# Helicopter Optimal Control in OEI Low Speed Flights

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## Abstract

The objective of the research on helicopter optimal control was to develop a control method that would allow for safe landing of multiengine helicopter in one engine failure at low velocity and low height flight. To apply the control method, a simulation model of a single rotor helicopter was developed. The model contains 6 DoF of rigid fuselage and the angular speed of the main rotor as the additional DoF. For helicopter control an optimal control method was developed, where the quadratic cost function was used with assumed constrains. Both the cost function and the constrains may vary during the helicopter flight. In the paper the method and selected simulation results are also presented.

## Nomenclature

| | |
|---|---|
| $h$ | height above the ground |
| $J$ | cost function |
| $P, Q, R$ | angular rates |
| $P_s$ | power available |
| $U, V, W$ | velocity components |
| $U_{OEI}, W_{OEI}$ | horizontal and vertical speed |
| $t$ | time |
| $\mathbf{u}$ | control vector |
| $W_{(\,)}$ | weighting factor |
| $\mathbf{x}$ | state vector |
| $x, y$ | distance from point of power failure |
| $\Phi, \Theta, \Psi$ | fuselage attitude angles |
| $q_{0W}, q_{1W}, q_{2W}$ | main rotor control angles |
| $q_S$ | tail rotor collective angle |
| $\Omega_0$ | rotor angular nominal speed |
| $\Omega_W$ | main rotor angular speed |
| | of best OEI ascend |
| $(\,)_{norm}$ | normalization factor |

## Introduction

Helicopters from the beginning of their history were designed for missions impossible to fulfill for fixed wing aircraft. These missions cover also low-speed flight, performed very often at low heights above the ground. Because of helicopters principle of operation, these parts of flight envelopes unfortunately are often hazardous due to the lack of possibility for safe landing after power failure. The possibility for performing a safe landing is determined by helicopter height above the ground and its horizontal velocity, and thus the dangerous regions are plotted on height-velocity diagrams and are called the H-V zones Fig. 1.

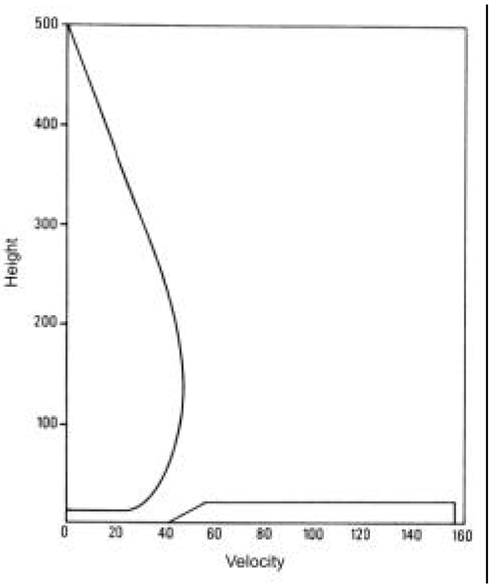


**Fig. 1 Sample of the H-V zones of a single engine helicopter**

Minimization of the H-V zones is a subject of the research of many helicopter manufacturers, since it may lead to enlargement of the safe flights envelope. Some of the methods include constructional improvements – specific rotor design, stronger power plant etc. – and application of them is limited for helicopters already being used. For such reasons different control strategies following power failure have been researched recently (1). Most of the emergency procedures and control techniques used during engine failure were developed more than half a century ago. It was unveiled that applying some changes to them may lead to enhancement of helicopter performance. Development of new control strategies is also related to unmanned vehicles, for which certain limitation of manned aircraft do not apply, and so the control techniques may be very different.

For purposes of the research of helicopter control methods simulation models are also being developed. Such models are also intended to be used in helicopter flight simulators for crew training in emergency flights (2),(3).

## Previous work

Several research of helicopter control in power failure situations were performed in the past. Fundamental work was done by Johnson (4), who developed a 2DoF (horizontal and vertical velocity) point-mass model of an OH-58A helicopter for investigation of a helicopter control in autorotation from hover. The model included simple ground effect formula as well as the expression for vortex ring modeling. Control algorithm applied by Johnson was based on the nonlinear optimal control theory. Cost function in this algorithm was a sum of squares of horizontal and vertical velocities at touchdown, both elements had weighting factors. The control variables were horizontal and vertical elements of rotor thrust.

In (5) the model developed in (4) was enhanced by control variables inequalities constrains and was used in research of helicopter control in autorotation from H-V zones. Both research (4) and (5) investigated the helicopter control in autorotation.

In (6)-(10) Zhao used enriched 2DoF model of UH-60 helicopter by engine dynamics to investigate control also in OEI operations. Zhao also investigated several cost functions applied in various situations.

In the series of work done by Carlson (11),(12) minimization of helicopter optimal trajectories and H-V zones was investigated using helicopter model similar to one used by Zhao.

Okuno and Kawachi (13)-(15) used 4 DoF helicopter model for helicopter trajectory optimization during takeoff and landing in the power failure situations. The degrees of freedom of the helicopter model were horizontal and vertical translations, fuselage pitch angle and rotor angular speed.

The 4 DoF helicopter model was also used in (16) for analytical studies of helicopter control in power failure situation. Recently 4 DoF helicopter and tiltrotor models were used in (17) for trajectory optimization. In that paper, development of numerical tools for trajectory optimization is presented.

Also in Poland research of helicopter control in autorotation were performed. In (18) Nelder-Mead optimization procedure was applied to 3 DoF helicopter model to predict the optimal trajectory in vertical descent. The DoF were helicopter vertical translation, rotor angular velocity and blade flapping angle.

As it can be seen from the short review above and other reference studies, that during last few decades a lot of work was devoted to helicopter control in autorotation. In majority of the presented research, an optimal control theory was applied to quite simple aeromechanical models of the helicopters and tiltrotors.

Few years ago In Warsaw University of Technology (WUT) the research was initiated to develop an optimal control method that would allow for helicopter control in power failure situations. After literature survey it was also decided to develop a more rich helicopter simulation model that would include 6DoF to model 3D motion of the helicopter. Additional DoF were rotor angular speed and engine power available.

## WUT Helicopter Model

A single rotor helicopter was chosen for modeling (19), (20). The model covers 6 DoF of the rigid fuselage, one DoF main rotor rotor angular speed and one DoF for engine power available. The control of the helicopter as a standard form includes collective and cyclic pitch of the main rotor and collective pitch angle of the tail rotor. The helicopter model was composed of: fuselage, main rotor, tail rotor, horizontal stabilizer and engine. All elements of the helicopter structure were modeled as rigid.

The analytical expressions for main rotor flapping coefficients calculation were derived including all degrees of freedom of the fuselage. Aerodynamic loads were calculated using blade element theory with quasi-steady aerodynamics. The induced velocity of main and tail rotors were modeled using Glauert formulae, and the main rotor downwash was taken into account in calculating flow over fuselage, horizontal stabilizer and tail rotor.

The helicopter equations of motion were derived using d'Alembert methodology of summing up all the loads acting on the helicopter elements.

For simulations the data of the PZL Mi-2 Plus helicopter (Fig. 2.)were used.

**Fig. 2 PZL Mi-2Plus helicopter**

The numerical model was programmed in Matlab environment. The model was validated using flight test data. The application developed allow for trim calculations as well as the calculations of the helicopter motion in time domain. To the model described the control algorithm was applied.

## Control Method

For controlling helicopter in a low speed flight an optimal control method was selected. The control objective was to calculate the helicopter control signals that assures the state variables to be within assumed constrains.

The main element of the optimal control problem is a selection of the cost function which is will be minimized:

$$J = \min\left[ f\left(\mathbf{x}_{\mathbf{k+1}}\right)\right] \tag{1}$$

This cost function depends of the state variables in the assumed time step. The state vector in time k+1 is calculated using the discredited equations of motion in form:

$$\mathbf{x}_{\mathbf{k+1}} = \mathbf{x}_{\mathbf{k}} + f\left(t_k, \mathbf{x}_{\mathbf{k}}, \mathbf{u} + \mathbf{?u}\right)\Delta t\,. \tag{2}$$

Both control and state variables are subject to constrains in inequalities form:

$$\mathbf{x}_{\mathbf{k+1}} \in \left(\mathbf{x}_{\mathbf{min}}, \mathbf{x}_{\mathbf{max}}\right), \tag{3}$$

$$\mathbf{u}_{\mathbf{k+1}} \in \left(\mathbf{u}_{\mathbf{min}}, \mathbf{u}_{\mathbf{max}}\right). \tag{4}$$

The cost function was assumed as a weighted sum of squared state variables. For simulation of OEI flights two cost functions were developed,

- for OEI landing (5)

$$\begin{aligned} J = {} & W_\Omega\left(\frac{25\left(\Omega - \Omega_0\right)}{\Omega_{norm}}\right)^2 \\ & + W_U\left(\frac{U}{U_{norm}}\right)^2 + W_V\left(\frac{V}{V_{norm}}\right)^2 + W_W\left(\frac{W}{W_{norm}}\right)^2 +, \\ & + W_P\left(\frac{P}{P_{norm}}\right)^2 + W_Q\left(\frac{Q}{Q_{norm}}\right)^2 + W_R\left(\frac{R}{R_{norm}}\right)^2 \end{aligned} \tag{5}$$

- for OEI rejected landing (6):

$$\begin{aligned} J = {} & W_\Omega\left(\frac{25\left(\Omega - \Omega_0\right)}{\Omega_{norm}}\right)^2 + W_U\left(\frac{U - U_{OEI}}{U_{norm}}\right)^2 + W_V\left(\frac{V}{V_{norm}}\right)^2 + W_W\left(\frac{W + W_{OEI}}{W_{norm}}\right)^2 + \\ & + W_P\left(\frac{P}{P_{norm}}\right)^2 + W_Q\left(\frac{Q}{Q_{norm}}\right)^2 + W_R\left(\frac{R}{R_{norm}}\right)^2 \end{aligned} \tag{6}$$

The weighting coefficients were obtained by simulation trials. It was assumed that the weighting factors may depend on simulated maneuver (OEI landing or rejected landing) and during the simulation vary depending on the helicopter height above the ground. The variation of the weighting factors should assure fulfilling the different control objectives in different flight phases. To ensure the similar influence of each element of the cost variables in the cost function were normalized.

The main goal of control during OEI operations in both landing and rejected landing is to ensure the proper horizontal and vertical speed in the final flight phase. The values should be within the limits required in the flight manual for each type of a helicopter. In simulation of OEI landing the goal is to reduce to zero both horizontal and vertical speeds while in simulation of OEI rejected landing the goal is to have horizontal and vertical speed of best OEI climb. The rotor should have enough kinetic energy not to allow the rotor to slow down below certain limits. The first part of the cost function (5) reflects this fact. In selected cost functions (5) and (6) , the weighting factor of the lateral flight speed should limit horizontal translations of the helicopter during the flight and the helicopter angular rates should provide slow changes of attitude.

The limitations imposed on the control and the state variables is derived by analyzing the helicopter design, taken from flight manuals of from preliminary simulations performed to investigate physical behavior of the model helicopter. The limitations of control variables were not changes during simulations while the constrains of the state variables varied during the simulation, depending on helicopter height above the ground. Above certain "decision" height, the helicopter control variables were constant, and below that height they varied with as a square of helicopter height to assure that the final values of state variables would be within the assumed limits (for instance those provided by a flight manual.)

Limits on the horizontal and the vertical flight velocities were restricted by the dangerous vortex-ring zone. It was assumed that it was forbidden for a helicopter to fly inside the vortex-ring zone. In the future this restriction may be released after including the vortex-ring state in the model of the main rotor induced velocity

For numerical simulations it was assumed that there is no need for real time simulation and algorithms available in Matlab toolboxes for solving equations of motion and for minimizing the cost functions were used.

## Sample results of simulation

The simulations were performed for various sets of input parameters, especially initial flight conditions.

To investigate whether the H-V zone of the Mi-2Plus helicopter could be minimized for OEI operations the starting points of the simulation were the values of the height above the ground and the forward velocity of the helicopter. It was assumed that at the moment of engine failure helicopter is in a steady flight. After the "engine failure", during the first 0.5s, modeling the pilot reaction time, the controls are kept constant. After that the operating engine starts to increase power to OEI value, and the control of the helicopter is calculated using optimization algorithm at each time step.

The OEI landing simulations were performed for the horizontal flight velocities from 0 m/s to 20 m/s and for heights ranging from 5 m to 50 m.. The final conditions of the safe landing are horizontal velocity below 8.3 m/s, vertical velocity below 2.1m/s and pitch attitude below 8 deg Ref. (21). Sample results from these simulations are presented below

In Fig.3 the simulation results are shown for landing from flight at 25 m above the ground with different horizontal velocities.

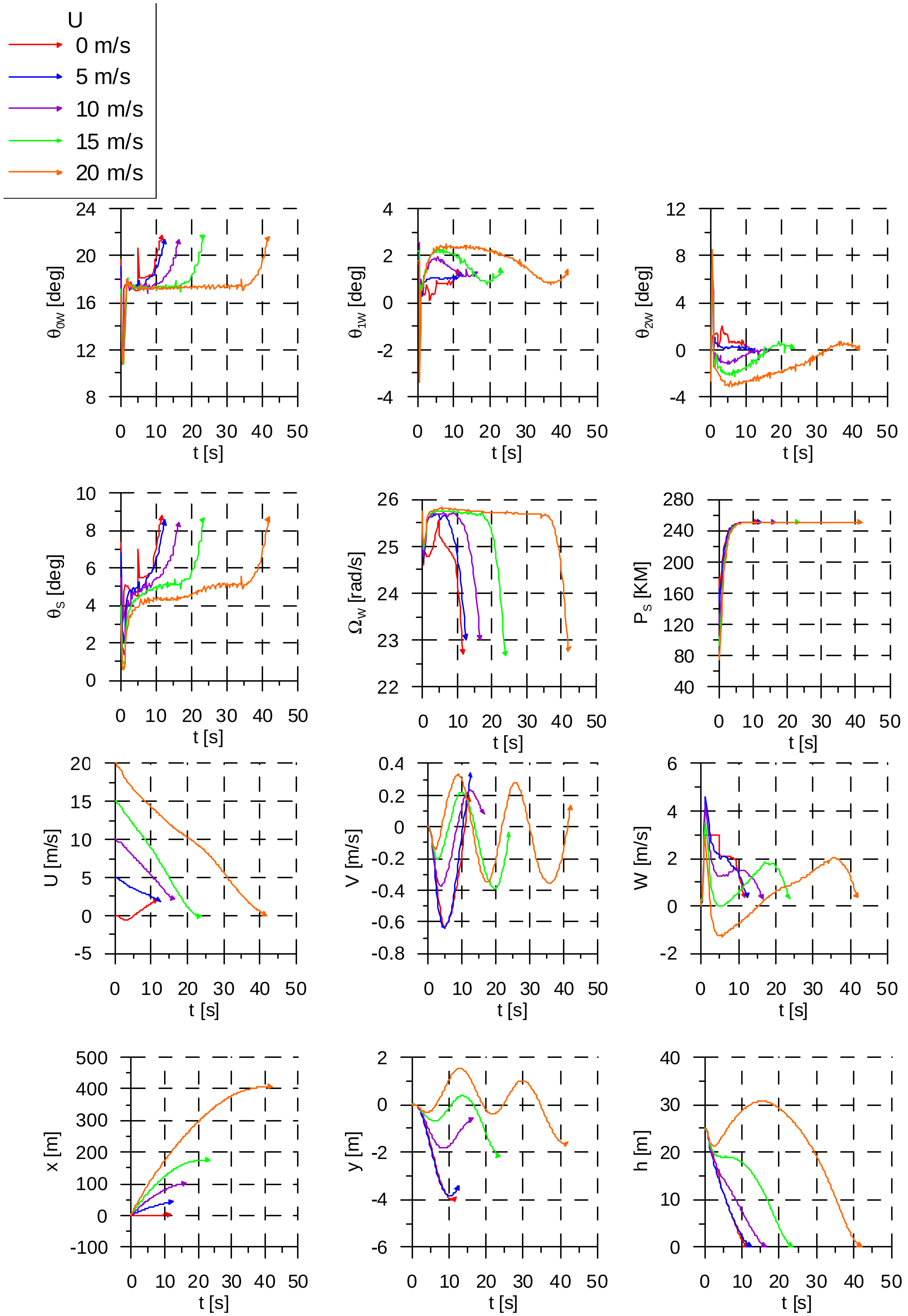
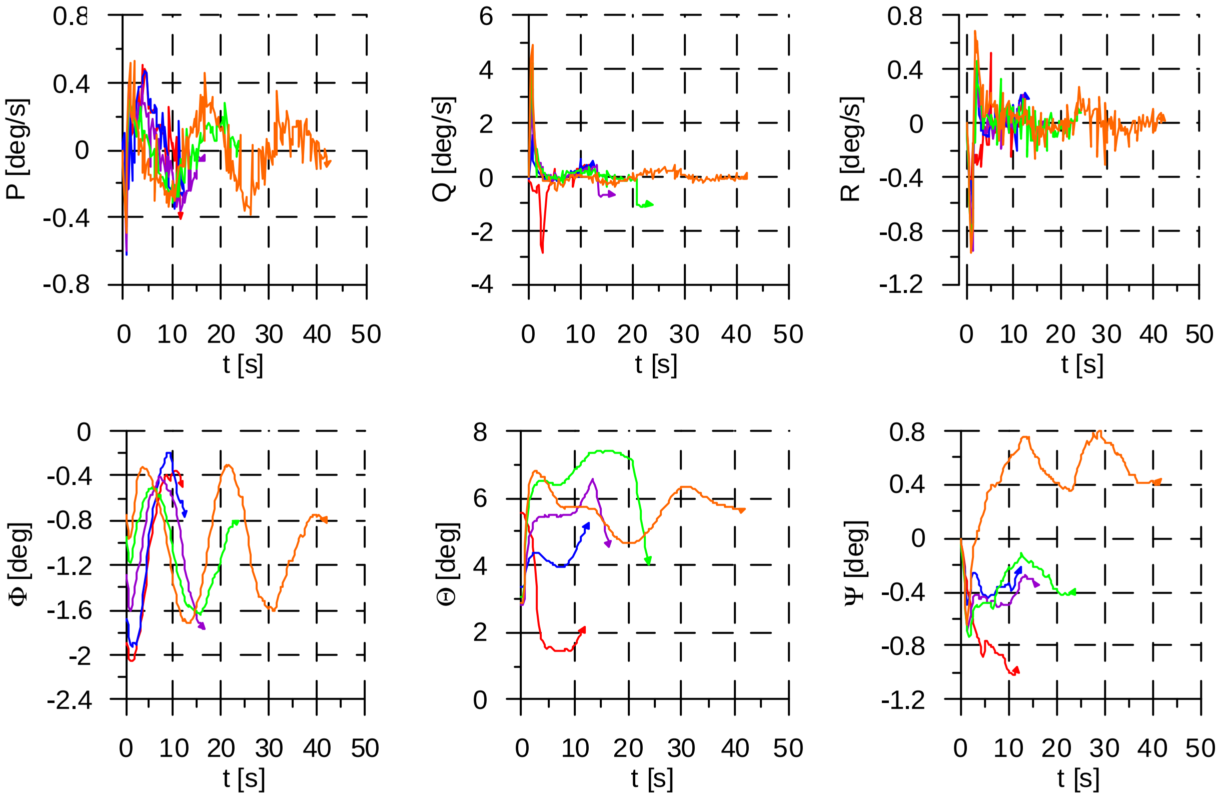


Fig. 3 OEI landing from horizontal flight at 25m above the ground



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It can be concluded (**Error! Reference source not found.**) that OEI landing fom 25 m above the ground can be performed safely for all flight velocities between 0 m/s and 20 m/s. The control variations for each situation are similar, first the collective pitch is decreasing to ensure descending and just before the touch down collective pitch is increased to slow down the horizontal velocity. Also the control in other channels are similar to each other.

To investigate minimization of the H-V zones the possibility for safe landing from hover is investigated , and the results are given in Fig. 4. The safe OEI landing is possible from all investigated heights. It also can be seen, that the landings are not purely vertical. For Mi-2Plus helicopter the vertical descent is limited to 3 m/s due to avoiding vortex-ring zone.

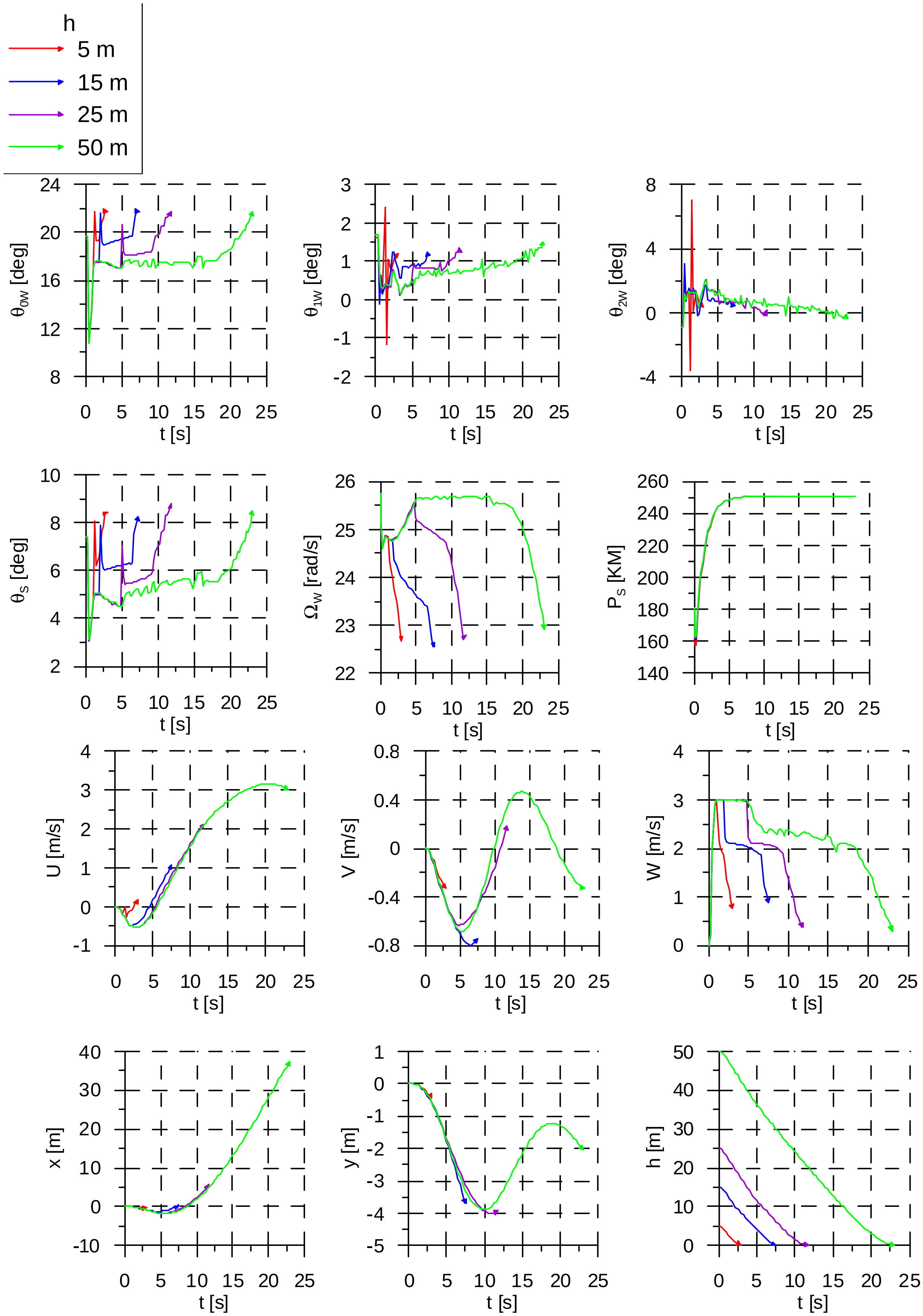

**Fig. 4 OEI landing from hover at different heights**

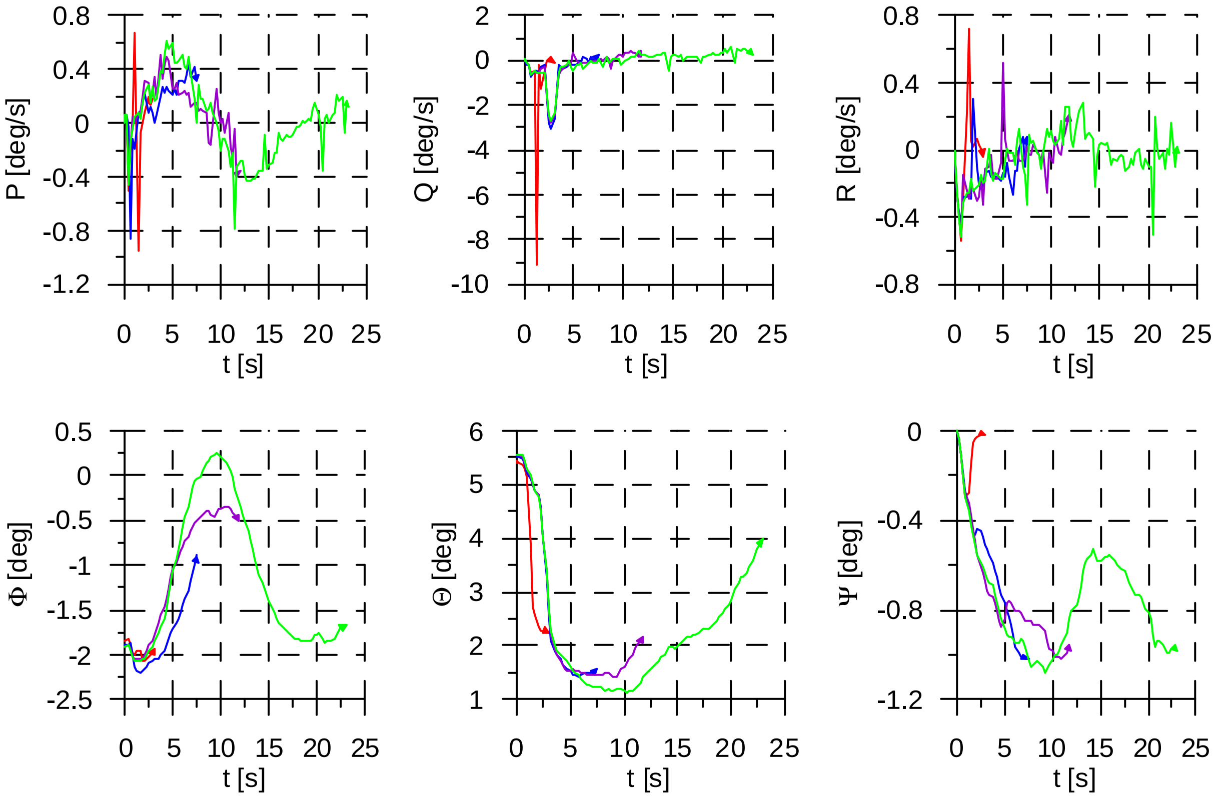

**Fig. 4 continuation**

## Conclusions

From results of simulation it can be concluded, that the model and the control method developed give rational results. Simulations performed show the influence of the weighting factors in the cost functions on the results obtained. A methodology for selection of weighting factors is the prospective objective of the research. Also some research on helicopter mass parameters influence should be performed.

It is very important to include the fuselage 6 DoF in some cases for instance landing in confined areas. The lateral movement of the helicopter cannot be neglected in such flights. It justifies application of a more complex model then I previous studies. But if the model and the method are to be applied in automatic flight control or in flight simulators, the model optimized and more time efficient numerical algorithms applied.

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