

A NEW KIRCHHOFF FORMULATION FOR TRANSONIC ROTOR NOISE

Paolo di Francescantonio
Acoustics and Vibration dept., Agusta
21017 Samarate (VA), Italy

Abstract

A new boundary integral formulation is presented for the evaluation of the noise radiated in an uniform medium by generic sources. The method requires the knowledge of pressure, velocity, and density disturbances on a smooth closed surface surrounding the source, and assumes that the propagation is linear outside the surface itself. When applied to the prediction of transonic rotor noise the method can be used in the same manner as Kirchhoff approach, but the new integral equations are derived releasing the non penetration condition in the Ffowcs-Williams Hawkins equation. The method is therefore referred as Kirchhoff-FWH. The main advantage of the proposed formulation in respect of Kirchhoff method is that it does not require the knowledge of the surface pressure normal derivative. Two different formulations are presented that differ in the way in which a time derivative is handled. Comparisons with experiment and with Kirchhoff method are presented for a hovering rotor in transonic conditions at various tip Mach numbers.

Introduction

The reduction of helicopter external noise has received in the last years a great attention from industries, both for the more stringent certification rules, and for the increased sensitivity of community and operators. The availability of fast and robust prediction codes is clearly a required step towards the development of quieter helicopters. Nowadays two different groups of methods are available, one based on the Computational AeroAcoustics approach (CAA), and one based on integral formulations. The first method permits to solve at the same time the aerodynamic and aeroacoustic problem, and is based on the solution of the fluid motion equations with classical field methods (finite volume, finite difference, finite elements) [1]. The main problem of CAA is that, in order to avoid the introduction of excessive dissipation, the required computer resources greatly increases with observer distance, and nowadays the solution can be obtained at a reasonable cost only for observers at a distance of about three times the rotor radius. The

distances that are usually required in realistic calculations are however 2 or 3 order of magnitude greater than the rotor radius, and, even considering an increase in computer speed, it is certainly not practical to apply directly CAA methods for these distances. The integral methods, instead, require the knowledge of the aerodynamic flowfield around the rotor, and permit to obtain the acoustic pressure in any point of the field executing a certain number of integrals. One of the interesting aspect of integral methods is that the required computational time is independent on the observer distance. Typical calculations of rotor noise are therefore executed in two steps, in the first one a CFD/CAA code is used to evaluate the aerodynamic field, and then an integral method is used to propagate the pressure disturbance in the far field. It is important to note that the computational time required by the integral methods is usually much lower than the time required to obtain the aerodynamic solution. Nowadays two different integral methods are available based respectively on Ffowcs-Williams Hawkins (FWH) and Kirchhoff equations. The FWH formulation is usually referred as a linear approach simply because in the great part of the implementations the volume quadrupole terms, that take in account for the non linearities, are neglected. However, introducing the volume terms, good results can be obtained below delocalization [2, 3, 4], and there are also some indications [5, 6] that, if the sonic singularity and the multiple emission times are correctly handled, good results can be obtained also at higher Mach numbers. On the other side the Kirchhoff formulation, obtained in its actual form by Farassat and Myers [15], permits to solve linear wave propagation problems once some flow quantities are given on a closed fictitious surface surrounding the source. In order to be applied to transonic rotor noise [10, 11, 12] the surface has to be placed at a sufficient distance from the rotor in order to ensure that the propagation be governed by the linear wave equation outside the surface itself. The main advantage in respect of FWH approach is that it is generally faster since only surface integrals have to be evaluated.

From a physical point of view it is important to realize that Kirchhoff formula is valid for any phenomenon (optics, acoustics, electromagnetism, ...) governed by

the linear wave equation, while FWH equation is specialised for aeroacoustics problems. As a consequence the Kirchhoff equation is written in term of a single fluid quantity (the pressure disturbance $p' = p - p_0$), while FWH require not only p' but also the fluid density ρ and the fluid perturbation velocity \mathbf{u} . Clearly in order to reconstruct the propagation the Kirchhoff formulation require some further information that is provided by the knowledge of the pressure normal derivative $\frac{\partial p'}{\partial n}$. The necessity of specifying $\frac{\partial p'}{\partial n}$ is certainly a disadvantage for rotorcraft problems, since, if discontinuities are present, the numerical evaluation of $\frac{\partial p'}{\partial n}$ can introduce undesidered smoothing. The other difference between the two formulations is that the surface integrals of FWH equation are executed on a well defined physical surface (the surface of rotor blades), while the Kirchhoff surface is completely fictitious being subject to the only restrictions of being smooth and of enclosing the source with all the non linear terms. Except from the above limitations, the surface can be placed anywhere in the field, and can have a generic motion eventually different from the motion of source itself. The degrees of freedom allowed in the definition of the Kirchhoff surface represent certainly an advantage in respect of FWH approach. For example, in calculation of High Speed rotor noise in delocalized condition, it is possible to use a non rotating Kirchhoff surface in order to avoid problems with surfaces in supersonic motion.

A question arise now spontaneously, if it is possible to develop an integral formulation specialised for aeroacoustics problems, but that permits the same flexibility of Kirchhoff formulation. The answer is yes, and in this work the new formulation is derived and applied to transonic rotor noise problems. Since the formulation combines aspects of both FWH and Kirchhoff approaches it is here referred as Kirchhoff-FWH formulation (KFWH). Two different formulation are presented that differs in the way in which time derivatives are handled. At the end some comparisons with classical Kirchhoff and experiments are shown for the UH-1H rotor in hover for tip Mach number up to 0.95.

The FWH Approach

In order to obtain the new formulation the derivation of FWH and Kirchhoff equation is here outlined trying to point out the differences and the similarities between the two approaches.

Consider a generic body immersed in a fluid, and whose surface S_b be described by the equation $f_b(\mathbf{x}, t) = 0$, being $f_b < 0$ for points inside the body (for semplicity we also assume that the function f_b be scaled in such a way that $|\nabla(f_b)| = 1$ for $f_b = 0$). The problem can be modelled replacing the body with fluid at rest ($p' = 0, \rho = \rho_0, \mathbf{u} = 0$), and the governing

equations can be written as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0 \quad (1)$$

$$\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_i} (P_{ij} + \rho u_i u_j) = 0 \quad (2)$$

Where P_{ij} is the fluid compressive stress tensor, ρ is the density, and u_i is the fluid perturbation velocity. The above equations represent respectively mass and momentum conservation, and are valid, with the respective boundary conditions, in the two regions separated by the surface S_b . In order to obtain a single equation valid both for $f_b < 0$ and $f_b > 0$ the surface S_b has to be considered as a discontinuity surface, and all the fluid quantities have to be regarded as generalized functions. Exploiting the properties of generalized derivatives we can obtain a non homogeneous version of the continuity equation that can be written as [9]:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) &= \rho_0 u_n \delta(f_b) \\ &+ (\rho - \rho_0) (u_n - v_n) \delta(f_b) \end{aligned} \quad (3)$$

The second term on the right hand side disappears in the classical formulation since the non penetrating condition states that $(u_n - v_n) = 0$. In a similar way the generalized version of the momentum equation can be obtained:

$$\begin{aligned} \frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_i} (P_{ij} + \rho u_i u_j) &= P'_{ij} n_j \delta(f_b) \\ &+ (\rho u_i) (u_n - v_n) \delta(f_b) \end{aligned} \quad (4)$$

Where $P'_{ij} = P_{ij} - p_0 \delta_{ij}$ is the perturbation stress tensor, and δ_{ij} is the Kronecker delta. Also in this case the second term on the right hand side vanishes since flow is not allowed accross S_b . It is now possible to assemble eqs. (3) and (4) following a standard procedure as outlined by Brandao [9]. The first step is to take the generalized derivative of eq. (4) with respect to x_i and to subtract the generalized time derivative of eq. (3). Then the term $c^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i}$ can be subtracted from the result of the previous operations. With some further manipulations, and considering that p_0 and ρ_0 are constant accross S_b the final form of FWH equation can be written as:

$$\begin{aligned} \square^2 [c^2 (\rho - \rho_0)] &= \frac{\partial}{\partial t} [\rho_0 u_n \delta(f_b)] \\ &- \frac{\partial}{\partial x_i} [P'_{ij} n_j \delta(f_b)] \\ &+ \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \end{aligned} \quad (5)$$

Where $T_{ij} = P'_{ij} + \rho u_i u_j - c^2 (\rho - \rho_0) \delta_{ij}$ is the Lighthill's stress tensor. If the perturbations are small the term $c^2 (\rho - \rho_0)$ can be substituted by p' and therefore eq. (5) can be used to evaluate the pressure disturbance. It must be pointed out that the hypothesis

of small perturbances has to be verified only at the observer location, while no restriction is posed near the body. Using standard Green function approach eq. (5) can be rewritten as an integral equation where the first two terms on the right hand side represent integrals on the surface S_b of the body (Thickness and Loading), while the last term generates a volume integral that describe the quadrupole contribution.

The Kirchhoff Approach

In order to better understand the common aspects of the two approaches we will start the derivation of Kirchhoff formulation a little upstream of what is usually done. Also in this case we consider a body B whose surface S_b is described by the equation $f_b = 0$, and immersed in a fluid medium. The motion is clearly governed by the continuity and momentum equations (1)(2). Let's now consider a generic closed and smooth surface S of arbitrary shape and motion, defined by $f(\mathbf{x}, t) = 0$ ($|\nabla(f)| = 1$ for $f = 0$), and try to evaluate the noise radiated by the body B for observers placed outside S . If the surface S is far enough from the body B , then the fluid outside S can be considered to be inviscid, the motion isentropic and irrotational, and the perturbances small. With these hypothesis eqs. (1)(2) can be rewritten as the standard wave equation:

$$\frac{1}{c} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' \equiv \square^2 p' = 0 \quad (6)$$

being c is the speed of sound in the undisturbed medium. The sound propagation outside S can therefore be modelled replacing the volume inside S with fluid at rest ($p' = 0$), and introducing a discontinuity surface across S . At this point, exploiting the properties of generalized derivatives, the non homogeneous version of eq. (6) can be obtained [15]:

$$\begin{aligned} \square^2 p' &= - \left(\frac{\partial p'}{\partial n} + \frac{M_n}{c} \frac{\partial p'}{\partial t} \right) \delta(f) \\ &- \frac{1}{c} \frac{\partial}{\partial t} [M_n p' \delta(f)] \\ &- \frac{\partial}{\partial x_i} [p' n_i \delta(f)] \end{aligned} \quad (7)$$

Where \mathbf{n} is the unit vector normal to the surface S and pointing outwards, $M_n = v_i n_i / c$ is the Mach number in the normal direction. The integral formulation can be easily obtained from eq. (7) using Green function approach.

The first KFWH Equation

From the above derivations it is clear that FWH and Kirchhoff can be seen as different descriptions of the same phenomenon since they can be obtained starting from the same physical problem described

with the same equations (1)(2). The differences between the two formulations are due to some choices that are made in the derivation process. The first choice is that for the Kirchhoff equation some simplifying hypothesis are introduced in the early stages of derivation, while no assumption is made for the FWH equation. The second difference is that the discontinuity surface S is imposed to be coincident with the surface S_b of the body in the FWH equation, while no limitation is given for S in the Kirchhoff method. A new formulation, that combines the positive aspects of FWH and Kirchhoff approaches, can at this stage obtained in a few steps, and the procedure for its derivation can be interpreted in two different ways. From one side one can think to follow the same approach used for the derivation of the FWH equation, using however a fictitious discontinuity surface S not necessarily coincident with S_b . On the other side one can think to start from the continuity and momentum equations and to follow the same procedure used in the derivation of Kirchhoff formulation with the difference that the simplifying hypothesis are no more introduced. Clearly from a practical point of view the approach is exactly the same. Starting from eqs. (1) (2), we introduce therefore a generic discontinuity surface S , and replace the volume inside S with fluid at rest ($p' = 0$, $\rho = 0$, $\mathbf{u} = 0$). The non homogeneous versions of eqs. (1) (2) are simply obtained from eqs. (3) (4) once f_b is replaced with f . It is however very important to note that, since the surface S is fictitious, the non penetration condition is no more verified, and, in order to obtain correct results, we have to allow a fluid flow across S . Equations (3) and (4) can therefore be assembled adopting the same procedure used above with the only attention that now the terms containing $(u_n - v_n) = 0$ can no more be neglected. The result can be written as:

$$\begin{aligned} \square^2 [c^2(\rho - \rho_0)] &= \frac{\partial}{\partial t} [\rho_0 u_n \delta(f)] \\ &- \frac{\partial}{\partial x_i} [P'_{ij} n_j \delta(f)] + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \\ &+ \frac{\partial}{\partial t} [(\rho - \rho_0) (u_n - v_n) \delta(f)] \\ &- \frac{\partial}{\partial x_i} [\rho u_i (u_n - v_n) \delta(f)] \end{aligned} \quad (8)$$

Where $T_{ij} = P'_{ij} + \rho u_i u_j - c^2(\rho - \rho_0)\delta_{ij}$ is the Lighthill's stress tensor. Equation (8) can be interpreted a modified version of FWH equation extended to the case in which flux flow is allowed on the discontinuity surface. Clearly if the surface S is coincident with the body surface S_b the flow is zero and the classical FWH equation is obtained.

It is interesting to note that eq. (8) can be rearranged in order to have the same formal aspect of the classical FWH equation. Defining the quantities U_i

and L_{ij} as:

$$U_i = u_i + \left(\frac{\rho}{\rho_0} - 1 \right) (u_i - v_i) \quad (9)$$

$$L_{ij} = P'_{ij} + \rho u_i (u_j - v_j) \quad (10)$$

eq. (8) can be rewritten as:

$$\begin{aligned} \square^2 [c^2(\rho - \rho_0)] &= \frac{\partial}{\partial t} [\rho_0 U_n \delta(f)] \\ &- \frac{\partial}{\partial x_i} [L_{ij} n_j \delta(f)] \\ &+ \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \end{aligned} \quad (11)$$

that is identical to the classical FWH equation if u_n is replaced with U_n and P'_{ni} with L_{ni} . The terms U_i and L_{ij} here introduced can be interpreted respectively as a modified velocity and a modified stress tensor, that take in account for the flux flow across S . It is so possible to conclude that FWH equation is still valid for permeable surfaces if the modified velocity and stress tensor are introduced.

The Green function G of the unbounded three dimensional space is defined as $G = \delta(g)/r$, where $r = \|\mathbf{x} - \mathbf{y}\|$, $g = t - \tau - r/c$, and where \mathbf{x}, \mathbf{y} represents respectively observer and source position and t, τ observer and source time. Executing a convolution of eq. (8) with the Green function G it is possible to recast the above equation in an integral form, that, for a not deformable surface S , can be written as:

$$\begin{aligned} &4 \pi c^2 (\rho - \rho_0) = \\ &+ \frac{\partial}{\partial t} \int_S \left[\frac{\rho_0 u_n + (\rho - \rho_0) (u_n - v_n)}{r |1 - M_r|} \right]_{ret} dS \\ &+ \frac{1}{c} \frac{\partial}{\partial t} \int_S \left[\frac{P'_{nr} + \rho u_r (u_n - v_n)}{r |1 - M_r|} \right]_{ret} dS \\ &+ \int_S \left[\frac{P'_{nr} + \rho u_r (u_n - v_n)}{r^2 |1 - M_r|} \right]_{ret} dS \quad (12) \\ &+ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int_V \left[\frac{T_{rr}}{r |1 - M_r|} \right]_{ret} dV \\ &+ \frac{1}{c} \frac{\partial}{\partial t} \int_V \left[\frac{3T_{rr} - T_{ii}}{r^2 |1 - M_r|} \right]_{ret} dV \\ &+ \int_V \left[\frac{3T_{rr} - T_{ii}}{r^3 |1 - M_r|} \right]_{ret} dV \end{aligned}$$

Where $M_r = v_i r_i / c$ is the Mach number in the observer direction, $T_{rr} = T_{ij} r_i r_j$, and $T_{ii} = T_{11} + T_{22} + T_{33}$. Besides V is the volume external to the surface S ($f > 0$), and the symbol \llbracket_{ret} means, as usual, evaluation at the retarded time $\tau^* = t - r/c$. In order to obtain eq. (12) the following formula [15]:

$$\frac{\partial}{\partial x_i} \left[\frac{\delta(g)}{r} \right]_{ret} = - \frac{1}{c} \frac{\partial}{\partial t} \left[\frac{r_i \delta(g)}{r^2} \right]_{ret} - \frac{r_i \delta(g)}{r^3} \quad (13)$$

has been used to transform space derivatives in time derivatives. Equation (12) can be seen as a boundary integral equation that, for any point external

to a generically moving surface S , relates the density disturbance with the values of pressure, velocity, and density on the surface itself, and with the Lighthill stress tensor in the volume external to the surface. This equation has been derived directly from the equation of conservation of mass and momentum without any further assumption and so can be applied to a generic surface independently if the propagation is linear outside the surface or not. If the surface S is placed on the body the classical FWH equation is obtained and the non linear propagation effects are taken in account by the quadrupole volume terms. Instead if the surface is far enough from the body, then the Lighthill stress tensor outside S can be neglected and, using the relation $c^2(\rho - \rho_0) = p'$, valid if perturbations are small, eq. (12) can be rewritten as:

$$\begin{aligned} 4\pi p' &= \frac{\partial}{\partial t} \int_S \left[\frac{\rho_0 u_n + (\rho - \rho_0) (u_n - v_n)}{r |1 - M_r|} \right]_{ret} dS \\ &+ \frac{1}{c} \frac{\partial}{\partial t} \int_S \left[\frac{P'_{nr} + \rho u_r (u_n - v_n)}{r |1 - M_r|} \right]_{ret} dS \\ &+ \int_S \left[\frac{P'_{nr} + \rho u_r (u_n - v_n)}{r^2 |1 - M_r|} \right]_{ret} dS \end{aligned} \quad (14)$$

This formula together with eq. (18) is the main results of this paper and is here referred as first Kirchhoff-FWH equation (KFWH).

It can be interesting to note that, if the surface S is placed near the body, then a sort of *mixed* formulation can be obtained, in which part of the non linearities are taken in account by the quadrupole volume terms, and part by the surface integrals.

Let's now compare the above equation with both FWH and Kirchhoff approaches. The advantages of eq. (14) in respect of the Kirchhoff formulation are due to the fact that KFWH approach is more closely related to the nature of the sound propagation, while the Kirchhoff formulation is valid for any phenomenon governed by the wave equation, independently on the nature of the phenomenon itself. This is the reason for which the Kirchhoff approach describes the sound propagation outside S using a single fluid quantity, namely the pressure p' , and requires over S the knowledge not only of p' , but also of its normal derivative. On the other side KFWH uses not only p' but also \mathbf{u} and ρ , and these quantities permit to reconstruct the sound propagation outside S without the need of any normal derivative. The main practical advantage is therefore that KFWH only contains quantities that are directly available from CFD codes, without the need of executing derivation of CFD data. This aspect can be of a certain importance if shocks are present in the field around the surface S , as happens in delocalized conditions. In this case, in fact, the evaluation of the pressure derivative can easily be a source of undesired smoothing that can degrade the quality of the acoustic result.

In respect of FWH the first clear advantage of KFWH is that, like the Kirchhoff method, it permits

to avoid the evaluation of the volume integrals, and therefore reduce the computational cost reducing a volume integration to a surface one. The other interesting aspect is that KFWH can be applied to any radiation problem independently if the source is a body in motion in the fluid, or any other mechanism. In fact, once p' , \mathbf{v} , and ρ are known on a proper surface surrounding the source, the method can be applied independently on the source itself.

The Second KFWH Equation

The presence, in the integrals of eq. (14), of time derivatives of quantities depending on the retarded time is a critical aspect that can generate problems if the numerical derivation is not executed with great care. In fact, in order to numerically execute the time derivative, there is the need to evaluate twice the retarded times, and this fact, joined with the higher accuracy required in each retarded time evaluation, almost double the computational time in respect of other methods in which the numerical derivative does not appear [7]. The time derivatives can however be easily moved inside the integrals following the same procedure used by Farassat in deriving his formulation 1A [17].

Taking in account that, for a generic function $Q = Q(\mathbf{y}, \tau)$:

$$\frac{\partial}{\partial t} [Q(\mathbf{y}, \tau)]_{ret} = \left[\frac{1}{1 - M_r} \frac{\partial Q}{\partial \tau} \right]_{ret} \quad (15)$$

and using the relations:

$$\dot{r} = -v_r \quad \dot{\hat{r}}_i = -v_i \quad (16)$$

$$\dot{\hat{r}}_i = \frac{\partial}{\partial \tau} \left(\frac{r_i}{r} \right) = \frac{-v_i + \hat{r}_i v_r}{r} \quad (17)$$

being $\hat{r}_i = r_i/r$, then eq.(14) can be rewritten as:

$$\begin{aligned} 4\pi p' &= \int_S \left[\frac{\rho_0 (\dot{U}_i n_i + U_i \dot{n}_i)}{r |1 - M_r|^2} \right]_{ret} dS \\ &+ \int_S \left[\frac{\rho_0 U_i n_i K}{r^2 |1 - M_r|^3} \right]_{ret} dS \\ &+ \frac{1}{c} \int_S \left[\frac{\dot{F}_i \hat{r}_i}{r |1 - M_r|^2} \right]_{ret} dS \\ &+ \int_S \left[\frac{F_i \hat{r}_i - F_i M_i}{r^2 |1 - M_r|^2} \right]_{ret} dS \\ &+ \frac{1}{c} \int_S \left[\frac{F_i \hat{r}_i K}{r^2 |1 - M_r|^3} \right]_{ret} dS \end{aligned} \quad (18)$$

Where :

$$K = \dot{M}_i \hat{r}_i r + M_r c - M^2 c \quad (19)$$

$$F_i = L_{ij} n_j \quad (20)$$

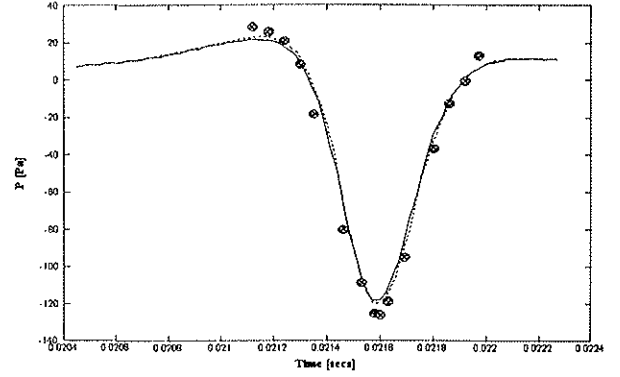


Figure 1: UH-1H $M=0.88$, Comparison of fixed surface KFWH (solid line), fixed surface Kirchhoff (dotted line), and experiments (dots).

Even if this formulation is more complex than eq. (14), it has the great advantage that it does not require any numerical evaluation of derivatives of quantities depending on the retarded time, and this is a great advantage from the computational point of view.

Another useful version of formula (14) can be obtained if the integration is executed on the acoustic surface Σ leading to:

$$\begin{aligned} 4\pi p' &= \frac{\partial}{\partial t} \int_{\Sigma} \left[\frac{\rho_0 u_n + (\rho - \rho_0) (u_n - v_n)}{r \Lambda} \right]_{ret} d\Sigma \\ &+ \frac{1}{c} \frac{\partial}{\partial t} \int_{\Sigma} \left[\frac{P'_{nr} + \rho u_i (u_n - v_n)}{r \Lambda} \right]_{ret} d\Sigma \\ &+ \int_{\Sigma} \left[\frac{P'_{nr} + \rho u_i (u_n - v_n)}{r^2 \Lambda} \right]_{ret} d\Sigma \end{aligned} \quad (21)$$

Being $\Lambda = \sqrt{1 + M_n^2 - 2M_n \cos \theta}$. It is possible to show that, with an appropriate numerical approach [6, 8, 16], this formula has the great advantage that can be applied when the surface S is moving supersonically, while eqs. (14), (18) presents a singularity in this case.

Results

In order to check the validity of the proposed formulation some calculations have been conducted, comparing the results of the Kirchhoff-FWH formulation with classical Kirchhoff and FWH approaches. The test case considered here is the well known UH-1H rotor in hover for tip Mach numbers equal to 0.88, 0.90, and 0.95. The aerodynamic data used as input were provided by DLR and were obtained using a finite volume Euler code [13]. In all the comparisons the geometry and the discretization of the Kirchhoff and Kirchhoff-FWH surfaces is exactly the same, and the same aerodynamic results are used to provide the different input data. The observer is always placed in the rotor plane at a distance of $3.09R$.

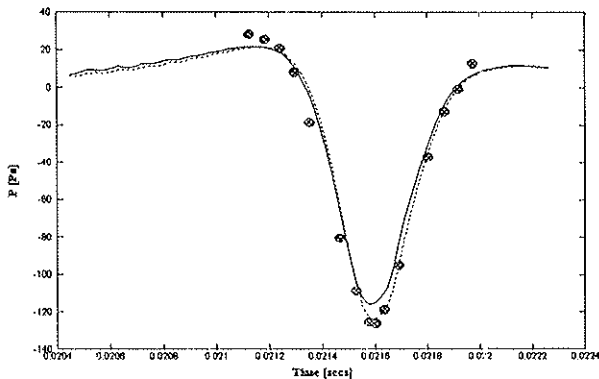


Figure 2: UH-1H $M=0.88$, Comparison of rotating surface KFWH (solid line), rotating surface Kirchhoff (dotted line), and experiments (dots).

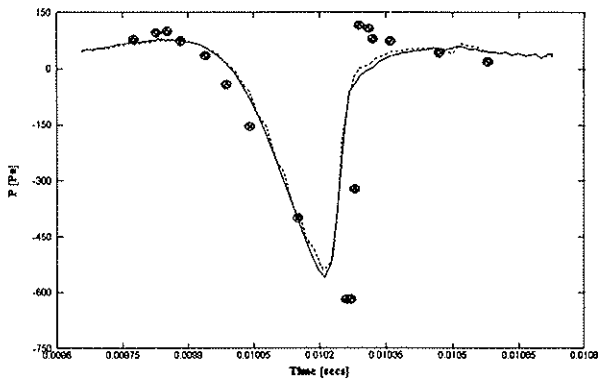


Figure 4: UH-1H $M=0.90$, Comparison of fixed surface KFWH (solid line), fixed surface Kirchhoff (dotted line), and experiments (dots).

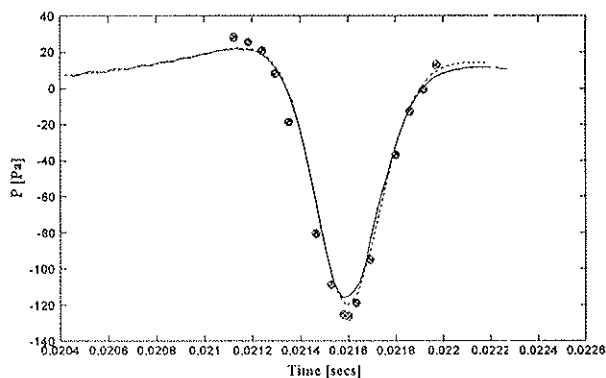


Figure 3: UH-1H $M=0.88$, Comparison of rotating surface KFWH (solid line), FWH Thickness + Loading + Quadrupole (dotted line), and experiments (dots).

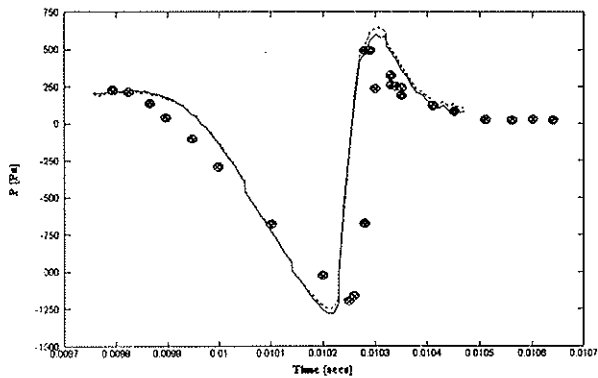


Figure 5: UH-1H $M=0.95$, Comparison of fixed surface KFWH (solid line), fixed surface Kirchhoff (dotted line), and experiments (dots).

In fig. (1) are reported the comparisons for $M = 0.88$. The results refer to a cylindrical surface kept fixed in respect of the undisturbed air and surrounding the entire rotor. The cylinder axis was coincident with the rotor axis of rotation, and the top and bottom surfaces of the cylinder were not considered since their contribution is negligible. In each point of the cylinder the aerodynamic quantities are unsteady due to the rotor rotation, and a bilinear interpolation was used to transform the aerodynamic results, originally given in the rotating frame.

The results given in fig. (2) refer instead to the same case evaluated with a rotating surface kept fixed in respect of the blade. The external surface radius is the same of this of the fixed surface used in fig. (1), and is equal to $1.15R$. In all the figures the continuous line is the KFWH approach, the dotted line is classical Kirchhoff, and the dots are the experimental measurements. The agreement between the two formulations and experiment is good and only small differences exist in the case of the rotating surface.

The same case is considered in fig. (3) where the sum of thickness, loading, and quadrupole terms of

the FWH equation is compared with the KFWH approach for $M = 0.88$. The KFWH surface S was in this case placed on the external surface of the volume used for quadrupole calculation, and, as it could be expected, the two formulations provide almost identical results, since they neglect exactly the same terms (the quadrupole sources outside S).

In figs. (4),(5) the results for $M = 0.90$ and $M = 0.95$ are given for a fixed surface of radius equal to $1.3R$. Also in this case the agreement with experiment is satisfactory, and the differences in the slopes of the pressure disturbance are probably due to an excess of dissipation introduced in the aerodynamic solution. What is however important here is that, also in these cases, KFWH and Kirchhoff produces almost the same results.

At the end in figs. (6),(7) a convergence test for $M = 0.90$ is showed respectively for the KFWH and Kirchhoff formulations. The different curves are obtained using different Kirchhoff cylinders placed at different radius. It can be seen that the behaviour of the results is similar for the two formulations. In particular for $r/R = 1.1$ the surface is too near to the blade and some non linear terms are neglected. On

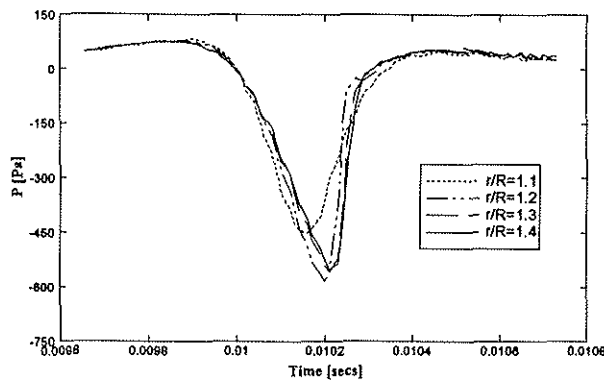


Figure 6: UH-1H $M=0.90$, Convergence test for KFWH method.

the other side for $r/R = 1.3$ the convergence is practically achieved since the results are almost identical to these ones obtained for $r/R = 1.4$ with both the methods.

Conclusions

A new boundary integral equation has been presented that permits the evaluation of the noise radiated by arbitrary sources once pressure, velocity, and density disturbances are known on a smooth closed surface surrounding the source.

The main advantage of the proposed approach in respect of Kirchhoff formulation is that it can be more easily interfaced with CFD codes. The new method in fact does not require the numerical evaluation of the surface pressure normal derivative, operation that can be source of problems if the aerodynamic grid is not sufficiently refined around the Kirchhoff surface. Two different formulations have been presented. In the first one a time derivative appears outside some of the integrals and has to be evaluated numerically. In the second one the derivative is taken inside the integrals and is evaluated analytically. Some calculations reveal that the KFWH method produces almost the same results than Kirchhoff method, and also the convergence properties in terms of surface distance from the source seem to be similar.

Further work has to be performed to assess the accuracy required by the two approaches in terms of grid definition for the aerodynamic calculation, in order to understand if the use of KFWH formulation could permit to use a less refined aerodynamic grid without affecting the accuracy in the acoustic solution.

Acknowledgements

The author wants to thank Dr. Massimo Gennaretti of Rome III University for the very useful discussions on the subject, and for firstly suggesting the idea of trying to use the FWH equation in order to obtain a Kirchhoff-like formulation. The author would like

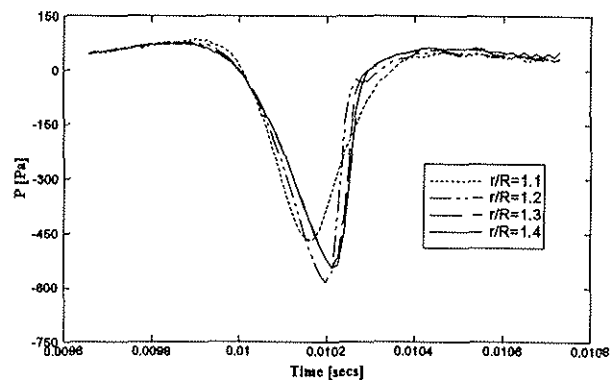


Figure 7: UH-1H $M=0.90$, Convergence test for Kirchhoff method.

also to thank Dr. N.Kroll and Dr.M.Kuntz of DLR for having kindly provided the Euler aerodynamic results, within the EU research project IMT AERO 2017-2060 (HELISHAPE).

References

- [1] J. D. BAEDER The Role and Status of Euler Solvers in Impulsive Rotor Noise Computations *AGARD Fluid Dynamics Panel Symposium on Aerodynamics and Aeroacoustics of Rotorcraft*
- [2] J. PRIEUR Calculation of Transonic Rotor Noise Using a Frequency Domain Formulation *AIAA Journal*, Vol. 26 (2) February 1988, pp 156-162.
- [3] S.IANNIELLO, E. DE BERNARDIS Calculation of High Speed Noise from Helicopter Rotors Using Different Description of Quadrupole Source *AGARD Symposium on Aerodynamics and Aeroacoustics of Rotorcraft*, Berlin, Germany, 10-13 October 1994.
- [4] K.S. BRENTNER, A.S.LYRINTZIS, E.K.KOUTSAVDIS A Comparison of Computational Aeroacoustic Prediction Methods for Transonic Rotor Noise *52th Annual Forum of American Helicopter Society*, Washington, D.C. June 1996
- [5] S.IANNIELLO, E.DE BERNARDIS Volume Integration in The Calculation of Quadrupole Noise From Helicopter Rotors *First CEAS-AIAA Aeroacoustics Conference*, June 12-15 1995, Munchen Germany.
- [6] P. DI FRANCESCANTONIO High Speed Rotational Noise: An Acoustic Volume Approach *Noise-93*, St.Petersburg Russia, May 31- June 3 1993
- [7] P. DI FRANCESCANTONIO A New Boundary Integral Formulation for the Prediction of Sound

radiation *Submitted to the Journal of Sound and Vibration June, 1996*

- [8] P. DI FRANCESCANTONIO A Supersonically Moving Kirchhoff Surface Method for Delocalized High Speed Rotor Noise Prediction. *CEAS-AIAA 2nd Aeroacoustics Conference June 1996 PA*
- [9] M.P.BRANDAO On the Aeroacoustics, Aerodynamics, and Aeroelasticity of Lifting Surfaces *PhD Thesis, february 1988, Department of Aeronautics and Astronautics , Stanford University.*
- [10] A.S.LYRINTZIZ Review: The Use of Kirchhoff's Method in Computational Aeroacoustics *Journal of Fluids Engineering Dec. 1994, Vol. 116*
- [11] R.C.STRAWN, R. BISWAS, A.S.LYRINTZIS Helicopter Noise Predictions Using Kirchhoff Methods *51st Annual Forum of American Helicopter Society, 9-11 May, 1995, Fort Worth, TX.*
- [12] Y.H. YU, S.LEE, M.P.ISOM Rotor High Speed Noise Prediction With A Combined CFD-Kirchhoff Method *50st Annual Forum of American Helicopter Society, 11-13 May, 1994, Washington, D.C.*
- [13] M.KUNTZ, D.LOHMANN, J.A.LIESER, K.PAHLKE Comparison of Rotor Noise Prediction obtained by a Lifting Surface Method and Euler Solutions using Kirchhoff Equation *First CEAS-AIAA Aeroacoustics Conference, June 12-15 1995, Munchen Germany.*
- [14] C.POLACSEK, J.PRIEUR High-Speed Impulsive Noise Computations in Hover and Forward Flight Using a Kirchhoff Formulation *First CEAS-AIAA Aeroacoustics Conference, June 12-15 1995, Munchen Germany.*
- [15] F.FARASSAT M.K.MYERS Extension of Kirchhoff's Formula to radiation from moving surfaces *Journal of Sound and Vibration, Vol. 123(3) pp. 451-460*
- [16] F.FARASSAT M.K.MYERS The kirchhoff Formula for a Supersonically Moving Surface *First CEAS-AIAA Aeroacoustics Conference, June 12-15 1995, Munchen Germany.*
- [17] K.S.BRENTNER Prediction of Helicopter Rotor Discrete Frequency Noise *NASA TM-87721, pp 6-7.*