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NON-LINEAR ANALYSIS OF THE STATIC BEHAVIOR OF SINGLE LAPPED JOINT

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Abstract

In this article, the stresses and strain distribution along the overlap of a single lapped joint between two adheredns of composite materials, is analysed. The study has been performed using an extension of Goland's and Reissner's study. The non-linear model here developement can be solved with a finite differences tecnique and then solving the non-linear algebric equations with a Newton Raphson procedure. The adherends are modelled as laminated bending plates applaing the Classical Lamination Theory. Using this theory, it is possible to analyse laminates whose arbitrarily oriented plies under a complex combinations of extensional and flexural load.

1 Introduction

As a consequence of the development of technology, in the recent years the use of composite materials in aerospatial industry is very common. The advantages of using composite materials are well known as are the problems which appear in joining different substructures made of composite materials. Bonding technology is considered a good solution for some mechanical joints. In some structures,such as aircraft and helicopters, also the problem of joining adherends of different materials exists (e.g. alluminium-fiber reinforced composite plates). It is also necessary to being in mind certain critical factors in joining fiber reinforced plates or metal and composite adherends. One critical factor is the initial bond strength, related to the presence of contaminants in the interlayer between adherend and adhesive. This can cause fracture in interlayer adhesive-adherends that it is not possible to prevent with mechanical tests. In the case of joining different materials it is necessary also to take into account their different thermal expansion coefficients to evaluate the stesses distribution along the overlap. The aim of this paper is to indicate a method which can be used to calculate the stress distribution in the overlap of the two adherends. In its simplest form an adhesively bonded structure consists in the single-lap joint. In this article a method of analysis is presented for the non-linear behaviour of two generally orthotropic adherends bonded together. General non-linear relations for the stress couple, transverse shear resultant, and in-plane stress resultant at any location in the adherends and adhesive are derived. Several theoretical analyses of this difficult problem have appeared in the literature such as the work by Goland and Reissner (Ref 1). Their study is about two limits cases: where the thickness and modulus of elasticity ratio of the adhesive is much less than that of the adherends and its effect on the flexibility of the joint may be neglected, where the joint flexibility is influenced from the presence of the adhesive layer. This approach has been used by Renton and Vinson (Ref 2) applied to a composite materials, or extend to all adhesive layer conditions by Chen and Cheng (Ref 3). Other authors have improved the model of joint using a viscoelastic behavior of the adhesive such as Delale and Erdogan (Ref 4), or non-linear one as Bigwood and Crocombe (Ref 5). In this paper the case where the two adherends are cylindrically bent plates is considered with the adhesive layer behaviour being modeled by an actual non-linear stress-strain curve. Using the experience of these later work, it is possible to envelope a mathematical model of a lapped joint with adherends of composite materials. The model consists in a system of six first order non-linear differential equations which can be solved using a finite different technique to reduce the differential equations to more simple algebric ones. Then the non-linear algebric equations are solved using the Newton-Raphson scheme. The program is been implemented using MATHEMATICA software. The method has been used to study the stress distribution in the overlap and its relations with the adhesive thickness layer, length of the overlap and different type of composite adherends.

2 Formulation of the Problem

Two rectangular fibre reinforced plates of equal thickness t and length (l+2c) are considered. The two sheets are lap-jointed over the length 2c and the bond between sheets is established by means of an adhesive layer with thickness h. At free edge of the adherends, a combination of tensile, transverse shear and bending moment loads is applied. The problem can be formulated under the following primary assumptions:

a) the adherends are treated as linear fiber-reinforced laminated plates using the Classical Lamination Theory. It is important to remember that the most important limitation of this theory is that each ply is assumed to be in a state of plane stress and that interlaminar stresses are neglected;

b) the adhesive is assumed to be non-linear using a realistic material model for the adhesive interlayer;

c) the problem is a plane strain, i.e. the bonded joint is very "wide" and undergoes cylindrical bending;

d) the adherends have the same thickness and are made of the same materials;

e) the structure is a single lap joint

Other elements of the theory such as the deformation hypothesis, the equilibrium equation and strain-displacemant relationships are as used in the classical plate theory.

Determination of Laminate Engineering Constants

The laminate is made up of multiple laminae and it is assumed that the individual laminae are perfectly bonded together so as to behave as a unitary, nonhomogeneous, anisotropic plate. The lamina is assumed to be in a simple two-dimensional state of stress. In this case the orthotropic stresses-strain relationships can be simplified by putting $\sigma_3 = \tau_{23} = \tau_{31} = 0$.

The lamina stresses in terms of tensor strains are given by:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{Bmatrix} \tag{1}$$

where Q_{ij} are the components of the lamina stiffness matrix.

In the analysis of laminates having multiple laminae, it is often necessary to know the stress-strain relationship for the generally orthotropic lamina in general system of coordinates referred to the entire laminate. As it is possible to see in Gibson (Ref 7) and in Jones (Ref 9) the stiffness matrix of laminate can be determined by multipling the stiffness matrix of a single lamina for the transformation matrix $[T]$, to give the constitutive relation for the composite panel.

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ K \end{Bmatrix} \tag{2}$$

Using the compliance matrix of laminate, the inverse of stiffness matrix which can be seen in equation [2], it is possible to derive the material constants of the laminate.

$$\begin{aligned} E_1 &= \frac{\sigma_x}{\varepsilon_x} = \frac{1}{tA'_{11}} \\ E_2 &= \frac{\sigma_y}{\varepsilon_y} = \frac{1}{tA'_{22}} \\ G_{12} &= \frac{\tau_{xy}}{\gamma_{xy}} = \frac{1}{tA'_{66}} \\ \nu_{12} &= -\frac{A_{12}'}{A'_{11}} \end{aligned} \tag{3}$$

which are necessary to solve the mathematical model.

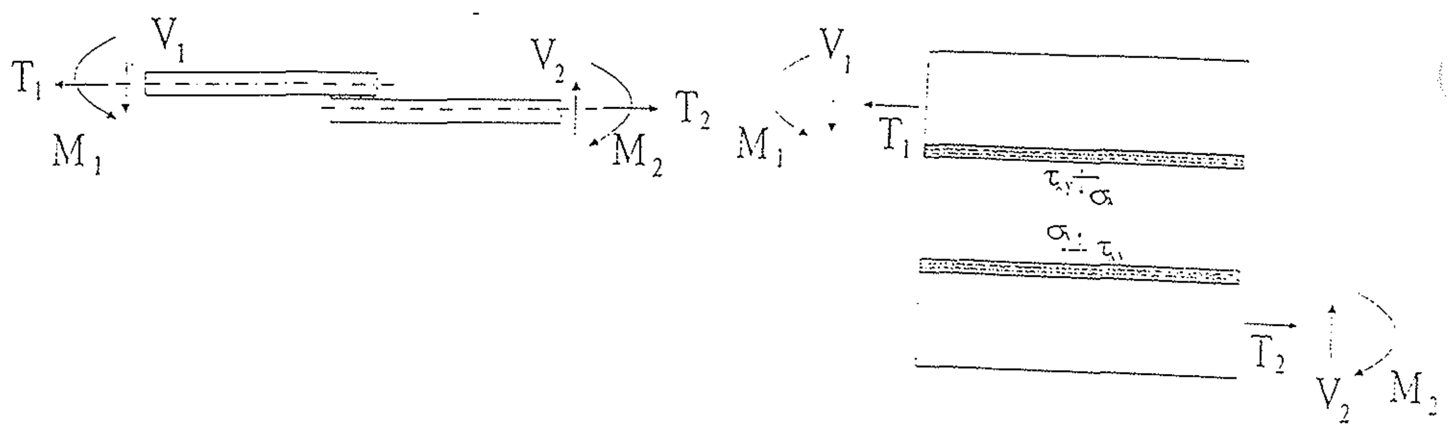

Figure 1: Geometry of the joint and loading conditions

Formulation of the Static Equations

To obtain the mathematical equations a single element of length dx at distance x from the edge of the junction is considered (Fig. 1). The equilibrium of moment for the element dx of the sheet given by:

$$\frac{dM_u}{dx} - V_u + \tau_{xy}\frac{t_u + h}{2} = 0 \tag{4}$$

$$\frac{dM_l}{dx} - V_l + \tau_{xy}\frac{t_l + h}{2} = 0 \tag{5}$$

The conditions of horizontal force equilibrium are:

$$\frac{dT_u}{dx} - \tau_{xy} = 0 \tag{6}$$

$$\frac{dT_l}{dx} + \tau_{xy} = 0 \tag{7}$$

and for vertical force:

$$\frac{dV_u}{dx} - \sigma_y = 0 \tag{8}$$

$$\frac{dV_l}{dx} - \sigma_y = 0 \tag{9}$$

From the equation of the elastic line it is possible to write, denoting by v_u and v_l the transverse deflection of the upper and lower sheets:

$$\frac{d^2 v_u}{dx^2} = -\frac{M_u}{D_u} \tag{10}$$

$$\frac{d^2 v_l}{dx^2} = -\frac{M_l}{D_l} \qquad (11)$$

where D are the flexural stiffness respectively:

$$D = \frac{E\,h^3}{12\,(1-\nu_p^2)} \qquad (12)$$

for the adhesive and

$$D_i = \frac{\sqrt{E_1\,E_2}\,t_i^3}{12\,(1-\nu_{12}\,\nu_{21})} \qquad (13)$$

for a composite plate and i are referred to the lower or upper adherend.

In the same way it is possible to consider the displacements along the x axis and obtain the sequent relations:

$$\frac{du_u}{dx} = \frac{1}{E}\left(\frac{T_{ux}}{t_u} - \frac{6(1-\nu_u^2)}{t_u^2}M_{ux}\right) \qquad (14)$$

$$\frac{du_l}{dx} = \frac{1}{E}\left(\frac{T_{lx}}{t_l} + \frac{6(1-\nu_l^2)}{t_l^2}M_{lx}\right) \qquad (15)$$

The previous relation considers the combinations of bending and direct stress. The system of equations is completed by the relations between the displacement and strain in the adhesive:

$$\varepsilon_y = \frac{v_1 - v_2}{h} \qquad (16)$$

$$\gamma_{xy} = \frac{u_1 - u_2}{h} \qquad (17)$$

The edge conditions for the two adherends are:

at $x = c$

$$\begin{cases} M_{lx} = T_{lx} = V_{lx} = 0 \\ M_{ux} = M_0,\ V_{ux} = 0,\ T_{ux} = T_0 \end{cases}$$

at $x = -c$

$$\begin{cases} M_{ux} = T_{ux} = V_{ux} = 0 \\ M_{lx} = M_0,\ V_{lx} = 0,\ T_{lx} = T_0 \end{cases}$$

The equations from [5] to [17] can be used to solve the non-linear problem where the nonlinearty is induced from the non-linear stress-strain relationship of the adhesive.

Non-Linear Analysis of the Adhesive

In the deformation theory of plasticity, it is possible to relate the plastic strain to the plastic stress component, such that:

$$\left\{ \begin{array}{c} \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{array} \right\} = \frac{1}{E_s} \left[\begin{array}{ccc} 1 & -\nu_p & 0 \\ -\nu_p & 1 & 0 \\ 0 & 0 & 2\,(1+\nu_p) \end{array} \right] \left\{ \begin{array}{c} \sigma_y \\ \sigma_z \\ \gamma_{zy} \end{array} \right\} \tag{18}$$

where ν_p is the plastic Poisson's ratio:

$$\nu_p = \frac{1}{2}\left[1 - \frac{E_s}{E}\,(1 - 2\,\nu)\right] \tag{19}$$

which varies between the elastic and its asymptote value for fully plastic case. E_s is the secant modulus, and can be defined such that:

$$E_s = \frac{\sigma_{idu}}{\varepsilon_{idu}} \tag{20}$$

where σ_{idu} are the ultimate stresses and ε_{idu} are ultimate strains from equation [18], assuming that $\varepsilon_z = 0$ its possible to have the stress-strain equation for the adhesive:

$$\sigma_y = \frac{E_s\,\varepsilon_y}{(1 - \nu_p^2)} \tag{21}$$

$$\sigma_z = \frac{E_s\,\nu_p\,\varepsilon_y}{(1 - \nu_p^2)} \tag{22}$$

$$\tau_{xy} = \frac{E_s\,\gamma_{xy}}{2\,(1 + \nu_p)} \tag{23}$$

From the definition of secant modulus it is possible to modelled the adhesive using the von Mises criterion for homogeneous materials:

$$\sigma_{idu} = \sqrt{\sigma_y^2 + \sigma_z^2 - \sigma_y\,\sigma_z + 3\,\tau_{xy}^2} \tag{24}$$

and substituting equation [21], [22] and [23] into [24] it is possible to express von Mises criterion using the strains:

$$\varepsilon_{id} = \frac{1}{1-\nu_p^2}\sqrt{(1-\nu_p+\nu_p^2)\,\varepsilon_y^2 + \frac{3}{4}(1-\nu_p^2)\,\gamma_{xy}^2} \qquad (25)$$

Using equations [5], [7], [9], [14], [21] and [23] it will be obtained a system of six first order non-linear differential equations depending from the variables T_{ux},V_{ux}, M_{ux},γ_{xy}, $\frac{d\varepsilon_y}{dx}$ and ε_y.

3 Numerical Example and Results

In order to investigate the potential of this model and the algorithm, numerical simulations on a single lap joint have been done (Fig 1). The adherend materials is constructed with laminae of $T300/934$ graphite/epoxy and the longitudinal modulus E_1, transverse modulus E_2, shear modulus G_{12} and Poisson's ratio ν_{12} of lamina are respectively $E_1 = 131GPa$, $E_2 = 10.3GPa$, $G_{12} = 6.9GPa$ and $\nu_{12} = 0.22$. The laminate constants have been calculated with program for different layouts of laminae each one with different fiber orientation. The length of overlap varies from $20mm$ to $30mm$ and different thickness of adhesive and adherends have been chosen. The results obtained can be compared to some derived by a finite element analysis and stress-strain curve of some experimental test can be approximated very well with an hyperbolic tangent $\sigma = A\tanh\left(\frac{E_s\varepsilon}{A}\right)$. In (Fig 4) and (Fig 5) it is possible to see the distribution along the overlap of the tensile, shear and moment load in the adherend. Using multiaxial strength criteria (Ref 7) it is possible to characterised the lamina failure. For the adhesive, a homogeneus isotropic material, the von Mises criterion can be used. This permits to the designer rapid estimation of when the joints failure will occur under complex loading conditions. In (Fig 9) it is possible to see a comparison between linear and non-linear case. When the non-linear behaviour of adhesive is considered, the distribution of stresses and strains along the overlap is better and the peaks values are lower than the non-linear case.

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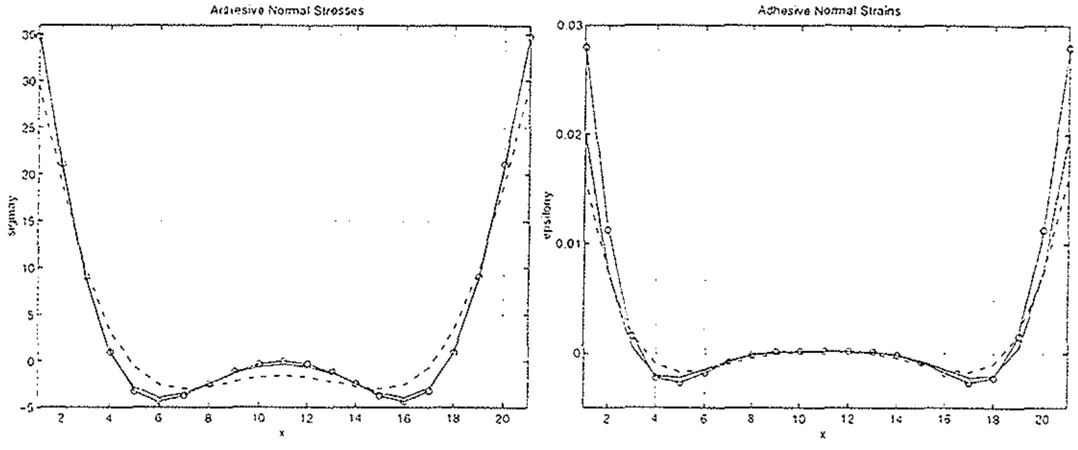

Figure 2: Adhesive σ_y and ε_y for different laminates layout with adherends thickness of 1 mm

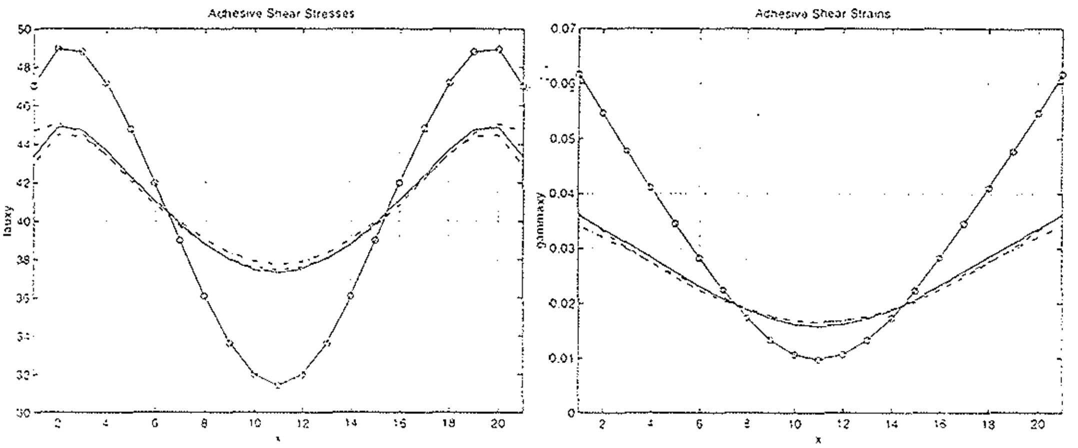

Figure 3: Adhesive τ_{xy} and γ_{xy} for different laminates layout with adherends thickness of 1 mm

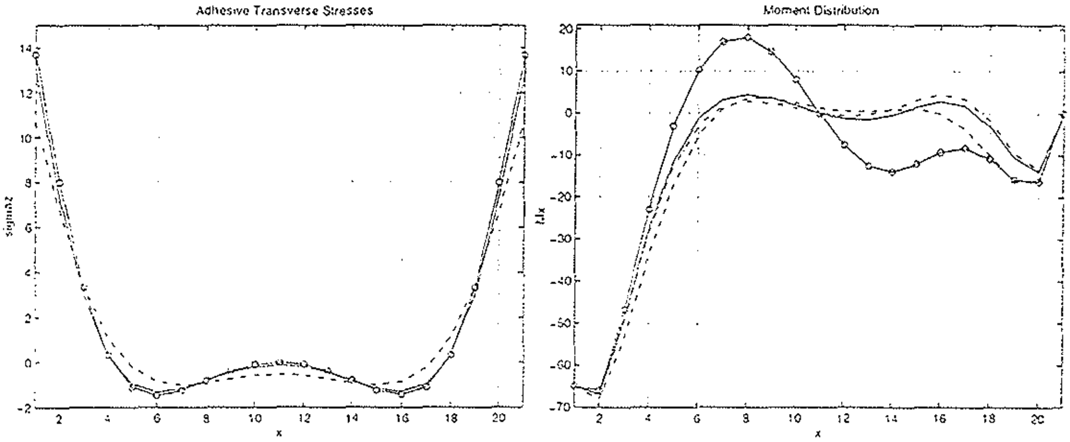

Figure 4: Adhesive σ_z in transverse direction and moment M_x distribution with adherends thickness of 1 mm

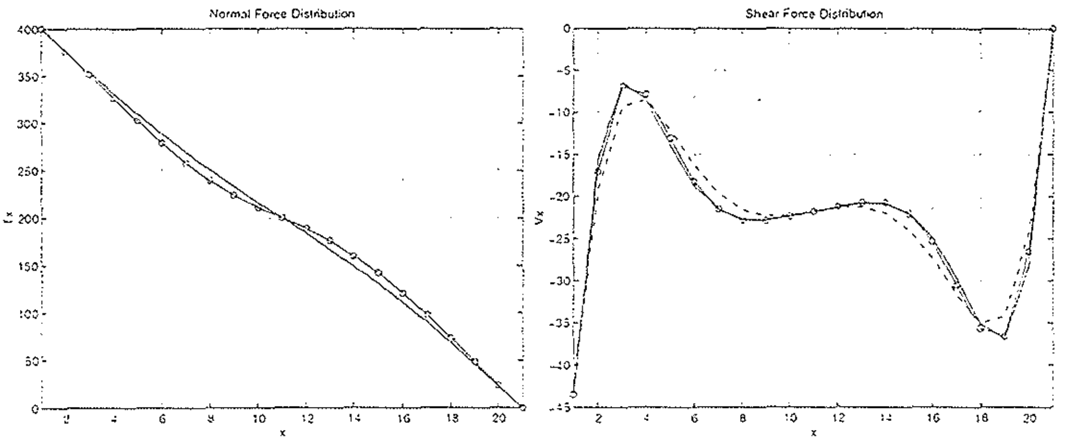

Figure 5: Adherend T_x and V_x for different laminates layout with adherends thickness of 1 mm

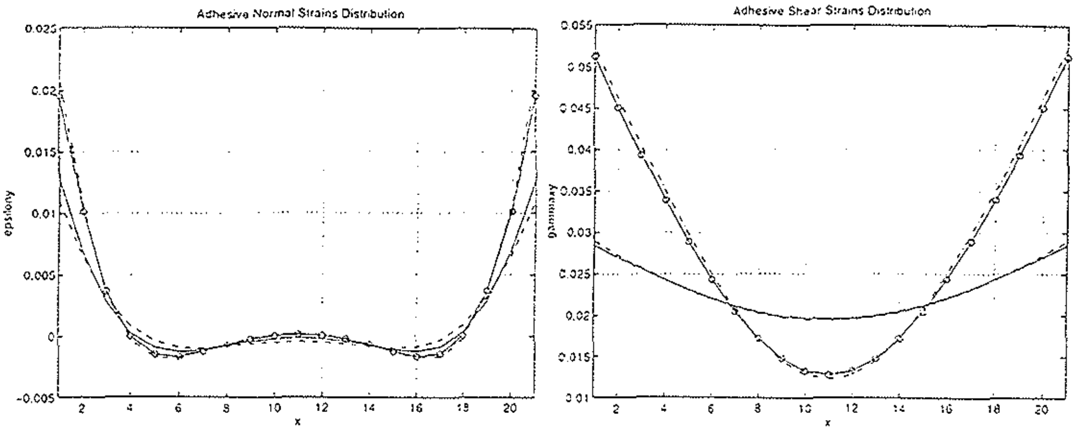

Figure 6: Adhesive ε_y and γ_{xy} for different laminates layout with adherends thickness of 2 mm

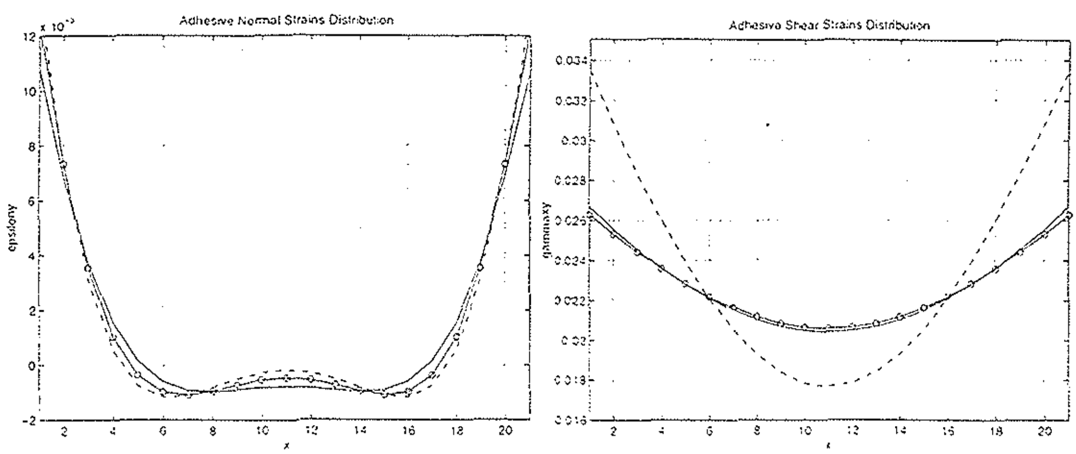

Figure 7: Adhesive ε_y and γ_{xy} for different laminates layout with thickness of adhesive 1.5 mm

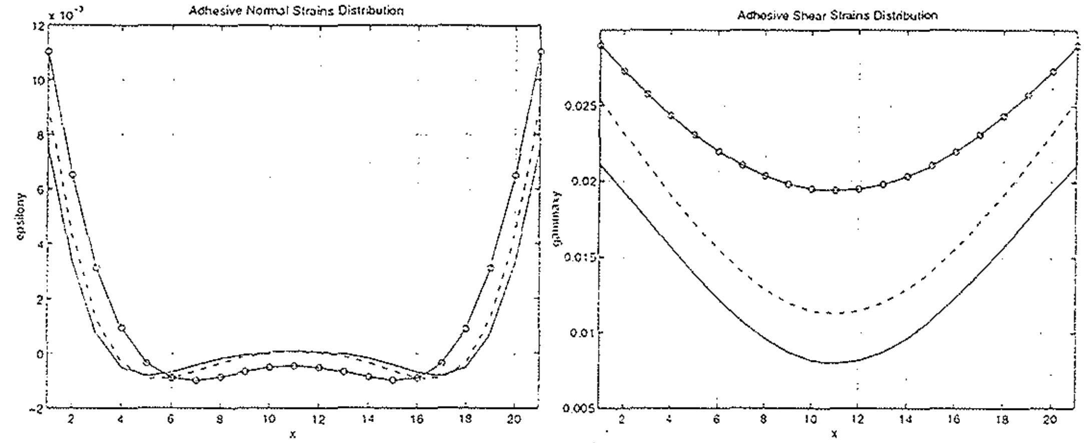

Figure 8: Adhesive ε_y and γ_{xy} for different laminates layout with different overlap of joint $(20-30mm)$

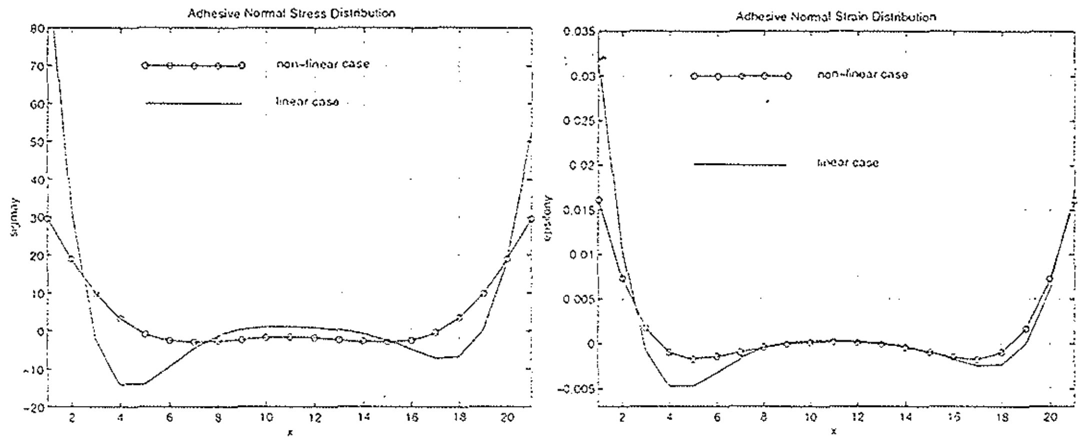

Figure 9: Comparison between linear and non-linear model adhesive σ_y and τ_{xy}

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