°	5.3°	10.2°	-20°	0°	20°
-20°	-1.06	-0.096	0.96	-7.07	-8.08	-6.43
0°	-0.336	0.96	2.30	-1.29	-0.12	-0.77
20°	-1.34	-0.29	0.384	5.01	9.37	5.6

(f) C_n ($\times 10^{-3}$)

$\Psi \diagdown \Theta$	0.4°	5.3°	10.2°	-20°	0°	20°
-20°	-3.55	-5.38	-5.95	1.95	-0.236	-1.24
0°	0.	0.	0.288	-0.819	-0.089	1.29
20°	3.98	4.99	5.38	-3.01	0.88	3.36



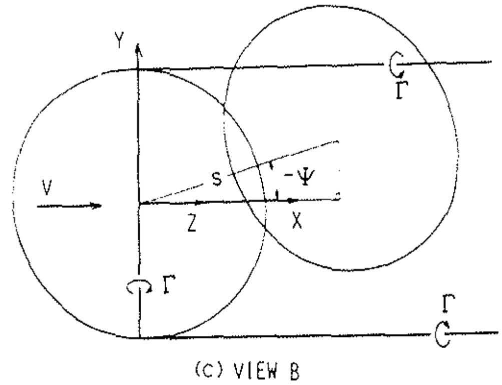


Figure 1. Vortex system of front rotor.

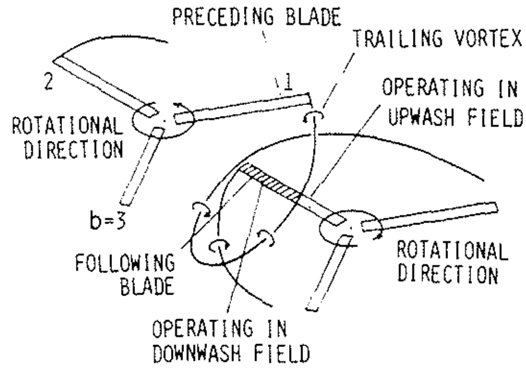


Figure 2. Interference between blades and tip vortices.

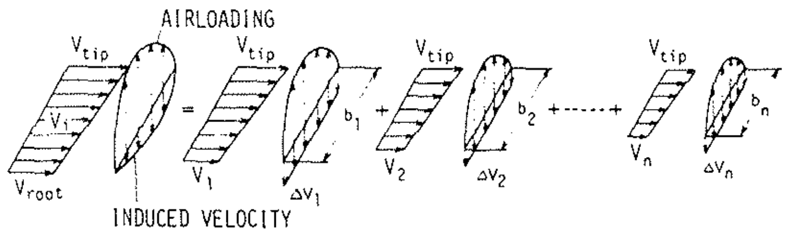


Figure 3. Decomposition of a rotary wing.

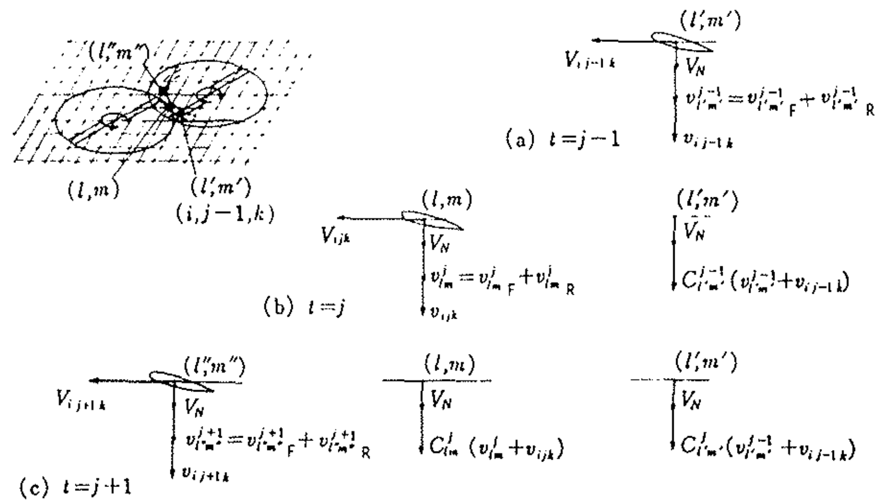


Figure 4. Successive change of the induced velocity in the rotational plane of a rotor.

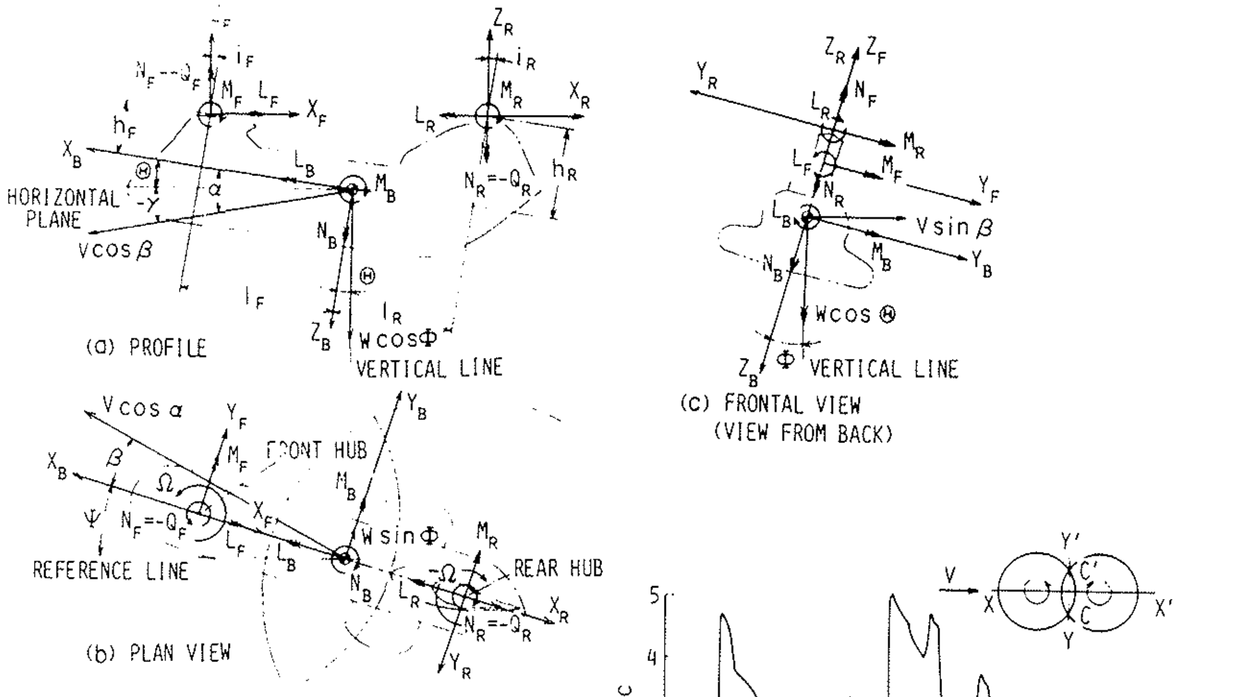


Figure 5. Forces and moments acting on a helicopter.

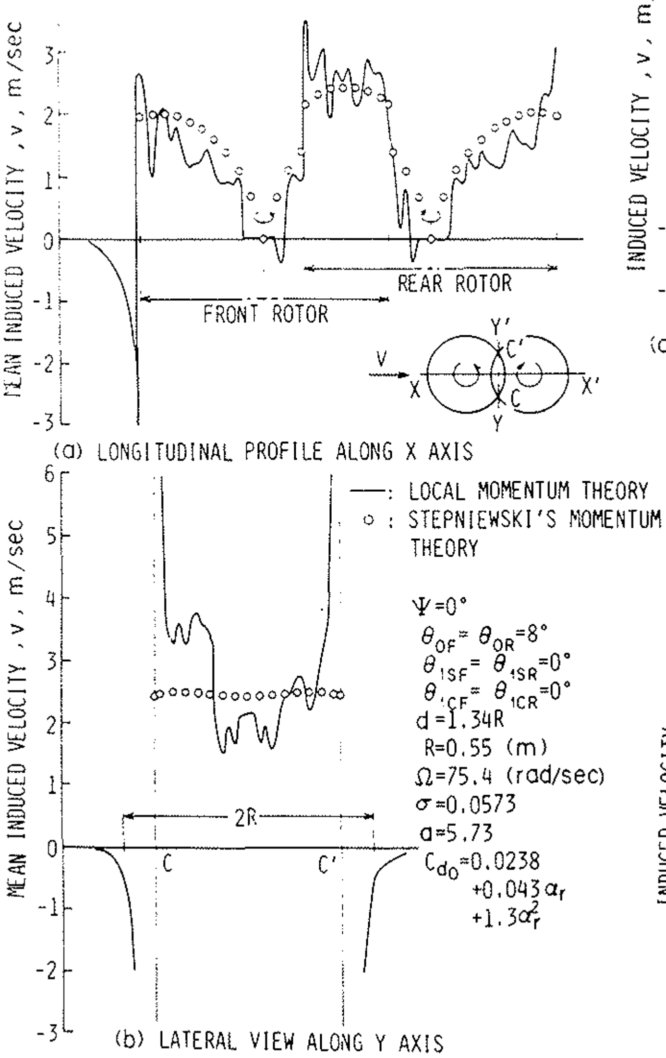


Figure 6. Induced velocity distribution of a hovering rotor. (μ=0,C=0.8)

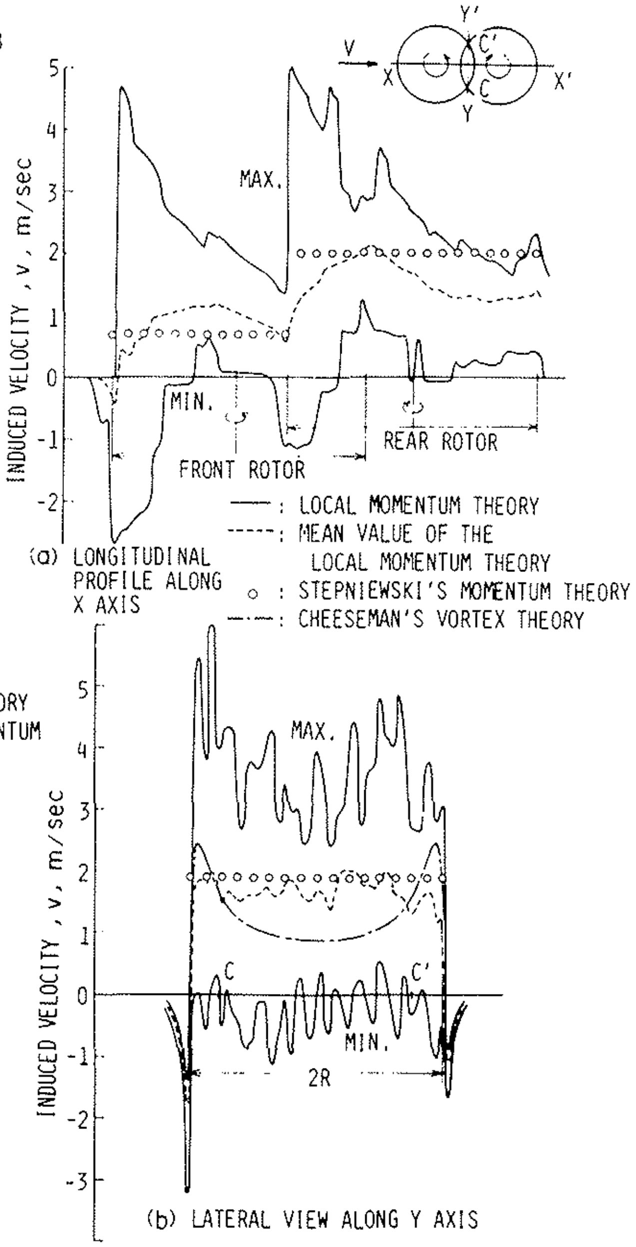


Figure 7. Induced velocity distribution of an advancing rotor. (μ=0.15,C=0.78)

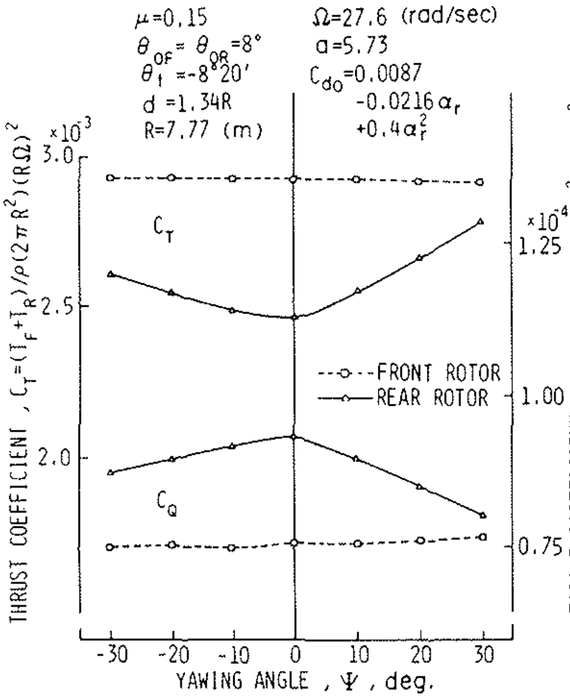


Figure 8. Performance of a tandem rotor in yaw. (No cyclic pitch inputs)

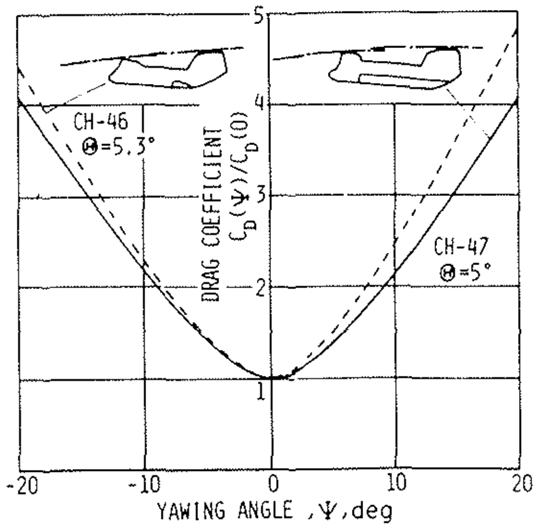


Figure 9. Parasite drag coefficients fuselage for CH-46 and CH-47 obtained from experimental data.

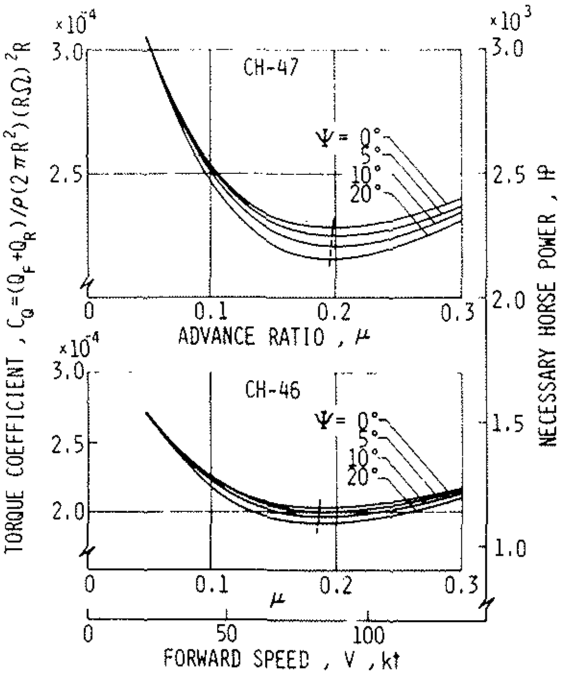


Figure 10. Necessary horse power in yawed flight. (C.G. position at center)

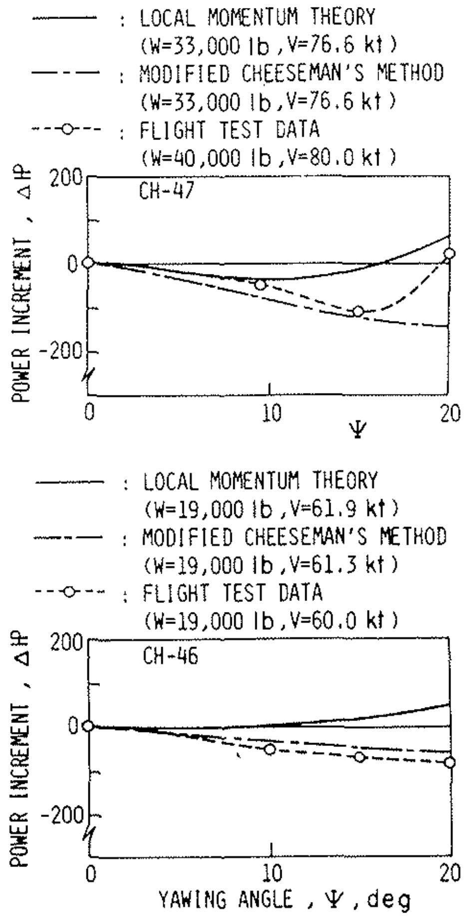


Figure 11. Comparison of theoretical results and flight test data in the power increment due to right yaw.