PAPER Nr. : 66

NONLINEAR ANALYSIS FOR DYNAMIC BEHAVIOURS OF A COUPLED ROTATIONAL VIBRATION SYSTEM NEAR ITS CRITICAL SPEED

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TENTH EUROPEAN ROTORCRAFT FORUM

AUGUST 28 – 31, 1984 – THE HAGUE, THE NETHERLANDS

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Abstract

From a viewpoint of the nonlinear mechanics, theoretical analyses for dynamic behaviours of an unbalanced rotor near its critical speed are carried out, while placing emphasis on the variation of rotor speed. A numerical method to construct solutions of autonomous, nonlinear, two degrees of freedom differential equations as trajectories upon the reduced phase plane is proposed. It is understood that basic features of dynamic behaviours near its critical speed should be attributed to properties of pumping phenomena, the relaxational energy transformation between rotational and translational motions. Periodic solutions to be in existence in this nonlinear system are determined numerically by the single and the multiple closed trajectories which could be referred as the specified space filling curves in ergodic space. Analytical approaches utilizing the Fourier expansion method are also presented and dependencies of periodic solutions on their initial conditions as well as δ, the nonlinearity parameter are clarified. The existence of another kind of periodic solutions will be suggested with relation to the diverged trajectories where the rotational motion of unbalanced rotor will be degenerated to the reciprocating one.

1. Introduction

It is well recognized that the rotor-fuselage coupling play an important role in vibration analysis of a helicopter as a whole. Typical example of this is the shaft critical speed of asymmetric rotor with elastic supports. The ground resonance for a soft-in-plane rotor helicopter is more popular example, though somewhat complicated due to its multi-degrees of freedomness, which is characterized by two different kinds of the mechanical instability, the critical speed and the self-excited instability.

Until now, a number of researches for these instabilities including the work by Coleman and Feigold have been conducted, however, almost of them are the linear analyses based on the assumption of constant rotor speed which could not always succeeded to clear out the detailed mechanism generating these instabilities, that is, by what kind of energy transfer between degrees of freedom, these instabilities could be maintained in an autonomous system.

It should be remembered that for a coupled rotational vibration system, the revolutional speed of rotor is no more than one of the state variables and the condition of constant rotor speed could not be realized unless the system to be concerned is under the quasi static equilibrium state and drived by unlimited power supply. Kononenko et al. studied the nonstationary vibration in passing through the critical speed of a motor with a limited power supply and showed the fact that the occurrences of jumping phenomena in their frequency responses are largely dependent on the interaction between a power source and a vibration system. The purpose of this paper is to give a clear picture for the detailed mechanism generating self-excited instabilities in an autonomous, coupled rotational vibration system from the viewpoint of nonlinear mechanics.

As mentioned before, the ground resonance is a most suitable subject for this, but to reduce mathematical difficulties, a simple conservative coupled rotational vibration system excited by an unbalance is taken up and its dynamic behaviours near the critical speed are analysed while emphasis placing on the variation of rotor speed due to the nonlinear coupling. Despite simplicity of the system, it still comprise a typical nonlinear, multi (at least two) degrees of freedom vibration problem. Various expressions for equations of motion as well as their solution methods are available, however, the phase space (plane) analysis seems to be most suitable.

From this point of view, an attempt is made to rewrite the equations of motion which originally given by autonomous, two degrees of freedom, second order nonlinear differential equations into nonautonomous, single degree of freedom, second order one. Adopting this apparent nonautonomous differential equation as a basic equation of motion, a new numerical method to construct its solutions upon the reduced phase plane is developed. Because of conservative, autonomous nature of the system, trajectories constructed on the reduced phase plane could be referred as one of alternative expressions of the space filling curves in ergodic space which, in contrast to properties of the limit cycle, suggest equal probability of occurrence for every state of the system and a great variety of trajectories can be obtained under various combinations of the initial conditions and δ, the parameter indicating nonlinearity of the system.

By observing diversity of behaviours of trajectories, it is clearly understood that fundamental features of the system dynamics near the critical speed should be attributed to inherent properties of the pumping phenomena which, from the energy point of view, imply relaxational energy conversions between rotational and translational motions while conserving the total energy of the system as a constant. Among these various trajectories, one can identify two distinctive classes of trajectories, one is a class of the single and the multiple closed trajectories where harmonic energy conversions between degrees of freedom could be realized, the other, the diverged ones where unidirectional transformations of the rotational energy to the translational energy would be occurred. Then along the later, several discontinuities of trajectories may be encountered repeatedly at each instant when the rotational energy will be converted completely to its counterpart and the rotational motion of unbalanced rotor will be degenerated into the reciprocating one. Periodic solutions to be able to exist in this nonlinear system are determined by the single and the multiple closed trajectories. It is shown that there are several kinds of closed trajectories, however, the number as well as the multipleness of closed trajectories are largely dependent on their initial conditions and δ.

Analytical studies for these periodic solutions are conducted by applying the Fourier expansion method and the effects of δ as well as the initial conditions on their amplitude - frequency relationships are clarified. It is also found that predominate frequencies of the periodic solutions corresponding to the multiple closed trajectories are given by the subharmonics of the fundamental frequency for that corresponding to the single closed trajectories with the order of subharmonics equal to that of multipleness of the closed trajectories. with respect to the diverged trajectories for which the angular motion of an unbalance are reduced to the pendulum motion, the existence of another kind of periodic solutions is ascertained.

2. Transformation of Equations of Motion

A mathematical model to be used is shown in Fig. 1 where a rotor with an unbalance, being supported by a linear spring, rotates freely with an angular velocity of $\dot{\theta}$. The translational motion of an unbalanced rotor occurs in the horizontal plane and is constrained to execute a rectilinear motion along the y axis. (In section 4, we will deal with the case where the translational motion of a rotor is constrained to a rectilinear motion along the x axis.) M, m, I and e in Fig. 1 are the mass of a rotor and an unbalance, the moment of inertia of a rotor and the offset between the center of gravity of M and m, respectively. For simplicity, the system is considered to be conservative and autonomous because there is no power supply.

Taking into account of the variation of rotor speed as one of the state variables, the equations of motion which describe the system dynamics near its critical speed can be obtained as follows

$$\ddot{y} + y = -\delta\,(\sin\theta)^{\cdot\cdot} \tag{1}$$

$$\ddot{\theta} = -\delta\,\ddot{y}\cos\theta \tag{2}$$

where y is the linear displacement of an unbalanced rotor nondimensioned by the equivalent radius of gyration, ke and θ, the angle of rotation of an unbalance measured from the x axis in direction of C.C.W.. ke, δ, the parameter indicating system nonlinearity and ν, the natural frequency are defined respectively by

$$ke = \frac{me^2 + I}{M + m} \quad , \quad \delta = \frac{me}{ke(M+m)} \quad , \quad \nu^2 = \frac{K}{M + m}$$

and dots denote the differentiation with respect to $\tau = \nu t$. To cope with large motions of the system, the order of δ, though it appears only in R.H.S of both equs. (1) and (2), does not always consider to be a small quantity compared with unity.

Owing to nonlinear natures of the system, various expressions for the equations of motion other than equs. (1) and (2) are available, however, for numerical analysis, it is convenient to rewrite them to a single degree of freedom, second order differential equation. Now, dealing with y and $\dot{\theta}$ in equs. (1) and (2) as the functions of θ, instead of τ and using the energy conservation theorem, we can obtain the following differential equation as an alternative expression for equs. (1) and (2)

$$\frac{1 - \delta^2 \cos^2 \theta}{1 + \delta y' \cos \theta} y'' + \frac{y'^2 + 2\delta y' \cos \theta + 1}{\lambda - y^2} y = \delta \sin \theta \tag{3}$$

where λ is an invariant of the system given by

$$\lambda = \dot{\theta}^2 (y'^2 + 2\delta y' + 1) + y^2 = \text{const.} \tag{4}$$

which is equal to twice of the total energy to be conserved and primes denote the differentiation with respect to θ.

Introducing a new variable $Y = dy/d\theta$, equ. (3) can be rewritten as

$$f_1(Y,\theta) - f_2(Y,\theta) \frac{dY}{dy} = F(\lambda,y) \tag{5}$$

where $f_1(Y,\theta)$, $f_2(Y,\theta)$ and $F(\lambda,y)$ are defined respectively by

$$f_1(Y,\theta) = \frac{\delta \sin \theta}{Y^2 + 2\delta Y \cos \theta + 1} \quad , \quad f_2(Y,\theta) = \frac{Y(1-\delta^2\cos^2\theta)}{(Y^2+2\delta Y\cos\theta+1)(1+\delta Y\cos\theta)}$$

$$F(\lambda,y) = \frac{y}{\lambda - y^2} \tag{6}$$

Although equ. (5) are rearranged as a nonautonomous, nonlinear differential equation, it still provides useful bases for the phase plane analysis because of its periodicity with respect to θ. Let $dY/dy = k$ and replacing the L.H.S of equ. (5) by $G(Y,\theta,k)$, this periodicity properties can be expressed as follows

$$\begin{aligned}
G(Y,\theta,k) &= F(\lambda,y) \qquad 0 \le \theta \le \pi/2 \\
G(Y,\pi-\theta,k) &= G(-Y,\theta,-k) \\
G(Y,\pi+\theta,k) &= -G(-Y,\theta,-k) \\
G(Y,-\theta,k) &= -G(Y,\theta,-k)
\end{aligned} \tag{7}$$

3. Determination of Periodic Solutions by Phase Plane Analysis

To clear out diversities of solution due to nonlinearity of the system and establish the conditions for the existence of periodic solutions, a numerical solution method for equ. (5) is developed. This is an extension of the isocline method, a typical graphical solution method for an autonomous nonlinear system, to a nonautonomous one.

From equ. (5), the slope of trajectory at an arbitrary point on the reduced phase plane $(y, Y = \dot{y}/\dot{\theta})$ are determined by

$$\frac{dY}{dy} = \frac{1 - \delta^2\cos^2\theta}{1 + \delta Y \cos\theta} \cdot \frac{\delta \sin\theta(\lambda - y^2) - (1 + 2\delta\cos\theta + Y^2)}{Y} \cdot \frac{1}{\lambda - y^2} \tag{8}$$

for each θ. After defining the slope of trajectory, dY/dy at $\theta = \theta_0$, a new position of trajectory which will be occupied with increased θ by small increment of $\Delta\theta$ can be determined by the following computational scheme

$$\begin{aligned} y &= y_0 + \Delta y = y_0 + Y_0\Delta\theta + \ldots \\ Y &= Y_0 + \Delta Y = Y_0 + (dY/dy)_0 Y_0 \Delta\theta + \ldots \\ \theta &= \theta_0 + \Delta\theta \end{aligned} \tag{9}$$

where (y_0, Y_0) is the predetermined position on the phase plane at $\theta = \theta_0$ for connecting them successively. As for as the accuracy of this procedure, it is ascertained that adequate accuracy can be assured using a linear approximation of equ. (9) with $\Delta\theta = 0.1^\circ$, by comparing the trajectories obtained by this method with those determined by Runge-Kutta-Gill method.

Several numerical results for the case of $\lambda = 1.0$ and $\delta = 0.2$ are shown in Fig. 2, each of which is the trajectory started from the neighbourhood of $y_0^* = 0.9149$, the initial amplitude of y at which the double closed trajectory will be anticipated. For these trajectories, the initial conditions other than y_0 which are indicated in each figure are given as $\dot{y}_0 = 0$, $\theta_0 = \pi/2$. $\dot{\theta}_0$ must be determined by equ. (4) which can be reduced in this case as $\dot{\theta}_0^2 = 1 - y_0^2$. Indeed, a great variety of trajectories can be obtained by incorporating other combinations for initial conditions, but for simplicity, the initial conditions of the system other than y_0 and θ_0 are restricted to the case of $\dot{y}_0 = 0$ and $\theta_0 = \pi/2$ in this paper.

By observing various behaviours of trajectories on the reduced phase plane, it is clearly understood that fundamental features of dynamic behaviours of an unbalanced rotor near its critical speed which, in the linear analysis, have been interpreted as an apparent resonance due to a perfunctional linearization, should be attributed to peculiarities of so called the pumping phenomena which imply a relaxational energy conversions between a rotational and a translational motions. Under these conditions, trajectories constructed on the reduced phase plane could be referred as a concrete expression of the space filling curves in ergodic space whose behaviours can be generally expressed by a generalized Fourier series as

$$y(t) = a_1 \cos pt + b_1 \sin qt + \ldots$$

where p and q are the incommeasurable numbers. However, among these trajectories, one can identify two distinctive classes of trajectories, one is a class of the single and the multiple closed trajectories which indicate the possible existence of periodic solutions and the other, the diverged ones along which the rotational motion of an unbalanced rotor will be degenerated to reciprocating one.

From the energy point of view, a harmonic energy conversion between degrees of freedom can be established for the closed trajectories, while for the later, in contrast with the former, an unidirectional transformation of a rotational energy to a translational one will be occurred. Then along the later, trajectories will become discontinuous at each θ corresponding to the time instant when a rotational energy will be converted completely into its counterpart. To investigate behaviours of the diverged trajectories near their discontinuities, it is convenient to rearrange equ. (8) as

$$\frac{y}{\lambda - y^2} + \frac{1 - \delta^2 \cos^2 \theta}{(1/Y + \delta \cos \theta)(1 + 2\delta \cos \theta/Y)} \frac{d}{dy} \left(\frac{1}{Y}\right) = 0 \qquad (10)$$

by dividing both sides of equ. (8) by Y^2 and neglecting $1/Y^2$ as a small quantity compared with unity. It is deduced that for the specified divergent trajectories, there may also exist another kind of periodic solutions for which the rotational motions of an unbalance will be degenerated into the reciprocating motions, but in the following, we shall be concerned with the periodic solutions defined by the closed trajectories on the reduced phase plane. It should be noted that the dependencies of trajectories on δ and their initial conditions are so severe that the number of the closed trajectories to be able to exist in equ. (5) are quite limited.

From the geometrical properties of closed trajectories, we can define the necessary and sufficient conditions for the existence of closed trajectories on the reduced phase plane as follows

$$\begin{aligned} &(a) \quad \dot{\theta}(\theta) > 0 \qquad , \quad \theta_0 \leq \theta \leq 2\pi + \theta_0 \\ &(b) \quad y(\theta_0+2\pi) = y(\theta_0) \quad , \quad Y(\theta_0+2\pi) = Y(\theta_0) \end{aligned} \qquad (11)$$

that is, the periodic solutions of equ. (5) could be established if and only when the trajectory started from arbitrary state on the reduced phase plane could return to and pass through the same state as initial one after an unbalance have been completed its one revolutional motion without reversing its direction of motion to the backward. The condition $\dot{\theta}(\theta) > 0$ in equ. (11) may be omitted if the scope of periodic solutions to be involved is extended to cope with these corresponding to the diverged trajectories.

According to the proposed numerical procedure (9), state variables of y as well as Y after θ have been elapsed by 2π are computed for the case of $\lambda = 1.0$ and $\delta = 0.2$ and plotted them as functions of $y(\theta_0)$, the initial amplitude of y, in Fig. 3 for $y(\theta_0) > 0$ and in Fig. 4 for $y(\theta_0) < 0$, respectively. Dotted lines in these figures show the additional relation $y(\theta_0) = y(\theta_0+2\pi)$. From the conditions given in equ. (11), it is easily understood that the closed trajectories could be obtained discretely at each $y(\theta_0)$ marked as A, B, C, D in Fig. 3 and E, F, G, H in Fig. 4. To indicate the effect of δ on the existence of closed trajectories, similar diagrams for the case of $\lambda = 1.0$ and $\delta = 0.5$ are also presented in Fig. 5 for $y(\theta_0) > 0$ and Fig. 6 for $y(\theta_0) < 0$, respectively.

It is surprised that the number of closed trajectories to be able to exist in equ. (5) are remarkably decreased with increasing δ and even for $\delta = 0.5$, as shown in Fig. 5 and Fig. 6, there may be only two different kinds of closed trajectories corresponding to each initial amplitude designated as I and J. Typical closed trajectories obtained by this numerical method for two different values of δ are shown in Fig. 7 and Fig. 9, respectively. Time histories of $\dot{\theta}$ corresponding these closed trajectories are also plotted in Fig. 8 and Fig. 10.

The following properties with respect to the closed trajectories are revealed.

(1) The single closed trajectories can be realized only for the case of $y(\theta_0) > 0$, that is, when an unbalance is initially arranged at the outerside of the center of rotation of a rotor.

(2) The larger δ, the lesser initial amplitudes of y for the single and the multiple closed trajectories are obtained.

(3) The possible appearances of the multiple closed trajectories more than triple are remarkably decreased with increasing δ.

(4) For a given δ, each closed trajectory is arranged regularly in the order of its multipleness.

(5) The predominate frequencies of periodic solutions corresponding to the multiple closed trajectories are given by the subharmonics of the fundamental frequency for periodic solutions corresponding to the single closed trajectories.

4. Determination of Periodic Solutions by Fourier Expansion Method

To supplement numerical results obtained in previous section, an analytical approach using the Fourier expansion method for determination of the periodic solutions which could be existed in this nonlinear system is described. In the remainder of this paper, we shall be concerned with the following equations of motion.

$$\ddot{x} + x = -\delta(\cos\theta)^{\cdot\cdot} \qquad (12)$$

$$\ddot{\theta} = \delta\,\ddot{x}\sin\theta \qquad (13)$$

It is understood that equs. (12) and (13) constitute the conjugate pair of equ. (1) and (2) and describe the motions of the system depicted in Fig. 1 where the translational motion of an unbalanced rotor is restricted to a rectilinear motion along the x axis.

Assume there may be certain kinds of periodic solutions in equs. (12) and (13) and expand them as follows

$$x = \sum_{i=1} x_i \cos \frac{i}{m} \omega\tau \tag{14}$$

$$\theta = \frac{\omega}{m} \tau + \theta_p \sin \frac{p}{m} \omega\tau + \sum_{i=1,\neq p} \theta_i \sin \frac{i}{m} \omega\tau \tag{15}$$

where ω is the fundamental frequency to be determined as a function of θ_p, the amplitude of pth component of θ with predominate frequency $p\omega/m$. As θ_p is considered not so small quantity compared with $\theta_i (i=1,2,\ldots,\neq p)$, the higher order terms of θ_p cannot ignore in approximation process. Using equs. (14) and (15), together with a proper assignment of m and p, we can obtain the analytical expressions for each periodic solution being able to exist in equs. (12) and (13) within the accuracies of approximations. Hereafter, as a typical example, let us consider the periodic solutions corresponding to the triple closed trajectories on the reduced phase plane as depicted in Fig. 7 (c) and (f).

Allotting as m = 3 and p = 2 and using the symmetrical property of motion, let the periodic solutions to be determined be expressed in the form

$$x = \sum_{i=1} x_{2i-1} \cos \frac{(2i-1)}{3} \omega\tau \tag{16}$$

$$\theta = \frac{\omega}{3} \tau + \theta_2 \sin \frac{2}{3} \omega\tau + \sum_{i=2} \theta_{2i} \sin \frac{2}{3} i\omega\tau \tag{17}$$

With assistance of the basic formula of the Bessel function and equ. (17), cos θ and sin θ can be expanded as

$$\cos \theta = \sum_{\substack{n=1\\ i=2}} C_{2n-1,2i} (\theta_2,\theta_{2i}) \cos (2n-1) \frac{\omega}{3} \tau \tag{18}$$

$$\sin \theta = \sum_{\substack{n=1\\ i=2}} S_{2n-1,2i} (\theta_2,\theta_{2i}) \sin (2n-1) \frac{\omega}{3} \tau \tag{19}$$

where $C_{2n-1,2i}(\theta_2,\theta_{2i})$ and $S_{2n-1,2i}(\theta_2,\theta_{2i})$ are given by respectively

$$\begin{aligned}
C_{2n-1,2} (\theta_2,\theta_{2i}) &= (-1)^n J_n(\theta_2) + J_{n-1}(\theta_2) \qquad (n = 1,2,\ldots)\\
C_{2n-1,2i}(\theta_2,\theta_{2i}) &= \frac{1}{2} [-(-1)^{n-i} G_{n-i} + G_{n-i-1} - G_{n+i-1} - G_{i-n}\\
&\quad + (-1)^{n+i} G_{n+i} + (-1)^{i-n+1} G_{i-n+1}] \theta_{2i}\\
&\qquad (n = 1,2,\ldots,\ i = 2,3,\ldots)\\
S_{2n-1,2}(\theta_2,\theta_{2i}) &= J_{n-1}(\theta_2) + (-1)^{n+1} J_n(\theta_2) \qquad (n = 1,2,\ldots)\\
S_{2n-1,2i}(\theta_2,\theta_{2i}) &= \frac{1}{2} [G_{n-i-1} + (-1)^{n-i} G_{n-i} - (-1)^{n+i} G_{n+i}\\
&\quad + (-1)^{i-n+1} G_{i-n+1} - G_{n+i-1} + G_{i-n}] \theta_{2i}\\
&\qquad (n = 1,2,\ldots,\ i = 2,3,\ldots)
\end{aligned} \tag{20}$$

In equ. (20), $J_n(\theta_2)$ is the Bessel function of order n with θ_2 as the argument and G_j indicate that $G_j = J_j(\theta_2)$ for $j \geq 0$ and $G_j = 0$ for $j < 0$.

Using equs. (12), (14) and (18), x can be expressed as

$$x = \delta \sum_{i=n=1} \frac{(2n-1)^2\omega^2}{9-(2n-1)^2\omega^2} C_{2n-1,2i}(\theta_2,\theta_{2i}) \cos \frac{(2n-1)}{3} \omega\tau \tag{21}$$

Rearranging equ. (13) with using equs. (17), (19) and (21) and equating the coefficient of both sides of equ. (13), we finally can obtain the simultaneous equations with respect to θ_{2j} ($j = 1,2,\ldots$), which are expressed in a matrix form as follows

$$\begin{pmatrix} \frac{2}{\delta^2} & -\frac{P_{2,4}}{4^2} & -\frac{P_{2,6}}{6^2} & \cdot \\ 0 & (\frac{2}{\delta^2} - \frac{P_{4,4}}{4^2}) & -\frac{P_{4,6}}{6^2} & \cdot \\ 0 & -\frac{P_{6,4}}{4^2} & (\frac{2}{\delta^2} - \frac{P_{6,6}}{6^2}) & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} 2^2\theta_2 \\ 4^2\theta_4 \\ 6^2\theta_6 \\ \cdot \end{pmatrix} = \begin{pmatrix} P_{2,2} \\ P_{2,4} \\ P_{4,6} \\ \cdot \end{pmatrix} \tag{22}$$

where $P_{2i,2j}$ ($i,j = 1,2,\ldots$) and $P_{2i,2}$ ($i = 1,2,\ldots$) are function of θ_2, being defined respectively as

$$P_{2i,2j} = \frac{1}{2} \sum_{n=1}^{(i,j)} \frac{(2n-1)^4}{9-(2n-1)^2\omega^2} \{C_{2n-1,2} (A_{2i-2n+1,2j} - A_{2n-1-2i,2j} + A_{2n-1+2i,2j}) + B_{2n-1,2j} (S_{2i-2n+1,2} - S_{2n-1-2i,2} + S_{2n-1+2i,2})\} \quad (i = 1,2,\ldots,\ j = 2,3,\ldots) \tag{23}$$

$$P_{2i,2} = \frac{1}{2} \sum_{n=1}^{i} \frac{(2n-1)^4}{9-(2n-1)^2\omega^2} C_{2n-2,2} (S_{2i-2n+1,2} - S_{2n-1-2i,2} + S_{2n-1+2i,2}) \tag{24}$$

The notation (i,j) in equ. (23) means to take either integer larger than the other and $A_{q,2j}$, $S_{q,2} = 0$ for $q < 0$.

By eliminating higher order terms from equ. (22), we can obtain the transcendal equation of θ_2 which define the fundamental frequency ω as a function of θ_2. At the same time, in addition to these, the constraint given by equ. (4) which is rewritten for this case as

$$\lambda = \dot{x}^2 + 2\delta\dot{\theta}\dot{x} + \dot{\theta}^2 + x^2 = \text{const} \tag{25}$$

must be consulted. With using equs. (21), (22) and (25), the analytical expressions of periodic solutions corresponding to the triple closed trajectories on the reduced phase plane can be obtained in a form of equs. (16) and (17). It should be noted that there is no restriction for applying this analytical determination method to other periodic solutions, though only the case of m = 3, p = 2 is treated in this paper and no limitation concerning the order of δ. Comparison studies with numerical results show that adequate accuracies can be assured with the second order approximation for δ up to 0.5, that is, for periodic solutions corresponding to the triple closed trajectories, they could be approximated by

$$
\begin{aligned}
x &= x_1 \cos \frac{\omega}{3} \tau + x_3 \cos \omega\tau \\
\theta &= \frac{\omega}{3} \tau + \theta_2 \sin \frac{2}{3} \omega\tau + \theta_4 \sin \frac{4}{3} \omega\tau
\end{aligned}
\tag{26}
$$

within the accuracy of 1.0% and usefulness and reasonability of this analytical method is ascertained.

5. Conclusions

Basic researches to clarify the dynamic behaviours of an unbalanced rotor near its critical speed are conducted from a viewpoint of the nonlinear mechanics. A new method to construct the solutions of an autonomous, nonlinear, two degrees of freedom differential equation upon the reduced phase plane is developed and the existence of periodic solutions is established in relation to the single and the multiple closed trajectories. The periodic solutions to be able to exist in the nonlinear system are also determined analytically by applying the Fourier expansion method. It is shown that the inclusion of a variation of rotor speed due to the nonlinear coupling is essential for dynamic analysis of a coupled rotational vibration system in the instability ranges because the origin of instability can be attributed to the relaxational energy conversion from a rotational to a translational degrees of freedom. Further researches along this direction are necessary for dynamic analyses of a helicopter after a power failure, survivability analyses for a damaged rotor and vibration analyses for a showed and a stopped rotor.

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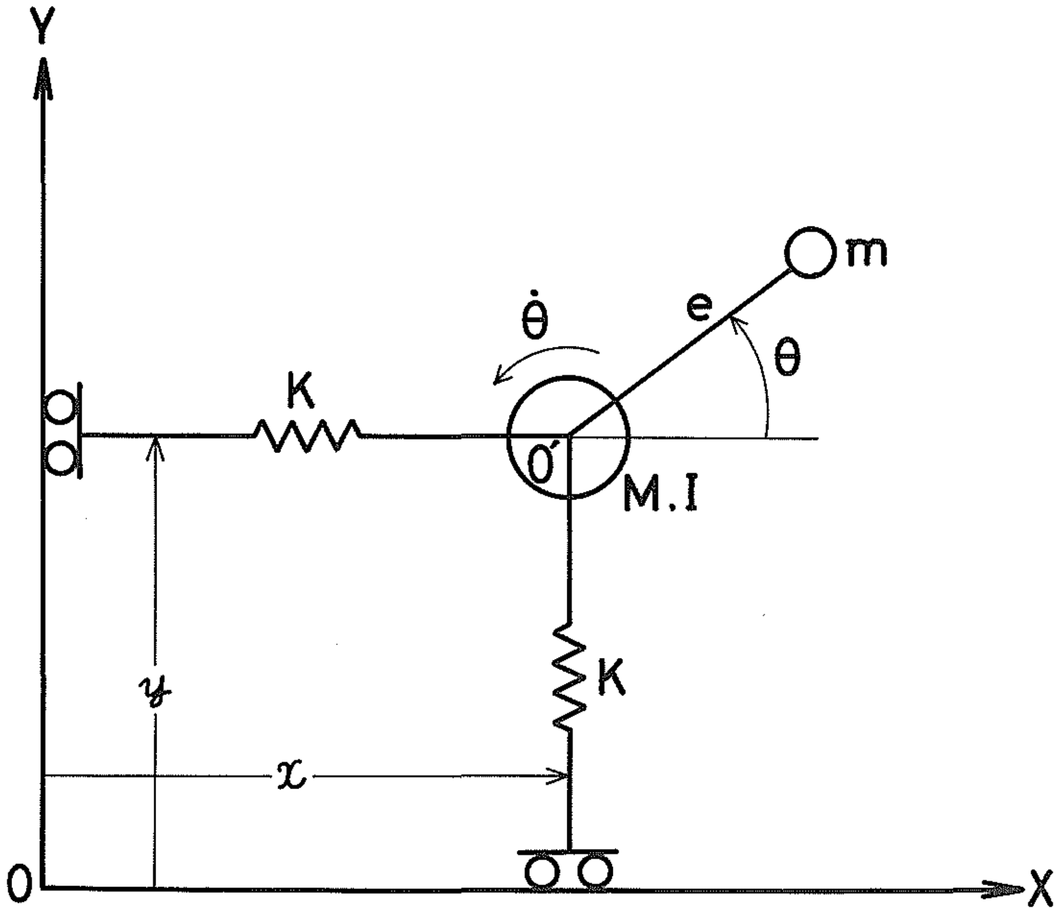

Fig.1 Mathematical Model

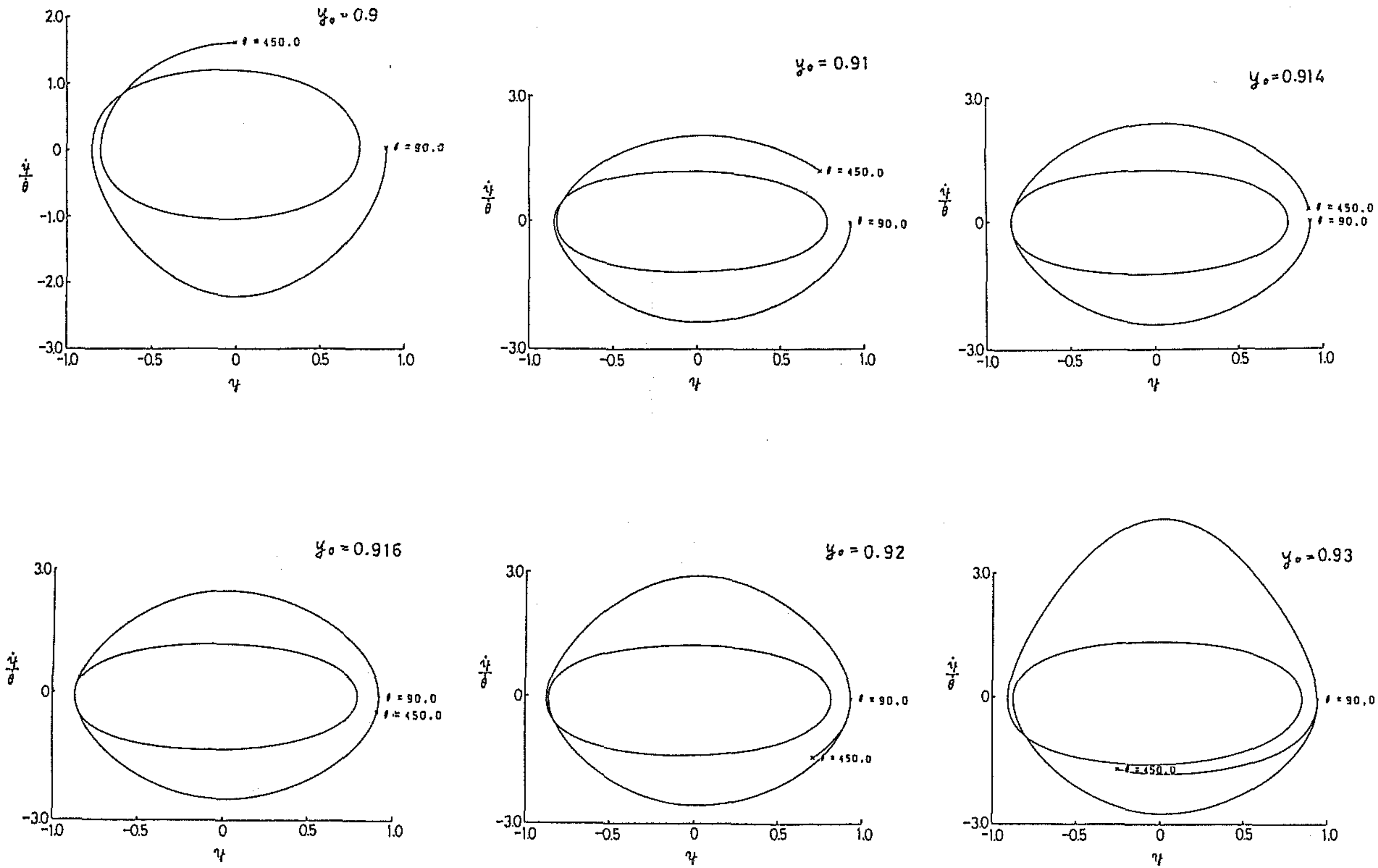

Fig.2 Trajectories for Various Amplitudes of y_0 (λ=1.0, δ=0.2, $\dot{y}_0$=0, θ_0=π/2)

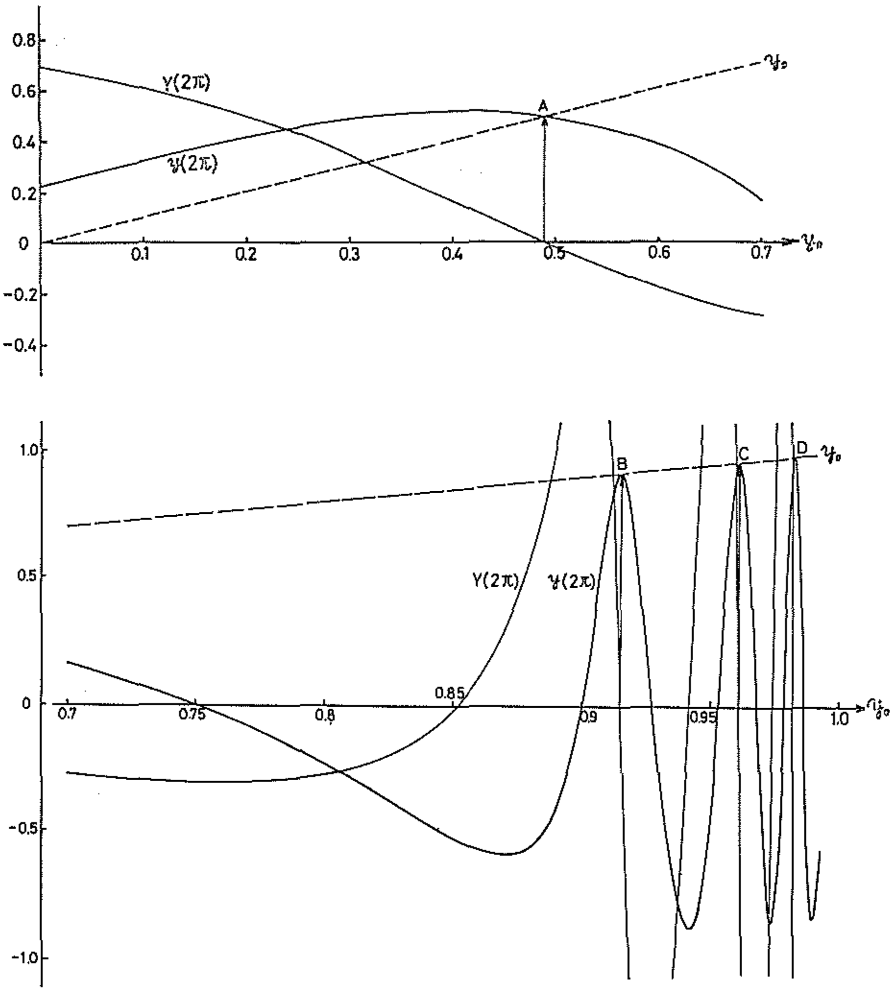

Fig.3 Plots of $y(2\pi)$ and $Y(2\pi)$ vs y_0($\lambda=1.0$, $\delta=0.2$, $\dot{y}_0=0, \theta_0=\pi/2$), $y_0>0$

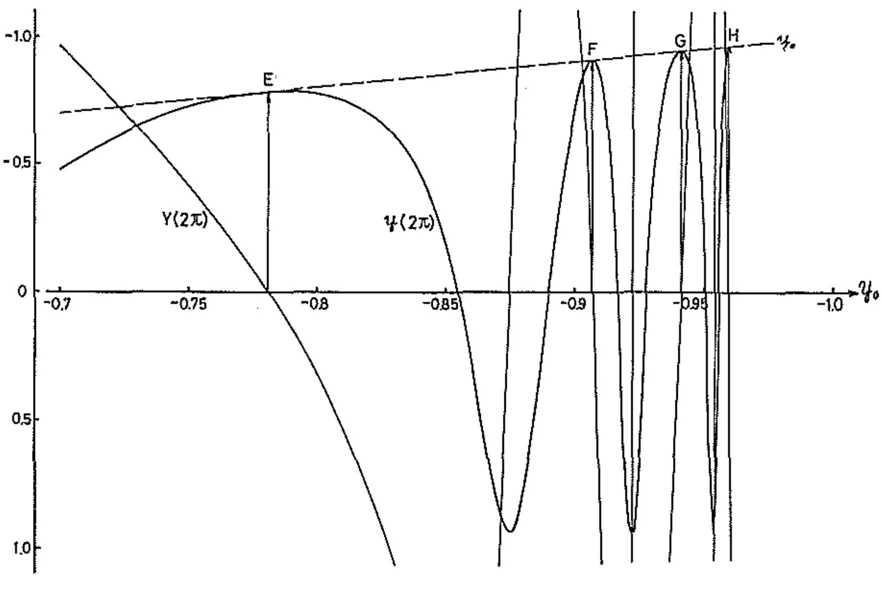

Fig.4 ($\lambda=1.0$, $\delta=0.2$, $\dot{y}_0=0, \theta_0=\pi/2$), $y_0<0$

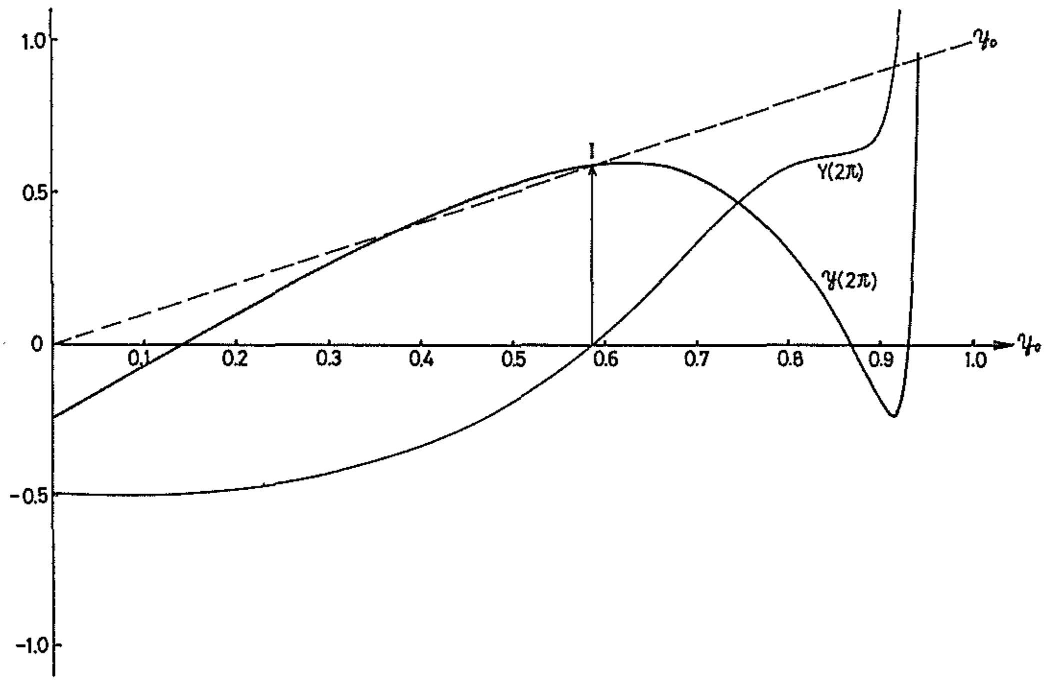

Fig.5 Plots of y(2π) and Y(2π) vs y_0(λ=1.0, δ=0.5, $\dot{y}_0$=0,θ_0=π/2), y_0>0

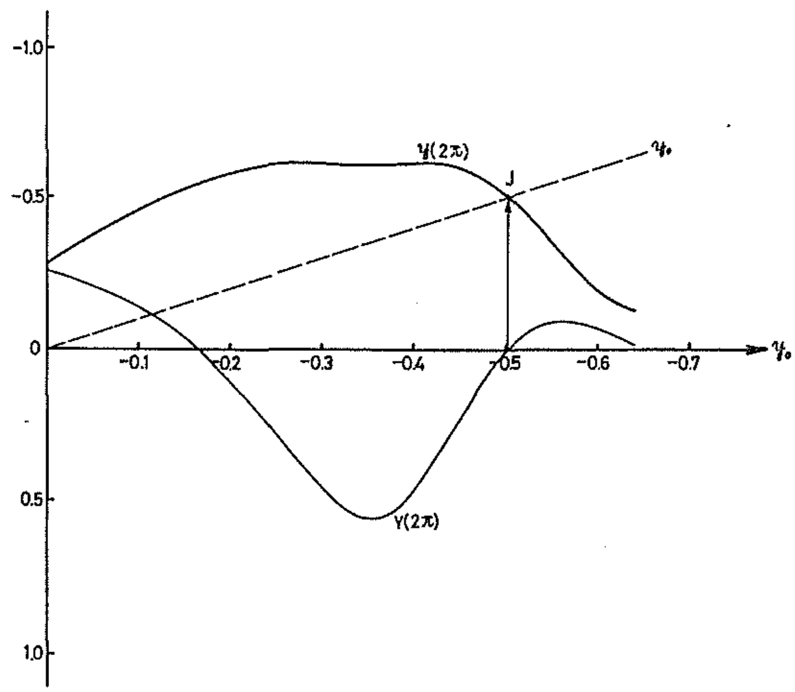

Fig.6 Plots of y(2π) and Y(2π) vs y_0(λ=1.0, δ=0.5, $\dot{y}_0$=0,θ_0=π/2), y_0<0

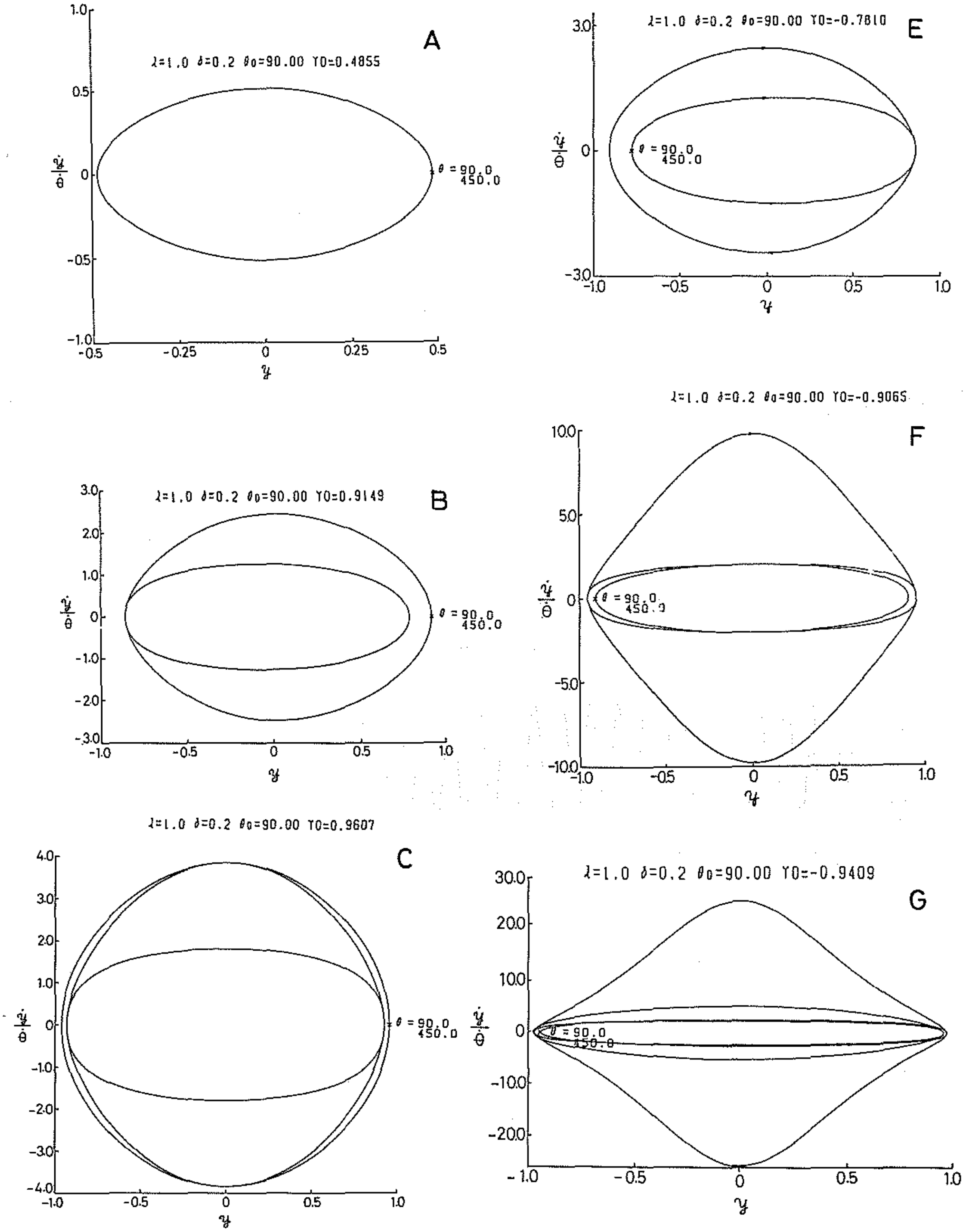

Fig.7 Typical Closed Trajectories for δ=0.2

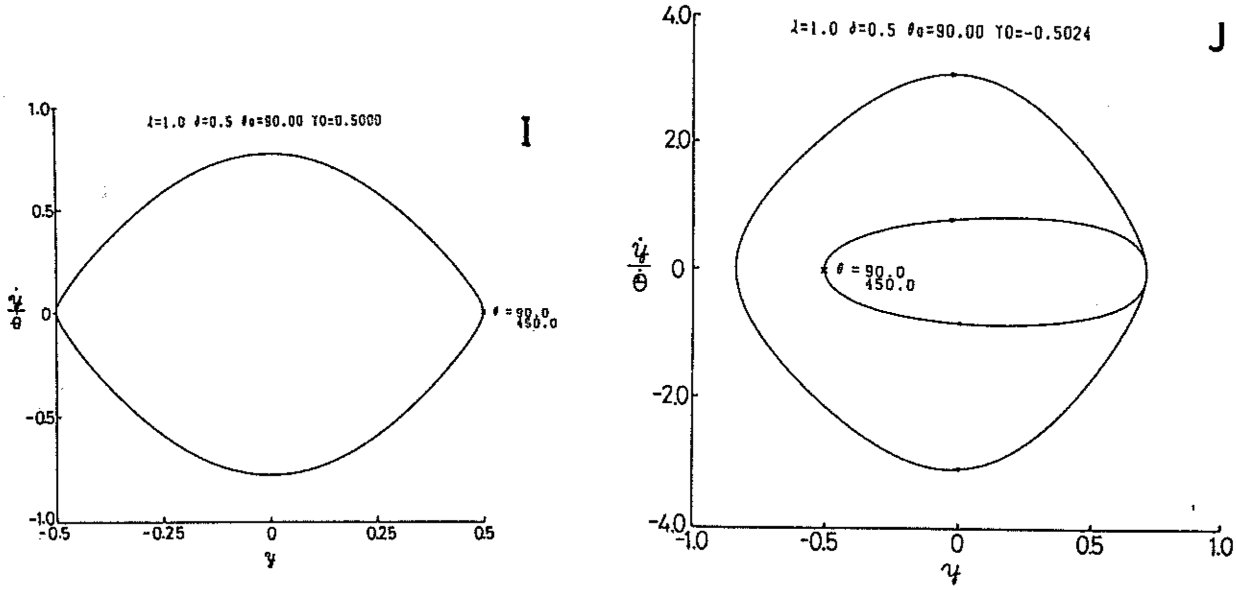

Fig.8 Typical Closed Trajectories for δ=0.5

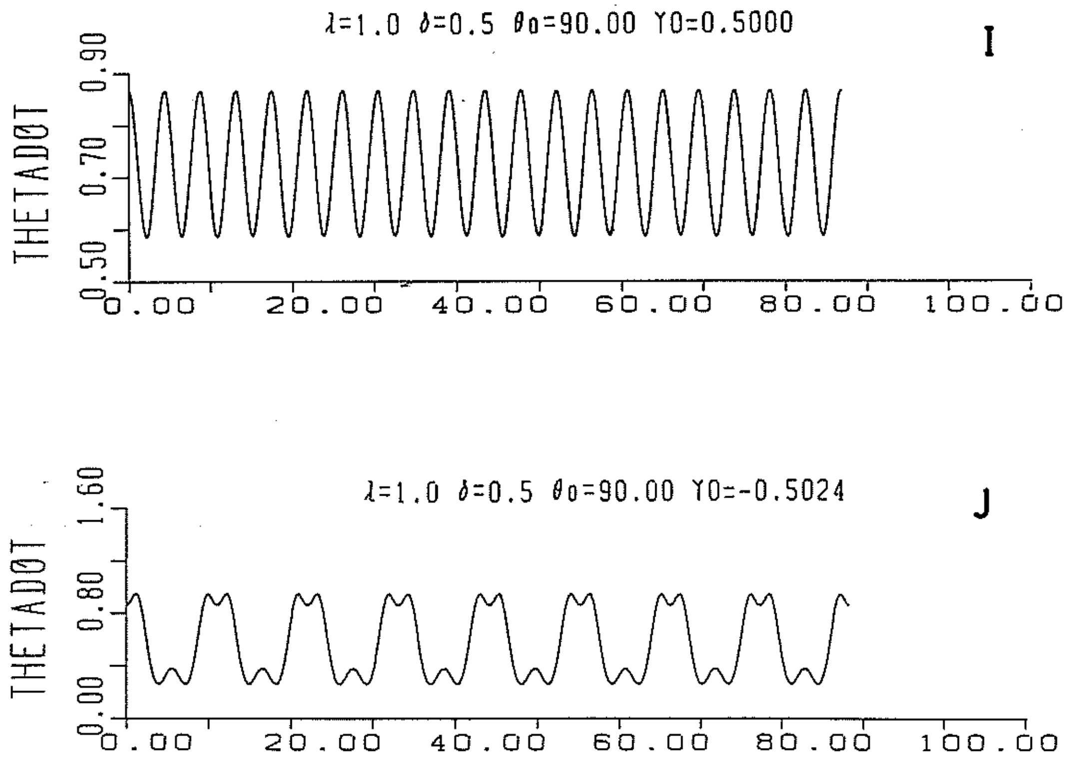

Fig.10 Time Histories of $\dot{\theta}$ corresponded to Closed Trajectories (δ=0.5)

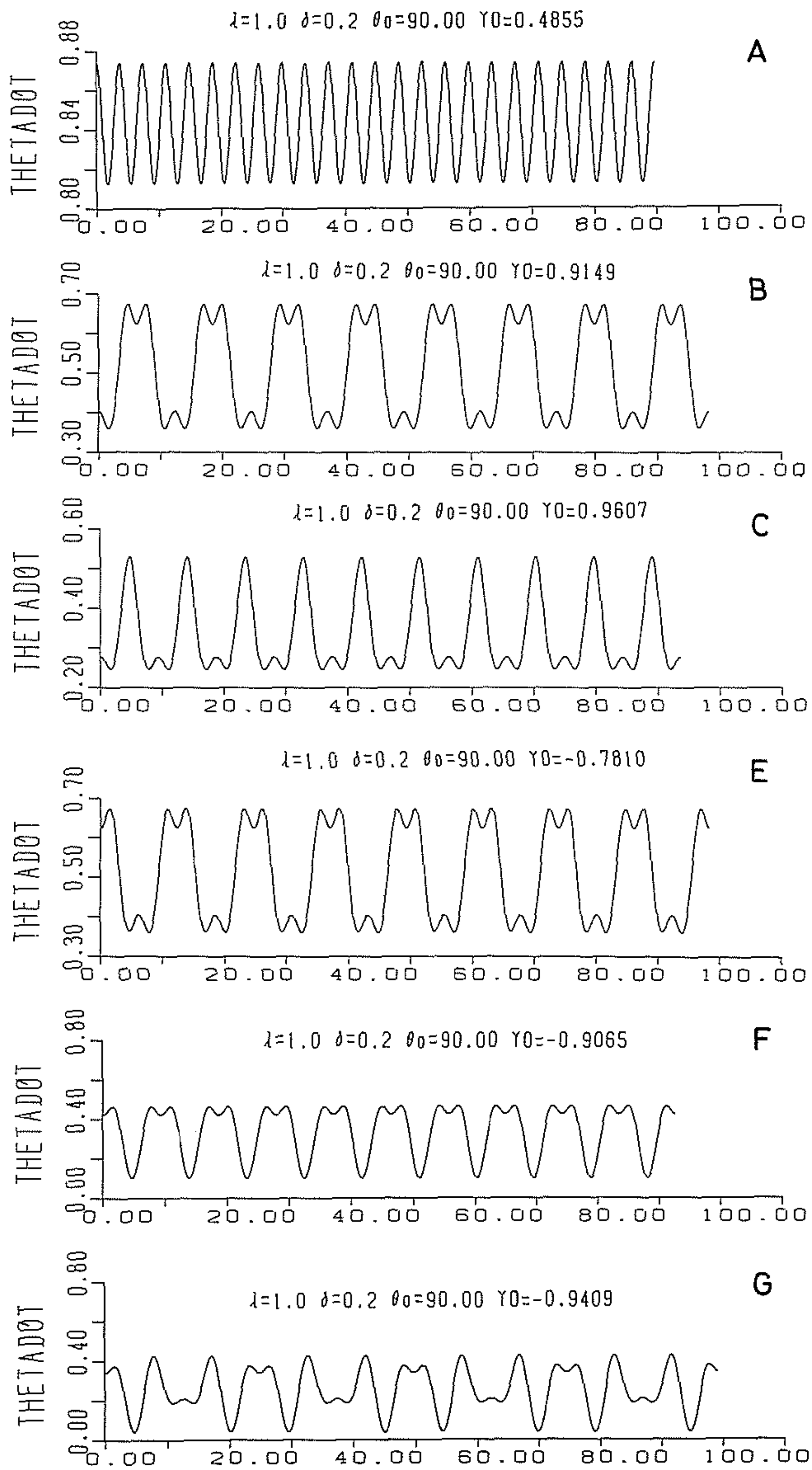

Fig.9 Time Histories of $\dot{\theta}$ corresponded to Closed Trajectories (δ=0.2)