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APPLICATION OF THE LOCAL CIRCULATION METHOD TO THE FLUTTER ANALYSIS OF ROTARY WINGS

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ABSTRACT

A simple method of numerical calculation to determine the critical torsional rigidity or the classical flutter boundary of rotary wings has been proposed as an extensive work of the Local Circulation Method (LCM). Exemplified calculations were performed for a helicopter rotor and a windmill rotor. The result for the helicopter rotor shows good correlation with that of the theoretical computation based on the lifting surface theory and with the experimental test. The result for the windmill rotor, which does not have any comparable subject of reference, shows a possibility of calculation for the windmill operating in yawed condition with respect to the wind direction.

NOMENCLATURE

a	nondimensional position of elastic axis based on the half chord
b	half chord length = $c/2$
$C(k)$	Theodorsen function = $F(k)+iG(k)$
C_{M_y}	nondimensional bending moment = $M_y/\rho S(R\Omega)^2R$
$C_\ell(\alpha)$	lift coefficient
c	blade chord
$F(k)$	real part of the Theodorsen function
$G(k)$	Imaginary part of the Theodorsen function

h normal displacement of a blade element, positive downward
I_θ moment of inertia about feathering hinge
$\imath$ inclination angle of tip-path-plane with respect to the general flow
i imaginary = $\sqrt{-1}$
k reduced frequency at flutter $k=b\omega/U$
k_β flapping stiffness about the flapping hinge
k_θ feathering stiffness about the feathering hinge
ℓ lift acting on a blade element=$\ell_1+\ell_2$
ℓ_1 apparent mass component of the lift ℓ
ℓ_2 circulatory component of the lift ℓ
M aerodynamic pitching moment acting on a blade
M_y flapwise bending moment of rotor blade
m mass of rotor blade or moment acting on a blade element=m_1+m_2
m_1 apparent mass component of the moment m
m_2 circulatory component of the moment m
N normal aerodynamic force acting on a blade
R rotor radius
r radius position of rotor blade
r_β radius position of flapping hinge
S rotor area=πR^2
$S(k)$ Sears function
U stationary inflow velocity=$\sqrt{U_{\overline{T}}^2+U_{\overline{P}}^2}$
U_T tangential component of the inflow velocity
$U_{\overline{T}}$ stationary component of U_T
U_P normal component of the inflow velocity
$U_{\overline{P}}$ stationary component of U_P
V wind velocity
v^j induced velocity generated by the preceding j-th blade
Δv induced velocity generated by the blade under consideration
x nondimensional distance=r/R
x_f nondimensional radius of gyration defined in equation (A.3)
x_β nondimensional radius of either actual or equivalent flapping hinge r_β/R
$x_{f,a}$, $\overline{x_\beta^n}$, $\overline{x^n}$, nondimensional quantity defined in equation (A.3)
y_{CG} nondimensional CG position based on the chord, positive forward from the elastic axis
$\bar{y}_{CG}$ nondimensional mean CG position
α angle of attack of a blade element=$\theta-\phi$
α_G angle of attack caused by the induced velocity=$-(\sum_j v^j+\Delta v)/U$
β flapping angle
β_0 coning angle
θ feathering angle
θ_0 initial feathering angle or pitch angle of a blade element
μ advance ratio = $V\cos\imath/R\Omega$
ρ air density

ϕ stationary inflow angle $=\tan^{-1}(-U_{\overline{P}}/U_{\overline{T}})$
Ψ yawing angle of the rotor shaft
ψ azimuth angle $=\Omega t$
Ω angular velocity of rotor or rotor speed
ω resonant frequency or flutter frequency
ω_θ undamped natural frequency of torsion
ω_1 undamped natural frequency of the flapwise bending
ω_1^N undamped natural frequency of the flapwise bending of non-rotating blade
$(\dot{\ })$ time differentiation of ( )

§1 INTRODUCTION

It has been paid attention to the classical flutter, coupled vibration of bending and torsional motions, of rotary wings as well as fixed wings in many years. Since the respective blade of rotary wings is operated in the field of large centrifugal force and of strong downwash left by the preceding blades, more sophisticated analysis is required than that of fixed wings.

Major differences of the dynamic characteristics of the propeller and windmill rotor from those of the helicopter rotor are as follows: (i) the inflow angle of the propeller and windmill blade is highly distorted along the blade span and requires to have large twist or washout in propeller and washin in windmill because they are principally operated in the axial flow, and (ii) the ratio of the aerodynamic force to the inertial force, typically represented by Lock number, is small.

The analysis for finding the critical speed of initiation of the blade flutter requires to know the time- and span-wise variation of the induced velocity precisely and to get the instantaneous airloading successively.

The Local Circulation Method (LCM)[1] has been developed as an extension of the Local Momentum Theory (LMT)[2] to calculate the dynamic airloading of the blade of rotary wings in highly distorted inflow angle. Since this method of calculation is based on the instantaneous circulation distribution of the blade the unsteady phenomena can be treated easily. In this paper the flutter of helicopter rotor and windmill rotor will be analyzed by the LCM.

§2 EQUATINS OF MOTION

The rotor blade treated here is, as shown in Fig. 1, assumed to have both flapping and feathering hinges and to be rigid other than these hinges. Thus equations of motion about these hinges can be given by

$$\left.\begin{aligned}&\int_{r_\beta}^{R}\{(r-r_\beta)^2\ddot{\beta}+r(r-r_\beta)\Omega^2\beta+cy_{CG}(r-r_\beta)\ddot{\theta}+cy_{CG}r\Omega^2\theta\}\left(\frac{dm}{dr}\right)dr\\&\qquad+k_\beta(\beta-\beta_0)=\int_{r_\beta}^{r}(r-r_\beta)\left(\frac{dN}{dr}\right)dr\end{aligned}\right\}\tag{2.1}$$

$$\left.\begin{aligned}&\int_{r_\beta}^{R}\left[(\ddot{\theta}+\Omega^2\theta)\left(\frac{dI_\theta}{dr}\right)+cy_{CG}\{(r-r_\beta)\ddot{\beta}+r\Omega^2\beta\}\left(\frac{dm}{dr}\right)\right]dr\\&\qquad+k_\theta(\theta-\theta_0)=\int_{r_\beta}^{R}\left(\frac{dM}{dr}\right)dr.\end{aligned}\right\}\tag{2.2}$$

When the blade has no flapping hinge actually, an equivalent hinge must be introduced as stated in APPENDIX.

If the rotor is operating without stall in axial flow only and if the induced velocity is assumed uniform, then the linear perturbation equations can be deduced from the above equations and thus the critical rotational speed of flutter can be found by solving the characteristic equations of the system.[3)]

If the rotor is operating in an inclined flow, then the coefficients of the above perturbed equation are periodic functions of azimuth angle and thus the solution can be given by using the Froquet's theorem.[4~6)]

When the effect of flow variation on the blade airloading is considered, the vortex theory is commonly used in order to estimate the chage of induced velocities generated by the blade itself and the preceding blades.[7~9)] In this case, however, the timewise trace of the blade motion is required to find whether the amplitudes of the motion for any mode are diverging or converging.[10~12)] Since this process of computation needs lengthy time for a great many cycles of motion if the effect of shed vortices is taken into account,[13,14] the reduction of computation time for one cycle of motion is desirable.

§3 LCM FOR UNSTEADY FLOW

The tangential and normal components of the inflow velocity with respect to a blade element are, as shown in Fig.1, given respectively by

$$\left.\begin{aligned} U_T &= R\Omega(x+\mu\sin\psi)\mp(\sum_j v^j+\Delta v)\sin\phi \\ U_P &= -R\Omega\mu\tan i\mp(\sum_j v^j+\Delta v)\cos\phi-\dot\beta(r-r_\beta)-R\Omega\beta\mu\cos\psi \end{aligned}\right\} \tag{3.1}$$

where

$$\left.\begin{aligned} \phi &= \tan^{-1}(-U_{\overline{P}}/U_{\overline{T}}) \\ U_{\overline{P}} &= -R\Omega\mu\tan i \\ U_{\overline{T}} &= R\Omega(x+\mu\sin\psi) \end{aligned}\right\} \tag{3.2}$$

and where $\mp$ sign of the induced velocities means to take negative for helicopter rotor and positive for windmill rotor respectively.

Then the lift and moment of the blade element can be given by

$$\left.\begin{aligned} \ell &= \ell_1+\ell_2 \\ m &= m_1+m_2 \end{aligned}\right\} \tag{3.3}$$

where $(\ )_1$ and $(\ )_2$ show the apparent mass components and circulatory components respectively, and are, hence, given by

$$\left.\begin{aligned} \ell_1 &= \rho\pi b^2(\ddot h+U\dot\alpha-ab\ddot\alpha) \\ m_1 &= \rho\pi b^3\{a\ddot h-U(\tfrac{1}{2}-a)\dot\alpha-b(\tfrac{1}{8}+a^2)\ddot\alpha\} \end{aligned}\right\} \tag{3.4}$$

$$\left.\begin{aligned} \ell_2 &= \rho U^2 b\{C(k)C_\ell(\alpha)+S(k)C_\ell(\alpha_g)\} \\ m_2 &= b(a+\tfrac{1}{2})\ell_2, \end{aligned}\right\} \tag{3.5}$$

and where

$$\left.\begin{aligned} U &= \sqrt{U_{\overline{T}}^2+U_{\overline{P}}^2} \\ \dot h &= \{-(r-r_\beta)\dot\beta-R\Omega\mu\beta\cos\psi\}\cos\phi \\ \alpha &= \theta-\phi=\theta-\tan^{-1}(-U_{\overline{P}}/U_{\overline{T}}) \\ \alpha_G &= -(\sum_j v^j+\Delta v)/U. \end{aligned}\right\} \tag{3.6}$$

The induced velocity mainly generated by the trailing vortices, $\sum_j v^j + \Delta v$, has been regarded as if there is a vertical gust for the blade element. The effect of shed vortices on the lift and moment has been represented by the Theodorsen and Sears functions. Actually, since the circulatory component ℓ_2 consists of many harmonics each of which has the reduced frequency of k, the above circulatory components can further be divided into low and high frequency components as stated later.

The induced velocity generated by the preceding blades and left on the rotor rotational plane, $\sum_j v^j$, can be calculated by the way proposed in Ref.1 and 2. The induced velocity generated by the blade element under consideration, Δv, may be calculated by the way written in Ref.15. Here, however, the Δv is assumed to be calculated by the momentum balance in quasi-steady flow for simplification of the computation.

§4 EXAMPLES

An exemplified calculation by means of the LCM was performed for the flutter boundary of a model helicopter rotor, the dimensions and operating conditions of which are given in Table 1. The calculation was proceeded as follows : (i) After attained a trimmed state of the rotor by performing several rotations with fixed feathering axis, a step disturbance of blade pitch angle was introduced and the feathering motion was released. (ii) By watching the behaviour of the feathering and flapping motions in several rotations, the flutter boundary could be determined and the computation was stopped.

The critical values in torsional rigidity versus advance ratio of the torsion-flapping flutter is presented as shown in Fig. 2 in comparison with theoretical calculation based on the vortex theory, and with experimental tests in the wind tunnel.[13,14]

In the calculation by the LCM, the Theodorsen and Sears functions are assumed to be that (i) $C(k)=S(k)=1.0$ for low frequency less than the order of the rotor rotational speed Ω, and (ii) $C(k)=S(k)$ for high frequency at critical condition of flutter $\omega \gg \Omega$.

Shown by a dotted line is the flutter boundary obtained from the quasi-steady calculation[14] in which the Theodorsen function was assumed to be C(k)=1.0. If either the torsional rigidity represented by the undamped natural frequency of the torsion ω_θ is smaller or the rotational speed Ω is larger than that specified by this line, the system will be unstable.

By considering the unsteady effect, which was obtained by multiplying the real part of the Theordorsen function into the quasi-steady lift and moment, and by introducing the blade cutoff of 3 percent radius at blade root and tip,[13] the flutter boundary shifts downward as shown by a chain line and hence the system increases the stability.

The lifting surface theory[14] with rigid wake shown by a double chain line gives a closer boundary with that of the experimental test,[14] which is given by a thin solid line with triangular marks, than other results based on the simplified vortex theory.

The present method of calculation, the LCM, gives more conservative result, as shown by a thick line, than that of the lifting surface theory. This result well coincides with the experimental result at hovering state and goes away as the advance ratio increases. The difference from either the experimental test or the lifting surface theory is, however, within allowable range in the practical application. For the study of the sensitivity of the Theodorsen function, more simplified calculations were performed by assuming that the C(k) was approximated by (i) C(k)=F(k) and (ii) C(k)=1. The results are respectively shown by (i) a hatched line for various advance ratios and (ii) a circle for hovering flight. The difference between the results of the simplified calculation (i) and of the LCM for fully unsteady flow with C(k) is essentially caused by the effect of shed vortices, whereas the difference between the results of the simplified calculation (ii) and the LCM for fully unsteady flow with C(k) is resulted from phase difference of the Theordorsen function. The above tendency is almost independent to the collective pitch of the rotor.

Fig.3 shows the critical torsional rigidity or flutter boundary of a windmill rotor (see Table 1) calculated by the present method , the LCM for fully unsteady flow with C(k), as a function of nondimensional center of gravity,

$$\bar{y}_{CG}=\int_{r_\beta}^{R} cy_{CG}(r-r_\beta)(dm/dr)dr \Big/ \int_{r_\beta}^{R}(r-r_\beta)^2(dm/dr)dr,$$

in comparison with a numerical result based on the blade element theory combined with the uniform inflow distribution.[16] It can be seen that the latter result gives very conservative boundary.

Shown in Fig.4 is an example of nondimensional bending moment at blade root of another windmill versus yawing angle of the rotor shaft with respect to the wind direction, calculated by the LCM. Since it was found that the bending moment was strongly affected by the yaw angle, the flutter analysis of the windmill was extended to the rotor in yawed state.

Fig.5 shows the effect of the yaw angle on the flutter boundary of the windmill rotor given in Table 1. As the yaw angle increases, the critical value of the torsional rigidity increases a little within the yaw angle from 0° to 30° specifically in higher wind speed.

CONCLUSION

A simple method of numerical calculation to determine the critical torsional rigidity for the initiation of classical flutter of rotory wings has been proposed as an extensive work of the Local Circulation Method, LCM. The method was initially applied to find the flutter boundary of a helicopter rotor for which the theoretical calculation based on the lifting surface theory and wind tunnel test were performed and approved to show good correlation with these results. Then, the method has been extended to analyze the flutter boundary of windmill rotor for which the inflow angle is highly distorted along the blade span. It is believed that the boundary was clearly specified more than that calculated by the blade element theory based on the simple momentum balance.

APPENDIX

By following Young's method[17] the equivalent stiffness and hinge offset of a flexible blade, both in nondimensional form, can be given by

$$k_\beta/m(R\Omega)^2 x_f{}^2=(\omega_1^N/\Omega)^2=(\omega_1/\Omega)^2-(x_{f,a}/x_f)^2 \tag{A.1}$$

$$\left.\begin{aligned} x_\beta &= \bar{x}_\beta\left[1-\frac{1}{2}\left(\frac{x_f}{x_{f,a}}\right)-\sqrt{\left\{1-\frac{\overline{x_\beta^2}}{(\bar{x}_\beta)^2}\right\}\left\{1-\left(\frac{x_f}{x_{f,a}}\right)\right\}+\left\{\frac{1}{2}\left(\frac{x_f}{x_{f,a}}\right)\right\}^2}\,\right] \\ &\approx \bar{x}\left[1-\frac{1}{2}\left(\frac{x_f}{x_{f,a}}\right)-\sqrt{\left\{1-\frac{\overline{x^2}}{(\bar{x})^2}\right\}\left\{1-\left(\frac{x_f}{x_{f,a}}\right)\right\}+\left\{\frac{1}{2}\left(\frac{x_f}{x_{f,a}}\right)\right\}^2}\,\right] \end{aligned}\right\} \tag{A.2}$$

where

$$\left.\begin{aligned} &\left(x_{f,a}/x_f\right)^2=1+x_\beta(\bar{x}_\beta-x_\beta)/x_f{}^2 \\ &x_f{}^2=\int_{x_\beta}^{1}\left(\frac{dm}{dx}/m\right)(x-x_\beta)^2dx\bigg/\int_{x_\beta}^{1}\left(\frac{dm}{dx}/m\right)dx \\ &\quad=\overline{x_\beta^2}-2x_\beta\bar{x}_\beta+x_\beta^2 \\ &\overline{x_\beta^n}=\int_{x_\beta}^{1}\left(\frac{dm}{dx}/m\right)x^n dx\bigg/\int_{x_\beta}^{1}\left(-\frac{dm}{dx}/m\right)dx \;;\; n=1.2 \\ &\overline{x^n}=\overline{x_\beta^n} \qquad \text{for } x_\beta=0 \end{aligned}\right\} \tag{A.3}$$

and where ω_1 and ω_1^N are undamped natural frequencies of rotating and nonrotating blade respectively.

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Table 1 Rotor parameters and operating conditions

Items	Helicopter rotor[13)]	Windmill rotor[16)]
Rotor radius, R, m	0.907	22.9
Blade chord at 0.75R, m	0.084	1.31
Blade section	0012	230XX
Blade twist from root to tip, deg.	0	-22∿2(nonlinear)
Collective pitch angle at 0.75R, deg.	4.0	0.0
Number of balde	1	2
Flapping hinge offset, m	0.047	—
Position of elastic axis (from leading-edge)	0.25c	0.25c
Non-dimensional position of C.G., y_{CG}	-0.20	—
Order of reduced frequency at 0.75R	0.2	0.4
Lock number	9.4	1.4

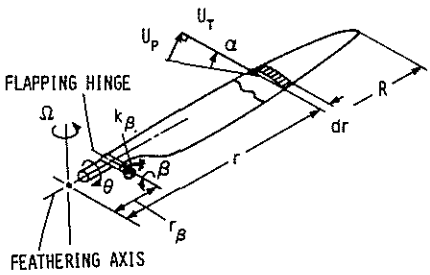

Figure 1 Blade configuration.

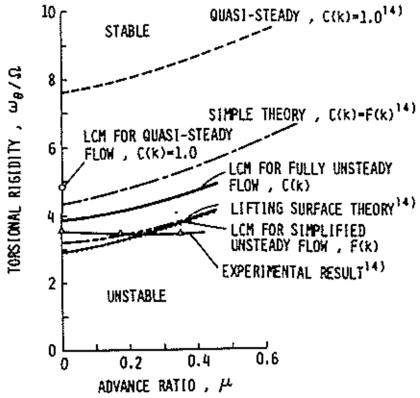

Figure 2 Flutter boundary of a helicopter rotor.

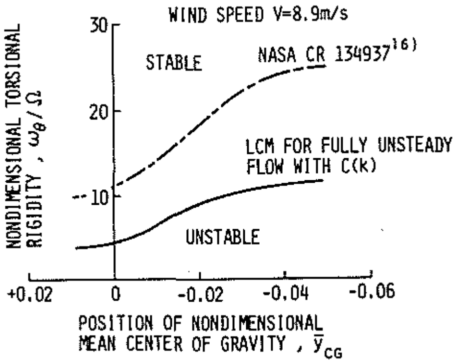

Figure 3 Flutter boundary of a windmill rotor.

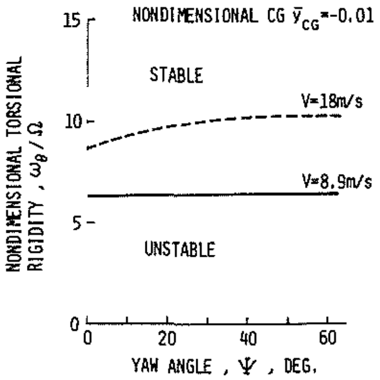

Figure 5 Flutter boundary of a windmill rotor versus yawing angle.

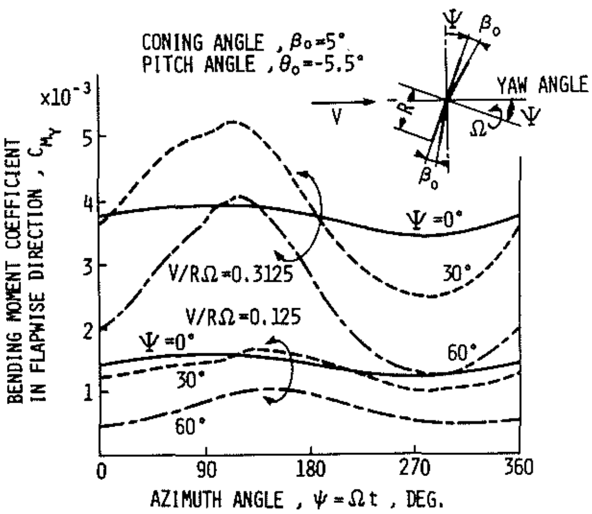

Figure 4 Variation of the bending moment of a windmill blade versus yawing angle.