GENERAL MODEL OF ISOLATED HELICOPTER BLADE FOR STABILITY INVESTIGATION

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Abstract

The method with computer code was developed for investigation of hub and blade arrangements influence on rotor blade motion. The isolated rotor blade of helicopter in steady flight is considered. The hub can be hingeless or composed with up to three hinges of arbitrary sequence. Stiffness and damping can be present in each hinge. The blade can be rigid or deformable (can twist around straight elastic axis and bend in two perpendicular directions). The structural viscous damping of blade deformations can be included into analysis.

The aerodynamic loading is calculated with strip theory using steady nonlinear airfoil characteristics with unsteady effects described by dynamic inflow model.

The equations of motion have been derived from Hamiltonian principle. The blade deformations were discretized at the very early stage of equations derivation which helped to express equations in matrix form and to do most of algebraic manipulations within the computer program.

The equations are included into the set of computer codes aimed to perform comprehensive stability analysis of helicopter motion.

Notation

$\mathbf{A}$ - matrix of rotation, (7a),
$\mathbf{A}(\alpha)$ - matrix of rotation in lagging hinge,
A - rotor disc area,
A_S - blade section area,
A_T - effective (tension) cross section area,
$\mathbf{B}(\mathbf{q})$ - inertia matrix,
$\mathbf{B}(\beta)$ - matrix of rotation in flapping hinge,
$\mathbf{C}(\mathbf{q})$ - Coriolis loading matrix,
$\mathbf{C}(\theta)$ - matrix of rotation in feathering hinge,
$c(x)$ - blade section chord,
c_{aMx} - rotor rolling moment coefficient, $c_{aMx} = 2M_{ax}/\rho ARV_T^2$,
c_{aMy} - rotor pitching moment coefficient, $c_{aMy} = 2M_{ay}/\rho ARV_T^2$,
c_D - drag force coefficient,
c_L - lift force coefficient,
c_M - aerodynamic moment coefficient,

$\mathbf{C}_{Fa}$ - vector of aerodynamic loading coefficients in dynamic inflow model,

C_T - rotor thrust coefficient, $C_T = 2T_s/\rho A V_T^2$,

dD - blade section drag force,

dL - blade section lift force,

dM - blade section aerodynamic moment,

$\mathbf{D}_k(\mathbf{q})$ - gyroscopic loading matrix,

$\mathbf{D}(\psi)$ - rotor shaft rotation matrix,

E - Young modulus,

EI - blade bending stiffness,

$\mathbf{e}$ - vector of first hub stiff element, $\mathbf{e} = [e_x, e_y, e_z]^T$,

$\mathbf{f}$ - vector of second hub stiff element, $\mathbf{f} = [f_x, f_y, f_z]^T$,

$\mathbf{f}(\mathbf{q})$ - vector in the expressions of inertia loading,

$\mathbf{f}_{A()}$ - blade section aerodynamic force in () coordinate system,

$\mathbf{F}_D$ - vector of damping moments,

$\mathbf{g}$ - vector of third hub stiff element, $\mathbf{g} = [g_x, g_y, g_z]^T$,

G - Kirchoff modulus,

GJ - blade torsional stiffness,

$\mathbf{h}$ - vector of fourth hub stiff element, $\mathbf{h} = [h_x, h_y, h_z]^T$,

K_L - coefficient in dynamic inflow model,

K_p - stiffness of blade pitch angle control system,

K_w - stiffness of rotor shaft and drive system,

$\mathbf{L}, \mathbf{M}$ - matrices in dynamic inflow equations,

M_{ax} - rotor aerodynamic rolling moment,

M_{ay} - rotor aerodynamic pitching moment,

$\mathbf{m}_{A()}$ - blade section aerodynamic moment in () coordinate system,

N_d - number of generalized coordinates, $N_d = N_e + N_r$,

N_e - number of "elastic" degrees of freedom, $N_e = N_v + N_w + N_\phi$,

N_r - number of "rigid" degrees of freedom, $N_r = 0,..,3$,

N_v - number of in-plane bending deflection modes,

N_w - number of out-of-plane bending deflection modes,

N_ϕ - number of torsion deflection modes,

$\mathbf{P}(\beta_1)$ - matrix of rotation in first hinge,

$\mathbf{q}$ - vector of generalized coordinates,

$\mathbf{Q}(\beta_2)$ - matrix of rotation in second hinge,

$\mathbf{Q}_A$ - vector of aerodynamic loading in equations of motion,

$\mathbf{Q}_D$ - vector of damping forces in equations of motion,

Q_N - nonconservative forces in Hamiltonian equations of motion,

Q_{si} - hinge spring moments, $(i=0,..,N_r)$,

$\mathbf{r}, \mathbf{r}_o$ - vectors given in expressions (7b) and (7c) respectively,

R - rotor radius,

$\mathbf{R}(\delta_3)$ - matrix of rotation in third hinge,

$\mathbf{s}$ - vector given in expression (7e),

t - time,

T - kinetic energy, $T = T(t,\mathbf{y},\dot{\mathbf{y}},\mathbf{y}',\dot{\mathbf{y}}')$,

T_s - rotor thrust,

$\mathbf{T}$ - matrix of blade section rotation due to elastic deformations,

U - potential energy, $U = U(\mathbf{y},\mathbf{y}',\mathbf{y}'')$,

$\mathbf{v}$ - vector of bending deformations, $\mathbf{v} = [0,v,w]^T$,

v_i - induced velocity,

v_k - in-plane bending deflection of the k-th blade,

V - flow velocity in the blade section, $V^2 = V_\eta^2 + V_\zeta^2$,

$\overline{V}$ - nondimensional velocity, $\overline{V} = V/V_T$,

$\mathbf{V}_L$ - flight velocity, $\mathbf{V}_L = [V_{Lx},V_{Ly},V_{Lz}]^T$,

V_T - rotor tip speed, $V_T = \Omega R$,

w_k - out-of-plane bending deflection of the k-th blade,

W - work of nonconservative forces, $W = W(t,\mathbf{y},\dot{\mathbf{y}},\mathbf{y}',\dot{\mathbf{y}}')$,

$\mathbf{x},\mathbf{x}_p$ - vectors given in expressions (5) and (6) respectively,

$\overline{x}$ - nondimensional coordinate along the undeformed blade,

$\mathbf{x}_1$ - blade cross section translation along undeformed elastic axis, $\mathbf{x}_1 = [x_1,0,0]^T$,

$\mathbf{y}$ - generalized coordinates in general form of equations of motion,

α - blade section angle of attack,

α_v - rotor vortex skew angle,

β_i - angle of rotation in i-th hinge,

β_{ik} - constant part of angle of rotation in i-th hinge,

β_{ip} - periodic part of angle of rotation in i-th hinge,

β_{iq} - unknown part of angle of rotation in i-th hinge,

Υ_i - blade viscous damping coefficients, $(i=1,\dots,N_d)$,

δ_3 - flap hinge axis skew angle,

η_a - distance between aerodynamic center and elastic axis in blade section,

η_i - blade normal modes of vibration, $(i=1,\dots,N_e)$,

$\theta_c(\psi)$ - blade pitch control angle, $\theta_c = \theta_0 + \theta_1\cos(\psi) + \theta_2\sin(\psi)$,

θ_g - blade geometrical twist angle,

$\varkappa$ - pitch-flap coupling coefficient,

λ - induced velocity coefficient,

$\boldsymbol{\lambda}$ - vector of induced velocity perturbations, $\boldsymbol{\lambda} = [\lambda_1,\lambda_2,\lambda_3]^T$,

λ_c - rotor axial flow velocity coefficient, $\lambda_c = V_{Lz}/V_T$,

λ_h - induced velocity coefficient in hover,

λ_i - induced velocity perturbation coefficient,
λ_0 - constant term of induced velocity coefficient,
μ - tip-speed ratio,
$\boldsymbol{\xi}$ - blade section point coordinates, $\boldsymbol{\xi} = [\xi, \eta, \zeta]^T$,
ρ - air density,
ρ_b - blade average density,
φ_0 - angle between blade section velocity component in the rotor plane of rotation and helicopter plane of symmetry,
ϕ_K - torsion angle of the k-th blade,
ψ - blade azimuth angle,
Ω - rotor shaft angular velocity,
$(\dot{\ })$ - derivation with respect to time,
$(\ ')$ - derivation with respect to $\overline{x}$.
Matrices and vectors are written bold.

Introduction

New rotor concepts have been put into production during the previous decade (Ref.1). This was proceeded by a considerable research and development effort. Each new product was carefully analyzed analytically and tested experimentally.

Parametric studies for the new rotor concepts evaluation have to be done at as early design stage as possible. The computer simulation is more flexible and cheaper than ground and flight tests, so it is applied widely in an early design work.

During this study the model and the set of computer programs have been developed for investigation of hub and blade arrangements influence on rotor blade motion. Hub and blade are modeled in general way to allow studying different rotor concepts.

The method concerns isolated rotor blade for helicopter in steady flight.

The hub can be composed in many different ways. It is possible to include into hub model up to three hinges of arbitrary sequence or to consider hingeless or bearingless rotor. Blades can bend in two planes and twist around elastic axis.

Equations of motion have been derived from Hamiltonian principle. The blade deformations were discretized by deflection modes computed for blade rotating in vacuum. This discretization was done at the very early stage of equations derivation, that helped to express equations of motion in matrix form and to do most of algebraic manipulations within the computer program. This procedure can be named "half automatic" generation of equations and is similar to that worked out by Done (Ref.2) and his co-workers.

The equations of motion were included into the set of computer codes aimed to perform comprehensive stability analysis of helicopter motion.

In the paper the model and the equations of motion derivation are described.

1. Rotor model

1.1. Hub

The isolated rotor blade of helicopter in steady flight is considered. The rotor shaft angular speed Ω is constant. The hub can be composed in many different ways, including up to three hinges of arbitrary sequence.

The most general hub model (Fig.1) consists of four stiff elements e, f, g and h linking the blade to the rotor shaft. Points 0_P, 0_Q and 0_R are the ends of the first three stiff segments. The fourth segment h extends to blade (point A). Locations of ends of stiff segments are specified by vectors: $\mathbf{e}$, $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{h}$. In each point: 0_P, 0_Q and 0_R one of three hinges (flapping, lagging or feathering) can be placed. Matrices of rotation in these hinges are given in Appendix 1.

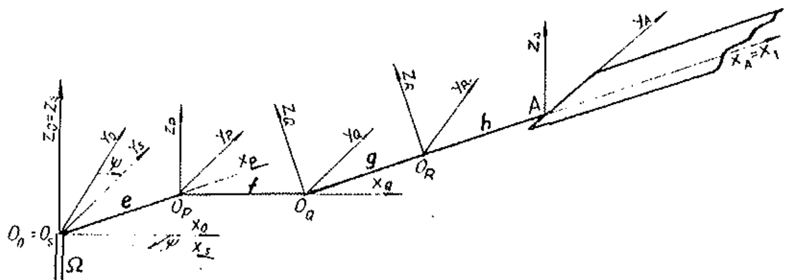

Fig.1

Angle of rotation β_i in i-th hinge consists of:
- constant part β_{ik} describing design angles like: precone, droop etc.,
- periodic function $\beta_{ip}(t)$ which describes the blade steady motion,
- unknown function $\beta_{iq}(t)$ related to disturbed motion of the blade,

$$\beta_i = \beta_{ik} + \beta_{ip}(t) + \beta_{iq}(t), \qquad (i=1,\ldots,N_r). \tag{1}$$

Blade pitch control angle θ_c is added to the feathering hinge rotation β_θ, if such hinge is included into the hub:

$$\beta_\theta = \beta_\theta + \theta_c(t). \tag{2}$$

If there are both feathering and flapping hinges in the hub, the kinematic pitch-flap coupling can be applied (β_f - flapping hinge rotation):

$$\beta_\theta = \beta_\theta - \varkappa\beta_f. \tag{3}$$

Damping and stiffness can be taken into account in each hinge.

In the shaft coordinate system the location of the end of the last hub segment h (point A) can be written in the form:

$$\mathbf{x}_p = \mathbf{D}(\psi)[\mathbf{e} + \mathbf{P}\mathbf{f} + \mathbf{P}\mathbf{Q}\mathbf{g} + \mathbf{P}\mathbf{Q}\mathbf{R}\mathbf{h} + \mathbf{P}\mathbf{Q}\mathbf{R}\mathbf{x}_1]. \tag{4}$$

1.2. Blade

The blade (Fig.2) is attached to the hub in the point A at the end of segment h. The blade can be rigid or deformable. In the later case the blade has straight elastic axis parallel to the axis of segment h. The blade is pretwisted by the angle $\theta_g(x)$ around elastic axis or, if is rigid, around feathering axis. The elastic properties of blade sections are symmetrical about chord.

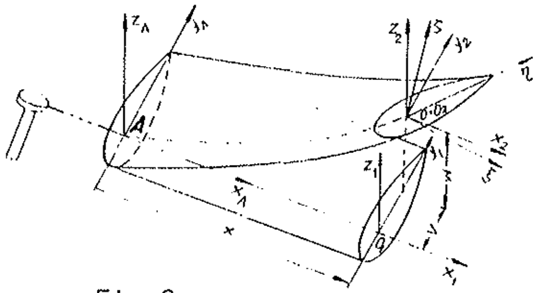

Fig.2

The viscous structural damping of blade deformations can be also taken into account.

The deformations of the blade are:

- in-plane bending $v(x)$,
- out-of-plane bending $w(x)$,
- torsion $\phi(x)$.

In the undeformed elastic axis coordinate system the vector of the deformed blade point coordinates can be written in the form:

$$\mathbf{x} = \mathbf{x}_1 + \mathbf{v} + \mathbf{T}\cdot\boldsymbol{\xi}. \tag{5}$$

Matrix $\mathbf{T}$ is given in Appendix 1.

The vector of deformed blade point coordinates is obtained form (4) and (5):

$$\mathbf{x}_p = \mathbf{D}(\psi)[\mathbf{e} + \mathbf{P}\mathbf{f} + \mathbf{P}\mathbf{Q}\mathbf{g} + \mathbf{P}\mathbf{Q}\mathbf{R}\mathbf{h} + \mathbf{P}\mathbf{Q}\mathbf{R}(\mathbf{x}_1 + \mathbf{v} + \mathbf{T}\cdot\boldsymbol{\xi})]. \tag{6}$$

The final expression for $\mathbf{X}_p$ can be written in the form:

$$\mathbf{x}_p = \mathbf{D}(\psi)\,\mathbf{r}_o = \mathbf{D}(\psi)(\mathbf{r} + \mathbf{A}\mathbf{x} + \mathbf{A}\mathbf{s}), \tag{7}$$

where:

$$\mathbf{A} = \mathbf{P}(\beta_1)\mathbf{Q}(\beta_2)\mathbf{R}(\beta_3), \tag{7a}$$

$$\mathbf{r} = \mathbf{e} + \mathbf{P}(\beta_1)\mathbf{f} + \mathbf{P}(\beta_1)\mathbf{Q}(\beta_2)\mathbf{g} + \mathbf{P}(\beta_1)\mathbf{Q}(\beta_2)\mathbf{R}(\beta_3)\mathbf{h}, \tag{7b}$$

$$\mathbf{r}_o = \mathbf{r} + \mathbf{A}\mathbf{x} + \mathbf{A}\mathbf{s}, \tag{7c}$$

$$\mathbf{x} = [x,y,z]^T, \quad y = \cos(\theta_g) + \sin(\theta_g), \quad z = -\sin(\theta_g) + \cos(\theta_g), \tag{7d}$$

$$\left.\begin{aligned} &\mathbf{s} = \mathbf{x}_1 + \mathbf{v} + \mathbf{T}\cdot\boldsymbol{\xi} = \sum \mathbf{s}_i, \qquad (i=1,2,3), \\ &\mathbf{s}_1 = [-v'y,\ v,\ 0]^T, \quad \mathbf{s}_2 = [-w'z,\ 0,\ w]^T, \quad \mathbf{s}_3 = [0, -z\phi,\ y\phi]^T. \end{aligned}\right\} \tag{7e}$$

2. Equations of motion

2.1. General remarks

The blade model described in Section 1.2 includes a variety of models for which equations of motion can have different forms. This was the reason why a special method for equations of motion derivation had to be elaborated.

The equations of motion have been derived by the method which can be named "half automatic" generation of equations, since algebraic manipulations are performed mainly by computer.

The method is based on utilizing the vector $\mathbf{x}_p$ (7) to the expressions for inertia and stiffness loadings and to the aerodynamic forces calculation. To avoid numerical difficulties, the derivatives of matrices and vectors are calculated analytically and included into computer code. The inertial and structural loadings need not be integrated along the blade span during the computation of right hand sides of equations. The blade generalized masses and stiffnesses are taken from the separate computer program before solving (or analyzing) the equations of motion.

The equations of motion derivation is based on Hamiltonian principle:

$$\int_{t_1}^{t_2} [\delta(U-T) - \delta W]\,dt = 0, \quad \text{or} \quad \int_{t_1}^{t_2} (\delta U)\,dt - \int_{t_1}^{t_2} (\delta T)\,dt = \int_{t_1}^{t_2} (\delta W)\,dt. \tag{8}$$

In the above equations the inertial loading I.L. results from variation of kinetic energy, the stiffness loading S.L. comes from variation of potential energy and the nonconservative loading N.L. arises from the work done by aerodynamic Q_A and damping Q_D forces.

The kinetic and potential energies are the functions of: generalized coordinates $\mathbf{y}$, generalized velocities $\dot{\mathbf{y}}$ and their derivatives with respect to length,

$$T = T(t,\ \mathbf{y},\ \dot{\mathbf{y}},\ \mathbf{y}',\ \dot{\mathbf{y}}'), \tag{9a}$$

$$U = U(\mathbf{y},\ \mathbf{y}',\ \mathbf{y}''). \tag{9b}$$

Variations of T and U in (8) were calculated in terms of the variations: $\delta\mathbf{x}$, $\delta\dot{\mathbf{x}}$, $\delta\mathbf{x}'$ and $\delta\dot{\mathbf{x}}'$, and after substituting $\delta d = d\delta$ and integrating by parts, the expressions containing the velocity variations $\delta\dot{\mathbf{x}}$ and $\delta\dot{\mathbf{x}}'$ were eliminated:

$$\left.\begin{aligned}&\int_{t_1}^{t_2}\left[\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\mathbf{y}}}\right] - \frac{\partial T}{\partial \mathbf{y}} + \frac{\partial U}{\partial \mathbf{y}}\right]\delta\mathbf{y}\,dt + \int_{t_1}^{t_2}\left[\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\mathbf{y}}'}\right] - \frac{\partial T}{\partial \mathbf{y}'} + \frac{\partial U}{\partial \mathbf{y}'}\right]\delta\mathbf{y}'dt + \int_{t_1}^{t_2}\left[\frac{\partial U}{\partial \mathbf{y}''}\right]\delta\mathbf{y}''dt\\ &= \int_{t_1}^{t_2} Q_N\delta\mathbf{y}\,dt.\end{aligned}\right\} \tag{10}$$

During the integration by parts the boundary conditions (not given here) have been also obtained.

At this stage of equations of motion derivation, the discretization of deflections v, w and ϕ was performed.

2.2. Generalized coordinates.

The blade motion can be described by:

- "elastic" degrees of freedom v, w, ϕ,
- "rigid" degrees of freedom β_i, $(i=0,..,N_r)$.

The blade deflections v, w and ϕ were discretized by free vibration modes:

$$v = \sum v_i = \sum \eta_i(x)q_i(t), \qquad (i=1,\dots,N_v), \tag{11a}$$

$$w = \sum w_i = \sum \eta_i(x)q_i(t), \qquad (i=N_v+1,\dots,N_v+N_w), \tag{11b}$$

$$\phi = \sum \phi_i = \sum \eta_i(x)q_i(t), \qquad (i=N_v+N_w+1,\dots,N_e). \tag{11c}$$

Vector of generalized coordinates for rotor blade motion is composed with (1) and (11) in the succession of deflections and angles:

$$\mathbf{q} = [q_i]^T = [v,w,\phi,\beta_j]^T, \qquad (i=1,\dots,N_d), \quad (j=0,\dots,N_r). \tag{12}$$

2.3. General form of equations of motion

Taking into account (10) and (12), two kinds of equations of motion have been obtained:

- for "elastic" degrees of freedom $(j=1,\dots,N_e)$:

$$\left.\begin{aligned}&\int_R\left[\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_j}\right] - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j}\right]\eta_j dR + \int_R\left[\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_j'}\right] - \frac{\partial T}{\partial q_j'} + \frac{\partial U}{\partial q_j'}\right]\eta_j' dR\\ &+ \int_R\left[\frac{\partial U}{\partial q_j''}\right]\eta_j'' dR = \int_R Q_{Nj}\eta_j dR,\end{aligned}\right\} \tag{13a}$$

integration in (13a) extends along the blade;

- for "rigid" degrees of freedom $(j=N_e+1,\dots,N_e+N_r)$:

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_j}\right] - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} = Q_{Nj}. \tag{13b}$$

3. Blade inertia loading

Kinetic energy T can be calculated in the form:

$$T = \frac{1}{2}\int_R\int_{A_s} \rho_b \dot{\mathbf{x}}_p^2 \, dA_s dR. \tag{14}$$

Derivation of vector $\mathbf{x}_p$ can be obtained from (7) in the form:

$$\dot{\mathbf{x}}_p = \dot{\mathbf{D}}\,\mathbf{r}_o + \mathbf{D}\sum \frac{\partial \mathbf{r}_o}{\partial q_i}\dot{q}_i, \qquad (i=1,\dots,N_d). \tag{15}$$

Inserting (15) into (14) and putting result into (13a) for "elastic" degrees of freedom and into (13b) for "rigid" degrees of freedom, the inertial loading in the equations of motion takes the form:

$$\left.\begin{aligned} \text{I.L.} = \int_R\int_{A_s} \rho_b \Big[&\sum_i \left[\frac{\partial \mathbf{r}_o}{\partial q_n}\right]^T \left[\frac{\partial \mathbf{r}_o}{\partial q_i}\right]\ddot{q}_i + 2\sum_i \left[\frac{\partial \mathbf{r}_o}{\partial q_n}\right]^T \mathbf{D}^T\dot{\mathbf{D}}\left[\frac{\partial \mathbf{r}_o}{\partial q_i}\right]\dot{q}_i \\ &+ \sum_i \sum_j \left[\frac{\partial \mathbf{r}_o}{\partial q_n}\right]^T \left[\frac{\partial^2 \mathbf{r}_o}{\partial q_i \partial q_j}\right]\dot{q}_i\dot{q}_j + \left[\frac{\partial \mathbf{r}_o}{\partial q_n}\right]^T \mathbf{D}^T\ddot{\mathbf{D}}\,\mathbf{r}_o\Big] dA_s dR, \end{aligned}\right\} \tag{16}$$

where n is the number of generalized coordinates considered, $(n \leqslant N_d)$.

In matrix form the inertia loading can be written as:

$$\text{I.L.} = \mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + 2\mathbf{C}(\mathbf{q})\dot{\mathbf{q}} + \dot{\mathbf{q}}^T\mathbf{D}_n(\mathbf{q})\dot{\mathbf{q}} + \mathbf{f}(\mathbf{q}). \tag{17}$$

Elements of matrices $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}_n$ and vector $\mathbf{f}$ are given in Appendix 2.

4. Blade stiffness loading

Stiffness loading is caused by deformations of hinge springs and/or blade deflections.

Hinge springs stiffness can be an arbitrary function of hinge angles of rotation, i.e. in the "rigid" degrees of freedom the stiffness loadings can be expressed as:

$$\text{S.L.} = Q_{si}(q_i), \qquad (i=0,\dots,N_r). \tag{18}$$

Blade deformation forces and moments are derived from model given in Ref.3. In the equations of motion the stiffness loading can be put in the form:

$$\text{S.L.} = \mathbf{Q}\mathbf{q} + \mathbf{h}. \tag{19}$$

Elements of matrix $\mathbf{Q}$ and vector $\mathbf{h}$ are given in Appendix 3.

5. Aerodynamic loading

Aerodynamic loading is calculated according to two-dimensional model with the steady nonlinear airfoil characteristics. The unsteady effects are included by applying dynamic inflow model with Pitt-Peters (Ref.4) coefficients. In each blade cross section (Fig.3) the 2D flow is assumed. The aerodynamic loading acting in section aerodynamic center A.C. consists of:

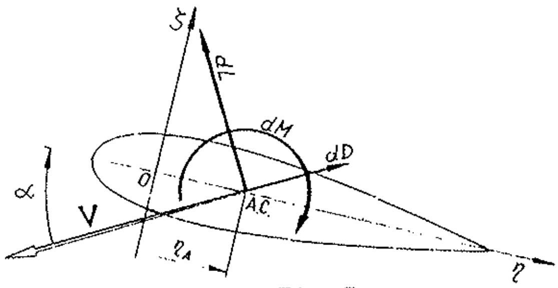

Fig.3

- drag $dD=\frac{1}{2}\varrho c(x)V^2 c_D(\alpha)dx,$ (20a)
- lift $dL=\frac{1}{2}\varrho c(x)V^2 c_L(\alpha)dx,$ (20b)
- moment $dM=\frac{1}{2}\varrho c^2(x)V^2 c_M(\alpha)dx.$ (20c)

The aerodynamic coefficients c_D, c_L and c_M are calculated from nonlinear characteristics for blade section instant angles of attack.

In the blade section coordinate system $O\xi\eta\zeta$ the flow velocity can be expressed as:

$$\mathbf{V} = (\mathbf{A}\mathbf{T})^{-1}(\mathbf{V}_L + \mathbf{v}_i + \dot{\mathbf{x}}_p). \tag{21}$$

The aerodynamic loadings in the blade section are expressed as force $\mathbf{f}_A$ and moment $\mathbf{m}_A$ vectors in $O\xi\eta\zeta$ coordinate system:

$$\begin{aligned}
&\mathbf{f}_A=[f_\xi,f_\eta,f_\zeta]^T, \quad f_\xi=0, \quad f_\eta=dD\cos(\alpha)-dL\sin(\alpha), \quad f_\zeta=dD\sin(\alpha)+dL\cos(\alpha),\\
&\mathbf{m}_A=[m_\xi,m_\eta,m_\zeta]^T, \quad m_\xi=-dM+\eta_A f_\zeta, \quad m_\eta=0, \quad m_\zeta=0.
\end{aligned} \tag{22}$$

Vector of blade aerodynamic loadings can be obtained from (22) by integration along the blade span and then vector of total rotor aerodynamic loadings

$$\mathbf{Q}_A = \mathbf{Q}_A(t,\mathbf{q},\dot{\mathbf{q}}) = (Q_{Ai}), \qquad (i=1,\dots,N_d) \tag{23}$$

can be calculated by successive transformations of loads from the blade root to the rotor shaft.

5.1. Dynamic inflow model

The dynamic inflow model allows to account into analysis the effects of unsteady flow. The total rotor aerodynamic loadings are utilized for calculations.

In this model there is only one component of induced velocity vector, perpendicular to the plane of rotation, i.e. the component Oz_s in the $O_s x_s y_s z_s$ (rotor shaft) system of coordinates. Induced velocity coefficient is obtained from:

$$\lambda = \lambda_o + \lambda_i, \qquad \lambda = v_i/V_T. \tag{24}$$

Constant part λ_o of induced velocity is calculated:

- for hover: $\lambda_h = \sqrt{C_T/2}$, (25a)

- for axial rotor flow: $\lambda_o = -\frac{\lambda_c}{2} + \sqrt{(\frac{\lambda_c}{2})^2 + \lambda_h^2}$, (25b)

- for forward flight: $\lambda_o = \sqrt{\frac{1}{2}(\sqrt{\overline{V}_P^4 + 4\lambda_h^4} - \overline{V}^2)}$, $\quad \overline{V}_P^2 = \overline{V}_{Lx}^2 + \overline{V}_{Ly}^2$. (25c)

Induced velocity perturbation λ_i is computed as:

$$\lambda_i = \lambda_1 + \lambda_2\overline{x}\sin(\psi - \varphi_o) + \lambda_3\overline{x}\cos(\psi - \varphi_o), \tag{26}$$

where

$$\varphi_o = \sin^{-1}(V_{Ly}/V_p). \tag{27}$$

Components λ_1, λ_2 and λ_3 of induced velocity perturbation are calculated from the system of ordinary differential equations:

$$\Omega^{-1}\mathbf{M}\dot{\boldsymbol{\lambda}} + \mathbf{L}^{-1}\boldsymbol{\lambda} = \mathbf{C}_{Fa}, \tag{28}$$

where

$$\boldsymbol{\lambda} = [\lambda_i]^T, \qquad (i=1,2,3), \qquad \text{and} \qquad \mathbf{C}_{Fa} = [C_T, c_{aMx}, c_{aMy}]^T. \tag{29}$$

Matrices $\mathbf{M}$ and $\mathbf{L}$ taken from Ref.4 are given in Appendix 4.

6. Damping loading

Damping forces can be expressed as a vector:

$$\mathbf{F}_D = \mathbf{F}_D(q_i, \dot{q}_i) = [F_{Di}]^T, \qquad (i=1,\ldots,N_d). \tag{30}$$

For hinges loading, the damping moments can be arbitrary function of hinge angles of rotation and angular velocities:

$$F_{Di} = F_{Di}(q_i, \dot{q}_i), \qquad (i=N_\phi+1,\ldots,N_\phi+N_r). \tag{31}$$

For blade deflection, the viscous damping loading is modeled in the form:

$$F_{Di} = \dot{q}_i \int_R \eta_i \gamma_i^* dR, \qquad (i=1,\ldots,N_e). \tag{32}$$

7. Final form of equations

Collecting together expressions: (17), (19), (23) and (30), equations of motion can be written in the form:

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} = -2\mathbf{C}(\mathbf{q})\dot{\mathbf{q}} - \dot{\mathbf{q}}^T\mathbf{D}_n(\mathbf{q})\dot{\mathbf{q}} - \mathbf{f}(\mathbf{q}) - \mathbf{Q}\mathbf{q} - \mathbf{h} - \mathbf{F}_D(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{Q}_A(t,\mathbf{q},\dot{\mathbf{q}}). \tag{33}$$

For numerical integration of (33) the Gear's algorithm has been used.

Examples of application of the described model will be shown during presentation of this paper.

References

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3. Houbolt J.C., Brooks G.W.: "Differential equations of motion for combined flapwise bending, chordwise bending and torsion of twisted nonuniform rotor blades", NACA Rep. 1346, 1957.
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Appendix 1. Matrices of rotation

Shaft rotation

$$\mathbf{D} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Lagging hinge rotation

$$\mathbf{A}(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Flapping hinge rotation

$$\mathbf{B}(\beta) = \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \\ 0 & 1 & 0 \\ \sin(\beta) & 0 & \cos(\beta) \end{bmatrix},$$

Feathering hinge rotation

$$\mathbf{C}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix},$$

Blade deformation

$$\mathbf{T} = \begin{bmatrix} 1 & -v'\cos(\theta_g)+w'\sin(\theta_g) & -v'\sin(\theta_g)-w'\cos(\theta_g) \\ v' & \cos(\theta_g)-\phi\sin(\theta_g) & \sin(\theta_g)+\phi\cos(\theta_g) \\ w' & -\sin(\theta_g)-\phi\cos(\theta_g) & \cos(\theta_g)-\phi\sin(\theta_g) \end{bmatrix}.$$

Appendix 2. Inertia matrices

$$B_{ni} = \iint\limits_{RA_T} \rho_b \left[\frac{\partial \mathbf{r}_0}{\partial q_n}\right]^T \left[\frac{\partial \mathbf{r}_0}{\partial q_i}\right] dA_s dR, \qquad C_{ni} = 2\iint\limits_{RA_T} \rho_b \left[\frac{\partial \mathbf{r}_0}{\partial q_n}\right]^T \mathbf{D}^T \dot{\mathbf{D}} \left[\frac{\partial \mathbf{r}_0}{\partial q_i}\right] dA_s dR,$$

$$D_{nij} = \iint\limits_{RA_T} \rho_b \left[\frac{\partial \mathbf{r}_0}{\partial q_n}\right]^T \left[\frac{\partial^2 \mathbf{r}_0}{\partial q_i \partial q_j}\right] dA_s dR, \qquad f_i = \iint\limits_{RA_T} \rho_b \left[\frac{\partial \mathbf{r}_0}{\partial q_n}\right]^T \mathbf{D}^T \mathbf{D}\, \mathbf{r}_0 dA_s dR.$$

Appendix 3. Elements of blade stiffness matrix

Matrix $\mathbf{Q}$ is symmetrical; the elements Q_{ij} are:

for $i = 1,\ldots,N_v$, for $j = 1,\ldots,N_v$,

$$Q_{ij}=\left[(EI_{\eta\eta}\cos^2(\theta_g)+EI_{\zeta\zeta}\sin^2(\theta_g)\right](\eta_i'')^2+t(x)(\eta_i')^2,$$

for $j = N_v+1,\ldots,N_v+N_w$,

$$Q_{ij}=-(EI_{\eta\eta}-EI_{\zeta\zeta})\cos(\theta_g)\sin(\theta_g)\eta_i'\eta_j',$$

for $j = N_v+N_w+1,\ldots,N_v+N_w+N_\phi$,

$$Q_{ij}=-\theta_g'C_\eta\cos(\theta_g)\eta_i'\eta_j''+t(x)\eta_s\sin(\theta_g)\eta_i\eta_j'',$$

for $i = N_v+1,\ldots,N_v+N_w$, for $j = N_v+1,\ldots,N_v+N_w$,

$$Q_{ij}= \left[EI_{\eta\eta}\sin^2(\theta_g)+EI_{\zeta\zeta}\cos^2(\theta_g)\right](\eta_i'')^2+t(x)(\eta_i')^2,$$

for $j = N_v+N_w+1,\ldots,N_v+N_w+N_\phi$,

$$Q_{ij}=\theta_g'C_\eta\sin(\theta_g)\eta_i'\eta_j''+t(x)\eta_s\cos(\theta_g)\eta_i\eta_j'',$$

for $i = N_v+N_w+1,\ldots,N_v+N_w+N_\phi$, for $j = N_v+N_w+1,\ldots,N_v+N_w+N_\phi$,

$$Q_{ij}=\left[GJ+\theta_g'L_K+t(x)k_E\right](\eta_i')^2.$$

Components of vector $\mathbf{h}$ are:

$$h_i=-t(x)\eta_s\cos(\theta_g)\eta_i'', \qquad (i=1,\ldots,N_v),$$

$$h_i=t(x)\eta_s\sin(\theta_g)\eta_i'', \qquad (i=N_v+1,\ldots,N_v+N_w),$$

$$h_i=t(x)k_E\theta_g'\eta_i', \qquad (i=N_v+N_w+1,\ldots,N_v+N_w+N_\phi),$$

where

$$A_E=\int_{A_T} E\,dA_s, \qquad C_\eta=\int_{A_T} E(\eta^2+\zeta^2-k_T)(\eta-\eta_s)\,dA_s, \qquad \eta_s=\int_{A_T} E\eta\,dA_s/A_E,$$

$$k_E=\int_{A_T} E(\eta^2+\zeta^2-k_T)\,dA_s/A_E, \qquad k_T=\int_{A_T} E(\eta^2+\zeta^2)\,dA_s/A_E, \qquad L_K=\int_{A_T} E(\eta^2+\zeta^2-k_T)\,dA_s,$$

$$t(x)=\int_{A_T} E\left[u'+\phi'\theta_g'(\eta^2+\zeta^2)-v''\left[\cos(\theta_g)+\sin(\theta_g)\right]+w''\left[\sin(\theta_g)-\cos(\theta_g)\right]\right]dA_s.$$

Appendix 4. Dynamic inflow elements

$$\mathbf{M} = \begin{bmatrix} \frac{128}{75\pi} & 0 & 0 \\ 0 & \frac{-16}{45\pi} & 0 \\ 0 & 0 & \frac{-16}{45\pi} \end{bmatrix}, \quad \sin(\alpha_V) = \frac{\lambda_c + \lambda_0}{\sqrt{\mu^2 + (\lambda_c + \lambda_0)^2}}, \quad K_L = \frac{\mu^2 + (\lambda_c + \lambda_0)(\lambda_c + 2\lambda_0)}{\sqrt{\mu^2 + (\lambda_c + \lambda_0)^2}},$$

$$\mathbf{L} = \frac{1}{K_L} \begin{bmatrix} \frac{1}{2} & 0 & \frac{15}{64}\pi\sqrt{\frac{1-\sin(\alpha_V)}{1+\sin(\alpha_V)}} \\ 0 & \frac{-4}{1+\sin(\alpha_V)} & 0 \\ \frac{15}{64}\pi\sqrt{\frac{1-\sin(\alpha_V)}{1+\sin(\alpha_V)}} & 0 & \frac{-4}{1+\sin(\alpha_V)} \end{bmatrix}, \quad \mu = \frac{\sqrt{V_{Lx}^2 + V_{Lz}^2}}{V_T}.$$

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