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# MODELLING, SIMULATION AND CHARACTERISATION OF HELICOPTER RESPONSE TO ATMOSPHERIC TURBULENCE

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## Abstract

This Paper describes the mathematical modelling required to facilitate the simulation of rotorcraft response to atmospheric turbulence. The response of a single main and tail rotor helicopter to a simple sharp-edged vertical gust profile is predicted by an individual blade/blade element mathematical model for various airspeeds, and the frequency content of the airframe acceleration response analysed. Cases for instantaneous and penetrative immersion are compared. The well-known effect of the former type of simulation over-predicting the initial response is confirmed, while the latter type is shown to propagate vibratory response at frequencies other than n/rev. Describing functions are discussed as a suitable way to characterise the vibratory response, thereby enhancing understanding of a physically and mathematically complex problem.

## Nomenclature

| | |
|---|---|
| $M$ | Transformation matrix - Earth to airframe axes (rad) |
| $R$ | Rotor radius (m) |
| $R_i$ | Radial position of ith blade element (m) |
| $T_1$ | Transformation matrix - airframe body axes to non-rotating hub axes (rad) |
| $T_2$ | Transformation matrix - non-rotating hub axes to rotating hub axes (rad) |
| $T_3$ | Transformation matrix - hinge axes to blade axes (rad) |
| $e$ | Offset hinge radial distance from hub centre (m) |
| $\underline{u}^g_{a/c}$ | Gust velocity vector in airframe body axes (m/s) |
| $\underline{u}^g_{bl}$ | Gust velocity vector in blade axes (m/s) |
| $\underline{u}^g_{E}$ | Gust velocity vector in Earth axes (m/s) |
| $\underline{u}^g_{hg}$ | Gust velocity vector in rotating hub axes (m/s) |
| $\underline{u}^g_{hub}$ | Gust velocity vector in non-rotating hub axes (m/s) |
| $\underline{x}_{cg}$ | Aircraft centre of mass position vector in airframe reference axes (m) |
| $\underline{x}_{E}$ | Aircraft position vector in Earth axes (m) |
| $\underline{x}_{fin}$ | Fin position vector in airframe reference axes (m) |
| $\underline{x}_{tp}$ | Tailplane position vector in airframe reference axes (m) |
| $\underline{x}^{bl}_{a/c}$ | Blade element position vector w.r.t. centre of mass in airframe reference axes (m) |
| $\underline{x}^{bl}_{E}$ | Blade element position vector in Earth axes (m) |
| $\underline{x}^{cg}_{a/c}$ | Centre of mass position vector w.r.t. ith blade element in airframe reference axes (m) |
| $\underline{x}^{fin}_{E}$ | Fin position vector in Earth axes (m) |
| $\underline{x}^{g}_{E}$ | Gust front position vector in Earth axes (m) |

| | |
|---|---|
| $\underline{x}_{bl}^{hg}$ | Hinge position vector w.r.t. ith blade element in blade axes (m) |
| $\underline{x}_{hg}^{hg}$ | Hinge position vector w.r.t. ith blade element in hinge axes (m) |
| $\underline{x}_{a/c}^{hub}$ | Hub position vector w.r.t. ith blade element in airframe reference axes (m) |
| $\underline{x}_{hg}^{hub}$ | Hub position vector w.r.t. ith blade element in hinge axes (m) |
| $\underline{x}_{hub}^{hub}$ | Hub position vector w.r.t. ith blade element in non-rotating hub axes (m) |
| $\underline{x}_{E}^{tp}$ | Tailplane position vector in Earth axes (m) |
| $\beta$ | Blade flap angle (rad) |
| $\delta R_i$ | Length of ith blade element (m) |
| $\phi$ | Aircraft roll attitude (rad) |
| $\phi_s$ | Rotor shaft lateral tilt (rad) |
| $\varphi$ | Aircraft yaw attitude (rad) |
| $\theta$ | Aircraft pitch attitude (rad) |
| $\theta_s$ | Rotor shaft longitudinal tilt (m) |
| $\psi$ | Blade azimuthal position (rad) |
| $\zeta$ | Blade lag angle (rad) |
| $^T$ | Matrix transpose |

## Introduction

Helicopter response to atmospheric turbulence is important from the viewpoint of handling qualities as well as passenger comfort. In addition, any physical source that contributes to vibratory loading has the potential for reducing airframe and component lives. Simulating the response of helicopters to gusts will lead to enhanced understanding in this area, that can be applied to the design process. This Paper shows that the mathematical modelling required to allow simulation is relatively straightforward if an individual blade/blade element model is available.

## Background

The literature contains many references to helicopter flight in turbulence, separating mainly into two categories: those that deal with aircraft response (e.g. Refs. 1 and 2); and those that deal with modelling the structure of the turbulence itself, e.g. Ref. 3. The literature also suggests that over the past ten years, there has been little activity by the community at large in this field, with a few notable exceptions e.g. Ref. 4. Recently however papers have been published that indicate perhaps a re-awakened interest in the modelling and simulation of rotorcraft flight in turbulence (Refs. 3 and 5). This Paper describes in detail the mathematical modelling approach taken to incorporate the effect of atmospheric disturbances in a rotorcraft simulation model (Ref. 6).

## Mathematical modelling

Table 1 highlights the principal features of the rotorcraft model used, Ref. 6. An individual blade/blade element model allows easy implementation of atmospheric disturbances. The approach taken is to specify a region in a flat-Earth frame of reference, beyond which the air has a spacially-varying velocity. Since this velocity is specified in Earth axes, there are two modelling requirements.

First, the position of each blade element on both rotors has to be calculated in Earth axes. Second, the velocity of the moving atmospheric air mass has to be transformed into local blade axes. It is then a simple case of modifying the local blade element velocities with the atmospheric disturbance velocity at that point in space.

A similar process is used for the non-rotating aircraft components i.e. the fuselage, tailplane and fins.

**Table 1 Mathematical model description**

| Model item | Characteristics |
|---|---|
| Rotor dynamics (both rotors) | • up to 10 individually-modelled rigid blades<br>• fully-coupled flap, lag and feather motion<br>• blade attachment by offset hinges & springs<br>• lag damper |
| Rotor loads | • aerodynamic and inertial loads represented by up to 10 elements per blade |
| Blade aerodynamics | • lookup tables for lift and drag as function of angle-of-attack and Mach number |
| Wake model | • momentum-derived dynamic wake model<br>• uniform and harmonic components of inflow<br>• rudimentary interaction with tail surfaces<br>• ground effect |
| Transmission | • coupled rotorspeed and engine dynamics<br>• up to 3 engines<br>• geared or independently-controlled rotor torque |
| Airframe | • fuselage, tailplane and fin aerodynamics by lookup tables or polynomial functions |
| Atmosphere | • International Standard Atmosphere<br>• provision for variation of sea-level temperature and pressure |

a/. <u>derivation of blade element position in Earth axes</u>

The position of a co-located flap, lag and feather hinge with respect to a blade element i is given in blade axes as

$$\underline{x}_{bl}^{hg} = [\,0 \;\; (R_i + \delta R_i / 2) \;\; 0\,]^T$$

The position of the hinge with respect to the blade element i in hinge axes is

$$\underline{x}_{hg}^{hg} = T_3^T \, \underline{x}_{bl}^{hg}$$

where $T_3$ transforms a vector from hinge to a set of blade axes. (Since the transformation is orthogonal, the inverse of the matrix is simply its transpose). This transformation is given by

$$T_3 = \begin{bmatrix} \cos\zeta & \sin\zeta & 0 \\ -\cos\beta\sin\zeta & \cos\beta\cos\zeta & \sin\beta \\ \sin\beta\sin\zeta & -\cos\zeta\sin\beta & \cos\beta \end{bmatrix}$$

The position of the hub with respect to the blade element i in hinge axes is

$$\underline{x}_{hg}^{hub} = \underline{x}_{hg}^{hg} + [\,0 \;\; eR \;\; 0\,]^T$$

The position of the hub with respect to the blade element i in hub (i.e. non-rotating axes) is

$$\underline{x}_{hub}^{hub} = T_2^T \, \underline{x}_{hg}^{hub}$$

where $T_2$ is the transformation from the hub non-rotating to hub rotating frame of reference and is given by

$$T_2 = \begin{bmatrix} \sin\psi & -\cos\psi & 0 \\ \cos\psi & \sin\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The position of the hub with respect to the blade element i in airframe axes is then

$$\underline{x}_{a/c}^{hub} = T_1^T \, \underline{x}_{hub}^{hub}$$

where $T_1$ is the transformation from airframe body axes to non-rotating hub axes and is given by

$$T_1 = \begin{bmatrix} \cos\theta_s & 0 & -\sin\theta_s \\ \sin\theta_s \sin\phi_s & \cos\phi_s & \sin\phi_s \\ \sin\theta_s \cos\phi_s & -\sin\phi_s & \cos\theta_s \cos\phi_s \end{bmatrix}$$

Now, the aircraft equations of motion are referred to an axes system that is aligned with the aircraft body axes but whose origin is at the aircraft centre of mass. The position of the centre of mass with respect to the blade element i in aircraft axes is

$$\underline{x}_{a/c}^{cg} = \underline{x}_{a/c}^{hub} - \underline{x}_{cg}$$

The position of the blade element i with respect to the centre of mass in aircraft axes is then simply

$$\underline{x}_{a/c}^{bl} = -\underline{x}_{a/c}^{cg}$$

Finally, the position of the blade element i with respect to the Earth origin in Earth axes is

$$\underline{x}_{E}^{bl} = M^T \, \underline{x}_{a/c}^{bl} + \underline{x}_E$$

where $M$ is the Euler angle transformation

$$M = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ (\sin\phi\sin\theta\cos\varphi - \cos\phi\sin\varphi) & (\sin\phi\sin\theta\sin\varphi + \cos\phi\cos\varphi) & \sin\phi\cos\theta \\ (\cos\phi\sin\theta\cos\varphi + \sin\phi\sin\varphi) & (\cos\phi\sin\theta\sin\varphi - \sin\phi\cos\varphi) & \cos\theta\cos\phi \end{bmatrix}$$

Back-substitution then gives

$$\underline{x}_{E}^{bl} = M^T \, [\, \underline{x}_{cg} - T_1^T \, [\, T_2^T \, [\, T_3^T \, [\,0 \;\; (R_i + \delta R_i) \;\; 0\,]^T + [\,0 \;\; eR \;\; 0\,]^T \,]\,]\,] + \underline{x}_E$$

A blade element will be immersed in the gust if

$$\underline{x}_E^{bl} \geq \underline{x}_E^g$$

b/. <u>derivation of gust velocity in blade axes</u>

The gust velocity components in airframe axes are given by

$$\underline{u}_{a/c}^g = M\underline{u}_E^g$$

The gust velocity components in non-rotating hub axes are then given by

$$\underline{u}_{hub}^g = T_1\underline{u}_{a/c}^g$$

Similarly, the gust velocity components in rotating hub axes are given by

$$\underline{u}_{hg}^g = T_2\underline{u}_{hub}^g$$

and the gust velocity components in a blade axes set are

$$\underline{u}_{bl}^g = T_3\underline{u}_{hg}^g$$

Back-substitution then gives

$$\underline{u}_{bl}^g = T_3T_2T_1M\underline{u}_E^g$$

c/. <u>non-rotating components</u>

The corresponding equations for the fuselage, tailplane and fins are derived in a similar manner. The fuselage will be immersed in the gust if

$$\underline{x}_E \geq \underline{x}_E^g$$

The tailplane will have penetrated the gust front if

$$\underline{x}_E^{tp} \geq \underline{x}_E^g$$

where

$$\underline{x}_E^{tp} = M^T(\underline{x}_{tp} - \underline{x}_{cg}) + \underline{x}_E$$

and similarly each fin will be immersed in the gust if

$$\underline{x}_E^{fin} \geq \underline{x}_E^g$$

where

$$\underline{x}_E^{fin} = M^T(\underline{x}_{fin} - \underline{x}_{cg}) + \underline{x}_E$$

The gust velocity at the fuselage, tailplane and fins in aircraft body axes is given by

$$\underline{u}_{a/c}^g = M\underline{u}_E^g$$

## Results

The rotorcraft model was configured as a medium single main and tail rotor helicopter. Figure 1 shows the predicted vertical acceleration response at the airframe centre of mass at an airspeed of 10 knots, when instantaneously encountering a 2.5 m/s up-gust. The entire main rotor is subjected to an instantaneous increase in angle-of-attack, giving an increase in thrust and hence vertical acceleration. The increased loading on the blades then causes them to flap up, reaction to which is a download on the fuselage, seen as the trough in the response. The blades rapidly attain their new steady state, the increased loading lifting the entire aircraft. This gives the second peak in acceleration. The upwards motion of the aircraft reduces the blades' angle-of-attack, tending to reduce the thrust and hence the acceleration decreases gradually as vertical velocity increases.

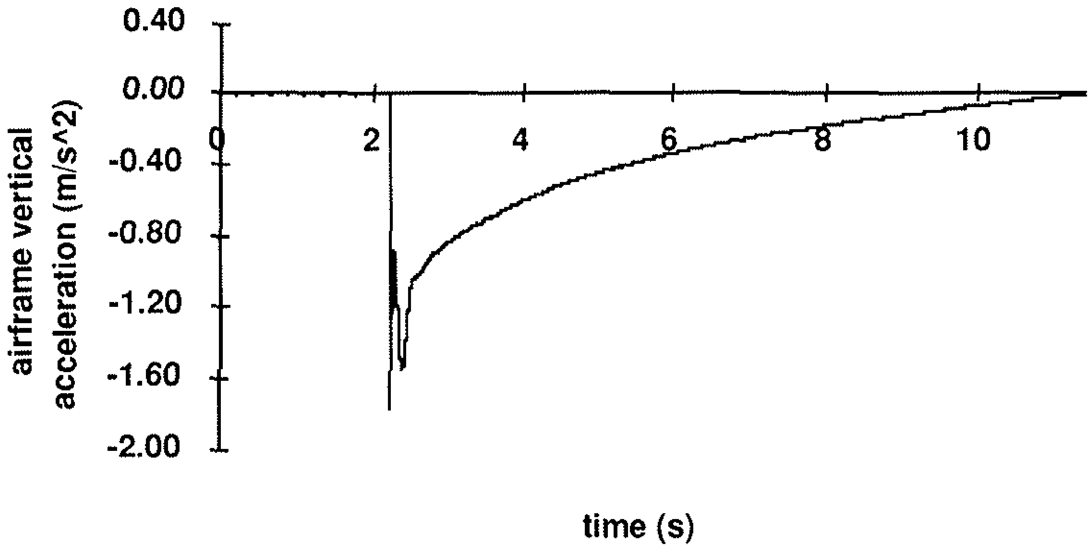

Figure 1 Vertical acceleration response to instantaneous immersion in vertical gust

Figure 2 shows the helicopter penetrating the gust front gradually. The peak vertical acceleration is some 30% less than the instantaneous case, and occurs after a gradual build-up taking about 3 s instead of occurring instantaneously. There is also a vibratory component to the

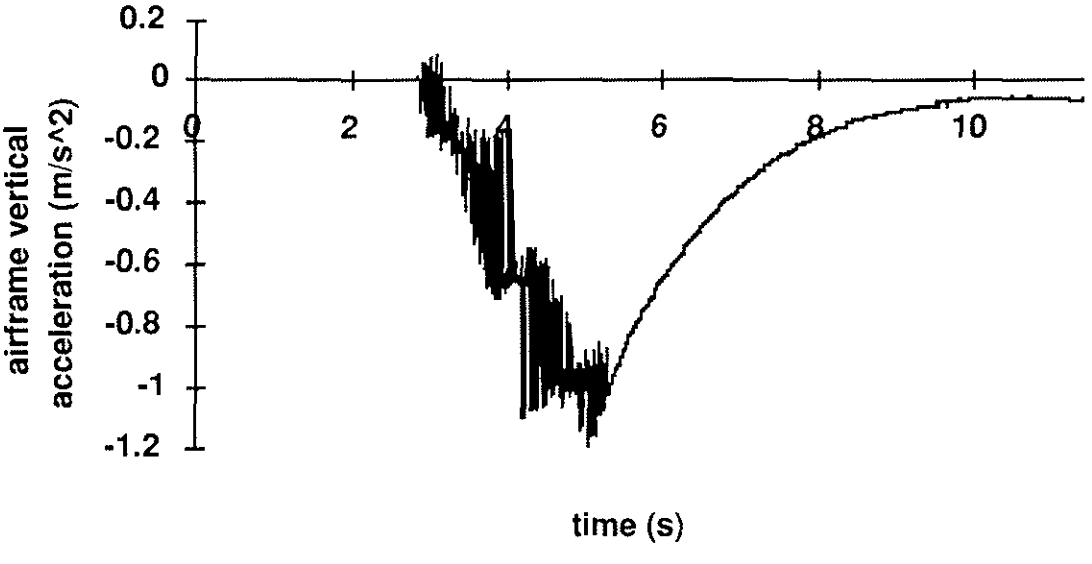

Figure 2 Vertical acceleration response to progressive immersion in vertical gust

response which has a peak amplitude of about 0.025 g. It grows to this value during immersion of the front half of the main rotor disc, and then diminishes during immersion of the rear half. It disappears once the main rotor is fully immersed in the gust, and is absent in the case of instantaneous immersion.

Figure 3 shows the vertical acceleration response when gradually penetrating the gust at an airspeed of 80 knots. The shape of the response is similar to instantaneous immersion at low speed, not surprising given that the main rotor takes only 1/3 s to become fully immersed in the gust at 80 knots. The vibratory component of the response is that associated with trimmed flight.

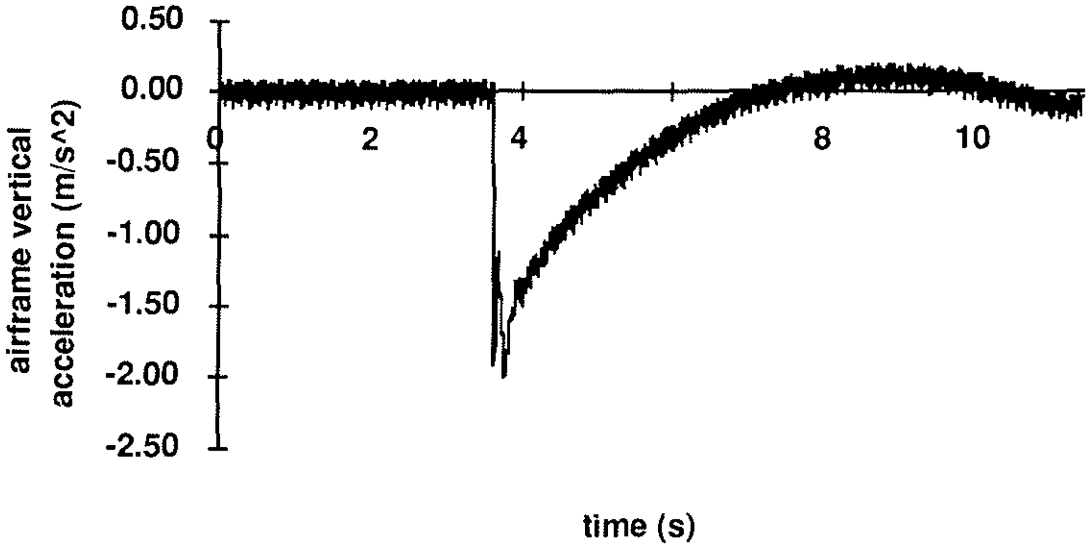

Figure 3 Vertical acceleration response to progressive immersion in vertical gust

Figure 4 presents a comparison of the power spectra of the responses to flight through the 2.5 m/s up-gust at 5, 10, 15 and 20 knots. The results are plotted between frequencies of 3 and 5 per rev, since the dominant result was found to exist at about 4 per rev. It can be seen that there are two peaks in each response either side of the blade passing frequency. These peaks diminish in magnitude and move further apart with increasing airspeed.

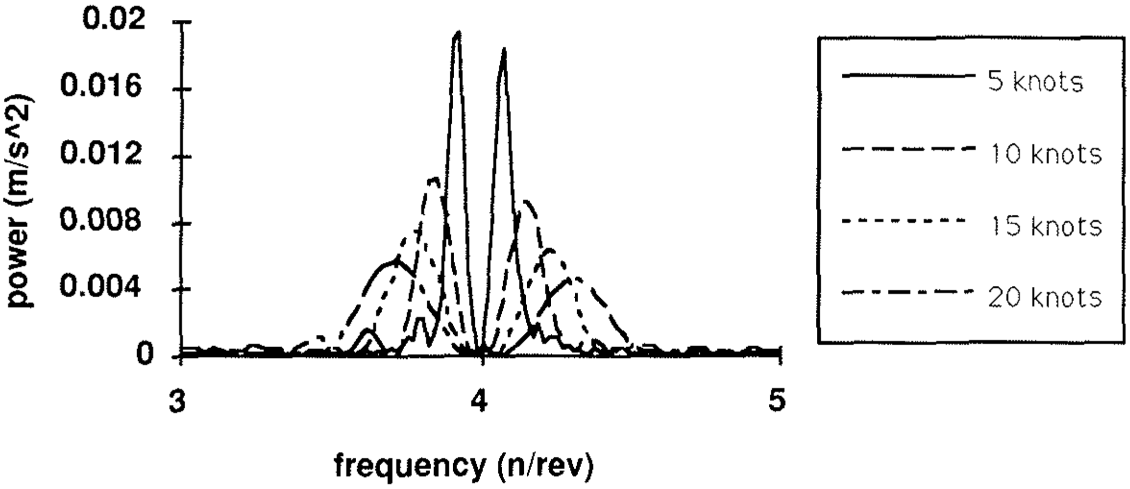

Figure 4 Power spectra of responses to progressive immersion in vertical gust

Finally, the potential use of a describing function is explored. This is a function that simply describes a complex, usually non-linear, process. Its application here is to describe the vibratory loading experienced flying through the sharp-edged gust. The previous result showed that there were two dominant frequencies in the response, and the approach taken was to specify the describing function

$$y(t) = a_0 + a_1 \sin(\omega_1 t + \phi_1) + a_2 \sin(\omega_2 t + \phi_2)$$

as a suitable expression of the main features of the response during each revolution of the main rotor. Values of the constants $a_0$, $a_1$ and $a_2$ were obtained by a least-squares fitting process over the time interval of one main rotor revolution, and updated for each turn of the rotor. For the 11 s interval shown in Figure 5a, there are 44 describing functions, each with a different value of the constants $a_0$, $a_1$ and $a_2$.

The 5 knot case was chosen, since the vibratory loading is greater at this airspeed and the peaks in the frequency content of the response are relatively sharp - the function chosen is likely to be most appropriate under this condition. Figure 5a shows the vertical acceleration response, and Figure 5b shows the response with the describing function removed from it. Although the function is not an entirely accurate representation of the response, it does represent a significant amount of the response at any given time.

The application of a describing function in this context might be as part of an adaptive disturbance rejection algorithm, the function, once identified, being used as the basis for a compensation signal applied by the rotor controls to negate that part of the helicopter's response that is approximated by the describing function. Figure 5b could be viewed as the improvement

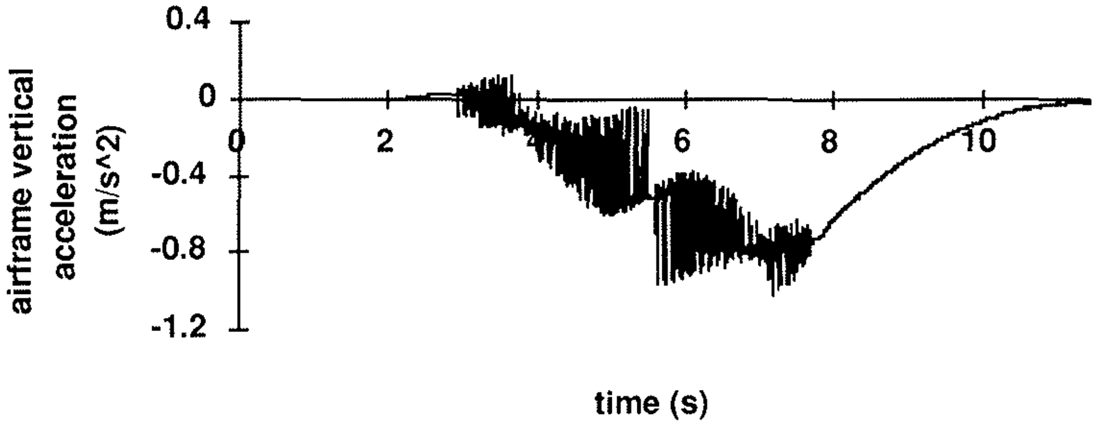

**Figure 5a Vertical acceleration response to progressive immersion in vertical gust - 5 knots**

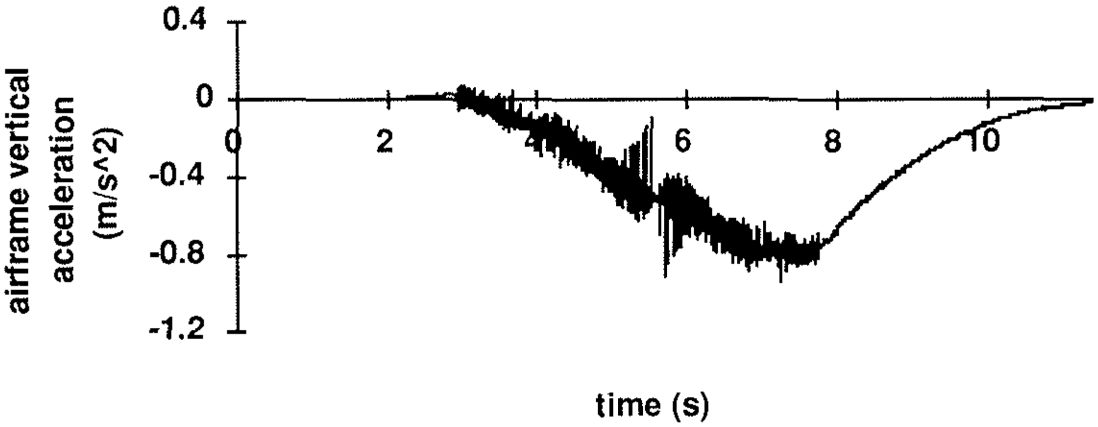

**Figure 5b Vertical acceleration response with describing function component removed**

in ride quality experienced if a compensation signal were applied by the flight controls to exactly cancel out that part of the response represented by the describing function.

## Discussion

The responses of the simulation during flight through a simple gust front highlight an aspect of rotorcraft flight dynamics in turbulence at low speed that has been discussed elsewhere (Refs. 3 and 5). Specifically, there is a vibratory component to the response at close to the blade passing frequency which diminishes with increasing airspeed. Spectral analysis showed that this loading is composed of two responses, one below blade passing frequency and one above it. Subjective consideration of the situation will confirm this to be correct. If the helicopter were at rest relative to the gust front, but with a part of the disc immersed in the gust, it is clear that a loading would be experienced at the blade passing frequency. Now, if the helicopter is moving relative to the gust front, the advancing blades will "see" the gust sooner than they would if the gust and the blades were relatively at rest - the frequency with which advancing blades meet the gust front is therefore greater than the passing frequency. Conversely, the retreating blades pass out of the gust front "later" than they otherwise would - the frequency with which retreating blades leave the gust front is therefore less than the passing frequency. Increased speed relative to the gust front will accentuate these effects.

The simulation predicts this vibratory loading to be fairly significant below about 20 knots. It might be expected that a gust alleviation control system should address this aspect of flight in turbulence, as well as the non-vibratory loading that will give rise to flight path and attitude changes. The vibration is however at a high frequency for a conventional flight control system. There may be a degree of risk in attempting to use feedback control, since the high frequency rotor modes may be adversely affected. Simple open-loop compensation may be appropriate, and the use of a describing function to characterise the high frequency response may have some use in this regard. Figure 5b could be viewed as the improvement in ride quality experienced if a compensation signal were applied by the flight controls to exactly cancel out that part of the response represented by the describing function. The simple describing function could be improved by targeting a few additional frequencies around the significant peaks. This would appear to be necessary in general, since the vibratory response is more widely distributed around the peak frequencies with increased speed.

Finally, although obtained for flight through a simple sharp-edged vertical gust, the results are generally applicable since the underlying physical behaviour of blades penetrating gust fronts while moving relative to them is the same.

## Conclusions

- The mathematical modelling required to simulate rotorcraft flight in turbulence is straightforward if the vehicle model is of the individual blade/blade element type.
- Instantaneous and gradual immersion of the main rotor will give distinctly different predictions of airframe acceleration response to gusts at low speed.
- Gradual penetration of a sharp-edged vertical gust gives rise to a vibratory component in the vertical acceleration response at low speed that is not at the blade passing frequency.
- The vibratory response to a sharp-edged vertical gust is most pronounced at low speed, and decreases in magnitude with increasing speed.

- Describing functions provide a useful means of simply characterising the response to gusts at low speed, and may form a suitable basis for the design of an adaptive gust rejection algorithm.

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