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A COMPUTER BASED STUDY OF HELICOPTER AGILITY, INCLUDING THE INFLUENCE OF AN ACTIVE TAILPLANE.

by

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Abstract

A computer-based study of helicopter agility in longitudinal manoeuvres has been made, comparing the relative agility capabilities of four helicopters with the same stiff-flapwise rotor, but different horizontal tailplane configurations. It is assumed that the flight paths are generated by an unconventional flight control system that demands changes in the velocity vector directly. Agility is quantified by applying ratings to paths in space, and the agility capability of each helicopter is determined by whether or not it can fly the manoeuvre represented by the rating, subject to the limiting hub moment. The vehicle equations of motion are solved using an inverse method which allows calculation of the attitude and control time histories given the helicopter's trajectory and speed. It was found that the inverse method is algebraic only for certain classes of problem, and that more generally the resulting system of equations is represented by a set of differential equations in state-space form. The most agile configuration at the speed studied (185 kmh^{-1}) is the helicopter with a controllable tailplane, the least agile that with a fixed tail. The former can fly bobups to 30m up to 10% more quickly than the latter, and as a result requires less airspace to manoeuvre. It is suggested that the tailplane would need to be actively controlled, and an integrated element of the helicopter FCS if improved agility in this type of manoeuvre is required. This is because the tailplane control law is a function of the three rotor controls and the pitch rate, and requires full control authority.

List of Symbols

A	system matrix
B	control matrix
C,D,E,F	matrices of coefficients
V	helicopter velocity along flight path (ms^{-1})
V_e	helicopter trim velocity (ms^{-1})
u,w	perturbation velocity components along body axes (ms^{-1})
p,q	perturbation roll and pitch rates about body axes ($degs^{-1}$)
h,s	manoeuvre height and distance (m)
k_1,k_2, etc	gains in tailplane control law
t,t_f	time and total manoeuvre time (sec)
L_u, etc	derivative with respect to perturbations in u, etc.
α	angle of attack (deg)
α_s	tailplane control angle (deg)
γ	flight path angle (deg)
θ	pitch attitude perturbation (deg)
θ_e	trim pitch attitude (deg)
θ_{1s},θ_{1c}	longitudinal and lateral cyclic pitch (deg)
θ_0	collective pitch (deg)

1. Introduction

The emerging requirements of future combat helicopters have resulted in studies to investigate ways in which agility, and therefore survivability, can be improved. Future *modes* of operation have been identified, ref. [1], two of which will involve manoeuvres in the vertical plane: high speed, low level transit to the operating zone; and nap-of-earth (NOE) flight when there, at speeds considerably greater than currently possible. This could place heavy demands on the airframe (in terms of speed and "g" capability) and the pilot. Design studies undertaken in the context of these requirements have demonstrated the suitability of the stiff-flapwise rotor configuration, ref. [2], and the desirability of advanced, integrated flight control systems (FCS) for reducing pilot workload, refs. [3], [4]. A disadvantage of this type of rotor system is that it can pose metal fatigue problems due to potentially large hub moments associated with blade flapping; as will be shown however, the hub moment limitation can be made less restrictive by use of an actively controlled horizontal tailplane. This paper assesses the implications for helicopter agility of using the stiff rotor configuration along with different levels of tailplane control. It is assumed that some kind of advanced FCS is available of the type that dispenses with the traditional pattern of control to give the pilot direct control of flight path parameters such as speed, load factor or climb angle. The present study makes use of an inverse solution of the helicopter equations of motion, which obviates the need to consider the design and implementation of the FCS in detail.

2. Agility and the Flight Path

The question of what agility is must be addressed before any progress can be made with its analysis. When something is described as agile, the intuitive idea is that it can change *speed* and *position* rapidly, even violently, but with absolute precision, in order that its task may be fulfilled in the shortest possible time. This is generally true of aerospace vehicles and so agility, as pointed out by Tomlinson and Padfield, ref. [3], embraces aspects of two, sometimes separate, areas of aircraft design, namely performance and handling qualities. In this paper the emphasis is on performance, in its widest sense, rather than handling qualities. In particular, agility is evaluated through consideration of longitudinal manoeuvring performance, which may be limited not simply by installed power or rotor thrust, but by parameters like rotor hub moment or blade flapping angle. In general, then, given that agility is limited by the need to keep a number of performance-related parameters within bounds, the aim is firstly to quantify agility and secondly to determine its value for several helicopter configurations.

For this work, agility was quantified through use of an *agility rating*, which was based on features the authors consider to be fundamental to the concept of agility, namely the geometry of the manoeuvre and the time taken to manoeuvre. The former reflects the tightness of the manoeuvre while the latter is a measure of how swiftly the manoeuvre is performed, and the two together can be a good guide to the overall loading on the helicopter. In effect the path in space is assigned a rating, *not* the helicopter, and the relative capabilities of several configurations are reflected in whether or not they can execute the path without exceeding a performance limitation. The concept of applying ratings to paths in space as a means of quantifying agility is discussed briefly in ref. [3]. Here the agility rating was defined as

$$AR = t_f \int_{s_0}^{s_f} h.ds, \qquad \text{figure 1.}$$

The rating was assigned to each member of a family of paths in space. It needs to reflect the increased level of agility required to fly a path that is more demanding than another, in that it occupies less space and is flown in less time. It can be seen that with this rating, as the required level of agility increases, then the rating will tend to zero. The family of paths cannot be *completely* general, but rather must be associated with a fairly well defined task so that the ratings assigned to different paths are directly comparable. In the present study it was sufficient to define the task in terms of a set of boundary conditions

$$\gamma_0 = \gamma_f = 0$$

$$h_0 = 0, \ h_f = 50$$

Speed is not included as a condition because incorporating time into the rating takes account of variations in speed. The paths can be seen to represent obstacle-clearing manoeuvres, or popups. It is important to investigate measures of agility associated with this type of manoeuvre as it is a basic element of manoeuvring flight, especially in an NOE environment. Quantifying agility in geometric terms is not new. Brotherhood and Charlton, ref.[5] for example, define a turn agility factor in terms of speed and geometric features of the turns flown in a series of flight experiments.

The geometry of the paths and the way in which they were assumed to be flown were selected with a particular FCS in mind. It was assumed that the FCS provided the pilot with the capability to command speed and flight path angle independently - ie. it was a "manoeuvre demand" system. Thus the flight paths are flown at constant speed and their geometry is such that they are piecewise-linear in the rate of change of flight path angle γ. A typical function of $\dot{\gamma}$ is shown in figure 2a with the resulting time history of γ in figure 2b. The trajectory defined by this function of γ is given in figure 3. It is feasible that the FCS could schedule control inputs to the helicopter in such a way that movement of a single control inceptor by the pilot would result in the helicopter flying the specified path. Of course, this is an over-simplified representation of obstacle clearance in that it ignores the detailed dynamics of the pilot/ autopilot/ system interaction, which would depend both on the pilot's perception of the task and on the design of the FCS. But these factors impinge more on handling qualities than performance. In this study, it is taken for granted that the FCS confers handling qualities which allow maximum advantage to be taken of the available performance.

Forcing a mathematical model of a helicopter to fly desired manoeuvres has been undertaken successfully in the past. Wood et al, ref.[6], describe a *Maneuver Criteria Evaluation Program* which models the execution of certain manoeuvres by a helicopter, based on general features of the manoeuvres that are specified. Haverdings, ref.[7], defines idealised manoeuvres in which the trajectories are tightly constrained, but allows for deviation from the ideal in the execution of the manoeuvres. The approach taken here is different from both of these: not only is the geometry of each flight path exactly specified, but the helicopter is assumed to stick rigidly to the desired path. While this is not achievable in practice, it is adequate for the purpose of assessing the relative agility of different helicopter configuration.

3. Inverse Solution of the Vehicle Equations of Motion

The vehicle equations of motion used in this study were of the linearised rigid-body derivative type. The rotor model is based on that of Bramwell, ref.[8] where the contributions of blade flapping to rotor hub moment are assumed to come only from the first blade mode shape. Previous studies had indicated that a very good approximation to the longitudinal modes of the helicopter configurations tested here could be obtained by adding the rolling moment equation and associated cross-coupling derivatives to the 4th. order system that represented purely longitudinal motion. The resulting system of order 5 can be written in state-space form as

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \text{ ----- (1), where}$$

$$x = [\ u\ w\ q\ \theta\ p\ ]^T$$

$$u = [\ \theta_{1s}\ \theta_0\ \theta_{1c}\ ]^T$$

Equation (1) is normally solved for the vector of state variables x given the control vector **u**. Assuming that the trim state of the helicopter is known, the flight path and attitude time histories that result from the inputs u can be constructed. The inverse solution consists essentially of calculating **u** given **x**. At first sight, the inverse solution appears to be algebraic in nature, but this is only the case for special classes of problem - where the number of independent controls is equal to the number of degrees of freedom, which is clearly not the case here. The case of the helicopter with a tailplane, *independently* controllable, is different: here the number of independent controls equals the number of degrees of freedom. Correspondingly the pitch attitude time history can be specified *a priori*. In this study, the tailplane is not independently controllable and only the velocity vector, ie. a combination of trajectory and speed, is specified. It might then appear that an inverse solution of equation (1) poses an intractable problem, having algebraically more unknowns than equations. The system can then be recast not algebraically but as a set of differential equations in state-space form. This is now described for the system given in equation (1).

A formalised statement of the inverse procedure is as follows: for purely longitudinal motion,

$$\dot{p} = p = 0$$

So the rolling moment equation is

$$L_u u + L_w w + L_q q + L_\theta \theta + L_{\theta_{1s}} \theta_{1s} + L_{\theta_0} \theta_0 + L_{\theta_{1c}} \theta_{1c} = 0$$

An expression for θ_{1c} may be obtained from this which can be substituted into the other equations in (1). The state-space description becomes on rearranging,

$$\dot{\mathbf{x}} = A_1 \mathbf{x} + B_1 \mathbf{u}, \text{ where}$$

$$\mathbf{x} = [\ q\ \theta\ u\ w\ ]^T \text{ and } \mathbf{u} = [\ \theta_{1s}\ \theta_0\ ]^T$$

Further, partitioning the resulting system gives

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} \mathbf{u}$$

where $\mathbf{x}_1 = [\ q\ \theta\ ]^T$ and $\mathbf{x}_2 = [\ u\ w\ ]^T$. Then

$$\dot{\mathbf{x}}_1 = A_{11}\mathbf{x}_1 + A_{12}\mathbf{x}_2 + B_{11}\mathbf{u} \text{----- (3)}$$

$$\dot{\mathbf{x}}_2 = A_{21}\mathbf{x}_1 + A_{22}\mathbf{x}_2 + B_{21}\mathbf{u} \text{----- (4)}$$

Equation (4) can be written as

$$\mathbf{u} = B_{21}^{-1}\dot{\mathbf{x}}_2 - B_{21}^{-1}A_{21}\mathbf{x}_1 - B_{21}^{-1}A_{22}\mathbf{x}_2 \text{ ----- (4a)}$$

so (3) becomes

$$\dot{\mathbf{x}}_1 = (\ A_{11} - B_{11}B_{21}^{-1}A_{21}\ )\mathbf{x}_1 + B_{11}B_{21}^{-1}\dot{\mathbf{x}}_2 + (\ A_{12} - B_{11}B_{21}^{-1}A_{22}\ )\mathbf{x}_2 \text{ --- (5)}$$

Figure 4 shows the relationship between sets of body-fixed, flight path and earth axes, from which expressions for u and w are obtained, viz.

$$u = V\cos\alpha - V_e\cos\theta_e$$

$$w = V\sin\alpha - V_e\sin\theta_e$$

$$\alpha = \theta + \theta_e - \gamma$$

Thus both x_2 and $\dot{x}_2$ can be expressed in terms of x_1 and the variables that define the flight path at time t, giving

$$x_2 = Cx_1 + D$$

$$\dot{x}_2 = Ex_1 + F$$

Then on substitution into (5) the new state-space description is

$$\dot{x}_1 = A'x_1 + B' \text{ ----- } (6), \text{ where}$$

$$A' = A_{11} - B_{11}B_{21}^{-1}A_{21} + B_{11}B_{21}^{-1}E + (A_{12} - B_{11}B_{21}^{-1}A_{22})C$$

$$B' = B_{11}B_{21}^{-1}F + (A_{12} - B_{11}B_{21}^{-1}A_{22})D$$

Note that the resulting system, while still described by a set of linear ordinary differential equations, may no longer be *invariant*, as A' can be a function of the velocity vector, itself some prescribed function of time. The system will be invariant if it is assumed that $\dot{\gamma}$ and γ are small. Then we will have

$$x_2 = D$$

$$\dot{x}_2 = F$$

Equation (6) can be solved by using a numerical integration technique, and the control time histories then obtained explicitly from (4a).

4. Results

The configuration of the baseline helicopter is given in table 1. A set of attitude and control time histories are presented in figure 5 for this helicopter flying one of the family of popup manoeuvres. The main feature to note is that the collective pitch time history corresponds in form at least, to that of the flight path angle. Lateral cyclic pitch inputs are not insignificant, which is not surprising considering the strong pitch/roll cross-coupling with this rotor configuration. The longitudinal cyclic pitch controls the pitching motion of the helicopter;

in the pullup segement of the manoeuvre, attitude changes are fairly small, caused by the cyclic moving forwards opposing the sizeable pitching moment from collective inputs. In the pushover segement the cyclic moves relatively far forwards, creating a large nose-down pitching moment, as is reflected in the pitch attitude response. In figure 6 the longitudinal component of the total hub moment is given, and in figure 6a is broken down to show the contributions of the three rotor controls. It can be seen that all three contribute significantly to the overall hub moment.

The horizontal tailplane will produce principally a pitching moment, and therefore influence the position of the longitudinal cyclic during the manoeuvre. The control strategy for the tailplane is then to produce a lift and pitching moment, in addition to the rotor, that will cause the longitudinal cyclic to move in such a direction that the hub moment will be reduced. The general form of the control law investigated was

$$\alpha_s = k_1 q + k_2 \theta_{1s} + k_3 \theta_{1c} + k_4 \theta_0$$

Generally the gains k_i, i=1,4 could be varied individually. However in this study it was found that adequate results were obtained when the gains were such that the tail control angle α_s was proportional to the hub moment. It may be important to reduce hub moment, as it can be considered a performance parameter that can reach a limit in manoeuvres and therefore by definition limit agility. For a given rotor, the use of the tailplane as a control is the only obvious way of reducing hub moment. As noted by Hohenemser, ref.[9], incorrect use of the tailplane can lead to excessive hub moments, the corollary being that correct use can reduce it.

The family of popup manoeuvres were flown at 185 kmh^{-1} (100knots), and the limiting manoeuvres for each helicopter configuration obtained. Agility ratings for these cases are shown in the "agility diagram" of figure 7, where the agility ratings are plotted against the respective values of h/s. There are several interesting features of this diagram - firstly, the limiting manoeuvres for all four configurations lie on a locus of points. This should not be surprising as the manoeuvres are all geometrically similar, with the same boundary conditions. Secondly, moving along this locus to the right requires higher levels of agility, as the rating is tending to zero and the ratio h/s is increasing. Thirdly, each helicopter configuration has several limiting manoeuvres - this point is expanded in the discussion but for now it is the ultimate level of agility that each configuration can achieve that is desired. The area of points which are of interest are shown in the exploded view of figure 7. In this sense then the least agile configuration is the helicopter with a fixed tailplane, the most agile that with the active tailplane. Of the other two configurations, the tailless helicopter is only slightly less agile than the active tail case, while the configuration with the tail geared to the longitudinal cyclic is only marginally more agile than the helicopter with the fixed tailplane. A physical interpretation of the differences in agility is given in figure 8, where the limiting trajectories of the least and most agile helicopters are shown. The most agile helicopter can fly a popup to 50m that intuitively requires more agility than the least agile configuration - the manoeuvre is tighter, requiring less airspace and can be started about 35m closer to the obstacle. This represents a saving in manoeuvre distance and time of about 10%. The tailplane control input during the limiting manoeuvre in figure 8 is given in figure 9, and it is obvious that the tail control system requires high authority, in order that the benefits in agility are achieved.

5. Discussion

The three main aspects of this paper require further discussion. They are however sufficiently self-contained to be dealt with separately.

It has been assumed that helicopter agility can be assessed by examining features of the flight path (the geometry and time taken to fly) which can then be combined in an "agility rating" that quantifies the level of agility required to fly a manoeuvre. The rating is not simply the time to perform the manoeuvre specified by the boundary conditions, but includes an assessment of the "tightness" of a specific path. What results is a measure of helicopter agility that quantifies some intuitive idea of the level of agility needed to fly a given path. The rating in this form has advantages over others in terms of uniqueness, giving a measure of agility that is not qualified by speed, for example. This type of analysis seems particularly amenable to a computer-based study where performance limitations as they pertain to the kinematics of agility are examined. However, as was noted, each helicopter has a series of limiting manoeuvres, and each one is flown differently - those to the left on the locus in figure 7 are flown with gentle, relatively lengthy pullups and severe pushovers, while those to the right are the opposite. This tends to suggest that the *style* of the manoeuvre then becomes important to the analysis, if each helicopter is to be represented on the locus by a single point, and this will depend on the pilot's perception of the task and his interaction with the helicopter/FCS combination. In a wider analysis of helicopter agility then, handling qualities considerations should probably be included, and a kinematics-based study such as this will probably not be sufficient, although likely to be necessary. In this case the agility rating may be based on features more removed from the actual geometry of the flight path and the kinematics of the manoeuvre, and closer to features important to the pilot such as achievable pitch and roll rates, time constants, stability and control power. The resulting agility diagram may then look like figure 10, reproduced from ref. [3].

By viewing the solution of the helicopter equations of motion as an inverse problem, different helicopters can then be made to fly identical paths since the velocity vector in each case is a precisely defined input to the system, equation (6), with the attitude and control time histories the output. The inverse solution may at first sight appear algebraic in nature, but this is true only for special classes of problem. In any case, the differential equation form of the inverse method is neater, in that it allows an analytical, rather than numerical, study of the stability and dynamics of the solution. The principal advantage of inverse methods for generating control inputs is simplicity; no assumptions are necessary about the form of the control system or the control strategy required. As a result it can give a significant insight into control strategies required to fly specific manoeuvres in any manner. What is in some sense a limiting feature of the inverse method as formulated in this paper is that the inverse of the matrix B_{21} must exist. This can be overcome as in this paper by use of a valid reduced order model. In the general case of motion in three dimensions where there are 4 controls and 3 velocity components, it will be necessary to impose an additional constraint equation eg. a condition of zero sideslip (assuming that the 3 attitude variables remain unspecified).

It is shown that using a controllable horizontal tailplane to reduce hub moment in popups on a helicopter with a stiff rotor requires a control law that is a function of the three rotor controls and pitch rate. In the past on helicopters with articulated rotors, the tailplane has been geared to the longitudinal cyclic to reduce blade flapping during manoeuvres, refs. [10], [11] for example. This does not appear to be sufficient on stiff-rotor helicopters, and seems to be the case because the hub moment is a function of, among other things, all three rotor controls; and during popups flown in the style presented in this paper, the rotor controls all vary, contributing different proportions of the total hub moment at different times during the manoeuvre. In absolute terms, the improvements in agility attainable with the controllable tail do not seem significant - reducing manoeuvre time by about 0.7 of a second. However relative to the least agile configuration, this is an improvement of about 10%.

6. Conclusions

The kinematic definition of helicopter agility based on the geometry and time taken to fly a specific path in space, together with the inverse solution of the vehicle equations of motion, has provided a fruitful means of comparing the relative agility capabilities of several helicopter configurations. It is however suggested that some consideration needs to be taken of the pilot's perception of the task.

The inverse solution is only algebraic for certain classes of problem. Otherwise manipulation of the state-space description of the helicopter allows the system to be recast as a set of differential equations. In this form, the resulting system may no longer be invariant, depending on assumptions about the velocity vector and its rate of change with time.

The theoretical studies of agility in bobup manoeuvres were made for four similar helicopter configurations. The measure of agility adopted was consistent in use, reflecting the need for higher levels of agility to fly paths in space that are intuitively more severe than others. The most agile helicopter configuration was that with a moveable horizontal tailplane which produced, for a given manoeuvre, some reduction in rotor hub moment. Correspondingly for a given limiting hub moment, the helicopter with the moveable tail could fly tighter popups. Although the controllable tailplane offered improved agility over the other configurations, the nature of the control algorithm and the control authority required suggest that the benefits would be achieved only if the tailplane was actively controlled and a fully integrated element of the vehicle FCS.

A more detailed analysis of inverse methods in studies of helicopter flight mechanics is required to increase the level of experience with what appears to be a very useful tool for investigations of helicopter agility.

7. References

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Flap freq. ratio --- 1.095	Blade lift slope --- 6.0 /rad
Rotor radius ------- 6.4 m	Tailplane lift slope 3.5 /rad
Rotor solidity ----- 0.0778	
Aircraft mass ------ 4518 kg	
Tailplane area ----- 1.197 m	
C.g. location -------0.127 m (aft)	
Rotor speed -------- 35.63 rad/s	

Table 1

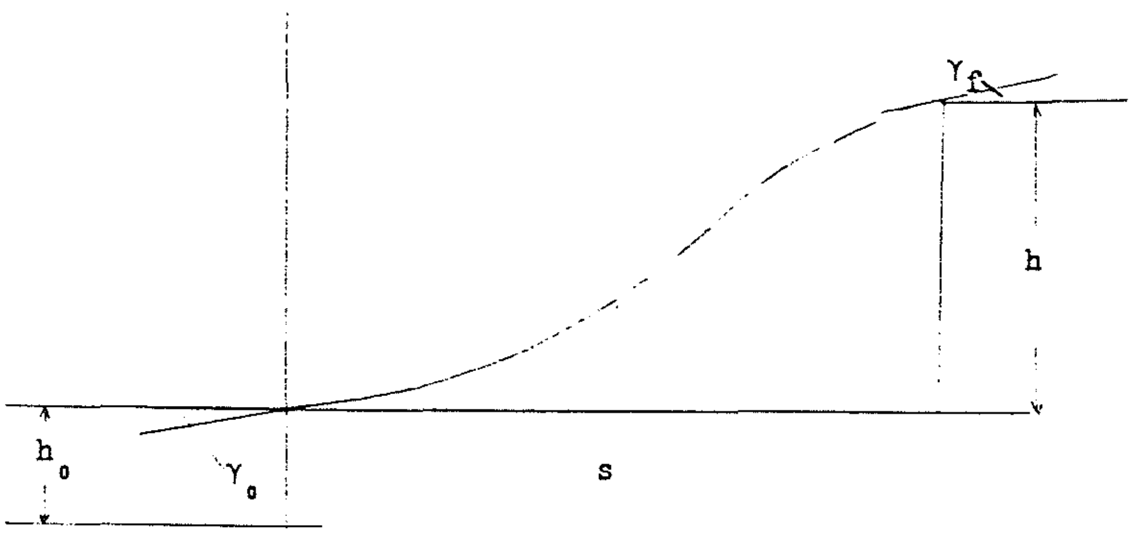


Figure 1

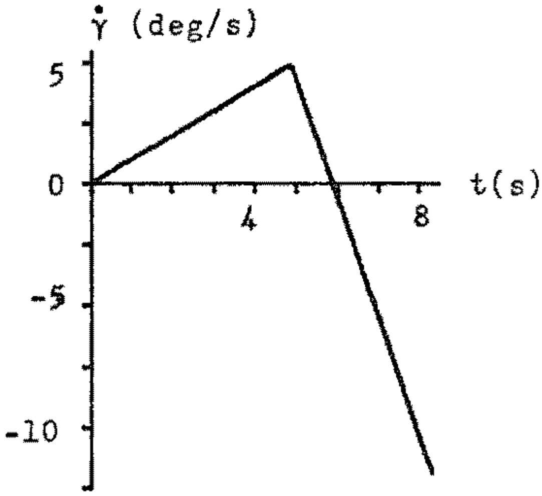
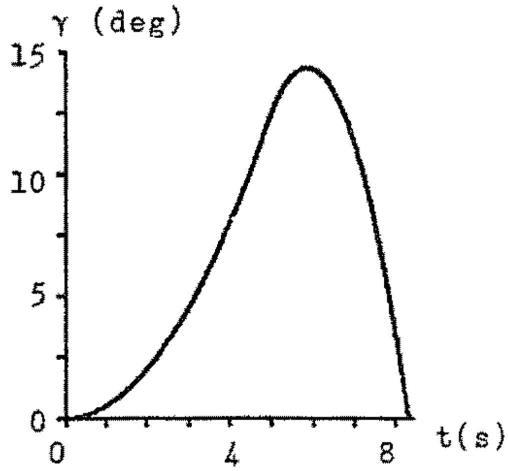


Figure 2a



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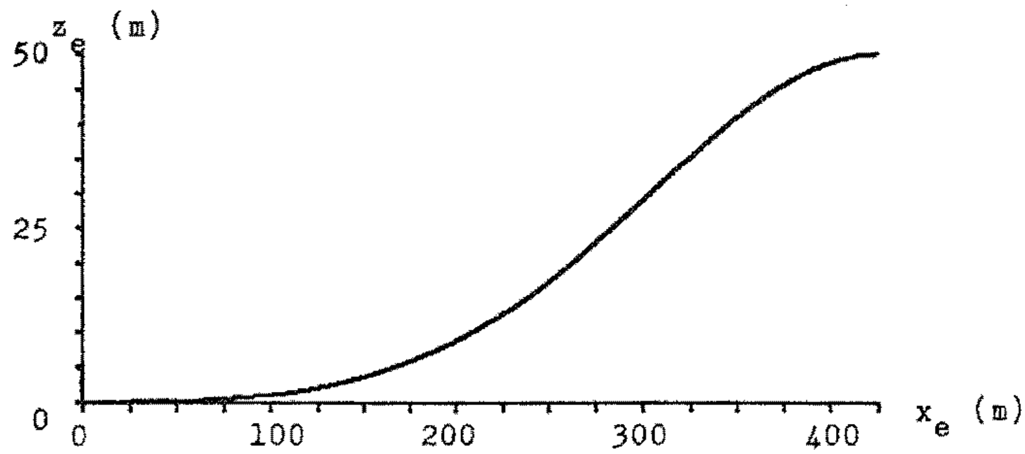

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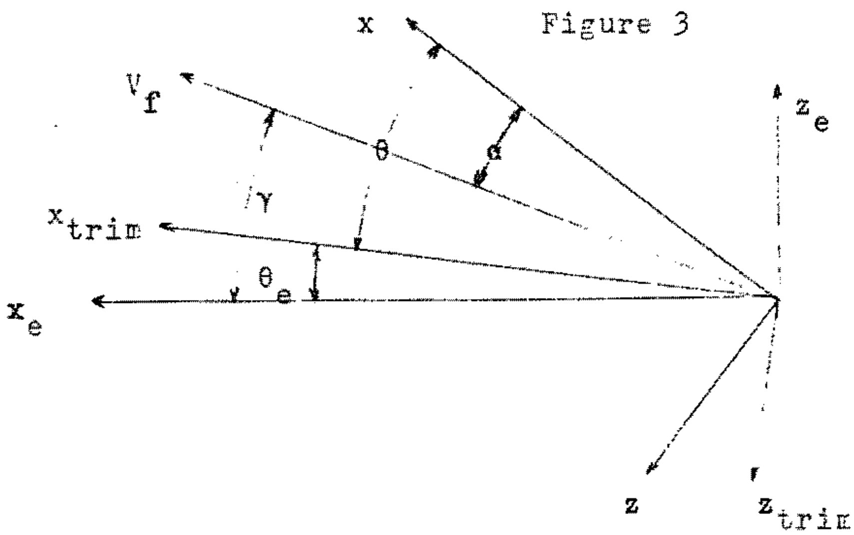

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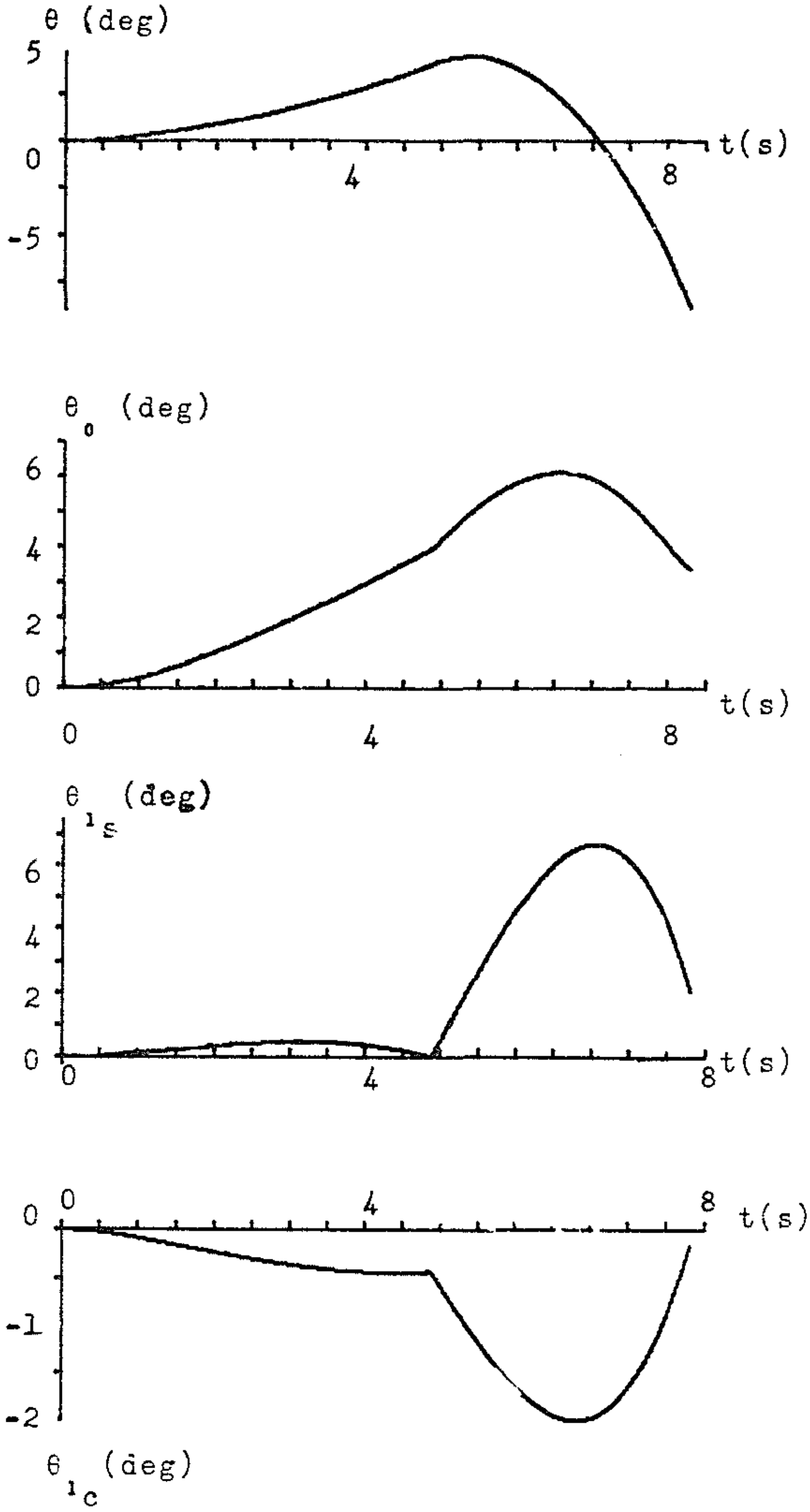
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0
-5
4
8
t(s)
θ0 (deg)
6
4
2
0
0
4
8
t(s)
θ1s (deg)
6
4
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t(s)
0
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4
8
t(s)
-1
-2
θ1c (deg)

Figure 5

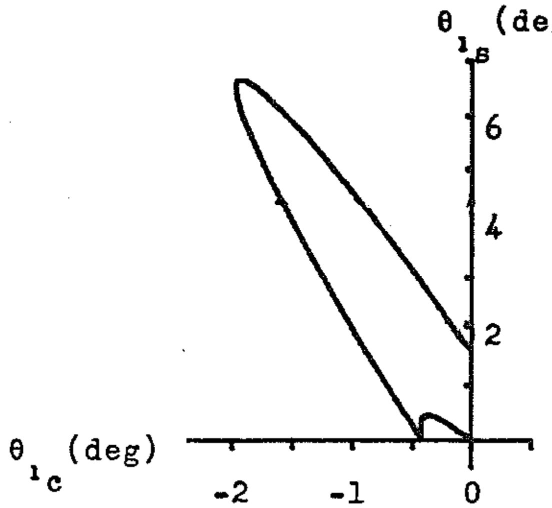


Figure 5a

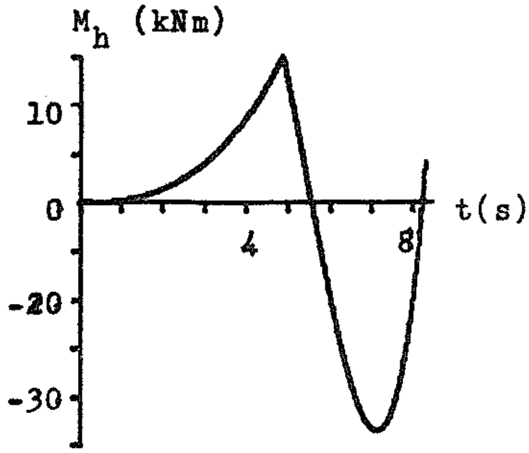


Figure 6

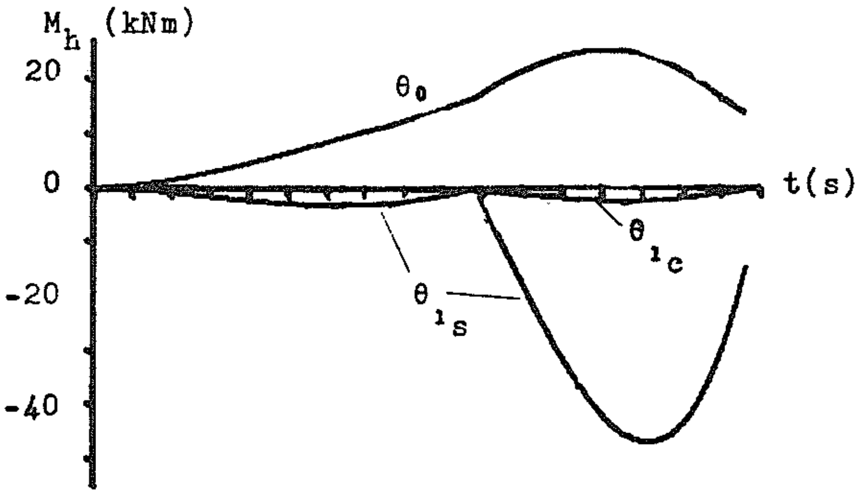


Figure 6a

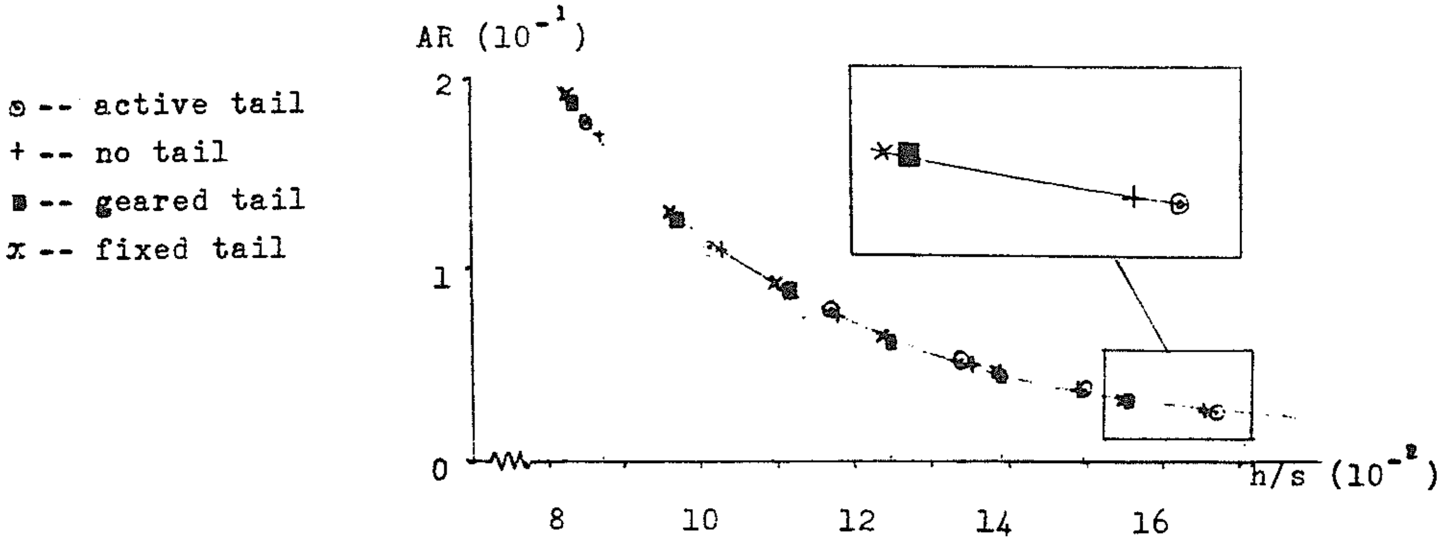
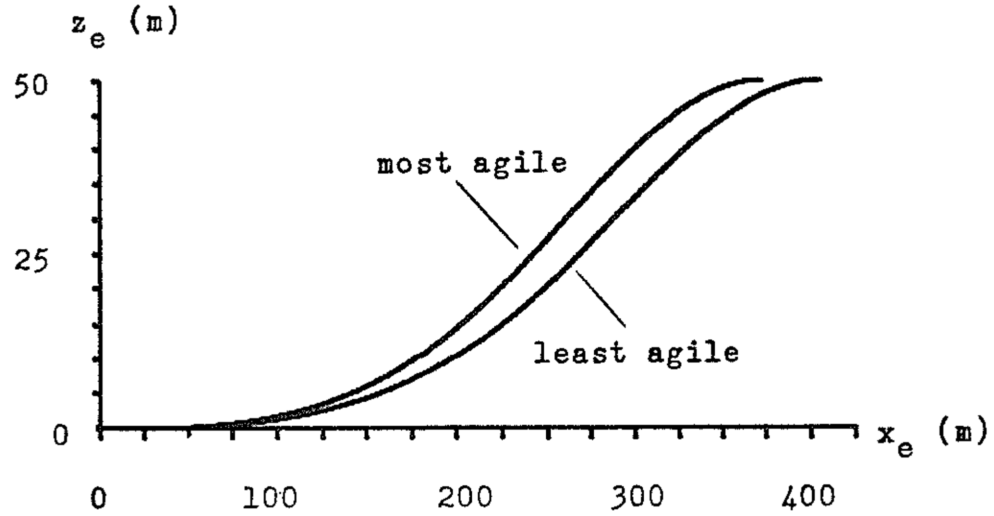


Figure 7



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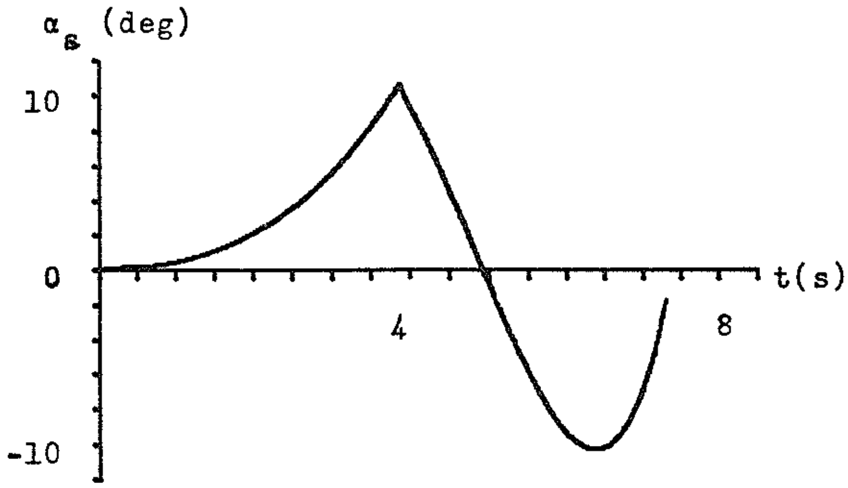

Figure 9

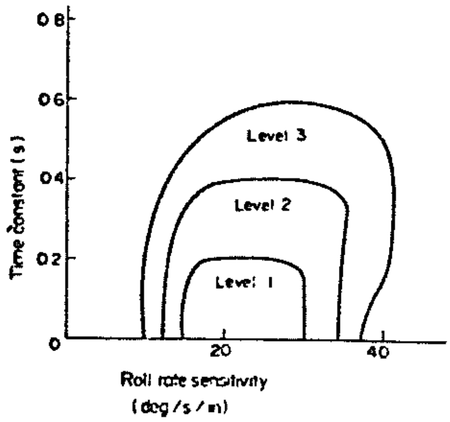

Figure 10