MULTILEVEL MATHEMATICAL MODEL OF ROTORCRAFT AERODYNAMICS

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Abstract

The subject of this work is the development of Mathematical Model (MM) of rotorcraft aerodynamics, which allows to model airflow about its different parts such as single main rotor and propeller, wing and tail unit with deflected control surfaces, coaxial main rotor, rotors for tandem, side-by-side and multirotor configurations, as well as combination of rotors and various rotorcraft airframe parts.

MM is based on nonlinear nonstationary theory of a lifting surface. Fluid is considered incompressible and non-viscous. Changes of rotorcraft shape and kinematic parameters of its motion are taken as prescribed. The general equations describing MM, which uses numerical method of discrete vortices as a basis, are presented. Using of MM the total and distributed loads, velocity, fields and unsteady vortex wake can be gained.

As an illustration the results of rotors, their combinations and rotorcrafts airflow modeling at various operating conditions are presented.

1.Introduction

Numeric modeling [1,2,3] offers large potential in detailed investigation of rotorcraft aerodynamics specific nature. The need for improvement of aircraft aerodynamics investigation techniques including transient processes, complex interaction between rotors and other aerodynamic surfaces including influence of separation boundaries, clearly manifests itself. The aerodynamic model based on nonlinear nonstationary rotor theory looks preferable[2,9].

From the theoretical point of view, development of one global MM suitable for investigation of all specific rotorcraft aerodynamic features is difficult in principle and it is difficult to implement it technically. So for various practical applications it is suitable to develop an MM containing elements of different complexity.

Such multilevel MM allowing for investigation of practically all rotorcraft operation conditions can be used for researching the following:

- helicopter main rotor aerodynamic characteristics in main, transient and limit conditions;
- aerodynamic characteristics of various rotor combinations;
- aerodynamic characteristics of convertible aircraft with rotating rotor and X-wing that can be stopped in flight;
- aerodynamic characteristics of a fixed wing aircraft with propellers;
- trailing vortex behind a rotorcraft.

A multilevel MM of rotorcraft aerodynamics is developed on a common algorithm basis, i.e. discrete vortex technique [4] and allows to investigate practically the whole range of rotorcraft/its components operational conditions.

2.Formulation of problem

In a general case a rotorcraft performing unrestricted motions is examined. The rotorcraft has rotors that can turn with respect to its body, wing and horizontal tail. However, the mathematic formulation of the problem can be examined with an example of a propeller/lifting surface combination [6] arbitrarily oriented with respect to the mainstream.

The blades of rotating lift/thrust rotor turn around axis $O_H Y_H$ that, in its turn, rotate with respect to longitudinal axis OX connected with the lifting surface by α_p angle and the whole combination translates at an average speed of $\vec{V}$ and rotates with respect to all axises of the body-axis coordinate system $OXYZ$ at a rate of $\vec{\Omega}$. The main rotor radius R is taken as the typical linear dimension and the main rotor square $F_H = \pi R^2$ is taken as a typical square area. No limitations are posed on the rotor blade configuration, lifting surface and the character of their motion.

The rotor blades and other lifting surfaces are replaced with infinitely thin basic surfaces S_i, (i – number of basic surface). The flow behind the combination demonstrates a developed wake in form of free vortex sheets σ_j (j – number of free vortex sheet)(fig.1).

An ideal incompressible medium is assumed. Everywhere beyond the rotor blades, other lifting surfaces S_i and their wakes σ_j the flow is consi dered to be nonvortex, i.e. for disturbance velocity potential $\Phi(x,y,z,t)$ Laplace equation is true:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0,\ (x,y,z)\notin\left(S_i\cup\sigma_j\right) \quad (1)$$

If $\overline{W}^*$ is lifting surface point velocity caused by translational, rotational and flapping motion and deformation, no-penetration boundary condition is observed in these points:

$$\left(\nabla\Phi - \overline{W}^*\right)\overline{n} = 0 , \ (x, y, z) \in S_i \qquad (2)$$

At transfer through the vortex trail surface σ_j pressure continuity and normal velocity component conditions are observed:

$$p_- = p_+ ; \ (\nabla\Phi\overline{n})_- = (\nabla\Phi\overline{n})_+ , \quad (x, y, z) \in \sigma_j . \qquad (3)$$

Here indexes "–" and "+" refer to different sides of surface σ_j .

For those lifting surface edges from which vortex sheets σ_j flow, Chaplygin-Zhoukovsky hypothesis of finite velocities is true:

$$p_- = p_+ ; (\nabla\Phi\overline{n})_- = (\nabla\Phi\overline{n})_+ , (x, y, z) \in L_j \qquad (4)$$

Here L_j - line of velocity tangential jump surface flow off.

At an infinite distance from rotor/lifting surface combination and its wake disturbances attenuate, hence

$$\lim_{R\to\infty} \nabla\Phi = 0 , \quad R = \sqrt{x^2 + y^2 + z^2} . \qquad (5)$$

Now the problem is to find potential $\Phi(x, y.z.t)$ of distributed velocities on blades and lifting surfaces and in the whole space [2,5].

In the examined model the lines of vortex sheet flow-off are always postulated on sharp trailing edges of rotor blades and other lifting surfaces. Root and leading blade edges are considered to be rounded. So the position of a flow separation zone is determined in the process of calculating the flow around the combination.

Time deformation of the vortex sheet modeling the wake can be defined by the following equality:

$$(\xi, \eta, \varsigma) = (\xi_1, \eta_1, \varsigma_1) + \int_{\tau_1}^{\tau_2} w_0(\xi, \eta, \varsigma) d\tau , \qquad (6)$$

where ξ, η, ς – sheet point coordinates at time moment τ; $(\xi_1, \eta_1, \varsigma_1)$ – at time moment τ_1 ; $w_0(\xi, \eta, \varsigma)$ – components of non-dimensional relative velocity of the medium.

Mathematical setting of the problem for rotating rotor/lifting surface combination is common for all elements of a multilevel MM.

To define the loads affecting lifting surfaces S_i , Cauchy-Lagrange integral is used.

3.Numerical method

Numerical method of the main rotor problem solution in nonlinear nonstationary setting using discrete vortices method involves space and time discretization. Continuous vortex layers, used to model basic surfaces of the rotorcraft, other lifting surfaces and their vortex sheets, are replaced with discrete vortex systems and time continuous process of boundary conditions/flow parameters changing, that is continuous in time, is replaced with a step process.

Sheet vortex model proves to be functionally connected with the rotor blade vortex model that demands for elaboration of specific vortex systems for various rotor blade flow conditions. For example, a vortex system based on discrete vortex segments is conceived to be justified for modeling of the main rotor aerodynamic characteristics in blade nonstalling flow conditions. That allows to use available computing power more efficiently. One of plausible blade vortex systems is presented in fig. 2. Lateral vortex filaments are numbered from the leading to trailing edge and longitudinal filaments from root to tip. The system of equations allowing to determine unknown vortex intensities at every specified moment r looks like:

$$\sum_{m=1}^{K}\sum_{k=1}^{N+1}\sum_{\mu=1}^{n} \Gamma_{\Sigma m\mu k}^{\mu k-1r} a_{m\mu k\nu}^{\mu k-1pp-1} + \sum_{m=1}^{K}\sum_{k=1}^{N+1} \delta_{mkk-1}^{(1)r} a_{mn+1k\nu}^{n+1k-1pp-1} = H_{m\nu}^{pp-1r} \qquad (7)$$

$$\sum_{\mu=1}^{n} \Gamma_{\Sigma m\mu k}^{\mu k-1r} + \delta_{mkk-1}^{(1)r} = -\sum_{s=1}^{r-1} \delta_{mkk-1}^{(1)s} , \qquad (8)$$

where ν=1, 2,...n; p=1, 2,..., N+1; m=1, 2,..., $K_л$.

Functions a under the sign of sums in equation (7) present known analytical expressions and depend upon the coordinates of vortex segments and points where these functions are calculated. The first equations (7) of the system represent a no-penetration condition (2) in reference points marked in fig. 2 by crosses and equation (8) represents a condition of closed loop circulation constancy. The right parts $H_{m\nu}^{pp-r}$ of equation (7) are functions of the rotorcraft geometric characteristics and kinemetic parameters of its motion, wake shape and motion background.

The values of kinematic parameters remain unchanged within the frame of one time step. At each time step starting from the first one after solution of equation system (7,8) all vortex segment intensities for the system of blades and their wake are determined. Blade loads are obtained using Cauchy-Lagrange integral over defined total vortex intensities. Distributed and total load characteristics are obtained by summing up aerodynamic loads by panels.

Another approach to investigation of rotorcraft rotor aerodynamic characteristics including those in stalling conditions is based upon direct modeling of rotor operation with allowance for a blade stall. Since a stall propagates rather fast along the blade chord an assumption is introduced that in those blade sections where the stall is identified it starts from the leading edge and propagates along the blade radius.

Fig. 3 presents the calculation pattern used. Kinematic characteristics of blade section motion $\omega_H r$, $\varphi(r)$, $\alpha(r,\psi)$, $V_{nep}(r)$, $\mathrm{Re}(r)$ are defined from a nonstalling flow model. Using these parameters a two-dimensional separated flow of a main rotor blade section is calculated and dependence of the stall position of blade section load $\bar{x}_{sep}\left(C_y{'}\right)$ is obtained allowing to define a load value at the beginning and at the end of the stall in this blade section and, hence, to define the blade stall area. Next the main rotor is calculated with consideration to its stalling flow. Peculiarity of a vortex system modeling the blade and its wake consists in introduction of a closing vortex sheet located at the inner boundary of separation zone.

In order to model rotor blade stalling flow when blade separation zones can appear and disappear in the process of blade motion, it seems more rational to use a modular principle of vortex system generation when the blade basic surface presents an individual module having an independent vortex system and free vortex sheets are attached along the flow separation lines and connected with each other also as modules. All modules are modified by four quadrangular closed vortex frames with permanent circulation along the parameter (fig. 4).

System of linear equations algebraic over unknown Γ_μ circulations of attached vortex frames expressing no-penetration condition of the rotor basic surfaces in reference points ν with radius-vector $\vec{r}_{Sk\nu}$ is written as:

$$\sum_{k=1}^{K}\sum_{\mu=1}^{N_{Sk}} a_{\nu\mu}(\vec{r}_{Sk\nu},\tau)\Gamma_\mu + \sum_{k=1}^{K}\sum_{p=1}^{K_p}\sum_{i=1}^{N\sigma_{pk}} a_{pi}(\vec{r}_{Sk\nu},\tau)\delta_{pi} = -4\pi\vartheta_\Pi(r_{Sk\nu},\tau) \qquad (9)$$

Here $a_{\nu\mu}$ и a_{pi} - influence functions calculated as velocity normal components at reference point ν;

ϑ_Π - normal component of point ν translational speed; δ_{pi} - circulation of vortex sheet frame.

Aerodynamic blade loads are calculated using Cauchy-Lagrange integral.

Dependence $\bar{x}_{sep}\left(C_y{'}\right)$ for a helicopter profile is defined using nonstationary boundary layer theory. A technique for calculation of solid profile two-dimensional separated flow is presented [7]. It is based on synthesis of incompressible non-viscous fluid parameter calculation under discrete vortex method and numeric integration of plane nonstationary boundary layer equations.

Separation of a boundary layer is defined thus: boundary layer is calculated at each section of its existence from critical point K (fig.5) to a point where either (1) surface friction becomes extremely small or (2) iteration process becomes divergent in solution of the boundary layer equations. In separation points R all boundary layer vorticity is ablated. The wake is further modeled by free vortices the intensity of which is determined by this vorticity.

As the profile angle of attack α increases with time bringing about an increase of load C_y, at a certain moment the flow gets separated from the trailing edge ($C_{y_{HC1}}$ in fig.6) and the stall moves to leading edge ($C_{y_{HC2}}$) and covers the whole upper part of the profile. Later on, as α decreases and, consequently, as C_y stalling point moves to the trailing edge at a practically stable value of C_y (C_{ykc} in fig. 6), two C_y values are introduced, i.e. $C_{y_{HC}}$ and C_{ykc}. When the $C_y = C_{y_{HC}}$ blade section value is reached in calculations vortex sheet is introduced in this section and when C_y reduces to C_{ykc} the sheet disappears, i.e. blade flow becomes nonstalling.

The mathematical model was validated in several stages. Analytical results were compared with ADT and flight test results and those obtained by other researches. Analytical results correlate well with the data obtained in physical experiments undertaken to evaluate total and distributed aerodynamic characteristics of blade sections, main rotor blades, rotor/lifting surface and main/tail rotor combinations

4.Results of modeling

While examining aerodynamic characteristics of an isolated main rotor it is necessary to note that within the frames of the present paper the limit conditions of the main rotor operation are of the most interest. A helicopter main rotor flow in conditions described by such parameters as $\bar{V}_н = 0.25$; $\alpha_н = 15^\circ$; $\varphi_0 = 13.5$ [8] was modeled. The calculations were performed according to the pattern presented in fig.3. It is found that the flow separated from the blades at azimuth $\psi_л$=210° and the stall persisted to $\psi_л$=50°. Comparative analysis of $C_т=f(\psi_л)$ curves in stalling and nonstalling blade conditions (fig.7) showed that, due to stalling flow, within $\psi_л$=210°...50° azimuth range the blade thrust coefficient $C_т$ is considerably less than that possible in nonstalling flow conditions. The largest drop of $C_т$ amounting to 35% was noticed at $\psi_л$=250°. The largest amplitude of $C_т$ variation occurred at $\psi_л$=320°...350°. To find the reason for such running of $C_т=f(\psi_л)$ curve, configurations of vortex sheets and distributes speed field were obtained for the main rotor stalling flow. Analysis of the vortex sheet configuration (fig.8) showed that starting with $\psi_л$=210°, the stalling zone gradually expands to the blade root sections. Simultaneously a separation flow develops over its upper surface that explains stable decrease of the aerodynamic load. This is confirmed by the normal force $C'y=f(\bar{r})$ coefficient spanwise distribution in the blade sections (fig.9).

In flight operations very often the helicopter has to fly near an arbitrarily oriented boundary surface. Position of the boundary surface (inclination angle γ and relative distances $\overline{H} = H/R$) have a considerable effect upon the main rotor thrust and moment characteristics. Fig. 10 presents thrust and overturning moment of a rigid main rotor versus the position of a boundary surface. The velocity field in the vicinity of the main rotor (fig. 11) allows to analyze the run of the rotor aerodynamic characteristic curves.

Operation of a rotor combination at the boundary surface is characterized by increased mutual influence. In fig. 12a,b the results of modeling combined main/tail rotor operation near a horizontal surface and influence of the main rotor rotation direction upon its thrust is shown.

Fig. 12,c presents influence of the upper rotor (1) upon the lower rotor (2) in a coaxial combination of rotors (3) when they operate near a horizontal surface as compared with a single rotor (4).

A flow around convertible aircraft with tilt rotors was also modeled. Complicated interaction between tilt rotors and fuselage was investigated in transient flight conditions. Fig. 13 presents a trailing vortex configuration that allows to analyze aerodynamic loading of its components. In fig. 14 distribution of thrust coefficient along the wing and horizontal empennage of a convertible aircraft is shown for tilt rotor axis positioned at $\alpha_p = 60^\circ$.

5.Conclusion

A multilevel mathematical model of rotorcraft aerodynamics is designed for investigations in a wide range of flying conditions with the aim to expand the rotorcraft application sphere and to increase its flight safety. It allows to model both everyday operation flight conditions and more complicated and specific conditions. A mathematical model presents new additional capabilities for aerodynamic design and modernization of rotorcraft.

References

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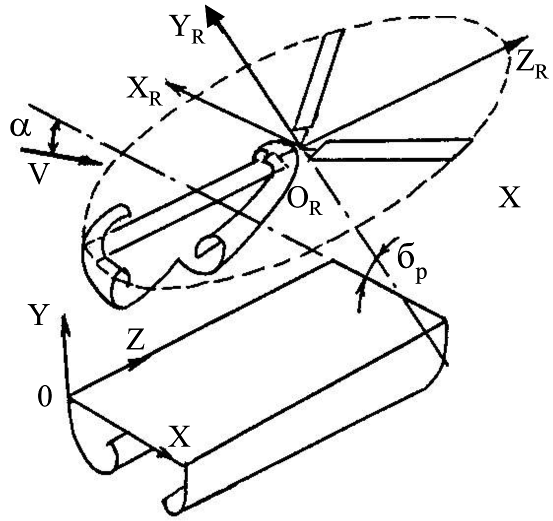


Fig. 1. Modeling of rotor/lifting surface combinations

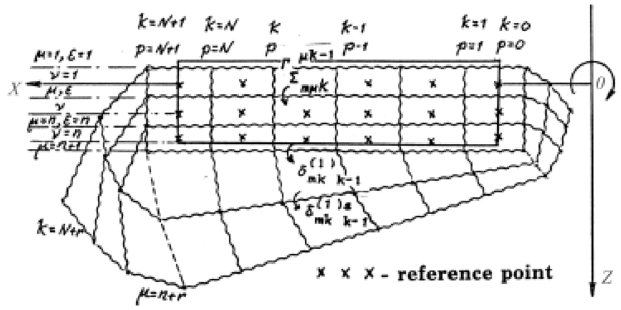


Fig. 2. The vortex system of a blade

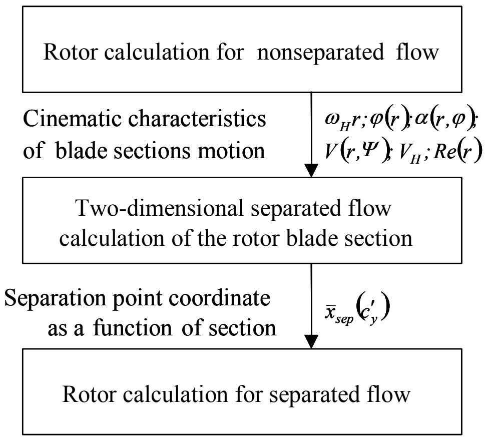


Fig. 3. Calculation pattern

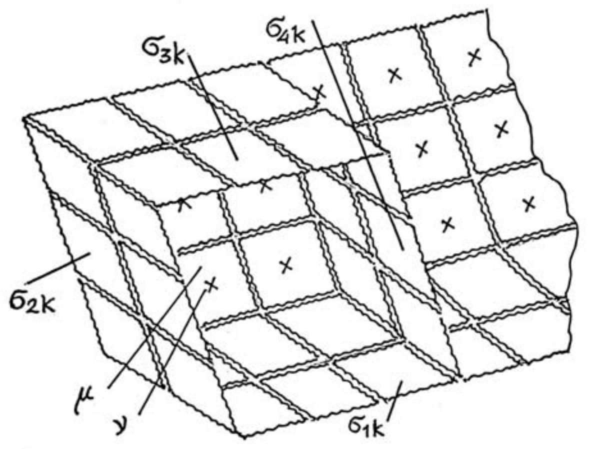


Fig. 4. The vortex system of a blade for separated flow

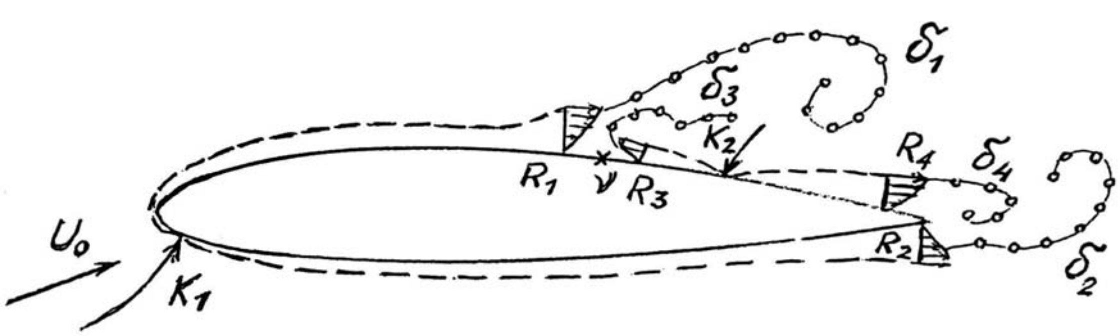


Fig. 5. Calculation pattern

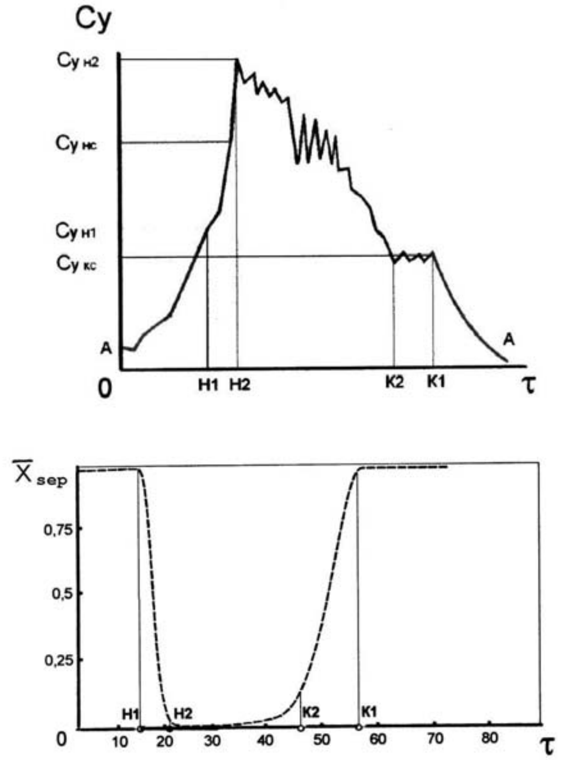


Fig.6. A time variation of the load C_y of an airfoil and separation point coordinate $\bar{x}_{sep}$

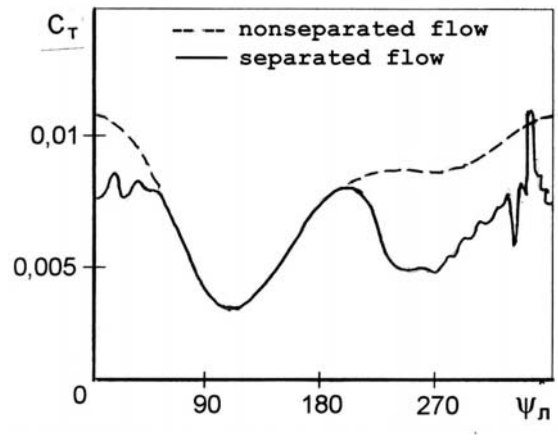


Fig. 7. Azimuth angle variation of thrust coefficient of the blade

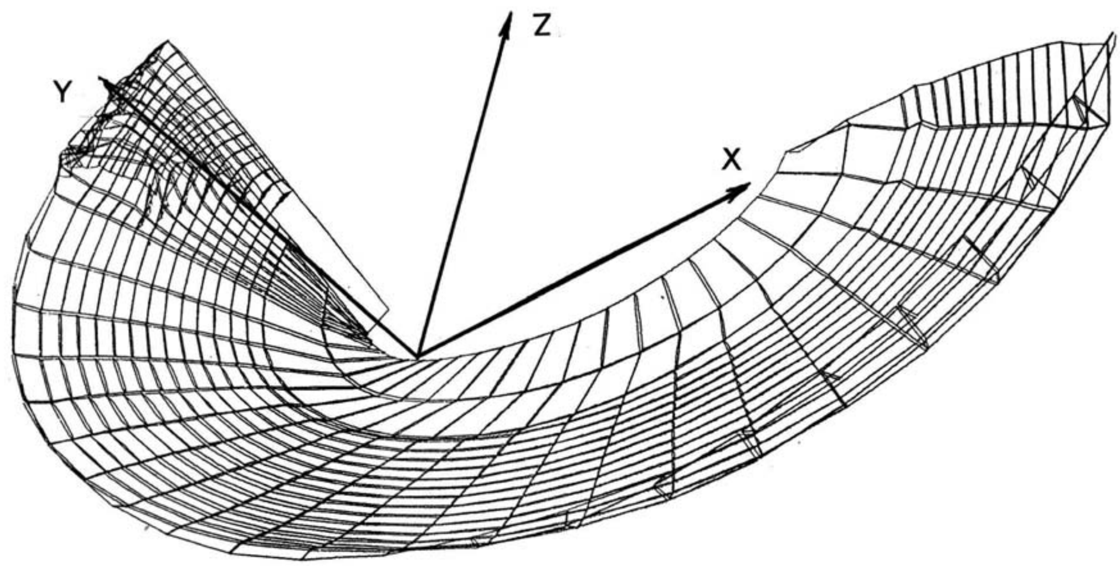
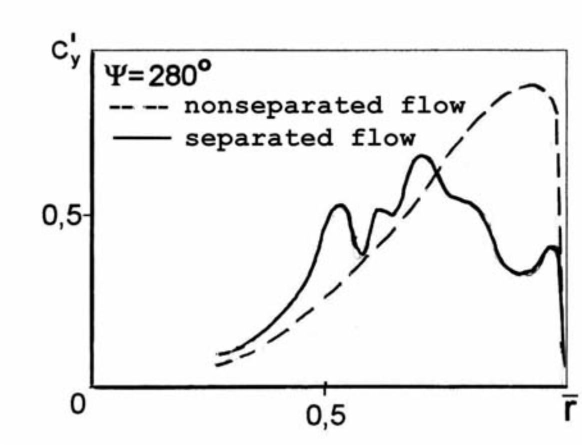


Fig. 8. Development of vortex wake behind a blade



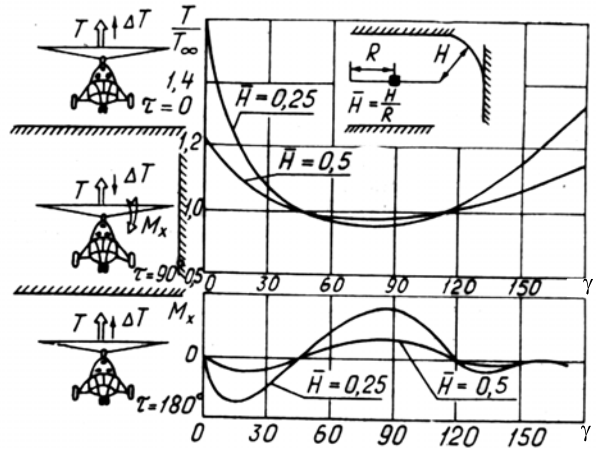


Fig. 9. Spanwise lift coefficient distribution

Fig. 10. Influence of boundary surface position on the helicopter characteristics

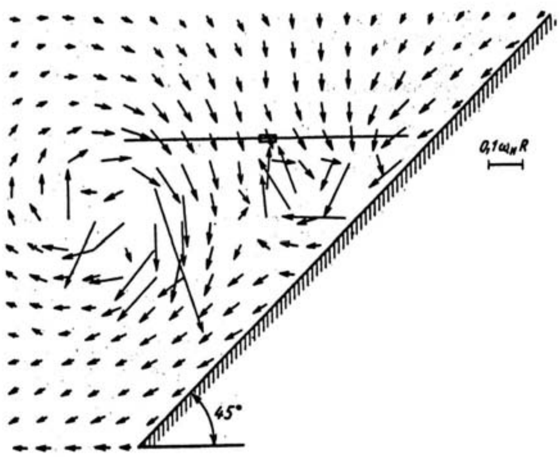


Fig. 11. Velocity field around a rotor

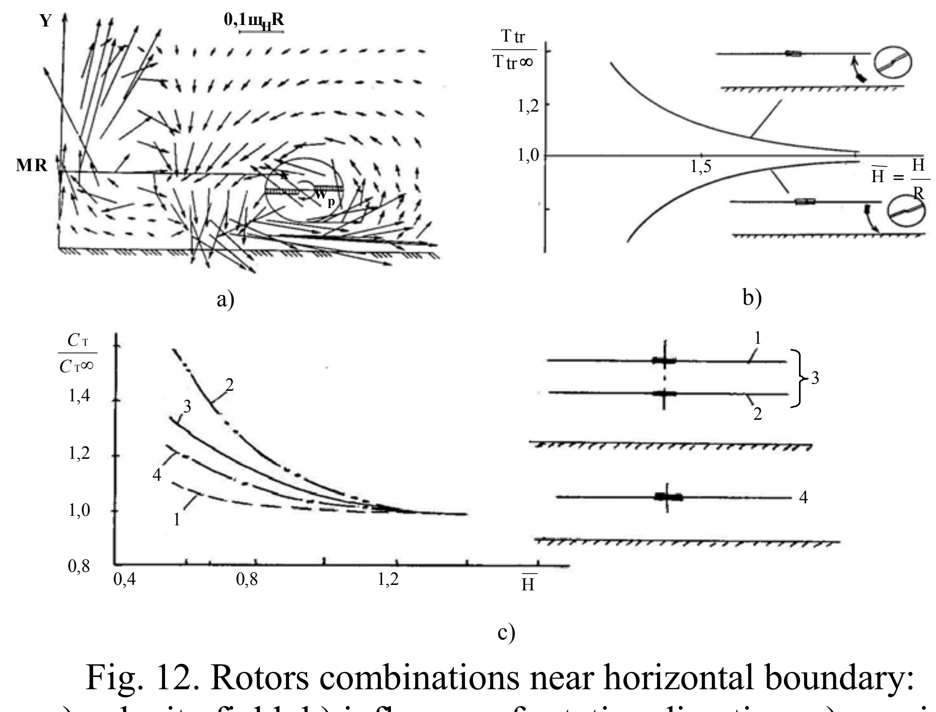


Fig. 12. Rotors combinations near horizontal boundary: a) velocity field; b) influence of rotation direction; c) coaxial rotors

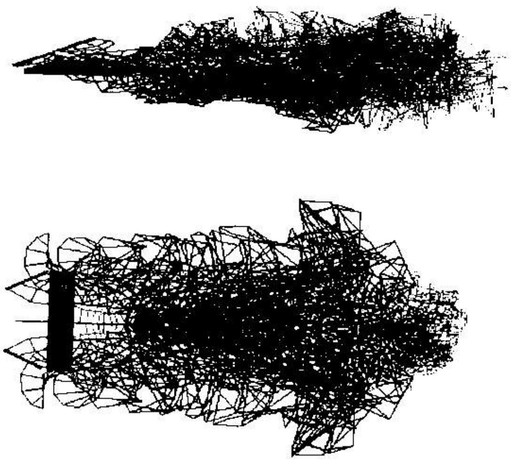


Fig. 13. Configuration of the vortex wake behind a convertible rotorcraft

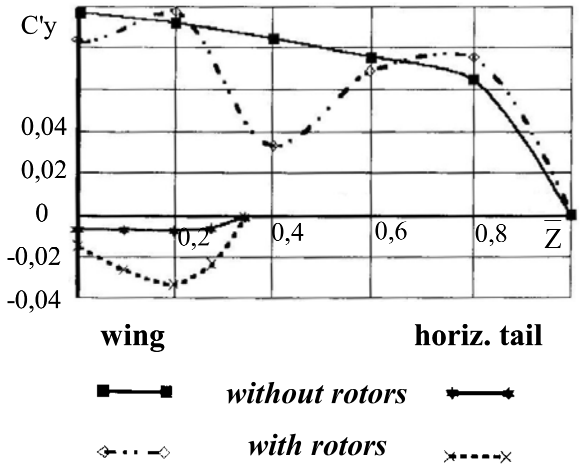


Fig. 14. Lift coefficient distribution on the wing and horizontal tail of the rotorcraft