# THE INFLUENCE OF FLEXIBLE BLADES ON THE CHARACTERISTICS OF THE ORNICOPTER

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## Abstract

The Ornicopter is a single rotor helicopter without a reaction torque. By forcing the blades of the Ornicopter to flap up and down, both a lifting force and an average propulsive force can be generated. Because of this average propulsive force the blades will propel (i.e. rotate) themselves and there will no longer be a need to transfer torque from the fuselage to the rotor. If there is no longer a torque transferred from the fuselage to the rotor there will neither be a reaction torque.

The theory and calculations performed in previous publications [1][3][4] were based on the assumption of rigid rotor blades. This paper will concentrate on the effects that flexible blades will have on the key characteristics of the Ornicopter. It will be investigated whether any power will be lost due to the flexibility of the blades, what the effects of a flexible blade are on the required flapping moment, the torque about the rotor hub and the vertical shear force fluctuations in the root of the blade. Additionally the difference between the root angle $\varepsilon$ of a flexible blade and the flapping angle $\beta$ of a rigid blade will be addressed and to conclude the phase difference between the flapping moment and the deflection of the flapping mechanism will be calculated.


## Notations

| | |
|---|---|
| $c$ | Blade chord |
| $c_l$ | Lift coefficient of a blade element |
| $c_{l\alpha}$ | Derivative of $c_l$ with respect to $\alpha$ $dc_l/d\alpha$ |
| $c_n$ | Constant of normalization for a particular mode shape |
| $f(n)$ | Generalized moment of inertia, value of the integral $\int_0^R mS_nS_m dr$ for $n=m$ |
| $k$ | $k^2=EI/m\Omega^2R^4$ |
| $m$ | Mass per unit length |
| $m_{fl}$ | Non-dimensional flapping moment |
| $\hat{m}_{fl}$ | Complex amplitude of the non-dimensional flapping moment |
| $r$ | Radius of blade element |
| $v_i$ | Induced velocity |
| $v_0$ | Non-dimensional vertical shear force in the root of the blade |
| $x$ | Non-dimensional radius of blade element $r/R$ |
| $A_{ij}$ | Coefficient of the equations of motion for a flexible blade |
| $B_{ij}$ | Coefficient of the equations of motion for a flexible blade |
| $C_l$ | Lift coefficient of a blade |
| $C_{nm}$ | $\frac{1}{R^4}\int_0^R S_nS_m dr$ |
| $E$ | Modulus of elasticity |
| $E_n$ | $S_n'(0)$ |
| $F$ | Excitation force |
| $F_n$ | $f_n/I$ |
| $G$ | Centrifugal force |
| $I$ | Mass moment of inertia of the rotor blade about the rotor hub |
| $K$ | Spring stiffness |
| $L$ | Lift |
| $M$ | Bending moment |
| $M_{fl}$ | Mechanical flapping moment |
| $P_a$ | Power available to drive the rotor |
| $P_{eng}$ | Engine power, power transmitted by the engine to the spring of the mechanical flapping mechansim |
| $P_{fl}$ | Mechanical flapping power, power exerted by the flap forcing mechanism on the blade |
| $Q$ | Torque about the rotor hub |
| $Q_n$ | Generalized force or moment for the generalized coordinate $\phi_n$ |
| $R$ | Rotor radius |
| $S_n$ | Mode shape for a particular eigenfrequency |
| $S'_n$ | Derivative of $S_n$ with respect to the azimuth angle $dS_n/d\psi$ |
| $S_m$ | Mode shape for a particular eigenfrequency |
| $S'_m$ | Derivative of $S_m$ with respect to the azimuth angle $dS_m/d\psi$ |
| $T$ | Kinetic energy |
| $U$ | Potential energy |
| $U_b$ | Strain energy due to bending |
| $U_{cf}$ | Strain energy due to the centrifugal force |
| $U_{spring}$ | Potential energy due to the deflection of the spring |
| $V_0$ | Vertical shear force in the root of the blade |
| $W_n$ | Work done in the direction of the generalized coordinate $\phi_n$ |

$Z$ Vertical displacement of the flexible rotor blade

$\alpha$ Angle of attack
$\alpha$ Deflection angle
$\hat{\alpha}_i$ Amplitude of the generalized coordinate $\phi_i$
$\hat{\alpha}_n$ Amplitude of the generalized coordinate $\phi_n$
$\beta$ Flapping angle of the blade
$\hat{\beta}$ Complex amplitude of the flapping angle
$\beta'$ Derivative of β with respect to the azimuth angle $d\beta/d\psi$
$\beta''$ Second derivative of β with respect to the azimuth angle $d^2\beta/d\psi^2$
$\dot{\beta}$ Derivative of β with respect to time $d\beta/dt$
$\ddot{\beta}$ Second derivative of β with respect to time $d^2\beta/dt^2$
$\delta$ Deflection of the mechanical flapping mechanism
$\dot{\delta}$ Derivative of $\delta$ with respect to time $d\delta/dt$
$\varepsilon$ Angle between the horizontal and the line tangent to the blade curvature at the blade root, or $\varepsilon = dZ(r=0)/dr$
$\dot{\varepsilon}$ Derivative of $\varepsilon$ with respect to time $d\varepsilon/dt$
$\hat{\varepsilon}$ Complex amplitude of the angle between the horizontal and the line tangent to the blade curvature at the blade root
$\gamma$ Lock number $\rho C_{l\alpha} c R^4/I$
$\gamma_i$ *i*-th displacement function
$\gamma'_i$ Derivative of the displacement function with respect to the radius $d\gamma_i/dr$
$\gamma''_i$ Second derivative of the displacement function with respect to the radius $d^2\gamma_i/dr^2$
$\varphi$ Inflow angle
$\varphi$ Angle of rotation of the flexible rotor blade
$\phi_i$ Generalized coordinate for a displacement function
$\phi'_i$ Derivative of the generalized coordinate for a displacement function with respect to the azimuth angle $d\phi_i/d\psi$
$\phi''_i$ Second derivative of the generalized coordinate for a displacement function with respect to the azimuth angle $d^2\phi_i/d\psi^2$
$\phi_n$ Generalized coordinate for an mode shape
$\phi'_n$ Derivative of the generalized coordinate for an mode shape with respect to the azimuth angle $d\phi_i/d\psi$
$\phi''_n$ Second derivative of the generalized coordinate for an mode shape with respect to the azimuth angle $d^2\phi_i/d\psi^2$
$\dot{\phi}_n$ Derivative of the generalized coordinate for an mode shape with respect to time $d\phi_i/dt$
$\ddot{\phi}_n$ Second derivative of the generalized coordinate for an mode shape with respect to time $d^2\phi_i/dt^2$
$\kappa$ $K/I\Omega^2$
$\lambda_i$ Non-dimensional induced velocity $v_i/\Omega R$
$\lambda_n$ Non-dimensional *n*-th eigenfrequency $\omega_n/\Omega$
$\theta$ Pitch angle
$\rho$ Air density
$\psi$ Azimuth angle
$\omega_n$ Eigenfrequency
$\Omega$ Rotational speed of the rotor

Subscripts

*Rigid* For a rigid rotor blade
*Flex* For a flexible rotor blade

## Introduction

A short introduction into the basic principles of the Ornicopter is given in an accompanying paper (Ref 1). The theory and calculations that have been presented in previous publications (Ref 2, 3, 4, and 5) were all based on the assumption of rigid rotor blades. The question this paper will provide an answer to is whether the introduction of flexible blades will have an effect on any of the key characteristics of the Ornicopter.

To be able to do so the equations of motion for a flexible blade during forced vibration by the forced flapping mechanism (Ref 1) will have to be derived. This will be done by using Lagrange's equations based on the theory as explained in (Ref 6). The following section will start with the derivation of the potential and kinetic energy of a blade in free vibration. Using these expressions the equations of motion for a blade in free vibration will be derived in the next section. Subsequently the excitation force will be added to the equation which will result both in the equations of motion for a blade during forced vibration by the flapping mechanism and in the bending function for a blade during forced vibration. This bending function will then be used to analyse the effects that flexible blades have on the characteristics of the Ornicopter.

## Potential and kinetic energy of a blade in free vibration

The lift force acting on the blade will bend the blade, for example in the way that is shown in figure 1. The differential angle of rotation ($d\varphi$) of an element of the blade with length *dr* can be expressed as:

$$d\varphi = \frac{Mdr}{EI} \quad (1)$$

In which *M* is the bending moment and *EI* is the flexural rigidity of the blade (*E* the modulus of elasticity and *I* the area moment of inertia of the cross sectional shape).

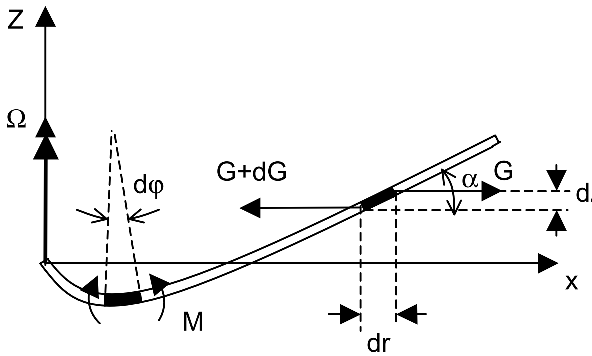


**Fig. 1: Bending of and forces acting on a rotating flexible blade**

The strain energy that is stored in this element is given by:

$$dU_b = \frac{1}{2} M d\varphi \quad (2)$$

Which, using equation (1) can also be written as:

$$dU_b = \frac{1}{2}\frac{M^2 dr}{EI} = \frac{1}{2} EI \left(\frac{d\varphi}{dr}\right)^2 dr \quad (3)$$

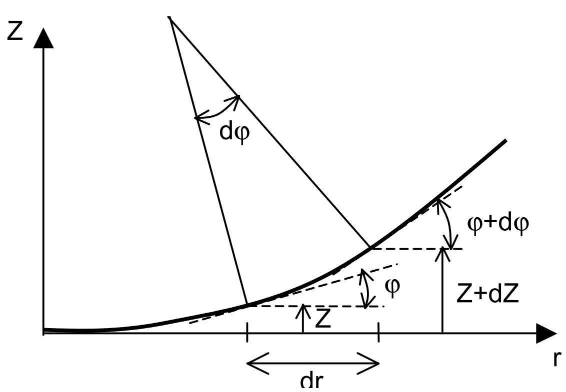


**Fig. 2: Relation between the vertical displacement and the angle of rotation**

Assuming small angles (see figure 2):

$$\tan\varphi \approx \varphi = \frac{dZ}{dr} \quad (4)$$

Substitution of equation (4) into equation (3) and integration yields a general expression for the total strain energy due to bending as a result of the lift force that is acting on the blade:

$$U_b = \frac{1}{2}\int_0^R EI \left(\frac{\partial^2 Z}{\partial r^2}\right)^2 dr \quad (5)$$

The centrifugal force (G) that is acting on the blade also causes potential energy. The magnitude of the moment that is caused by the centrifugal force can be calculated using figure 1:

$$M = GdZ + dG\frac{dZ}{2} \approx GdZ = \int_r^R mr\Omega^2 dr dZ \quad (6)$$

Where m denotes the mass per unit length and $\Omega$ the rotational velocity. The potential energy of the entire blade due to the centrifugal force can be calculated as:

$$U_{cf} = \int_r^R \frac{1}{2} M\alpha = \int_r^R \frac{1}{2} G \left(\frac{\partial Z}{\partial r}\right)^2 dr \quad (7)$$

The total potential energy is now obtained by summing equation (5) and (7):

$$U_{total} = \frac{1}{2}\int_0^R G\left(\frac{\partial Z}{\partial r}\right)^2 dr + \frac{1}{2}\int_0^R EI\left(\frac{\partial^2 Z}{\partial r^2}\right)^2 dr \quad (8)$$

To conclude the total kinetic energy of a flexible blade due to bending is equal to:

$$T = \frac{1}{2}\int_0^R m\left(\frac{\partial Z}{\partial t}\right)^2 dr \quad (9)$$

Equations of motion for a blade in free vibration

As a starting point to derive the actual bending of the flexible blade the displacement of the blade is expressed as a function of both the distance to the rotor hub and time:

$$Z(r,t) = \sum_i \gamma_i(r)\phi_i(t) \text{ for } i = (0,1,2,...) \quad (10)$$

In this equation $\gamma_i$ are functions used to approximate the blade shape and $\phi_i$ is the normal coordinate. The normal coordinates $\phi_i$ determine in which proportions the functions $\gamma_i$ have to be summed to arrive at the actual bending of the blade.

Using the displacement function (equation (10)) and the expressions for the potential and kinetic energy (8) and (9), the equations of motion for a bending blade will be derived. Substitution of equation (10) into equation (8) yields the potential energy:

$$U = \frac{1}{2}\sum_i \sum_j \phi_i(t)\phi_j(t)\int_0^R \left[G\gamma_i'(r)\gamma_j'(r) + EI\gamma_i''(r)\gamma_j''(r)\right]dr \tag{11}$$

With:

$$\gamma_i'(r) = \frac{d\gamma_i}{dr} \tag{12}$$

$$\gamma_i''(r) = \frac{d^2\gamma_i}{dr^2} \tag{13}$$

In which *j* obviously has the same properties as *i*. The kinetic energy is obtained by substitution of equation (10) into equation (9):

$$T = \frac{1}{2}\sum_i \sum_j \dot{\phi}_i(t)\dot{\phi}_j(t)\int_0^R m\gamma_i(r)\gamma_j(r)dr \tag{14}$$

The equations of motion can be derived by using Lagrange's equations and equations (11) and (14):

$$0 = \sum_j \ddot{\phi}_j(t)\int_0^R m\gamma_i(r)\gamma_j(r)dr + \\ + \sum_j \phi_j(t)\int_0^R \left[G\gamma_i'(r)\gamma_j'(r) + EI\gamma_i''(r)\gamma_j''(r)\right]dr \tag{15}$$

Equation (15) can be simplified by defining:

$$A_{ij} = \int_0^R m\gamma_i(r)\gamma_j(r)dr \tag{16}$$

$$B_{ij} = \int_0^R \left[G\gamma_i'(r)\gamma_j'(r) + EI\gamma_i''(r)\gamma_j''(r)\right]dr \tag{17}$$

Which results in the following equations of motion for free vibration:

$$\sum_j \left(\ddot{\phi}_j(t)A_{ij} + \phi_j(t)B_{ij}\right) = 0 \text{ for } i = (0,1,2,...) \tag{18}$$

To solve the equations of motion the following harmonic function is adopted:

$$\phi_j = \hat{\alpha}_j \cos(\omega t + \varepsilon_j) \tag{19}$$

In which $\omega$ is the eigenfrequency. Substitution of expression (19) into the equations of motion for every generalized coordinate as given by equation (18) gives:

$$\sum_j \left(-\omega^2 A_{ij} + B_{ij}\right)\hat{\alpha}_j = 0 \text{ for } i = (0,1,2,...) \tag{20}$$

Arranging the equations in matrix notation and putting the determinant to zero will give the values for the eigenfrequencies of the system. With the eigenfrequencies known, the relative values of $\hat{\alpha}_i$ can be determined:

$$\hat{\alpha}_i = \left[\frac{\hat{\alpha}_i}{\hat{\alpha}_0}\right]_n \hat{\alpha}_0 \tag{21}$$

Using equations (10), (19) and (21) this will result in the following displacement for a particular eigenfrequency:

$$Z_n = \sum_i \gamma_i \left[\frac{\hat{\alpha}_i}{\hat{\alpha}_0}\right]_n \hat{\alpha}_0 \cos(\omega_n t + \varepsilon_n) \tag{22}$$

Introducing the constant of normalization $c_n$ gives:

$$Z_n = \sum_i \gamma_i \left[\frac{\hat{\alpha}_i}{\hat{\alpha}_0}\right]_n c_n \hat{\alpha}_n \cos(\omega_n t + \varepsilon_n) \tag{23}$$

Define a new generalized coordinate $\phi_n$:

$$\phi_n = \hat{\alpha}_n \cos(\omega_n t + \varepsilon) \tag{24}$$

And define the normalized mode shape for a particular eigenfrequency as:

$$S_n = c_n \sum_i \gamma_i \left[\frac{\hat{\alpha}_i}{\hat{\alpha}_0}\right]_n \tag{25}$$

Combining equations (23), (24) and (25) yields the normalized displacement of a blade for one eigenfrequency:

$$Z_n = \phi_n S_n \tag{26}$$

And the complete solution for the bending of the blade in free vibration for all eigenfrequencies is given as:

$$Z = \sum_n \phi_n(t) S_n(r) \tag{27}$$

An important characteristic of mode shapes is that they are orthogonal which means that:

$$\int_0^R m S_m S_n dr = 0 \qquad \text{if } n \neq m \tag{28}$$

$$\int_0^R m S_m S_n dr = f(n) \qquad \text{if } n = m \tag{29}$$

The equations of motion can now also be derived in terms of the mode shapes, instead of in terms of the functions used to approximate the blade shape ($\gamma_i$). The potential energy due to the bending of the blade, the potential energy due to the centrifugal force and the kinetic energy of the flexible blade can be expressed in terms of the mode shapes by using equations (5), (7), (8), (9) and (29) and by using the orthogonality of the mode shapes (equations (28) and (29):

$$U_b = \frac{1}{2}\sum_n \sum_m \phi_n \phi_m \int_0^R EIS''_n S''_m dr \tag{30}$$

$$U_{cf} = \frac{1}{2}\sum_n \sum_m \phi_n \phi_m \int_0^R GS'_n S'_m dr \tag{31}$$

$$U_{total} = \frac{1}{2}\sum_n \phi_n^2 \omega_n{}^2 f(n) \tag{31}$$

$$T = \frac{1}{2}\sum_n \dot{\phi}_n^2 f(n) \tag{32}$$

Using Lagrange's equations, the following equations of motion for free vibration in terms of the mode shapes result:

$$\left(\ddot{\phi}_n + \phi_n \omega_n^2\right) f(n) = 0 \tag{32}$$

<u>Equations of motion for a blade during forced vibration by the flapping mechanism</u>

A general expression for the generalized force or moment per blade element ($dQ_n$) for each generalized coordinate is given by:

$$dQ_n = \frac{\delta dW_n}{\delta \phi_n} = \frac{\delta Z_n dF}{\delta \phi_n} = \frac{\delta \phi_n S_n \frac{\partial F}{\partial r} dr}{\delta \phi_n} \tag{33}$$

In which $W_n$ is the work done in the direction of the generalized coordinate $\phi_n$. Integration yields the generalized force or moment caused by the entire rotor blade:

$$Q_n = \int_o^R S_n \frac{\partial F}{\partial r} dr \tag{34}$$

A specific expression for the generalized forces that occur due to the flapping of the blade can be derived by using the formula for the total lift on a blade element (see also figure 3):

$$dL = c_{l_\alpha}\left[\theta - \frac{\lambda_i R}{r} - \beta'\right]\frac{1}{2}\rho(\Omega r)^2 c dr \tag{35}$$

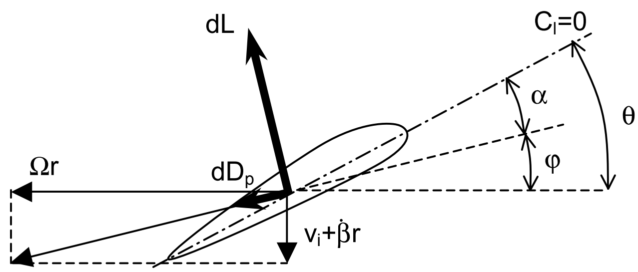


**Fig. 3: Aerodynamic forces and velocities on a blade element at distance r from the rotor hub**

With $\beta'$ the derivative of the flapping angle with respect to the azimuth angle and the non-dimensional induced velocity ($\lambda_i$) given by:

$$\lambda_i = \frac{v_i}{\Omega r} \tag{36}$$

This equation is valid for a rigid blade. If the term $\beta'$ is replaced by $\frac{dZ}{dt}\Big/\Omega r$ (since a displacement of the air downwards with $\dot{\beta} r$ corresponds with an upwards displacement of the blade element with $dZ/dt$, see figures 3 and 4) this expression would be valid for a flexible blade. The influences of pitch angle and induced velocity will be neglected since we are only interested in the fluctuating part of the lift force.

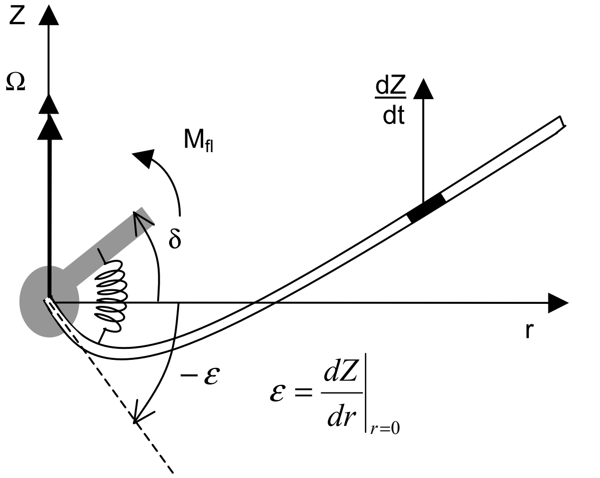


**Fig. 4: Schematic representation of a flexible blade with forced flapping mechanism**

The expression for the part of the lift on a blade element due to the flapping of the blade becomes:

$$dL = -C_{l_\alpha} \frac{\frac{\partial Z}{\partial t}}{\Omega r} \frac{1}{2} \rho (\Omega r)^2 c dr \tag{36}$$

The minus sign is due to the fact that a positive displacement $Z$ results in a difference in lift which is directed downwards. With reference to equation (34) it can be stated that:

$$\frac{\partial F}{\partial r} dr = dL = -C_{l_\alpha} \frac{\frac{\partial Z}{\partial t}}{\Omega r} \frac{1}{2} \rho (\Omega r)^2 c dr \tag{37}$$

Using the Lock number ($\gamma$) the mass moment of inertia ($I = 1/3mR^3$) and substituting equation (27) for the displacement $Z$ yields:

$$\frac{\partial F}{\partial r} dr = -\frac{\gamma}{2} \frac{I\Omega}{R^4} \sum_m \dot{\phi}_m S_m r dr \tag{38}$$

The reason that equation (38) is summed over $m$ instead of over $n$ has to do with the fact that equation (38) will be substituted into equation (34). This equation already contains the variable $n$ which denotes that this is the generalized force or moment for the generalized coordinate $\phi_n$. If equation (38) would be summed over $n$ and would be substituted into equation (34), this would imply that the mode shape $S_n$ already present in equation (34) would also be summed over $n$, which is not correct.

Note that since the excitation forces are included, this implies that the mode shapes are now forced to vibrate with the excitation frequency. This means that the generalized coordinate for each mode shape is from now on given by:

$$\phi_n = \hat{\alpha}_n \cos(\omega t + \varepsilon_n) \tag{39}$$

In which $\omega$ is the excitation frequency. Substitution of equation (38) into equation (34) yields the generalized force or moment due to the damping effect of the lift for a flexible rotor blade:

$$Q_n = -\frac{\gamma}{2} \frac{I\Omega}{R^4} \sum_m \dot{\phi}_m \int_0^R S_n S_m r dr \tag{40}$$

Now the flap forcing mechanism will be included in the equations of motion, an additional potential energy term emerges due to the torsion spring that is part of the flapping mechanism (see figure 4):

$$U_{spring} = \frac{1}{2} K (\delta - \varepsilon)^2 \tag{41}$$

$$U_{spring} = \frac{1}{2} K \left( \delta - \left.\frac{\partial Z}{\partial r}\right|_{r=0} \right)^2 \tag{42}$$

$$U_{spring} = \frac{1}{2} K \left( \delta - \sum_n \phi_n S_n'(0) \right)^2 \tag{43}$$

$$U_{spring} = \frac{1}{2} K \left( \delta^2 - 2\delta \sum_n \phi_n S_n'(0) + \sum_n \sum_m \phi_n \phi_m S_n'(0) S_m'(0) \right) \tag{44}$$

In which K is the spring stiffness. For Lagrange's equations it is necessary to calculate:

$$\frac{\partial U_{spring}}{\partial \phi_n} = -K \left( \delta - \sum_m \phi_m S_m'(0) \right) S_n'(0) \tag{45}$$

The equations of motion for a flexible blade in terms of the mode shapes including the excitation forces in terms of the mechanical flapping mechanism can now be composed by incorporating the effects of the spring (equation (45)) and the general expression for the generalized forces and moments (equation (40)) in equation (32):

$$\left( \ddot{\phi}_n + \phi_n \omega_n^2 \right) f(n) - K \left( \delta - \sum_m \phi_m S_m'(0) \right) S_n'(0) =$$

$$-\frac{\gamma}{2}\frac{I\Omega}{R^4}\sum_m \dot{\phi}_m \int_0^R S_n S_m r dr \quad \text{for } n=(0,1,2,...) \qquad (46)$$

The displacement of a blade during forced vibration by the flapping mechanism

The above means that the complete solution for the displacement of the blade during forced vibration is now known. Analogous to equation (29) the complete solution is given by:

$$Z=\sum_n \phi_n(t)S_n(r) \qquad (47)$$

with:

$$\phi_n=\hat{\alpha}_n\cos(\omega t+\varepsilon_n) \qquad (39)$$

$$S_n=c_n\sum_i \gamma_i\left[\frac{\hat{\alpha}_i}{\hat{\alpha}_0}\right]_n \qquad (25)$$

- $\gamma_i$ is a set of chosen displacement functions
- $\omega_n$ is determined by setting the determinant of equation (20) equal to zero.
- With the eigenfrequencies known, the relative values of $\hat{\alpha}_i$ for each eigenfrequency can be determined using equations (20) and (21)
- $c_n$ is the constant of normalization
- $\omega$ is the excitation frequency, equal to $\Omega$
- $\hat{\alpha}_n$ and $\varepsilon_n$ are calculated using equation (46)

Power balance for flexible blades: is any power lost due to the flexibility of the blades?

Now the bending of the blade can be calculated, the question this section will provide an answer to is whether any power that is provided by the engine is lost due to the flapping of the flexible blades or due to the transmission of the power to the mechanical flapping mechanism. As a consequence this power will not be available as a propulsive force to drive the rotor. To determine whether or not this is the case, the power available to drive the rotor, the mechanical flapping power and the power that is provided by the engine are calculated and compared in the following paragraphs.

The power available to drive the rotor

The power per blade that is available to drive the rotor due to the forced flapping of the blade ($P_a$) is equal to the forward component of the lift multiplied by $\Omega r$, integrated over the rotor blade and averaged over one revolution, see also figures 3 and 4:

$$P_a=\frac{1}{2\pi}\int_0^{2\pi} d\psi \int_o^R -dL\sin\varphi\Omega r \qquad (48)$$

$$P_a=\frac{1}{2\pi}\int_0^{2\pi} d\psi \int_o^R -dL\frac{\frac{\partial Z}{\partial t}}{\Omega r}\Omega r \qquad (49)$$

Using equation (47) it follows that:

$$P_a=-\frac{1}{2\pi}\int_0^{2\pi} d\psi \int_0^R \sum_n \dot{\phi}_n S_n dL \qquad (50)$$

Combining equations (46) (37) and (38) gives:

$$\int_0^R S_n dL=\left(\ddot{\phi}_n+\phi_n\omega_n^2\right)f(n)+ \\ -K\left(\delta-\sum_m \phi_m S_m'(0)\right)S_n'(0) \qquad (51)$$

Which can be simplified by introducing the following relation for the flapping moment $M_{fl}$ (figure 4):

$$M_{fl}=K\left(\delta-\sum_m \phi_m S_m'(0)\right) \qquad (52)$$

Substitution of equation (52) into equation (51), and subsequent substitution in equation (50) yields:

$$P_a=-\frac{1}{2\pi}\int_0^{2\pi} d\psi \sum_n \dot{\phi}_n\left[-M_{fl}S_n'(0)+ \\ +\left(\ddot{\phi}_n+\phi_n\omega_n^2\right)f(n)\right] \qquad (53)$$

The second part of the integral in equation (53) is equal to zero, which yields:

$$P_a=\frac{1}{2\pi}\int_0^{2\pi} M_{fl}\sum_n \dot{\phi}_n S_n'(0)d\psi \qquad (54)$$

Looking at figure 4 the following can be derived for the flexible blade root angle $\varepsilon$:

$$\varepsilon=\frac{dZ(r=0)}{dr}=\sum_n \phi_n S_n'(0) \qquad (55)$$

$$\dot{\varepsilon} = \sum_n \dot{\phi}_n S_n'(0) \qquad (56)$$

Substitution of equation (56) into equation (54) gives the final expression for the average available power:

$$P_a = \frac{1}{2\pi}\int_0^{2\pi} M_{fl}\dot{\varepsilon} d\psi \qquad (57)$$

<u>The flapping power: the power transmitted by the spring to the root of the blade</u>

The average flapping power (the average power transmitted by the spring to the root of the blade) can be derived from figure 4 and the fact that power is equal to moment times angular velocity:

$$P_{fl} = \frac{1}{2\pi}\int_0^{2\pi} M_{fl}\dot{\varepsilon} d\psi \qquad (58)$$

Comparing equations (57) and (58) the conclusion thus is:

$$P_a = P_{fl} \qquad (59)$$

Or in words: the power that is transmitted to the root of the blade is also available to drive the rotor, hence no power loss occurs at these points in the transmission.

<u>The engine power: the power transmitted by the engine to the spring</u>

The average engine power can again be derived from figure 4 and the fact that power is equal to moment times angular velocity:

$$P_{eng} = \frac{1}{2\pi}\int_0^{2\pi} M_{fl}\dot{\delta} d\psi \qquad (60)$$

Combining equations (52) and (55) gives:

$$M_{fl} = K(\delta - \varepsilon) \qquad (61)$$

Which substituted into equation (60) results in:

$$P_{eng} = \frac{1}{2\pi}\int_0^{2\pi} K(\delta - \varepsilon)\dot{\delta} d\psi \qquad (62)$$

$$P_{eng} = \frac{K\Omega}{2\pi}\int_0^{2\pi} \delta \frac{d\delta}{d\psi} d\psi - \frac{K}{2\pi}\int_0^{2\pi} \varepsilon\dot{\delta} d\psi \qquad (63)$$

$$P_{eng} = \frac{K\Omega}{2\pi}\frac{1}{2}\delta^2\Big|_{\psi=0}^{\psi=2\pi} - \frac{K}{2\pi}\int_0^{2\pi} \varepsilon\dot{\delta} d\psi \qquad (64)$$

And since $\delta$ is a periodic function it will have the same value for $\psi=0$ and $\psi=2\pi$, therefore equation (64) reduces to:

$$P_{eng} = -\frac{K}{2\pi}\int_0^{2\pi} \varepsilon\dot{\delta} d\psi \qquad (65)$$

To be able to compare the engine power to the power that is available to drive the rotor equation (57) is rewritten using equation (61):

$$P_a = \frac{1}{2\pi}\int_0^{2\pi} K(\delta - \varepsilon)\dot{\varepsilon} d\psi \qquad (66)$$

$$P_a = \frac{K}{2\pi}\int_0^{2\pi} \delta \frac{d\varepsilon}{dt} d\Omega t - \frac{K\Omega}{2\pi}\int_0^{2\pi} \varepsilon \frac{d\varepsilon}{d\psi} d\psi \qquad (67)$$

$$P_a = \frac{K\Omega}{2\pi}\int_0^{2\pi} \delta d\varepsilon - \frac{K\Omega}{2\pi}\frac{1}{2}\varepsilon^2\Big|_{\psi=0}^{\psi=2\pi} \qquad (68)$$

$$P_a = \frac{K\Omega}{2\pi}\left( \varepsilon\delta\Big|_{\psi=0}^{\psi=2\pi} - \int_0^{2\pi} \varepsilon d\delta \right) - \frac{K\Omega}{2\pi}\frac{1}{2}\varepsilon^2\Big|_{\psi=0}^{\psi=2\pi} \qquad (69)$$

And since $\varepsilon$ is also a periodic function:

$$P_a = -\frac{K\Omega}{2\pi}\int_0^{2\pi} \varepsilon d\delta \qquad (70)$$

or:

$$P_a = -\frac{K}{2\pi}\int_0^{2\pi} \varepsilon \frac{d\delta}{dt} d\Omega t = -\frac{K}{2\pi}\int_0^{2\pi} \varepsilon\dot{\delta} d\psi \qquad (71)$$

Comparing equations (71) and (65) it can be seen that the engine power and the available power are equal. This means that all the power that is provided by the engine is available to drive the rotor, hence that no power is lost due to the flexibility of the blades when the engine power is converted to flapping power.

A two mode (flexible) approximation

To calculate the influence of the flexibility of the blade on other important parameters, a two mode approximation will be used. In this section the expressions for the mode shapes and generalized coordinates of the first two modes will be calculated and the resulting bending motion will be presented. The following displacement function has been adopted (Ref 6):

$$\frac{\gamma_i}{R} = (i+2)(i+3)\frac{x^{i+1}}{6} - i(i+3)\frac{x^{i+2}}{3} + i(i+1)\frac{x^{i+3}}{6} \tag{72}$$

In which $x$ is the non-dimensional rotor radius:

$$x = \frac{r}{R} \tag{73}$$

This results in the following displacement functions which comply with the boundary conditions, i.e. the fact that the displacement is zero at r=0, and equal to R at r=R:

$$\gamma_0 = r \tag{74}$$

$$\gamma_1 = 2\frac{r^2}{R} - \frac{4}{3}\frac{r^3}{R^2} + \frac{1}{3}\frac{r^4}{R^3} \tag{75}$$

The coefficients $A_{ij}$ and $B_{ij}$ are now calculated:

$$A_{00} = \frac{1}{3} mR^3 \tag{76}$$

$$A_{01} = \frac{13}{45} mR^3 \tag{77}$$

$$A_{10} = A_{01} \tag{78}$$

$$A_{11} = \frac{104}{405} mR^3 \tag{79}$$

$$B_{00} = \frac{1}{3} m\Omega^2 R^3 \tag{80}$$

$$B_{01} = \frac{13}{45} m\Omega^2 R^3 \tag{81}$$

$$B_{10} = B_{01} \tag{82}$$

$$B_{11} = \frac{16}{5}\frac{EI}{R} + \frac{122}{405} m\Omega^2 R^3 = \left(\frac{16}{5}k^2 + \frac{122}{405}\right) m\Omega^2 R^3 \tag{83}$$

With $k$ given by:

$$k^2 = \frac{EI}{m\Omega^2 R^4} \tag{84}$$

In the remainder of this paper, $k^2$ is chosen to be equal to 1/270. Using equation (20) the eigenfrequencies are obtained:

$$\omega_0 = \Omega \tag{85}$$

$$\omega_1 = \sqrt{\frac{127}{13}}\Omega \tag{86}$$

And with the eigenfrequencies known, equation (20) can be used to calculate the mode shapes. For the first eigenfrequency ($\omega_0$) it follows that:

$$\frac{\hat{\alpha}_1}{\hat{\alpha}_0} = 0 \tag{87}$$

And with equation (25):

$$S_0 = c_0 (r \cdot 1 + \gamma_1 \cdot 0) = c_0 r \tag{88}$$

The normalization constant is chosen such that:

$$S_n(R) = R \tag{89}$$

Resulting in:

$$c_0 = 1 \tag{90}$$

$$S_0 = r \tag{91}$$

And for the second eigenfrequency:

$$\frac{\hat{\alpha}_1}{\hat{\alpha}_0} = -\frac{15}{13} \tag{92}$$

$$S_1 = c_1 \left( r \cdot 1 - \left( 2\frac{r^2}{R} - \frac{4}{3}\frac{r^3}{R^2} + \frac{1}{3}\frac{r^4}{R^3} \right)\frac{15}{13} \right) \tag{93}$$

$$S_1(R) = c_1 \left( R - \left( 2R - \frac{4}{3}R + \frac{1}{3}R \right)\frac{15}{13} \right) = R \tag{94}$$

$$c_1 = -\frac{13}{2} \tag{95}$$

$$S_1 = -\frac{13}{2} r + 15\frac{r^2}{R} - 10\frac{r^3}{R^2} + \frac{5}{2}\frac{r^4}{R^3} \tag{96}$$

The equations of motion (46) can be rewritten using equation (52):

$$\left(\phi_n'' + \phi_n \lambda_n^2\right) F_n = m_{fl} \mathrm{E}_n - \frac{\gamma}{2} \sum_m \phi_m' C_{nm} \tag{97}$$

in which:

$$\lambda_n = \frac{\omega_n}{\Omega} \tag{98}$$

$$F_n = \frac{f_n}{I} \tag{99}$$

$$m_{fl} = \frac{M_{fl}}{I\Omega^2} \tag{100}$$

$$\mathrm{E}_n = S_n'(0) = \frac{dS_n(0)}{dr} \tag{101}$$

$$C_{nm} = \frac{1}{R^4}\int_0^R S_n S_m r dr \tag{102}$$

For the first mode shape this results in:

$$\left(\phi_0'' + \phi_0 \lambda_0{}^2\right)F_0 = m_{fl}\mathrm{E}_0 - \frac{\gamma}{2}\left(\phi_0' C_{00} + \phi_1' C_{01}\right) \tag{103}$$

With the following quantities:

$$\lambda_0^2 = \frac{\omega_0^2}{\Omega^2} = 1 \tag{104}$$

$$F_0 = \frac{f(0)}{I} = 1 \tag{105}$$

$$E_0 = S_0'(0) = 1 \tag{106}$$

$$C_{00} = \frac{1}{R^4}\int_0^R S_0 S_0 r dr = \frac{1}{4} \tag{107}$$

$$C_{01} = \frac{1}{R^4}\int_0^R S_0 S_1 r dr = \frac{11}{168} \tag{108}$$

When these quantities are substituted, equation (103) can be written as:

$$\left(\phi_0'' + \phi_0\right) + \frac{\gamma}{2}\left(\frac{1}{4}\phi_0' + \frac{11}{168}\phi_1'\right) = m_{fl} \tag{109}$$

For the second mode shape, the equation of motion is given by:

$$\left(\phi_1'' + \phi_1 \lambda_1{}^2\right)F_1 = m_{fl}\mathrm{E}_1 - \frac{\gamma}{2}\left(\phi_0' C_{10} + \phi_1' C_{11}\right) \tag{110}$$

Substitution of the following quantities:

$$\lambda_1^2 = \frac{\omega_1^2}{\Omega^2} = \frac{127}{13} \tag{111}$$

$$F_1 = \frac{f(1)}{I} = \frac{\frac{13}{36} mR^3}{\frac{1}{3} mR^3} = \frac{13}{12} \tag{112}$$

$$\mathrm{E}_1 = S_1'(0) = -\frac{13}{2} \tag{113}$$

$$C_{10} = C_{01} \tag{114}$$

$$C_{11} = \frac{1}{R^4}\int_0^R S_1 S_1 r dr = \frac{25}{144} \tag{115}$$

gives:

$$\left(\phi_1'' + \phi_1 \frac{127}{13}\right)\frac{13}{12} + \frac{\gamma}{2}\left(\frac{11}{168}\phi_0' + \frac{25}{144}\phi_1'\right) = -\frac{13}{2} m_{fl} \tag{116}$$

Since the excitation frequency of the mechanical flapping moment is equal to the angular velocity *Ω*, the following expressions can be adopted for the generalized coordinates and the non-dimensional mechanical flapping moment (assume a phase difference equal to zero for the mechanical flapping moment):

$$\phi_0 = \hat{\phi}_0 e^{i\Omega t} = \hat{\phi}_0 e^{i\psi} \tag{117}$$

$$\phi_1 = \hat{\phi}_1 e^{i\Omega t} = \hat{\phi}_1 e^{i\psi} \tag{118}$$

$$m_{fl} = \hat{m}_{fl} e^{i\Omega t} = \hat{m}_{fl} e^{i\psi} \tag{119}$$

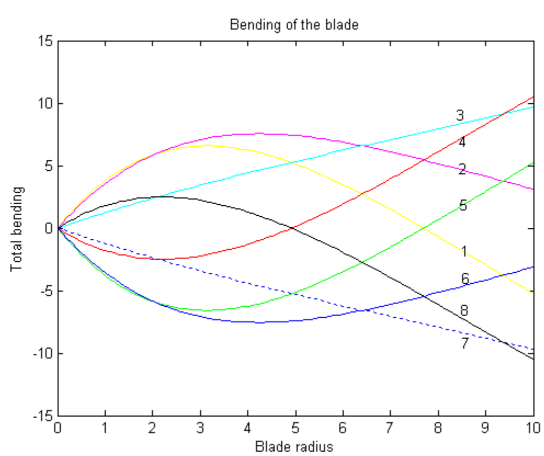


**Fig. 5: Bending of a flexible blade for $k^2$=1/270 and γ=8, normalized at S(R)=R. An azimuth angle difference of π/4 occurs between two successive plots.**

Substituting the above expressions into the equations of motion for the mode shapes (110) and (116), and assuming that the Lock number is equal to 8 will give, after some calculation, the following expressions for the generalized coordinates:

$$\phi_0 = 1.0291\hat{m}_{fl}\cos(\psi - 1.38944) \quad (120)$$

$$\phi_1 = -0.710241\hat{m}_{fl}\cos(\psi - 0.06578) \quad (121)$$

Combining equations (47), (91), (96), (120) and (121) finally gives the resulting bending motion for the two mode approximation. This resulting motion is shown in figure 5.

A one mode (rigid) approximation

The bending function of a rigid blade will also have to be derived in order to be able to compare the results of a flexible blade to the results of a rigid blade. A rigid blade can be simulated by using a one mode approximation. The displacement function for a one mode approximation is still given by equation (73) and the coefficients $A_{00}$ and $B_{00}$ are given by equations (76) and (80). This results in the eigenfrequency of equation (85) and the mode shape as given by equation (91). The corresponding generalized coordinate for the one mode (rigid) approximation can now be calculated as:

$$\phi_0 = \hat{m}_{fl}\cos\left(\psi - \frac{\pi}{2}\right) \quad (122)$$

Which results in the following bending function for a rigid blade:

$$Z = \hat{m}_{fl}\cos\left(\psi - \frac{\pi}{2}\right)r \quad (123)$$

The difference between the flexible blade root angle and the rigid blade flapping angle

The equation for the flexible blade root angle $\varepsilon$ is given by equation (55). For a two mode flexible blade approximation is thus follows that:

$$\varepsilon_{flex} = \phi_0 E_0 + \phi_1 E_1 \quad (124)$$

Substitution of equations (106), (113), (120) and (121) and some calculation yields:

$$\varepsilon_{flex} = \hat{m}_{fl}(4.79218\cos\psi + 1.31568\sin\psi) \quad (125)$$

Which results in the following amplitude:

$$\hat{\varepsilon}_{flex} = 4.96\hat{m}_{fl} \quad (126)$$

For the one mode approximation resembling a rigid blade, the 'flexible' blade root angle is equal to the flapping angle of the entire blade and can be calculated by:

$$\varepsilon_{rigid} = \beta = \phi_0 E_0 \quad (127)$$

Substitution of equations (106) and (122) then gives:

$$\varepsilon_{rigid} = \beta = \hat{m}_{fl}\cos\left(\psi - \frac{\pi}{2}\right) \quad (128)$$

And thus the following amplitude results for the rigid blade:

$$\hat{\varepsilon}_{rigid} = \hat{\beta} = \hat{m}_{fl} \quad (129)$$

Which is consistent with the findings in (Ref 2 and 4). When comparing equations (129) and (126) it can be seen that the amplitude of the flexible blade root angle is almost five times as large for the flexible blade as it is for the rigid blade. This is depicted in figures 6 and 7. Figure 6 is the same as figure 5 except for the fact that the two extreme positions of the movement of the rigid blade are added. Figure 6 shows a close up of the bending of the blades at the blade root, and clearly shows that the flexible blade root angle is five times as large. Physically this means that the up and down movement of the forced flapping mechanism at the blade root should be five times larger for a flexible blade than for a rigid blade.

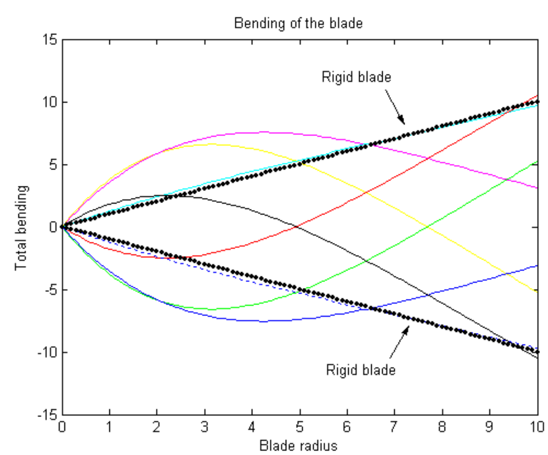


**Fig. 6: Bending of a flexible blade and a rigid blade for a given flapping moment**

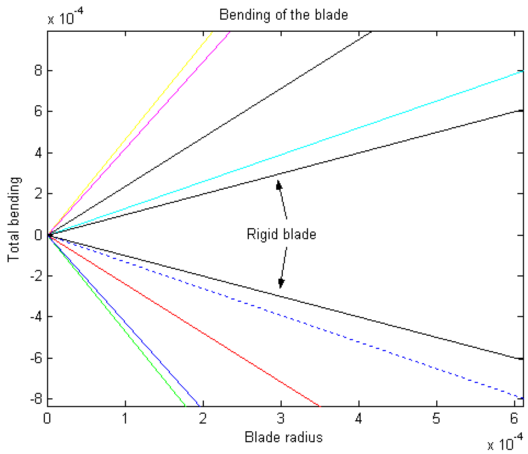


**Fig. 7: The root bending of a flexible blade and a rigid blade for a given flapping moment**

Windtunnel tests that have been performed (Ref 7) indeed showed that the amplitude of the forced flapping mechanism needed to be larger than what was expected from calculations based on the theory for rigid blades. However, these windtunneltests showed that the ratio of the amplitudes of the flexible blade and rigid blade ($\hat{\varepsilon}_{flex} / \hat{\varepsilon}_{rigid}$) was only equal to 1.5 which is lower than indicated by equations (126) and (129). This is due to the fact that the blades that were used for the windtunneltests had a larger stiffness.

## Effect of a flexible blade on the required flapping moment

The required flapping moment can be calculated by using the expression for the flapping power as given by equation (58). Using equations (56), (100) and (119) it can be derived for the two mode flexible blade approximation that:

$$P_{fl} = \frac{I\Omega^3}{2\pi} \int_0^{2\pi} \hat{m}_{fl} \cos\psi \left(\phi_0' + \phi_1' E_1\right) d\psi \tag{130}$$

Substitution of equations (113), (120) and (121) and some calculation yields:

$$P_{fl.flex} = 1.316 \frac{I\Omega^3}{2} \hat{m}_{fl}^2 \tag{131}$$

For the one mode approximation of the rigid blade, the flapping power can be expressed as:

$$P_{fl} = \frac{I\Omega^3}{2\pi} \int_0^{2\pi} \hat{m}_{fl} \cos\psi \left(\phi_0'\right) d\psi \tag{132}$$

Using equation (122) it follows that:

$$P_{fl,rigid} = \frac{I\Omega^3}{2} \hat{m}_{fl}^2 \tag{133}$$

Which is again consistent with the findings in (Ref 2 and 4). When comparing equations (131) and (133) it can be seen that to obtain the same amount of flapping power the amplitude of the mechanical flapping moment for the flexible blade only needs to be 76% of the amplitude of the mechanical flapping moment for a rigid blade. A smaller mechanical flapping moment is thus required for a flexible blade. The bending of a flexible blade resulting from this smaller mechanical flapping moment is compared to the flapping of a rigid blade resulting from a larger mechanical flapping moment in figure 8.

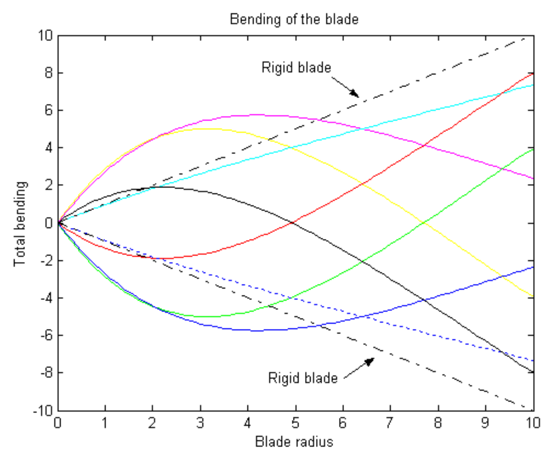


**Fig. 8: Bending of a flexible blade and a rigid blade for the same flapping power**

## Effect of a flexible blade on the torque about the rotor hub

Knowing that power is also given by:

$$P = \overline{Q}\Omega \tag{134}$$

in which $\overline{Q}$ is the average torque about the rotor hub, the torque about the rotor hub as caused by the flexible and rigid blade is readily calculated using equations (131) and (133):

$$\overline{Q}_{flex} = 1.316 \frac{I\Omega^2}{2} \hat{m}_{fl}^2 \tag{135}$$

$$\overline{Q}_{rigid} = \frac{I\Omega^2}{2} \hat{m}_{fl}^2 \tag{136}$$

Compared to the rigid blade, a smaller flapping moment is thus necessary for the flexible blade in order to generate a certain amount of torque around the rotor hub.

<u>Effect of a flexible blade on the vertical shear fluctuations in the root of the blade</u>

The vertical shear in the root of the blade can be expressed as follows:

$$V_0 = \int_0^R \left( \frac{\partial F}{\partial r} - m\ddot{Z} \right) dr \qquad (137)$$

Since we are only interested in the vertical shear fluctuations the influence of the pitch angle and induced velocity will be neglected and the first part of the integral in equation (137) is given by equations (37) and (38).
For the flexible blade two mode approximation the first part of the integral representing the shear force due to aerodynamic forces, when using equations (91), (96), (120) and (121) can be calculated as:

$$\left( \int_o^R \frac{\partial F}{\partial r} dr \right)_{flex} = 1.372 \frac{I\Omega^2}{R} \hat{m}_{fl} \cdot \cdot \sin(\psi - 1.38944) \qquad (138)$$

For the rigid blade the first part of the integral is calculated using equations (91) and (122):

$$\left( \int_0^R \frac{\partial F}{\partial r} dr \right)_{rigid} = -\frac{4}{3} \frac{I\Omega^2}{R} \hat{m}_{fl} \cos\psi \qquad (139)$$

The second term of the integral due to inertia forces can be expressed as:

$$\int_0^R - m\ddot{Z} dr = \int_0^R - m \sum_n \Omega^2 \phi_n'' S_n dr \qquad (140)$$

Using equations (91), (96), (120) and (121) it follows for the flexible blade that:

$$\left( \int_0^R - m\ddot{Z} dr \right)_{flex} = \frac{3}{2} \hat{m}_{fl} \frac{I\Omega^2}{R} (0.53996 \cos\psi + + 1.03556 \sin\psi) \qquad (141)$$

And the vertical force in the blade root due to inertia forces for the rigid blade can be expressed as (using equations (91) and (122)):

$$\left( \int_0^R - m\ddot{Z} dr \right)_{rigid} = \frac{3}{2} \hat{m}_{fl} \frac{I\Omega^2}{R} \sin\psi \qquad (142)$$

The total vertical shear force in the root of a flexible blade can now be calculated by adding equations (138) and (141):

$$V_{0,flex} = \frac{I\Omega^2}{R} \hat{m}_{fl} (1.801 \sin\psi - 0.540 \cos\psi) \qquad (143)$$

And the total vertical shear force in the root of a rigid blade is, when summing equations (139) and (142), given by:

$$V_{0,rigid} = \frac{I\Omega^2}{R} \hat{m}_{fl} \left( \frac{3}{2} \sin\psi - \frac{4}{3} \cos\psi \right) \qquad (144)$$

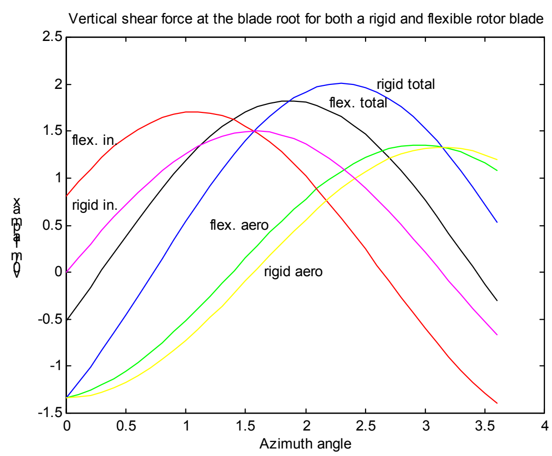


**Fig. 9: The total vertical shear force, the vertical shear force due to inertia forces (in.) and due to aerodynamic forces (aero) at the root of the blade for both a rigid blade and a flexible blade as a function of the azimuth angle**

This is also depicted in figure 9. In figure 9 the non-dimensional vertical shear force is used:

$$v_0 = V_0 \frac{R}{I\Omega^2} \qquad (145)$$

It can be seen that although the amplitudes of the separate contributions of aerodynamic forces and inertia forces to the vertical force in the root of the flexible blade are larger than those for the rigid blade, the amplitude of the total vertical force in the root of the flexible blade will be smaller than in the root of the rigid blade.

Effect of a flexible blade on the phase difference between the flapping moment and the deflection of the flapping mechanism

The mechanical flapping moment can be expressed by (see figure 4):

$$m_{fl} = \frac{M_{fl}}{I\Omega^2} = \frac{K}{I\Omega^2}(\delta - \varepsilon) = \kappa(\delta - \varepsilon) \tag{146}$$

In which κ is given by:

$$\kappa = \frac{K}{I\Omega^2} \tag{147}$$

Equation (146) can also be written as:

$$m_{fl} = \kappa\left(\delta - \frac{dZ(r=0)}{dr}\right) \tag{148}$$

As already shown in equation (43) this can be rewritten as:

$$m_{fl} = \kappa(\delta - \phi_0 S_0'(0) - \phi_1 S_1'(0)) \tag{149}$$

Using equations (91), (96), (120) and (121) and choosing κ equal to 0.2 finally gives:

$$\delta = \hat{m}_{fl}\, 9.880 \cos(\psi - 0.13356) \tag{150}$$

Equation (150) shows that a phase difference equal to –0.13356 rad indeed occurs between the mechanical flapping moment and the deflection of the flapping mechanism.

Conclusions

This paper has shown that the flexibility of the blades does not have an influence on the key characteristics of the Ornicopter. Despite the flexibility it is still possible to achieve a propulsive and lifting force by forced flapping of the blades, and it is thus still possible to realize a single rotor without reaction torque.

No power is lost due to the flexibility of the blades, which means that all the power that is provided by the engine is available to drive the rotor.

When the angle of the flexible blade at the blade root is compared to the flapping angle of a rigid blade it appears that the angle of the flexible blade is larger than that of the rigid blade at the blade root. As a consequence of the larger blade root angle (or flapping angle), the angular velocity of the flapping motion at the blade root will also be larger for a flexible blade. Since the flapping power is equal to the integral of angular velocity times flapping moment, it follows that for the same amount of power a smaller flapping moment will be necessary for a flexible blade than for a rigid blade since the angular velocity will be higher for a flexible blade.

Additionally it has been shown that the total vertical shear force in the root of the flexible blade will be smaller than the total vertical shear force in the root of a rigid blade. And that a phase difference occurs between the flapping moment and the deflection of the forced flapping mechanism.

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