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THE INFLUENCE OF THE INERTIA COUPLING ON THE STABILITY AND CONTROL OF THE HELICOPTER AND THE RESPONSE OF HELICOPTER GUST

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Abstract

The influence of the inertia coupling on the stability and control of the helicopter and the response of helicopter to dispersed gust are investigated. An articulated rotor with a blade hinge offset and with an elastic restraint about the flap hinge are used as the rotary wing dynamic model. The nonuniform distribution of induced velocity on the rotor disk derived from the generalized vortex theory is taken into account. The model of the dispersed gust is the sine-squared according to the demand of the specification.

A sample calculation of a typical helicopter has been made. Considering the inertia coupling and not and comparing the two states, the detail calculation and analysis of the stability, control and the response of helicopter to dispersed gust are given.

Notation

W	weight of helicopter
V_x, V_y, V_z	airspeed components of helicopter
$\omega x, \omega y, \omega z$	angular velocity components of helicopter
$\theta oc, \theta cc, \theta sc, (\theta_0)trc$	collective pitch control, longitudinal and lateral control, collective pitch control of tailrotor respectively
W_g	gust velocity
Hg	gust range
d	distance from considered point to origin point
d_1	distance from forward edge point to origin point
$\gamma x, \psi y, \vartheta z$	attitude angle of helicopter, roll angle, yaw angle and pitch angle
$C_T, C_H, C_S, C_{MX}, C_{MY}, C_{MZ}$	Coefficient of rotor force and rotor moment
β_s	slipping angle of helicopter
α_s	attack angle of helicopter

1. Introduction

The slender fuselage of helicopter and most of the mass concentrating on fuselage make the moment of inertia about the longitudinal x axis much more smaller than other two moments of inertia about the other axes, for example, a typical helicopter has the data about the moment of inertia, $I_x = 2945 m^2 Kg$, $I_y = 8578 m^2 Kg$, $I_z = 10409 m^2 Kg$ [1]. When the helicopter makes maneuver flight. The variations of the angular velocities are not small values. Therefore the moments of the inertia coupling items $\omega_z \omega_x (I_x - I_z)$, $\omega_y \omega_z (I_y - I_z)$ and $\omega_x \omega_y (I_x - I_y)$ can't be neglected, especially the first and second items. The moments of centrifugal inertia of helicopter are $I_{xy} = 726.8 m^2 Kg$, $I_{zx} = 61.7 m^2 Kg$ and $I_{yz} = 6.4 m^2 Kg$, in which I_{xy} value is larger than other, so the item $I_{xy}(\dot{\omega}_x + \omega_y \omega_z)$ must be also considered in study. In other words, there are twelve items of moments of inertia, which must be considered and dealed with respectively. Therefore the nonlinear differential equations of motion with six freedoms are used to analysis the property of helicopter motion.

Reference [2], [3] described the demand of inertia coupling but there is little papers investigating the inertia coupling of helicopter. A method to calculate and analysis the effects of inertia coupling on stability, control input response and gust response are studied in this paper. Obviously the quantitative analysis for the specification qualities and a certain reference of value for helicopter design are provided.

2. Dynamic Equations of Helicopter

2.1. Dynamic model of rotor

Rotor hub has a star-flexibility hub structure. The flapping deflection of rotor is only considered in this paper. The studied rotor may be equivalent to an articulated rotor with a blade hinge offset from the shaft and with an elastic restraint aboutthe flapping hinge. The condition of equivalent is the same natural property in flapping. ref. [1].

2.2. Aerodynamic model of rotor

The compression and stall are not considered. Using the steady theory to replace the unsteady theory, aerodynamic load is calculated. The distributions of the induced velocity over rotor disk are derived from generalized vortex theory. The influence of gust on vortex is neglected. ref. [4].

2.3. The dynamic equations of helicopter

The general motion of the helicopter in flight may be resolved into two motions, the motion of the center of mass and the rotation about the center of mass. Then using the theory of mechanic, the total dynamic equations of helicopter may be obtained as follows:

$$\frac{w}{g}(v_x + \omega_y v_z - \omega_z v_y) = X \qquad (1)$$

$$\frac{w}{g}(v_y + \omega_z v_x - \omega_x v_z) = Y \qquad (2)$$

$$\frac{w}{g}(v_z + \omega_x v_y - \omega_y v_x) = Z \qquad (3)$$

$$I_x\dot{\omega}_x - I_{xy}(\dot{\omega}_y - \omega_x\omega_z) - I_{xz}(\dot{\omega}_z + \omega_x\omega_y) - (I_y - I_z)\omega_y\omega_z$$
$$- I_{yz}(\omega_y^2 - \omega_z^2) = L \qquad (4)$$

$$I_y\dot{\omega}_y - I_{yz}(\dot{\omega}_z - \omega_x\omega_y) - I_{yx}(\dot{\omega}_x + \omega_y\omega_z) - (I_z - I_x)\omega_z\omega_x$$
$$- I_{zx}(\omega_z^2 - \omega_x^2) = M \qquad (5)$$

$$I_z\dot{\omega}_z - I_{zx}(\dot{\omega}_x - \omega_y\omega_z) - I_{zy}(\dot{\omega}_y + \omega_x\omega_z) - (I_x - I_y)\omega_x\omega_y$$
$$- I_{xy}(\omega_x^2 - \omega_y^2) = N \qquad (6)$$

In the equations (4), (5) and (6), except the first item respectively the else items are inertia coupling moments. Because the item I_x is much more smaller than the item I_y and I_z . So the I_x may be neglected. Therefore the pitch inertia coupling moment item $(I_y - I_x)\omega_x\omega_y$ and the yaw inertia coupling moment item $(I_z - I_x)\omega_z\omega_x$ are larger than other inertia coupling item relatively. The roll inertia coupling moment item $(I_y - I_z)\omega_y\omega_z$ and all of the centrifugal inertia coupling moment items are smaller. Within these centrifugal items the item $I_{xy}(\dot{\omega}_y - \omega_x\omega_z)$ is larger than other, its value is about the one-ten as large as the yaw inertia coupling moment. In other words, the study of the inertia coupling moment in practice is the study of the pitch and yaw inertia coupling moments which effects largely on the study.

Whether considering the inertia coupling items or not in mathematical is actually how to deal with the nonlinear items of the dynamic equations. Calculating the responses of the control input if the inertia coupling items are not considered, the method is conventional method used the small-disturbance theory and linearization. But if the inertia coupling items are considered, nonlinear motion equations are solved used the numerical method.

2.4. Disturbed equations of helicopter

Using the small-disturbance theory, the equations of the helicopter disturbed motion can be obtained. Using the linear transit of coefficient and treatment of reducing matrix rank, the standard form of equation is as follows:

$$(DE - A_s)X = B_s U$$

where $x = \left[\Delta v_x, \Delta v_y, \Delta v_z, \Delta\omega_x, \Delta\omega_y, \Delta\omega_z, \Delta\gamma_x, \Delta\psi_y, \Delta\vartheta_z\right]^T$

E is unit matrix by 9×9

$U = \left[\Delta\theta_{oc}, \Delta\theta_{cc}, \Delta\theta_{sc}, \Delta(\theta_o)_{trc}\right]^T$

A_s, B_s are coefficient matrix

3. The Influence of Inertia Coupling on Stability Roots of the Helicopter

In order to obtain the stability roots of the helicopter, put the matrix B_s to equal to zero, then the characteristic equation $\lambda E - A_s = 0$ is obtained and the roots of the characteristic equation may be solved, as follows, $\lambda_j (j = 1, 2, \ldots 9)$. Because of lineaization the items $I_{xy}\dot{\omega}_y$, $I_{yz}\dot{\omega}_z$ and $I_{zx}\dot{\omega}_x$ are remaineder. Relating to the the roots of stability, the above three items equal to zero when not considering the inertia coupling.

A sample calculation of stability roots for a typical helicopter is made when hovering and forward speed $\mu = 0.2$. Considering the coupling of the longitudinal and lateral motion at $\mu = 0.2$, the charactersitic roots of stability are shown in table (1). From the results of calculation the conclusions may be obtained as follows:

considering the inertia coupling		not considering the inertia coupling	
$-0.60710E^{00}$	$0.24840E^{01}$	$-0.61537E^{00}$	$0.25970E^{01}$
$-0.60710E^{00}$	$-0.24840E^{01}$	$-0.61537E^{00}$	$-0.25970E^{01}$
$-0.13158E^{00}$	$0.20292E^{01}$	$-0.12137E^{00}$	$0.20392E^{01}$
$-0.13158E^{00}$	$-0.20292E^{01}$	$-0.12137E^{00}$	$-0.20392E^{01}$
$-0.81721E^{00}$	0	$-0.81720E^{00}$	0
$-0.29844E^{00}$	0	$-0.26645E^{00}$	0
$0.39434E^{-2}$	$0.21309E^{00}$	$-0.16328E^{-2}$	$0.21268E^{00}$
$0.39434E^{-2}$	$-0.21309E^{00}$	$-0.16328E^{-2}$	$-0.21268E^{00}$
$-0.25362E^{-7}$	0	$-0.18086E^{-7}$	0

Table 1. The roots of stability

(1) Regardless of the coupling of the longitudinal and lateral motion, the inertia coupling moments have an effect on the roots of lateral stability, but have little effect on the roots of longitudinal stability. The effects in hovering state is greater than in forward speed $\mu = 0.2$.

(2) Considering the coupling of longitudinal and lateral motion, the inertia coupling has an effect on roots of stability more and less and the effects in hovering are smaller than in forward speed $\mu = 0.2$.

4. The Influence of Inertia Coupling on the Response to Controlled Input

The controlled inputs have three kinds, longitudinal, lateral and yaw controlled input. If the moments of inertia coupling are not considered, the responses of all controlled inputs may be calculated from the linearization equations:

$$(DE - A_s)X = B_s U \qquad (7)$$

where U is respectively as following:

longitudinal control $U = \left[0,\ 0,\ \Delta\theta_{sc},\ 0\right]^T$

lateral control $U = \left[0,\ \Delta\theta_{cc},\ 0,\ 0\right]^T$

yaw control $U = \left[0,\ 0,\ 0,\ \Delta(\theta_0)_{trc}\right]^T$

fig.1	the response of roll angle to longitudinal controlled input at $\mu = 0.2$
fig.2	the response of yaw angle to longitudinal controlled in input at $\mu = 0.2$
fig.3	the response of pitch angle to longitudinal controlled input at $\mu = 0.2$
fig.4	the response of roll angle to lateral controlled input at $\mu = 0.2$
fig.5	the response of yaw angle to lateral controlled input at $\mu = 0.2$
fig.6	the response of pitch angle to lateral controlled input at $\mu = 0.2$

In each figure , curve 1 is indicated the response regardless of inertia coupling and curve 2 is indicated the response considering the inertia coupling.

The conclusions may be obtained from the results of the calculation, as follows:

(1) The responses of the attitude angle γ_x, ψ_y, ϑ_z of helicopter to the longitudinal controlled input are greater when the inertia coupling moments are considered, and they are time-varying. But at 1 second or within 1 second moment, the responses are agreed with the demand of the flying quality specification of helicopter.

(2) The responses of the corresponding attitude angles to the coaxial controlled inputs are smaller than the non-coaxial controlled input in hovering, i.e. the response of the pitch angle to the longitudinal controlled input, the response of the roll angle to the lateral controlled input and the response of the yaw angle to the yaw controlled input are smaller. But there are no this phenomena in forward flight. It is described that the aerodynamics coupled with the inertia play an important role in the coupling.

(3) All of the responses of the attitude angle of helicopter to all controlled input at the $\mu = 0.2$ are larger than at the hovering state. There are no same variant laws within the response of the hovering state and forward flight state.

(4) In general, the influences of the responses of inertia coupling moments on all the attitude angles increase the responses, but in hovering the influence of the responses of inertia coupling on the attitude angles also have the decreased phenomena. For example, the response of yaw angle to the longitudinal controlled input, the response of roll angle to the lateral controlled input and the response of yaw angle to yaw controlled input.

5. The Influence of Inertia Coupling of Helicopter on the Response to Dispersed Gust

The total processes of penetrating gust, steeping gust and withdrawing gust are considered in this paper. The model of dispersed gust is determined from the demand of the flying quality specification which has more common sense and ability specified the essential questions. The distribution of the induced belocity over the rotor disk are nonuniform.

The model of dispersed gust have some assumptions as follows: the gust is non-anisotropic, the intensity of gust is equal in any direction and it has nothing to do with the selection of coordinate system. The gust field is taken to be "frozen". The variation of the gust is very small, which are considered as constant in any position later on the flight velocity of helicopter increased to a certain value and later on the flight range of helicopter increased to very longer distance.

Calculating the response of helicopter to the gust, the analysis of helicopter may be divided three parts, rotor, fuselage and tailrotor which enter to the total processes. The analysis of the rotor is the key of the analysis of helicopter passing through the total processes. The expressions of the sine-squared model of gust is as follows:

$$(Wg)_i = \frac{1}{2} (Wg_o)_i \left[1 - \cos \pi/(Hg)_i (d - d_1) \right] \quad , (i = 1, 2, 3) \tag{8}$$

where $i = 1, 2, 3$ is indicated longitudinal short period motion, up-down motion and Holland motion respectively.

Because the velocity of gust has an effect on motion equations of helicopter only by aerodynamic items. The influence of the aerodynamic items on the fuselage are the attack angle and slipping angle when the fuselage passes through the gust considered the total processes. Comparing the gust with the head-on velocity of blade section, the primary effect of the gust on the tailrotor is the head-on velocity of the blade section. The component of gust is so small that the influence of gust on the tailrotor can be neglected.

There are nine variables in dynamic equations of helicopter. In order to calculate the response of helicopter on gust, it is necessary to complete three kinematic equations as follows:

$$\frac{dr_x}{dt} = \omega_x - \omega_y \mathrm{tg}\,\vartheta_z \cos\gamma_x + \omega_z \sin\gamma_x \mathrm{tg}\,\vartheta_z \qquad (9)$$

$$\frac{d\psi_y}{dt} = \omega_y \cos\gamma_x / \cos\vartheta_z - \omega_z \sin\gamma_x / \cos\vartheta_z \qquad (10)$$

$$\frac{d\vartheta_z}{dt} = \omega_y \sin\gamma_x /+ \omega_z \cos\gamma_x \qquad (11)$$

Solving the above two sets of equations simultaneously, the dynamic responses of helicopter to gust may be obtained. The simultaneous equations are a set of nonlinear differential equations. When calculate the response of helicopter to gust, considering the influence of inertia coupling, the equations are nonlinear. But when the inertia of coupling is not considered, the equations are linear equations.

The influence of inertia coupling on the response of helicopter to gust are studied in this paper. The frequency of Holland roll motion is only selected as the frequency of gust and only the forward speed $\mu = 0.2$ is studied.

The results of calculation as follows:

$t = 1$ sec	roll angle γ_x	pitch angle ϑ_z	yaw angle ψ_y
considering the inertia coupling	1.32442°	0.74596°	0.04079°
not considering the inertia coupling	1.37404°	0.74102°	0.02943°

Table 2. The responses of áttitude angles at 1 second

	not considering the inertia coupling	considering the inertia coupling
$\gamma_{x\max}$	1.39269° , $t = 0.688$sec.	1.4033° , $t = 0.75$sec.
$\vartheta_{z\max}$	0.76697° , $t = 0.0859$sec.	0.76697° , $t = 0.1075$sec.
$\psi_{y\max}$	0.04079° , $t \fallingdotseq 1$ sec	0.02943° , $t \fallingdotseq 1$ sec.

Table 3. The time when the max. response of the attitude angle is created respectively

	Response not considering inertia coupling	Response considering inertia coupling
c_T	13.04298×10^{-3}	13.00801×10^{-3}
c_H	0.31153×10^{-3}	0.30984×10^{-3}
c_s	0.04368×10^{-3}	0.04787×10^{-3}
c_{Mx}	0.02807×10^{-3}	0.02789×10^{-3}
c_{My}	0.58699×10^{-3}	0.58600×10^{-3}
c_{Mz}	0.07055×10^{-3}	0.069988×10^{-3}
ω_x	0.00023	0.00013
ω_y	0.00005	0.00003
ω_z	0.00001	0
V_x	0.2	0.1999
V_y	-0.00252	-0.00254
V_z	0.00015	0.00013
β_s	0.04342	0.03617
α_s	0.7223	0.72798

Table 4. The responses of the else physical parameters at 1 sec.

From the above tables the conclusions may be obtained as follows:

(1) The response of attitude angle to the gust, the max. value is the roll angle and the min. value is the yaw angle.

(2) The max. influence of the inertia coupling on the response of attitude angle to the gust is the roll angle. The min. is the yaw angle. In general, all of the value of influences are more smaller.

(3) Considering the inertia coupling and not, the max. responses of attitude angles to the gust may be created non simultaneously. But the two times are very nearly.

(4) The influences of inertia coupling on the response of lateral motion parameters to the gust are larger than else parameters, for example, C_s , C_{Mx} , β_s , V_z and ω_x .

6. Synthetic Conclusions

(1) Respect to a typical helicopter, whether considering the inertia coupling or not the influences of the attitude angles on the response to controlled input are obvious, and they are time-varying. But at 1 second or within 1 second moment, the responses are agreed with the demand of the flying quality specification of helicopter.

(2) Whether considering the inertia coupling or not, in general, the influence of coupling on the response of the gust are smaller. Whether consider this factor or not is depended on the demand of the accuracy in engineering.

(3) The gust responses are different from the control responses. But there are same feature which is that the influences of inertia coupling on the responses of the lateral motion parameters are larger than the responses of the pitch motion parameters.

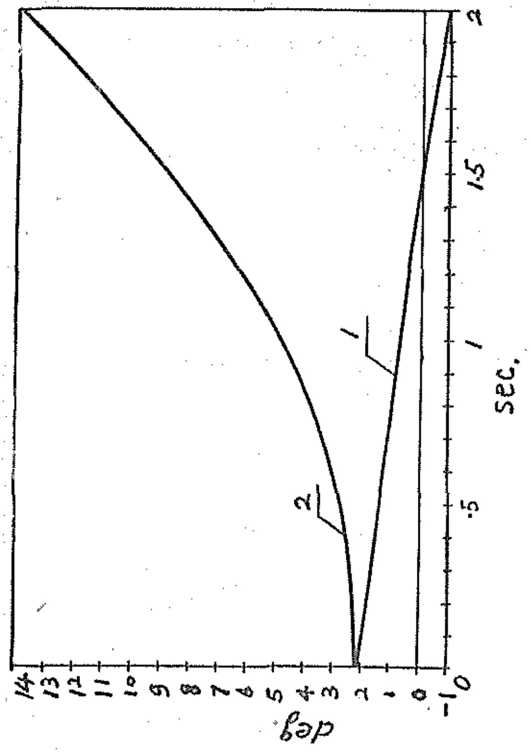

Fig.1

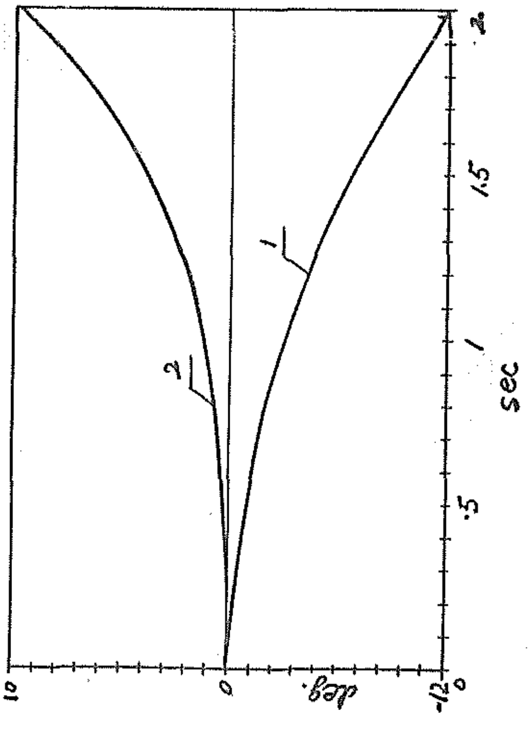

Fig.2

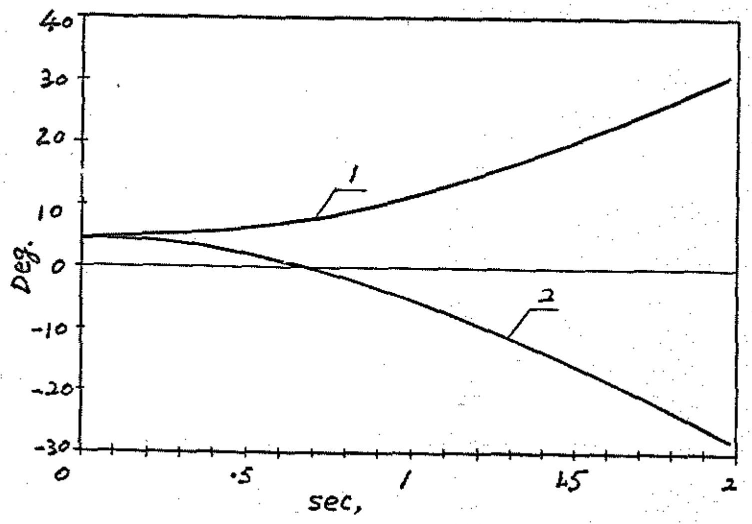

Fig.3

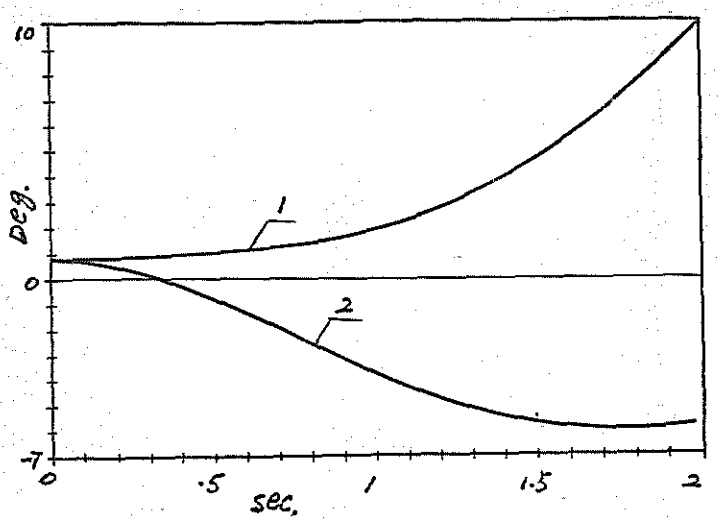

Fig.4

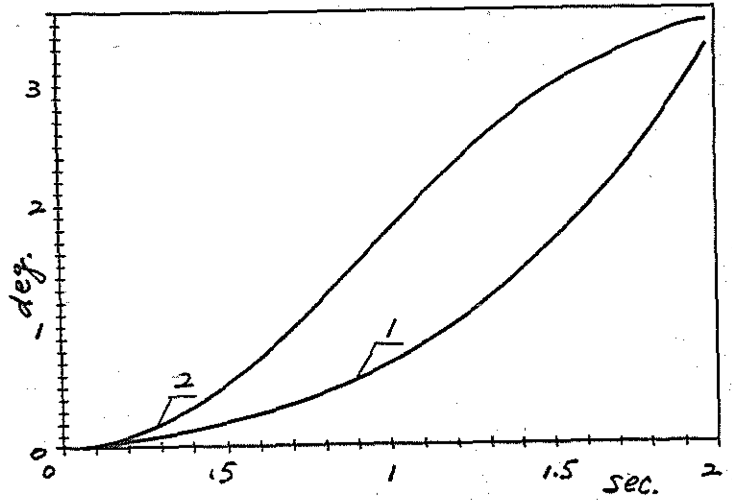

Fig.5

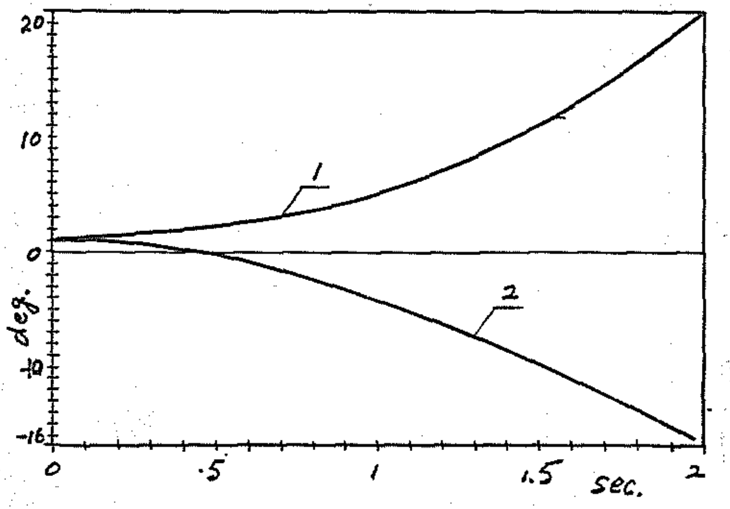

Fig.6

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