Multi-Fidelity Aerodynamic Optimization of a Helicopter Rotor Blade

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Abstract

A multi-fidelity optimization technique is applied to the design of a helicopter rotor blade, in order to improve its performance in forward flight. This optimization technique is based on surrogate models, which replace high fidelity CFD/CSD simulations, necessary to take into account the three-dimensional unsteady effects generated in the flowfield of a complex geometry blade planform. The single low fidelity model, based on Kriging methodology, and generated by lifting lines simulations leads to a power benefit of 2.5%, which is not reproducible by a posteriori high fidelity CSD/CFD computation. The optimization procedure based on two levels of fidelity (lifting line and CSD/CFD simulations), taken into account by Co-Kriging surrogate models, leads to a realistic blade planform, for which the power benefit is estimated at 2.2%. The advantages to use a Co-Kriging surrogate model in such aerodynamic optimization procedure are clearly shown.

Introduction

The aerodynamic optimization of helicopter rotor blades is a complex and challenging problem, due to unsteady flow phenomena. For instance, in forward flight, transonic effects on the advancing side of the blade, and dynamic stall phenomenon on the retreating side can be encountered. Furthermore, the effects between the aerodynamic behavior and the elastic response of an optimized rotor blade have to be taken into account thanks to a fluid-structure coupling, which requires a large amount of computational cost.

Historically, the aerodynamic optimization procedures were based on the coupling between low-fidelity, and fast computational codes (generally based on lifting line theory), and an optimizer. Two types of optimizer were generally used, either gradient-based (limited number of evaluations, but the risk to reach a local minimum of the objective function), or based on genetic algorithms (which require a large amount of evaluations, on a large research domain, which improves the capability to reach the global minimum of the objective function).

Single-objective optimizations with gradient-descent are among the first approaches to have been used, for their efficiency and rapidity. At NASA, Walsh and Bingham decomposed the problem of minimization of the power sequentially, first optimizing in hover, and then handling constraints in forward flight. The chosen optimizer was the CONMIN algorithm [1]. At ONERA, the CONMIN optimizer has been coupled with the R85 comprehensive code [2] to optimize the geometric laws of a rotor blade to reduce its required power in forward flight, while constraining the values of the pitch link loads [3].

Recently, the use of gradient-based algorithms formulated by discrete steady adjoint of the RANS equations has allowed high-fidelity models in hover optimizations, as the cost of the gradient evaluation becomes practically independent of the number of design parameters [4][5]. The problem is more complex for optimization in forward flight. The adjoint formulation for unsteady flows requires either considering the problem as periodic in order to use a steady adjoint formulation [6], or solving the unsteady adjoint equation backwards in time.

The second popular approach for optimization is the use of genetic algorithms. Results on aerodynamic optimizations based on the coupling between a comprehensive code and an evolutionary optimizer show actually the effectiveness of this type of optimization procedure to optimize the twist law of a helicopter rotor blade [7]. The main advantage of this method is the reduction of the risk to obtain a local optimum of the objective function, to the detriment of a high cost of computational time.

It is now well known that surrogate models are well adapted to reduce the computational cost due to large number of high fidelity evaluations, and to allow the use of high fidelity simulations in the optimization loop. For instance, CSD/CFD simulations are necessary to take into account fluid-structure interactions, especially for complex geometry blades. First studies performed by Collins [8] proposed a multi-fidelity framework combining both low and high fidelity tools. The connection between the two models was operating by a scaling operator that multiplies the value obtained by the low fidelity model. Recent studies performed by Wilke are based on a variable level surrogate model, using the Hierarchical Kriging [9]. The low and high fidelity models were built thanks to dynamic inflow models and Euler equations respectively. The research of the minimization of the required power led to optimized blade planforms in hover and in forward flight, showing a significant computational cost reduction. This single-objective optimization procedure has been extended to a three levels fidelity model [10], and to a multi-objective optimization in hover and forward flight [11]. It was shown that this technique allows being closer to a reference Pareto front than single fidelity optimization procedure.

The optimization procedures presented in this paper rely on Kriging and Co-Kriging based optimization of rotor blades using multi-fidelity methods. These methodologies are described in the first part of the paper. Then, these optimization procedures are applied to define the sweep law of a helicopter rotor blade to improve its performance in forward flight. The analysis of the origin of the power benefit, as well as the aero-elastic behavior of the optimized blades is then performed in details.

Surrogate models methodology

Surrogate modeling plays an important role in many areas of aerospace engineering, like aerodynamic design optimization, structural design and multidisciplinary optimization. Many methods have been studied like polynomial models (RSM), moving least-squares (MLS), Radial Basis Function (RBF), Support Vector Machine (SVM), Kriging and multi-fidelity methods.

Surrogate-models based optimization is a numerical optimization approach which uses surrogate models to guide the research of the real model optimum but with a reasonable computational cost.

A model is developed which can approximate the objective function data throughout the parameter space. Sample points are generated based on a Design-of-Experiment technique, based on the Latin Hypercube Sampling (LHS) method [12]. In the case of a multi-fidelity model, a sample points set is defined for each fidelity level. The aerodynamic data at sample points are evaluated by the use of respective fidelity level CFD methods. The high-fidelity model is constructed by means of Kriging or Co-kriging approach. If termination criterion is not fulfilled, iterative refinement is performed by adding new samples points. These new data are expected to improve the model accuracy and accelerate the research of the optimum.

Kriging and EGO Methodology

Kriging is a statistical interpolation method suggested by Krige [13] and mathematically studied by Matheron [14]. Its estimation depends on spatial correlation between n sample points, $\boldsymbol{X}_1, \dots, \boldsymbol{X}_n$, for which the function values have been computed, $\boldsymbol{Y}_1, \dots, \boldsymbol{Y}_n$. The Kriging model is a zero-mean Gaussian process, $\boldsymbol{Z}$, with covariance function cov(.,.) modeled as:

$$\mathrm{Cov}\left(\boldsymbol{Z}_i, \boldsymbol{Z}_j\right) = \sigma^2 \boldsymbol{R}(\boldsymbol{Z}_\mathrm{i}, \boldsymbol{Z}_\mathrm{j}; \boldsymbol{\theta}).$$

where σ^2 is the variance of the process and $\boldsymbol{R}$ is a correlation function which depends of internal parameters, $\boldsymbol{\theta}$**,** that can be determined by optimizing the likelihood.

The optimal unbiased linear predictor provided by Kriging theory is expressed as:

$$\hat{Y}(\boldsymbol{X}) = m_Y + \boldsymbol{r}^T(\boldsymbol{X})\boldsymbol{R}^{-1}(\boldsymbol{Y} - \boldsymbol{1}.m_Y),$$

where

$$\begin{cases} R_{ij} = \mathrm{corr}(\boldsymbol{Z}_i, \boldsymbol{Z}_j) \\ \boldsymbol{r}(\boldsymbol{X}) = [\mathrm{corr}(\boldsymbol{X}, \boldsymbol{Z}_1), \dots, \mathrm{corr}(\boldsymbol{X}, \boldsymbol{Z}_n)]^T \\ \boldsymbol{Y} = [Y_{\boldsymbol{1}}, \dots, Y_{\boldsymbol{n}}]^{\boldsymbol{T}} \\ m_Y = \dfrac{\boldsymbol{1}^{\boldsymbol{T}}\boldsymbol{R}^{-\boldsymbol{1}}\boldsymbol{Y}}{\boldsymbol{1}^{\boldsymbol{T}}\boldsymbol{R}^{-\boldsymbol{1}}\boldsymbol{1}} \end{cases}$$

Kriging provides also an uncertainty estimator (variance) as:

$$\hat{\sigma}^2 = (\boldsymbol{Y} - \boldsymbol{1}.m_Y)^T\boldsymbol{R}^{-\boldsymbol{1}}(\boldsymbol{Y} - \boldsymbol{1}.m_Y)\ .$$

In order to improve the research of the optimum, it may be necessary to enrich the sampling by adding new points to improve the accuracy of the Kriging model. The selection of these points can be performed by different means: point of the minimum of the model, point of the maximum of uncertainty, EGO (Efficient Global Optimization) [15]. To improve the model accuracy, the new points must be selected by balanced exploitation and exploration. So, EGO uses both the predictor and the variance of the model to estimate the expected improvement (EI) defined by:

$$E[I(\boldsymbol{X})] = \left(Y_{min} - \hat{Y}(\boldsymbol{X})\right)\Phi\left(\frac{Y_{min} - \hat{Y}(\boldsymbol{X})}{\hat{\sigma}}\right) + \hat{\sigma}\phi\left(\frac{Y_{min} - \hat{Y}(X)}{\hat{\sigma}}\right).$$

where $\Phi(.)$ and $\phi(.)$ are the cumulative distributive function and probability density function respectively. A new selected point corresponds to the maximum of the EI.

Co-Kriging Methodology

The idea of Co-Kriging is to use all available information to estimate unknown high-fidelity information. The basic Kriging formulation has been extended by many authors ([16][17][18]) to combine multiple levels of simulation to create a more accurate or less expensive high-fidelity model. The Kennedy and O'Hagan approach, is based on an autoregressive model, and consists in approximating the high-fidelity model by multiplying the low-fidelity model, $\boldsymbol{Z_c}$, by a scaling factor ρ, and by adding a Gaussian process, $\boldsymbol{Z_d}$, representing the difference between the low and high fidelity data,

$$\boldsymbol{Z_e}(\boldsymbol{X}) = \rho \boldsymbol{Z_c}(\boldsymbol{X}) + \boldsymbol{Z_d}(\boldsymbol{X}).$$

where $\boldsymbol{X^T} = [\boldsymbol{X_c^T}, \boldsymbol{X_e^T}]$, $\boldsymbol{X_c}$ and $\boldsymbol{X_e}$ represent the low and high fidelity sampling locations. The covariance matrix $\boldsymbol{C}$ is defined by:

$$\boldsymbol{C} = \begin{pmatrix} \sigma_c^2 \boldsymbol{R_c}(\boldsymbol{X_c}, \boldsymbol{X_c}) & \rho\sigma_c^2 \boldsymbol{R_c}(\boldsymbol{X_c}, \boldsymbol{X_e}) \\ \rho\sigma_c^2 \boldsymbol{R_c}(\boldsymbol{X_e}, \boldsymbol{X_c}) & \rho^2\sigma_c^2 \boldsymbol{R_c}(\boldsymbol{X_e}, \boldsymbol{X_e}) + \sigma_d^2 \boldsymbol{R_d}(\boldsymbol{X_e}, \boldsymbol{X_e}) \end{pmatrix}.$$

The correlation functions have a similar writing as the ones for the Kriging methodology, and they depend on the double of internal parameters to be determined. As $\boldsymbol{Z_c}$ and $\boldsymbol{Z_d}$ are considered to be independent, the internal parameters of the low-fidelity model can be determined in a similar manner as the Kriging model. So, ρ and the internal parameters of the difference process can be determined by optimizing the likelihood, but using the difference data:

$$\boldsymbol{d} = \boldsymbol{y_e} - \rho \boldsymbol{y_c}(\boldsymbol{X_e}).$$

The predictor provided by Co-Kriging is now expressed as:

$$\hat{Y}_e(\boldsymbol{X}) = m_Y + \boldsymbol{c}^T(\boldsymbol{X})\boldsymbol{C}^{-1}(\boldsymbol{Y} - \boldsymbol{1}.m_Y),$$

where

$$\begin{cases} \boldsymbol{Y^T} = \left[Y_c{}^{\boldsymbol{T}}, Y_e{}^{\boldsymbol{T}}\right] \\ m_Y = \dfrac{\boldsymbol{1^T C^{-1} Y}}{\boldsymbol{1^T C^{-1} 1}} \end{cases}$$

Kriging provides also an uncertainty estimator (variance) defined as:

$$\hat{\sigma}^2 = (\boldsymbol{Y} - \boldsymbol{1}.m_Y)^T \boldsymbol{C^{-1}} (\boldsymbol{Y} - \boldsymbol{1}.m_Y).$$

Validation on analytical function

The advantages of the Co-kriging method are illustrated by an example with one variable optimization. The data are defined by the following analytic functions:

$$Y_{bf} = \frac{1}{2}Y_{hf} + 10(x-1)$$
$$Y_{hf} = (6x-2)^2 sin(12x-4)$$

The four high-fidelity data usually chosen for this test case are not sufficient to give an accurate Kriging model. But this same high fidelity data added to the low fidelity data are now sufficient for the Co-kriging theory to improve the model. It must be noted that, in this case, the very good accuracy obtained is due to the fact that the two functions correspond to the autoregressive theoretical model.

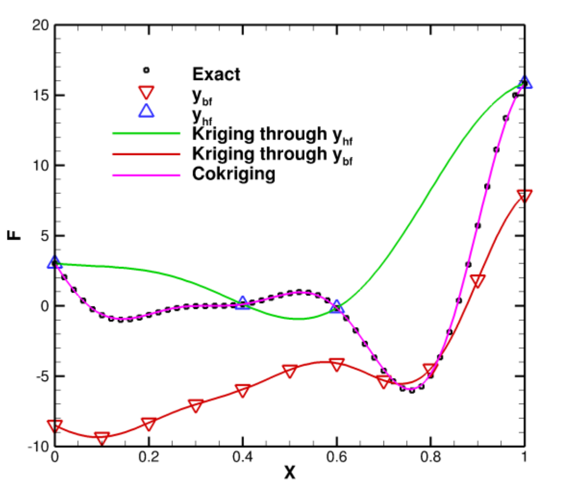


Figure 1: One-variable example of Kriging and Co-Kriging.

Application on Rotor Blade Optimization

The objective of this study is to apply the previously described Kriging and Co-Kriging optimization procedures, to optimize the sweep law of a reference rotor (defined by Airbus Helicopters Deutschland), in order to improve its performance in forward flight. This rotor is equipped with five blades, which are rectangular with a parabolic blade tip. The blade is equipped with the two airfoils OA312 and OA309. Linear interpolation is performed in the area between these airfoils. A linear geometric twist is defined, and no anhedral is applied at the blade tip. The planform of the reference blade is illustrated in Figure 2.

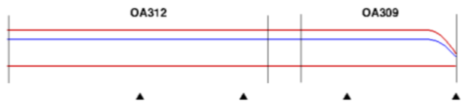


Figure 2: Reference blade planform and location of control points

Three active decision variables on the sweep law are chosen, located at 66%, 83% and 100% of the rotor radius R. These variables are parameterized by cubic splines, whose control points locations are shown in Figure 2. The first control point defining the cubic spline is located at 50% of the rotor radius, and is imposed at zero, in order to ensure a smooth transition between the initial area (up to 50%) and the optimized area (from 50%). The lower and upper bound values are respectively set at 0.20 m and 0.50 m.

The selected forward flight condition is the following: Vh=140 kts, C_T/σ=0.075, Ω=347 rpm.

Optimization chains

The research of the optimized solution, using a single fidelity model based on Kriging methodology is performed, following the optimization procedure represented in **Figure 3**.

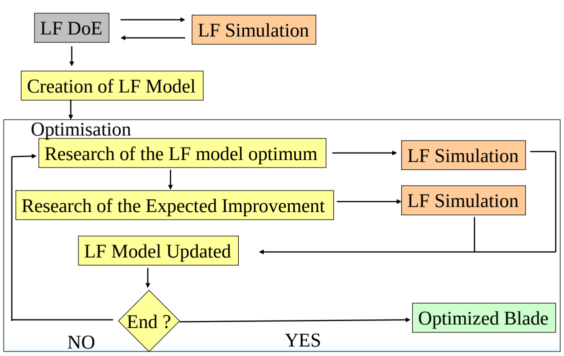


Figure 3: Chart of the optimization procedure by Kriging

The first step consists in building the Design of Experiment, based on a LHS spatial discretization of the design space. Then, the construction of the Kriging model is divided into two steps: the research of the minimum of the model (obtained thanks to a classical genetic algorithm optimizer), followed by the research of the maximum Expected Improvement. The Design of Experiment is enriched at each step by Low Fidelity (LF) simulations of these points. This procedure is repeated until a prescribed number of evaluations is completed, and the check that the power benefit cannot be improved.

The optimization procedure, based on the multi-level co-Kriging model, is shown in Figure 4.

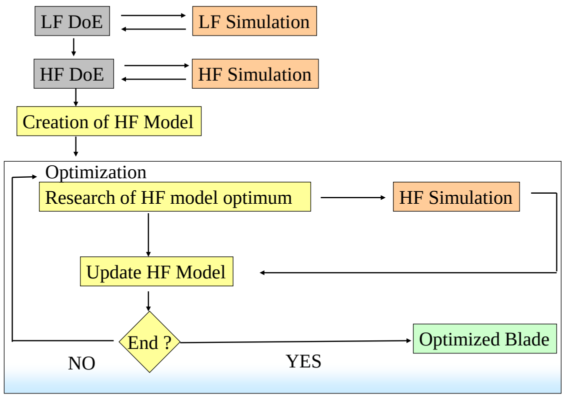


Figure 4: Chart of the optimization procedure by Co-Kriging

The first step consists in the generation of the two Design of Experiment data base: the first one defining the Low Fidelity (LF) model (the same as the one built for the Kriging model), built on a moderate number of points; the second one defining the High Fidelity (HF) model, built on a restricted number of points. Then, the building of the Co-Kriging model can start. The minimum point of the model is researched by the genetic algorithm, and then evaluated by a High Fidelity simulation. This new point is added and enriches the data base. Another optimization procedure can begin, as well as the research of the Expected Improvement point, until the convergence of the optimized solution (determined by the user).

Numerical Tools for Low Fidelity Simulations

The low fidelity simulations are performed with the HOST comprehensive rotor code, developed by Airbus Helicopters [19].

The structural model is based on a 1D Euler-Bernouilli beam model. The beam is discretized along the pitch axis as an assembly of rigid segments, with the elastic properties contained in the joints connecting them. The structural properties of the blade (lineic mass, inertia, stiffness …) are given as input data file for

HOST. During the aerodynamic optimization, the blade planform is modified, which leads to a change of these structural data. In 2011, ONERA has set up an updating procedure of the structural data used in the HOST code, to be integrated in an optimization loop [7]. This procedure is based on the definition of analytical polynomial laws which describe the evolutions of the stiffnesses, the lineic mass and the inertia with respect to the chord and the thickness laws of the profiles of the blade. Some analytical corrections are also performed to adjust the elastic axis and the gravity center axis with respect to the pitch axis. This procedure allows obtaining realistic blade planforms, with internal structural properties adapted to the new blade design. This procedure has been used in the framework of this study, for the optimization of the sweep law of the selected reference rotor.

The HOST comprehensive code is based on a lifting line approach to compute the blade aerodynamics loads. The blade is considered as a succession of 2D blade elements, each one shedding a vortex of bound circulation generated at its aerodynamic center (quart-chord). At a given Mach number, and for an equivalent angle of attack, the lift, drag and pitching moment coefficients can be obtained via 2D semi-empirical airfoil lookup tables. The compressibility effects and the viscosity are somewhat taken into account in these tables. Furthermore, some numerical corrections can be activated to include the effects of rotation and of the local blade geometry (like swept tips), and also the effects of stall and unsteadiness (unsteady Theodorsen corrections).

The rotor wake influences the rotor performance via the induced velocities it generates at the rotor disk. In the framework of this study, the METAR [20] prescribed wake model, (developed by Airbus Helicopters) is used. The wake geometry is prescribed, and considered helical. The induced velocities generated by a vortex segment of the wake are computed using the Biot-Savart law. This system is solved iteratively in the trim loop, until convergence is obtained, when the circulation of the wake is in accordance with the sectional blade lift forces, and the mean induced velocity reaches a threshold value.

Then, the HOST computations provide the rotor trim characteristics such as the control commands (pitch, flap and lag angles), the local aerodynamic loads, the blade elastic deformations, and the shaft power (which is the objective function of this study). The CPU cost of one evaluation is between 2 and 5 minutes on a local computer.

Numerical Tools for High Fidelity Simulations

The high fidelity simulations are performed using a loose coupling procedure [21] between the CSD code, HOST, and the CFD code developed at ONERA, *elsA* [22]. The three-dimensional unsteady Navier-Stokes equations are solved by the centered second-order Jameson's scheme. The time integration is performed by an implicit Euler scheme, with some Gear sub-iterations. The turbulence model is Kok k-ω, with SST corrections. The flow is supposed to be fully turbulent. The grids are generated thanks to Chimera technique. A multi-blocks, deformable mesh of O-H type is generated around each blade, containing 1.7 millions points. These blade grids are immersed into a Cartesian background grid, containing 13 millions points. The total mesh contains in total 21.5 million points, and can be considered as refined.

To obtain a satisfactory level of convergence of the coupling procedure between the CSD and CFD codes, 6 iterations are performed for each design point. The CPU cost for a converged coupling procedure requires about 90 hours, on 64 processors of the SGI parallel calculator, used at ONERA.

Analysis of Optimization Results with Kriging

The first step of the Kriging optimization procedure is to build the Design of Experiment data base. Over the initial 30 points defined by the Latin Hypercube Sampling procedure, 16 points have reached the numerical convergence of the Low Fidelity simulations (performed with the HOST code). Then, the data base has been enhanced by the research of the optimum points followed by the research of the Expected Improvement points. After 47 converged HOST evaluations, it is considered that the Kriging optimization process has converged to the global optimum solution (Figure 5).

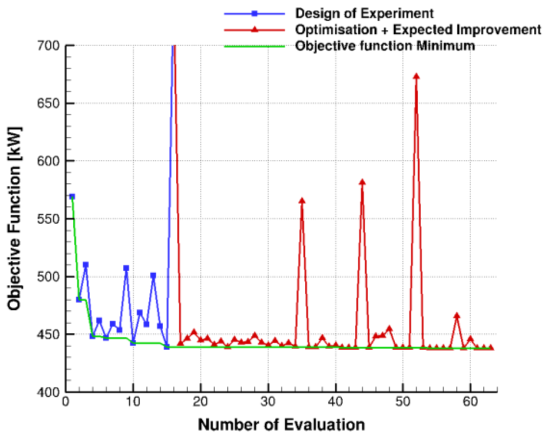


Figure 5: Convergence of Kriging optimization procedure

It has also been checked that this solution is similar to the one obtained with an evolutionary optimizer (CMA-ES), with a large reduction of HOST numerical evaluations. The cost of CPU is then reduced by a factor of 5.

The power benefit with respect to the reference blade is equal to 2.5%. The blade planform is visualized in Figure 6 .

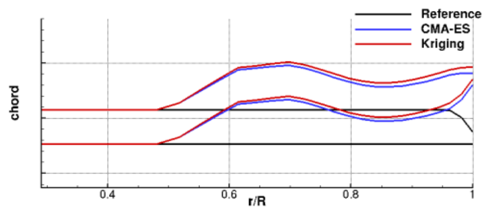


Figure 6: Optimized blade planform obtained by CMA-ES and Kriging procedures (with updated structural data)

The optimization procedures lead to a modification of the sweep law, the optimized blade has a forward sweep from 50% to 73% of span, followed by a backward sweep from 73% to the blade tip. It has been checked that this optimized blade planform is less sinuous, with reduced sweep angles in the backward and the forward directions in comparison with the one obtained without updating structural data during the optimization procedure.

The origin of the shaft power benefit can be analyzed by the distribution of the local lift coefficient on the rotor disk in Figure 7.

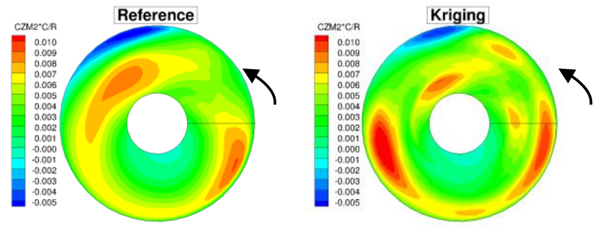


Figure 7: Distribution of the sectional lift coefficient of the reference and Kriging optimized blades

The HOST low fidelity calculations on the Kriging optimized blade predict a reduction of the lifting loads in the internal part of the blade, in the second quarter of the rotor disk (between 90° and 180° of azimuth) with respect to the reference, as illustrated in Figure 8.

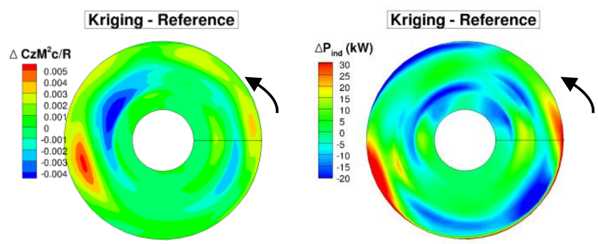


Figure 8: Distribution on the rotor disk of the difference between the Kriging optimized and the reference rotors of the sectional lift coefficient and the surfacic induced power (LF simulations)

It has been checked that the origin of the power reduction comes from a benefit of 11% of the induced power. The areas on the rotor disk where the reduction of the induced power is predicted are shown in blue in Figure 8. Three main regions can be noticed: the blade tip on the advancing and the retreating side, and the inner part on the advancing side.

The optimization of the blade sweep has an influence on the elastic torsion deformation and on the flap displacement at the blade tip, as shown in Figure 9.

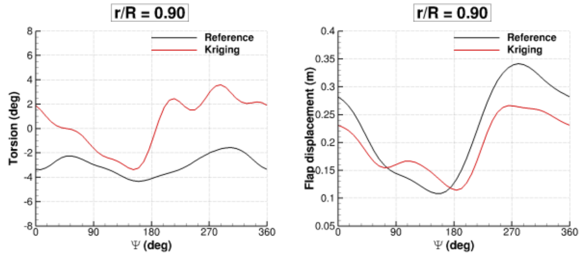


Figure 9: Torsion and flap deformations at the blade tip of the reference and the Kriging optimized blades

By its forwards sweep planform, the optimized blade obtained by the Kriging optimization procedure has a higher elastic torsional response than the reference in terms of amplitude (about 4° instead of 2°). The flap displacement is less sensitive then the torsion angle to the optimized blade planform (same evolution and same order of magnitude of the amplitude).

Analysis of Optimization Results with Co-Kriging

As for the Kriging optimization process, the first step is to evaluate the objective function (shaft power consumed by the main rotor) with the High fidelity numerical tool (weak coupling between HOST and *elsA*), on a very limited number of points, to build the High Fidelity Design of Experiment. Four points have been chosen among the 30 initial ones issued from the Latin Hypercube Sampling procedure.

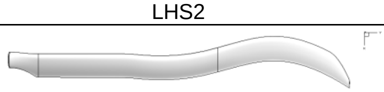
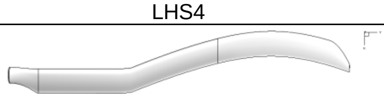
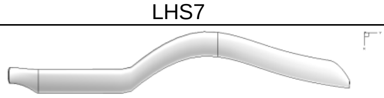
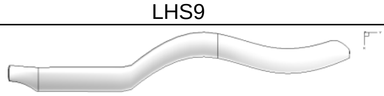
Blade planform	% (LF) / % (HF)
 LHS2	+14.2 / +36.7
 LHS4	+1.5 / -0.3
 LHS7	+2.7 / +2.4
 LHS9	+4.1 / +0.4
 Optim Kriging	-2.5 / +0.7

Figure 10: Blade planforms and power benefits estimated by LF and HF simulations for the HF DoE and the optimized rotor by Kriging

It is very interesting to notice that the hierarchy between the different rotors can be different with respect to the level of fidelity of the numerical tools (Figure 10). Especially, the HF simulations predict a loss of performance for the optimized blade issued from the Kriging procedure. Actually, the three-dimensional unsteady effects can have a major influence on rotor blades, especially designed with a double sweep. Hence the importance of taking into account these effects in the optimization procedure thanks to surrogate models, as Co-Kriging.

The convergence of the optimization procedure with Co-Kriging is obtained after 6 researches of the minimum point, as illustrated in Figure 11.

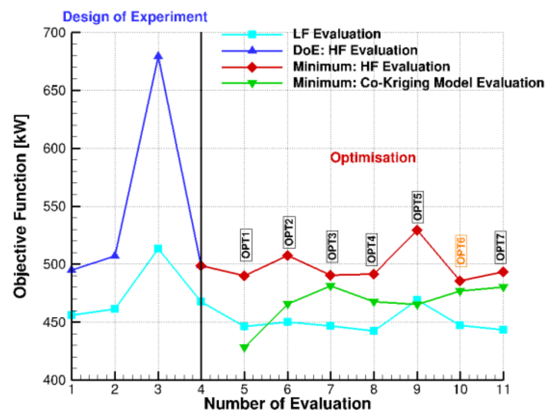


Figure 11: Convergence of optimization procedure with Co-Kriging

A limited number of HF simulations is required to obtain an optimized solution: 4 for the generation of the Design of Experiment, 6 for the research of the Minimum Point, and 1 for the research of the Expected Improvement point, that is to say 11 HF evaluations in total. This shows the efficiency of the Co-Kriging method to reach an optimum point that should have been obtained after (at least) 60 to 70 evaluations with the Kriging method. The gain in CPU time with the Co-Kriging method is about 6 with respect to the Kriging method.

For the OPT6 point, the difference between the minimum of the model and the estimation of this minimum point by HF calculations (weak coupling between HOST and *elsA*) is less than 2%, which is considered to be as satisfactory as possible.

Beyond the OPT6 point, the Expected Improvement point and the next OPT7 optimization point have been researched by the procedure, and then evaluated by HF calculations. As shown in Figure 12, the EI6 design does not provide any power benefit estimation. The estimated power benefit provided by the OPT7 optimized design is lower than the one obtained by the OPT6 point, which can be considered as the best optimization point of the Co-Kriging procedure.

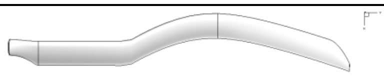
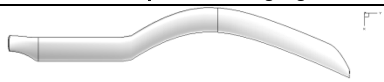
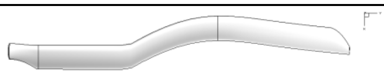
Blade planform	% (HF)
 **OPT6 = Optim Co-Kriging**	**-2.2**
 EI6	+0.02
 OPT7	-0.7

Figure 12: Blade planforms and power benefits estimated by HF simulations during Co-Kriging optimization procedure

The Co-Kriging optimization procedure results in a blade planform presenting a forward and then a backward sweep, evolving in a smoother manner than for the blade planform generated by the Kriging optimization procedure. The sweep angles are reduced with the Co-Kriging optimization procedure. So, this blade planform seems more realistic than the optimized Kriging blade. The break that can be noticed for the optimized blade planforms corresponds to the location of the first optimized control point (at mid-span), parametrizing the sweep law by a cubic spline. These designs have been generated in the framework of a theoretical study, and should be adapted if necessary.
The shapes of the models obtained by Kriging and Co-Kriging optimization procedures on the three decision variables, allowing to obtain their respective optimum point are shown in Figure 13.

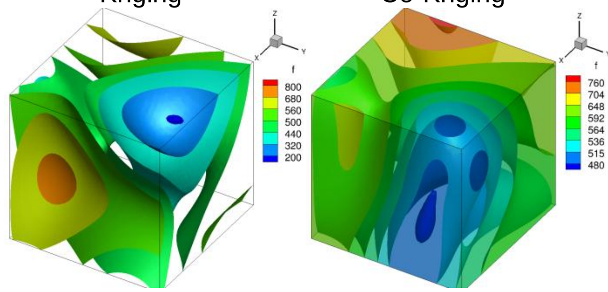


Figure 13: Shapes of the Kriging and the Co-Kriging models

Three zones on the Kriging and the Co-Kriging models can be detected: two with a maximum level, one with a minimum level. But, their shapes are quite different. For the Kriging model, the minimum point is inside an area close to the upper boundary of the research domain. For the Co-Kriging model, the minimum point is detected in a more extended region, inside the domain. The Co-Kriging domain brings a large modification of the shape of the surface response, and not only an improvement of the accuracy of the Kriging model.

The power benefit is estimated at 2.2% by High Fidelity computations, which is very encouraging The origin of this power benefit can be analyzed on the one hand by the distribution of the sectional lift estimated by the HF simulations, respectively for the reference, and the optimized rotors issued from the Kriging and the Co-Kriging procedures (Figure 14), and on the other hand, by the distribution of the difference of the induced power between the optimized Co-Kriging and Kriging rotors, obtained at the end of the coupling procedure of the HF simulations (Figure 15).

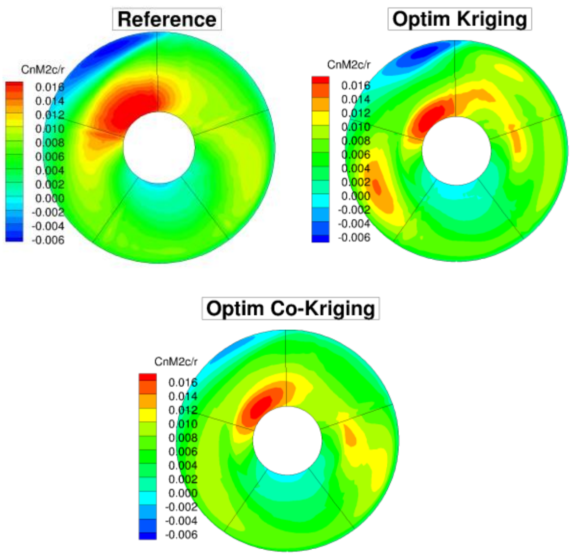


Figure 14: Distribution on the rotor disk of the sectional lift coefficient for the reference; and kriging and Co-Kriging optimized rotors (HF simulations)

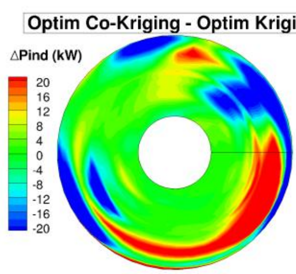


Figure 15: Distribution on the rotor disk of the difference of the surfacic induced power between the Co-Kriging and the Kriging optimized rotors

Both optimization procedures allow a decrease of the shaft power thanks to the reduction of the positive loads in the internal part of the advancing blade, in the second quadrant.
The smoother sweep evolution obtained with the Co-Kriging procedure with respect to the Kriging optimization results can explain the power benefit estimated by HF calculations (-2.2% for the Co-Kriging optimized blade, +0.7% for the Kriging optimized blade). The vanishing of the over-loaded area at the blade tip on the fore region, as well as the reduction of the negative peak at the blade tip in the second quadrant are the main reasons to explain the power benefit for the Co-Kriging optimized blade. These areas correspond to the ones where a gain on induced power is estimated (blue areas in Figure 15), showing that the main shaft power benefit comes from a reduction of the induced power.
The blade tip deformations in torsion and flap obtained by the two optimized rotors are shown in Figure 16. Thanks to its smoother and more realistic planform, the torsional deformation of the Co-Kriging optimized blade has an amplitude reduced by 3° in comparison with the Kriging optimized one, as well as a more regular shape.

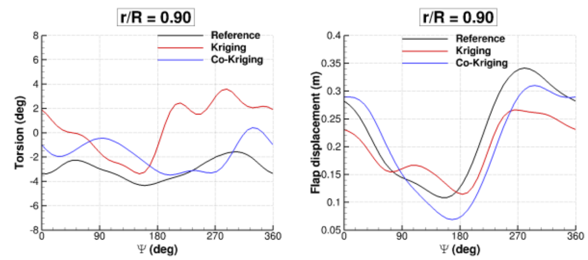


Figure 16: Torsion and flap deformations at the blade tip of the reference and the Kriging and Co-Kriging optimized blades

Concluding Remarks

The optimization of helicopter rotor blades is a challenging problem, as far as more and more complex blade planforms are considered. The main issue is an accurate simulation of the flowfield taking into account the three-dimensional unsteady effects in the optimization loop. By this way, one can assume to obtain realistic blade planforms and accurate estimation of the power consumed by the main rotor (generally considered as the objective function of the optimization procedure). The CFD codes being computationally expensive and time-consuming, it is well adapted to use surrogate models, which replace high order simulations.

In this paper, ONERA presented two aerodynamic optimization procedures, to define an optimized sweep law, by using low and high fidelity levels of surrogate models, respectively based on Kriging and Co-Kriging methodologies. The main results obtained in this study are the following:

- The optimization procedure in which the low fidelity HOST computations are replaced by a Kriging model provide an equivalent solution to the optimization procedure based on the coupling between HOST and the evolutionary CMA-ES optimizer, with a large gain on the CPU time consumption (factor of 5).
- A posteriori, the evaluation of the shaft power of this optimized Kriging rotor, by high

level computations (CSD/CFD weak coupling) actually shows a loss of about 1% on the shaft power with respect to the reference rotor. This shows that the accuracy of the HOST computations and the Kriging models is not sufficient. Taking into the high level evaluations in the optimization loop is mandatory, and can be effective by the use of a Co-Kriging model.

- The blade planform of the optimized rotor issued from the Co-Kriging optimization procedure is smoother and more realistic than the previous optimized rotor. The power benefit is estimated at about 2% by high level computations. A limited number of HF simulations is required to obtain a satisfactory optimized solution. A factor of 6 in CPU time can be estimated with respect to what would have required the Kriging optimization procedure based on HF simulations.

Perspectives of this study should consist in developing and validating different models to quantify the uncertainties propagation depending on selected uncertain and/or epistemic variables. This work should then lead to robust optimization procedures.

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