EIGHTH EUROPEAN ROTORCRAFT FORUM

Paper No 3.7

VIBRATION OF A STRUCTURE WITH A TANK CONTAINING FLUID

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August 31 through September 3, 1982

AIX-EN-PROVENCE, FRANCE

ASSOCIATION AERONAUTIQUE ET ASTRONAUTIQUE DE FRANCE

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SUMMARY

Modal tests have been performed on a specially-built test piece designed to characterise the vibration properties of typical helicopter structures. The effects of adding a tank containing some fluid to the test piece have now been studied to gain insight into the influence of fuel carried in a helicopter on its vibration modes.

Results from a comprehensive series of tests suggest that the fluid can be represented in a structural model by an effective added mass, the size of which depends on the dimensions of the fluid in the tank and on the mode of vibration. The measurements also suggest that no significant non-linear effects are introduced by the fluid, even though its displacements may be large in comparison with those of the structure itself.

1 INTRODUCTION

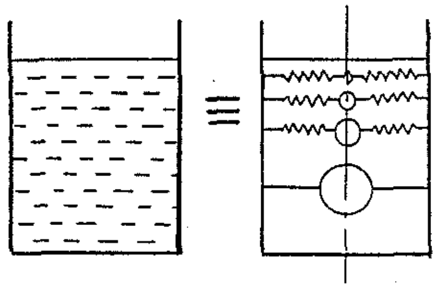
Some structures which are subject to vibration contain significant quantities of liquids, often in large containers such as fuel tanks. Low frequency vibrations may cause the whole mass of fluid to slosh and to generate large forces on the structure. This problem is well known, and by installing baffles inside the container, the forces on the structure may be reduced. However, when a structure including a container is partially filled with liquid and is subjected to vibration at frequencies above the fundamental sloshing frequency, a considerable proportion of the liquid mass may become decoupled from the vibrating container and effectively remain stationary, thus reducing the apparent mass of the liquid.

In helicopters for example, the mass of fuel at take-off is a significant fraction of the total aircraft mass. Structural vibration analysis based on representing the fuel as a solid mass will predict natural frequencies which are lower than the correct values, and will indicate greater increases in these frequencies as the fuel is consumed. It is clearly vital to be able to estimate the amount of decoupling which occurs if valid resonance predictions are to be obtained. The amount of decoupling may be expected to depend on the mode of vibration, and on the level of excitation. Fluid motion is probably larger than the container motion; this could also introduce non-linearity into the structure when it is subjected to vibration.

A previous paper (ref 1) described a series of tests performed on a special test structure which exhibited some non-linear behaviour when subjected to vibration. This same structure has now been modified by the addition of a tank (i) to enable the problems of testing structures containing liquids to be assessed (ii) to check whether additional non-linearity is introduced by the liquid and (iii) to investigate the amount of decoupling which occurs in the liquid for different modes of vibration.

2 THEORETICAL CALCULATIONS OF EFFECTIVE MASS OF FLUID IN A TANK

The dynamic behaviour of a fluid at its fundamental sloshing frequency may be modelled as an undamped spring-mass system (ref 2); the values of mass and stiffness being calculated to produce the same force on the tank and the same natural frequency as the sloshing fluid. The mass is some proportion of the total mass of fluid ; higher sloshing modes may be represented by other tuned spring-mass systems of progressively smaller mass. Physical equivalence with the fluid requires that the sum of the masses be equal to the total mass of fluid and this is assured by the inclusion of a mass fixed directly to the tank. The full model for the fluid behaviour over the whole frequency range therefore consists of a fixed mass and a series of spring-masses as shown in Fig 1.

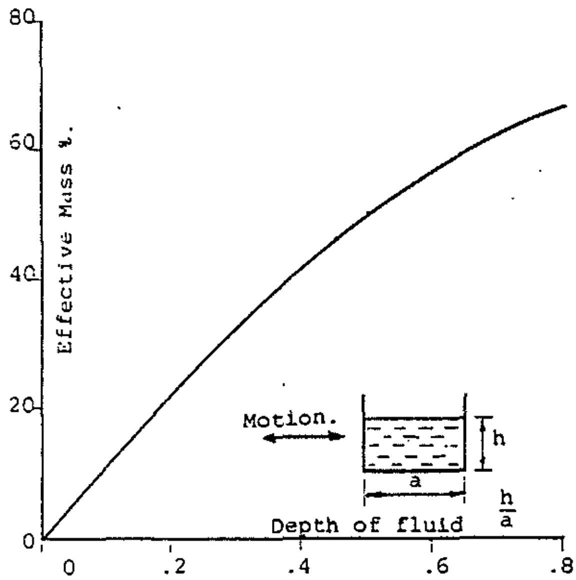


Equivalent spring and mass model of fluid for sway motion.

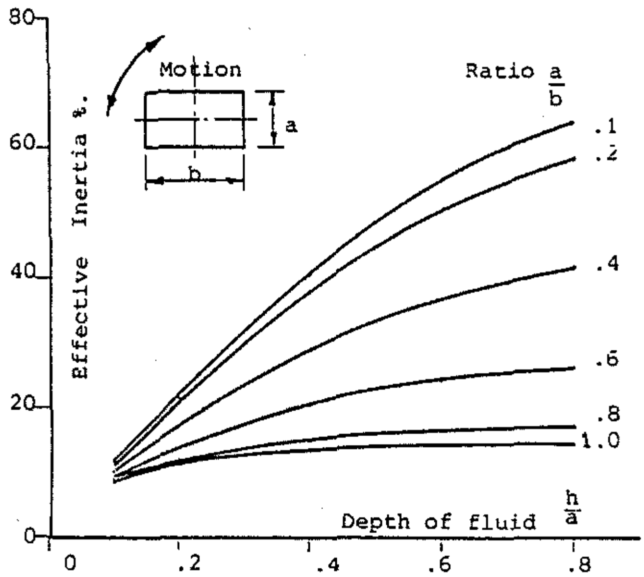
FIGURE 1

If the model is subjected to forced sinusoidal vibration then the sprung mases whose resonant frequencies are below the forcing frequency will be effectively decoupled. At frequencies considerably above the fundamental sloshing frequency, the majority of the spring masses will be thus decoupled, and then the effective mass of the fluid will closely approximate to the fixed mass.

Physical reasoning suggests that a mass of fluid in a shallow container would decouple more than the same mass of fluid in a tall container with a small plan area. For swaying modes the characteristic dimensions are the depth of fluid and the tank length in the direction of sway. In torsion modes about a vertical axis through the centre of the tank - YAW motion - the decoupling will depend upon the depth of fluid and upon the aspect ratio of the tank plan.



THEORETICAL effective mass for sway motion.
FIGURE 2.



THEORETICAL Inertia for yaw motion.
FIGURE 3.

Using the equations from ref 2 to calculate the sprung mass for each sloshing frequency, it is possible to determine the fixed mass as a function of the fluid dimensions for the sway and yaw motions. The ratio of fixed mass to total mass of luid, ie the effective mass is shown in Figs 2 and 3 for sway and yaw modes.

These predictions show that the effective mass is considerably less than the true mass of fluid except when the fluid depth is large compared with its length. The amount of decoupling is greater for yaw torsion modes than for sway modes, and for yaw modes, it is greater for square tanks than for rectangular tanks.

3 EXPERIMENTAL WORK

3.1 Test Structures

i. Tower structure The main structure used in these experiments is shown in Figure 4a. The structure has been described in detail elsewhere (refs 1 and 3); its main features are a lightweight aluminium tower fixed at the base to a massive concrete block, with a heavy steel angle plate fixed to the top of the tower using typical helicopter engine mountings. The whole structure is asymmetrical and has a mass of 74 kg.

A rectangular tank (30 cm x 22 cm, 30 cm high) of 20 litre capacity has been constructed from 1.6 mm mild steel sheet, welded and with reinforcing strakes bonded to the outside for stiffening, see Fig 4b. A removable lid and gasket enable the tank to be filled with liquid or with solid iron masses, and vents allow the tank to be completely filled with liquid and then sealed. The iron masses were bonded to the inside of the tank in positions which produced the same yaw moment of inertia as the same mass of "solid" water would have if in the tank.

The tank was bonded to the heavy steel angle plate on top of the tower in a position offset from the tower centre line, thereby increasing the aymmetry of the complete structure.

ii. Welded frame structure For a second series of tests the tank was removed from the tower structure and attached to a lightweight welded steel frame which was symmetrical and which could be expected to provide approximately linear motion with no rotation of the tank for the lower sway modes of vibration of the assembly. The arrangement of this frame and the tank is shown in Fig 4c. A steel plate could be attached to the lower rails of the frame and additional iron masses bonded to the plate. This arrangement allowed higher sway and torsion modes to be investigated.

3.2 Test Method

Previous tests (ref 1) indicated that the tower structure was nonlinear in behaviour, and that tests should be performed under controlled conditions if results were to be repeatable. All frequency response measurements were therefore made at constant force levels, as measured by the force gauge. Non-linear behaviour could be then investigated by repeating the measurements at several force levels.

The tower structure was lightly damped, with well separated modes

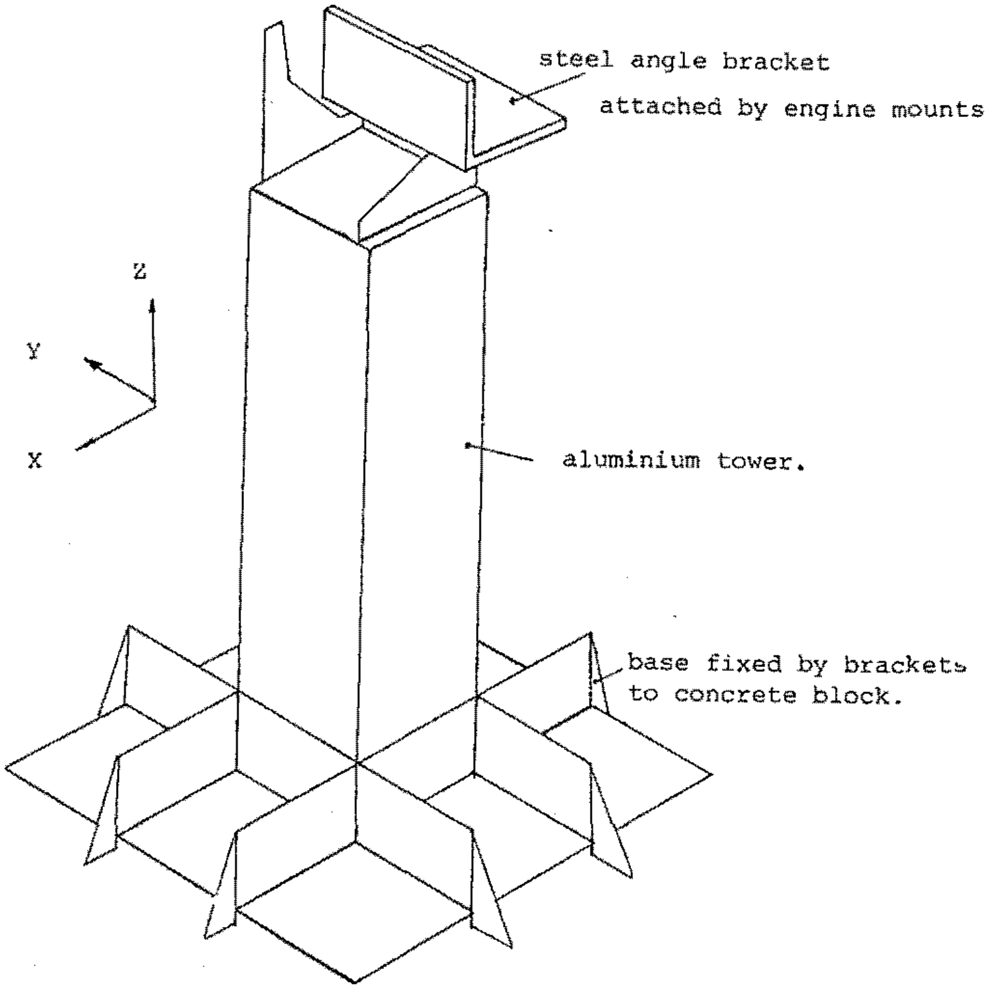


Diagram of TOWER STRUCTURE with steel angle bracket.

FIGURE 4 a.

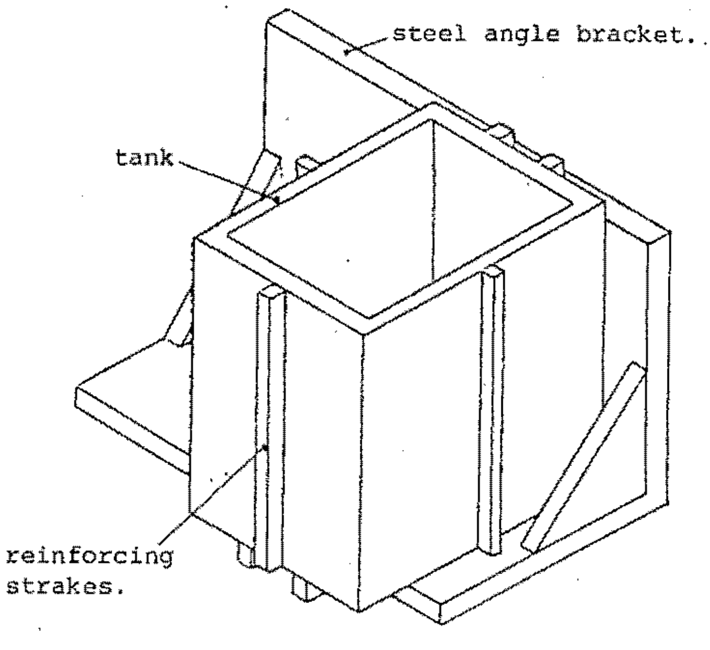


Diagram of steel tank bonded to steel angle bracket.

FIGURE 4 b.

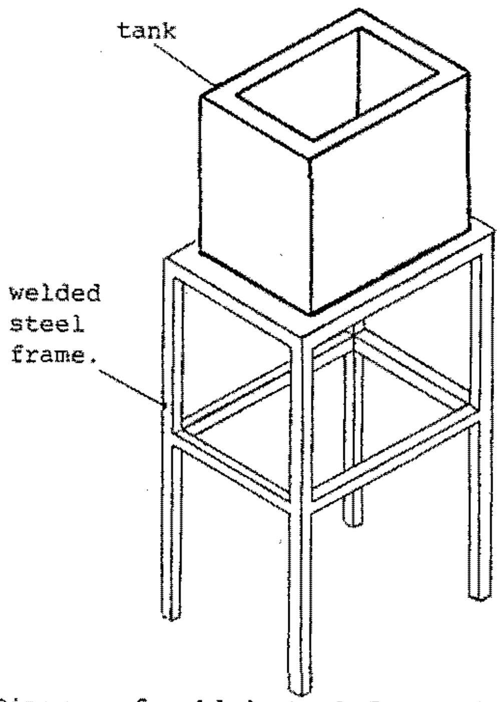


Diagram of welded steel frame with tank.

FIGURE 4 c.

and so a single degree of freedom curve-fit analysis was used to extract the modal data for each resonance in turn. Measurements were made at several points on the structure to define the type of motion occuring at the six resonances investigated.

Subsequent tests on the welded steel frame were concerned with changes in natural frequency, rather than with non-linear effects and consequently FFT measurements could be used to determine the natural frequencies. This rig was used to investigate the first two swaying modes and the first torsion mode, and to determine the effective mass of fluid on each case. Preliminary mode shape analysis indicated that the structure behaved in a symmetrical manner for these modes. However higher modes were not symmetrical and were therefore not investigated in detail.

Both structures were tested with several masses of water in the tank, and also with iron masses bonded to the inside of the tank to represent "solid" water.

4 RESULTS

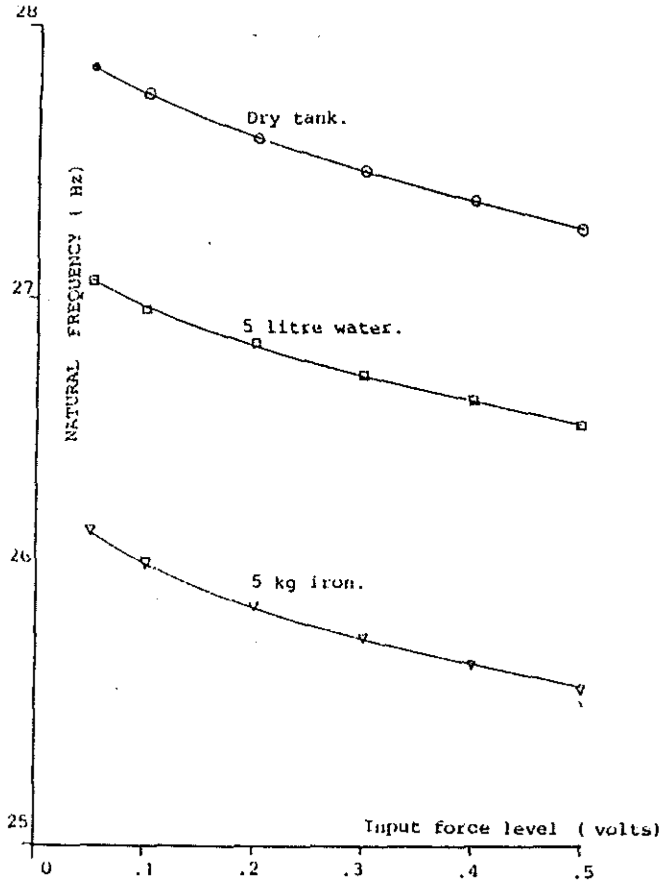
4.1 Problems of testing liquids in tanks

Doubts had been expressed about the effect of minute air bubbles trapped in the water, it was suggested that these would significantly alter the fluid behaviour and that the results would not be repeatable. However preliminary tests using constant force control gave very good repeatability, suggesting either that the water, from large storage tanks, was reasonably de-aerated, or that the effect of air bubbles was not significant.

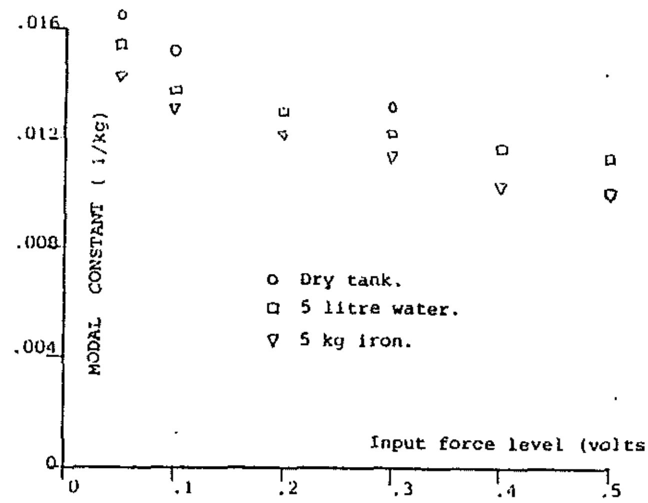
The fundamental sloshing frequency of the tank was calculated to be about 2 Hz, and the first structural mode of vibration was about 28 Hz. Visual examination of the water during preliminary testing showed that the surface sometimes formed many small waves at the higher harmonics of the sloshing frequency, but these were small in pitch and amplitude, although still large in comparison with the tank motion.

4.2 Linearity testing

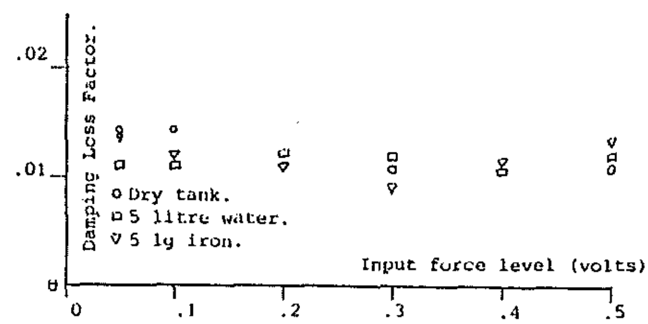
The tank was bonded to the angle plate on the tower. All tests were conducted at constant force levels, and these gave repeatable results and allowed the effects of changes in liquid mass or solid mass to be investigated accurately. Initial tests were made with the tank empty, for the first six modes of vibration. The tower was excited horizontally and the point mobility measured for each mode at different force levels, over a 10:1 range. These tests were repeated with 5 litres of water in the tank, and then with 5 kg of iron bonded to the inside of the tank. The estimates of natural frequency, damping loss factor and modal constant were extracted for each resonance using a single-degree-of-freedom curve fitting algorithm. The changes in natural frequency and modal constant for the first mode are shown in Figs 5 and 6 and for the damping loss factor in Fig 7. Similar results were obtained for the following five modes.



CHANGE of Natural Frequency with Force.
FIGURE 5.



CHANGE of Modal Constant with Force.
FIGURE 6.



CHANGE of Damping Loss Factor with Fo
FIGURE 7.

The results show that the basic structure with the empty tank is non-linear in behaviour, with the natural frequency dropping by about 2% and the modal constant dropping by about 40% as the force level is increased. Extrapolation of these curves to obtain the zero force value would then enable an estimate of the linear response to be deduced. Fig 7 shows that there is no significant pattern in the damping estimates, and that the fluid does not increase the damping significantly.

The addition of fluid to the tank does not appear to increase or to alter the non-linear behaviour of the dry structure; varying the force level again producing a large variation in the damping and modal constant values. Replacing 5 litres of water with 5 kg of iron, bonded to the tank base, lowers the natural frequencies, confirming that part of the water is decoupled from the tank motion.

4.3 Calculation of the effective mass of water

A very simple analytical model may be used to deduce the approximate amount of water decoupled from the vibrating structure. If the empty tank and structure is considered as a linear spring-mass system, and the 5 kg iron as an added point mass then the structure's stiffness and mass may be deduced, and hence the effective mass of water calculated from the relative changes in frequency. The effective mass was calculated from measurements at six force levels over a 10:1 range for the first six modes of vibration. For one mode of vibration the results are very similar showing that the amount of decoupling does not depend upon the force level. However the results are different for each mode; a series of measurements were therefore made at several points on the structure to determine the type of motion occuring in the tank. These measurements were again made at constant force, to minimise errors in the measured modal constants. The dominant motion for each natural frequency was identified as follows:

Table 1 - Identification of Dominant Motion and Effective Mass of Fluid

Mode	Resonant Frequency Hz	Damping	Effective Mass of Fluid*	Dominant Motion
1	27.85	.013	42%	Sway in X direction
2	29.8	.011	45%	Sway in Y direction
3	74.2	.008	28%	torsion about a vertical axis
4	89.8	.016	33%	torsion about a vertical axis
5	119.6	.015	94%	vertical bounce
6	164.5	.016	46%	torsion about a vertical axis

*Effective Mass calculated using spring-mass model

Insufficient transducer locations were used to identify fully the torsional movement of the steel mass and consequently the centre of rotation could not be identified. For the first five modes, and for the tank containing 5 kg of water, the results show that an approximate guide to the amount of decoupling for a given tank motion can be derived:

Table 2

MOTION	EFFECTIVE MASS
swaying about a horizontal axis	45%
vertical bounce motion	95%
torsion about a vertical axis	30%

These values would seem sensible for the type of tank motion experienced. Horizontal motion of the tank walls could be expected to cause more fluid to move than torsional movement of the tank about a vertical axis. However, the position of the pivoting axis is clearly crucial in the torsion mode; with a pivot axis near the centre of the tank little coupling may be predicted, but as the pivot axis moves away from the tank, it tends to a swaying rather than a torsion mode and the coupling could be expected to increase. This may well explain the effective mass estimation for Mode 6; insufficient measuring points on the steel mass do not allow a complete analysis of the tank motion.

When the tank is bouncing vertically, the whole mass of fluid should be directly coupled to the tank, unless the vertical acceleration is greater than 1g, when fluid could be expected to rise from the normal free surface. During the current tests, infufficient force was applied to the vertical axis to cause such decoupling.

4.4 Influence of depth of fluid on effective mass

The preceding tests were all made with 5 kg of iron or 5 litre of water, and the equivalent mass was deduced from the relative change in frequency, assuming that the structure stiffness could be represented by a single spring-mass system. In practice, as the mass in the tank is increased, the centre of gravity of the tank also rises, and consequently the stiffness will be reduced. The test structure is not of uniform stiffness, in particular the heavy angle plate is connected to the tower by light-weight mountings, consequently the tank may move significantly on top of the tower. In addition, the effective mass and its Centre of Gravity are expected to alter with the fluid proportions. Further tests were therefore performed to identify the change in natural frequency when different masses of fluid or iron were placed in the tank. The results for the first two modes are given in Fig 8. For any mass of fluid, the effective mass can be simply read from the graph, by finding the mass of iron required to produce the same natural frequency. The results for several fluid masses are given in Table 3 below.

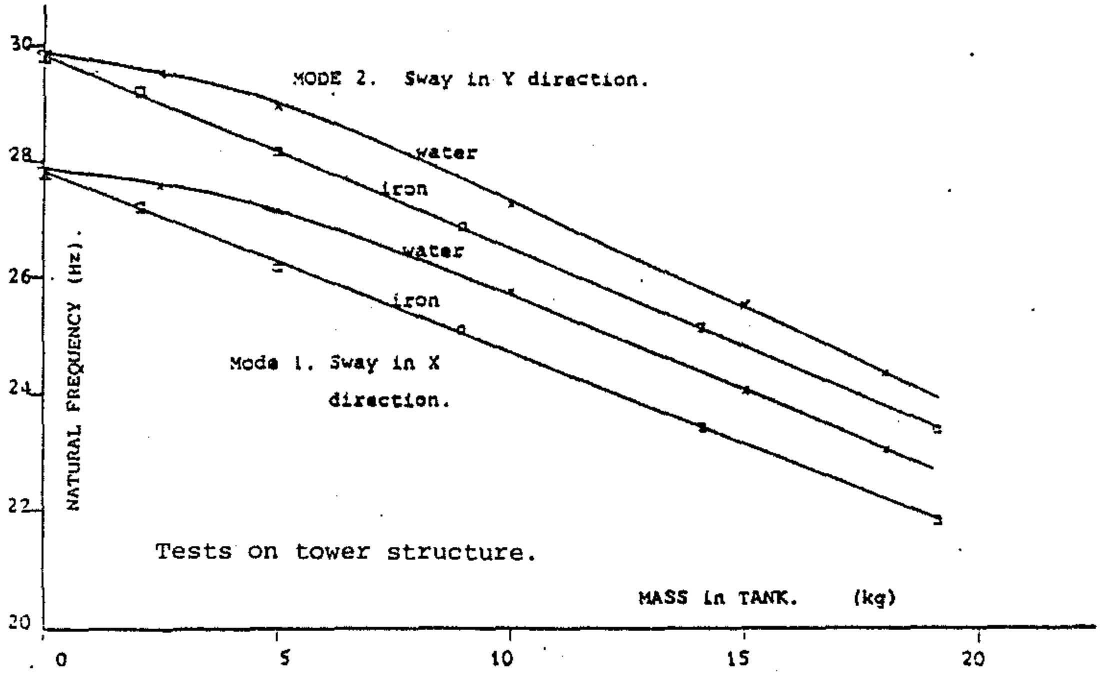
Table 3: Variation of Effective Mass of Fluid with Different Fluid Masses

MODE 1. Sway in X direction		MODE 2. Sway in Y direction.	
Mass of Fluid (kg)	Effective Mass (%)	Mass of Fluid (kg)	Effective Mass (%)
2.5	30	2.5	42
5.0	44	5.0	55
10.0	67	10.0	76
15.0	80	15.0	86
18.0	85	18.0	91
20.0	87	20.0	92

These results may be compared with the theoretical predictions based on the tuned vibrator model introduced in Section 2 and shown in Fig 9. The basic shapes of the curves are the same, showing that the amount of decoupling increases as the fluid depth is reduced. However the mode shape analysis shows that the tank is subjected to rotation about a horizontal axis in addition to swaying in the first two modes. This effect is probably increased by the lightweight mountings connecting the heavy steel angle plate to the tower. The position of the centre of gravity of the iron and the fluid, their effective moments of inertia about a horizontal axis and their effective masses should all be identical if a truly equivalent system is to be achieved.

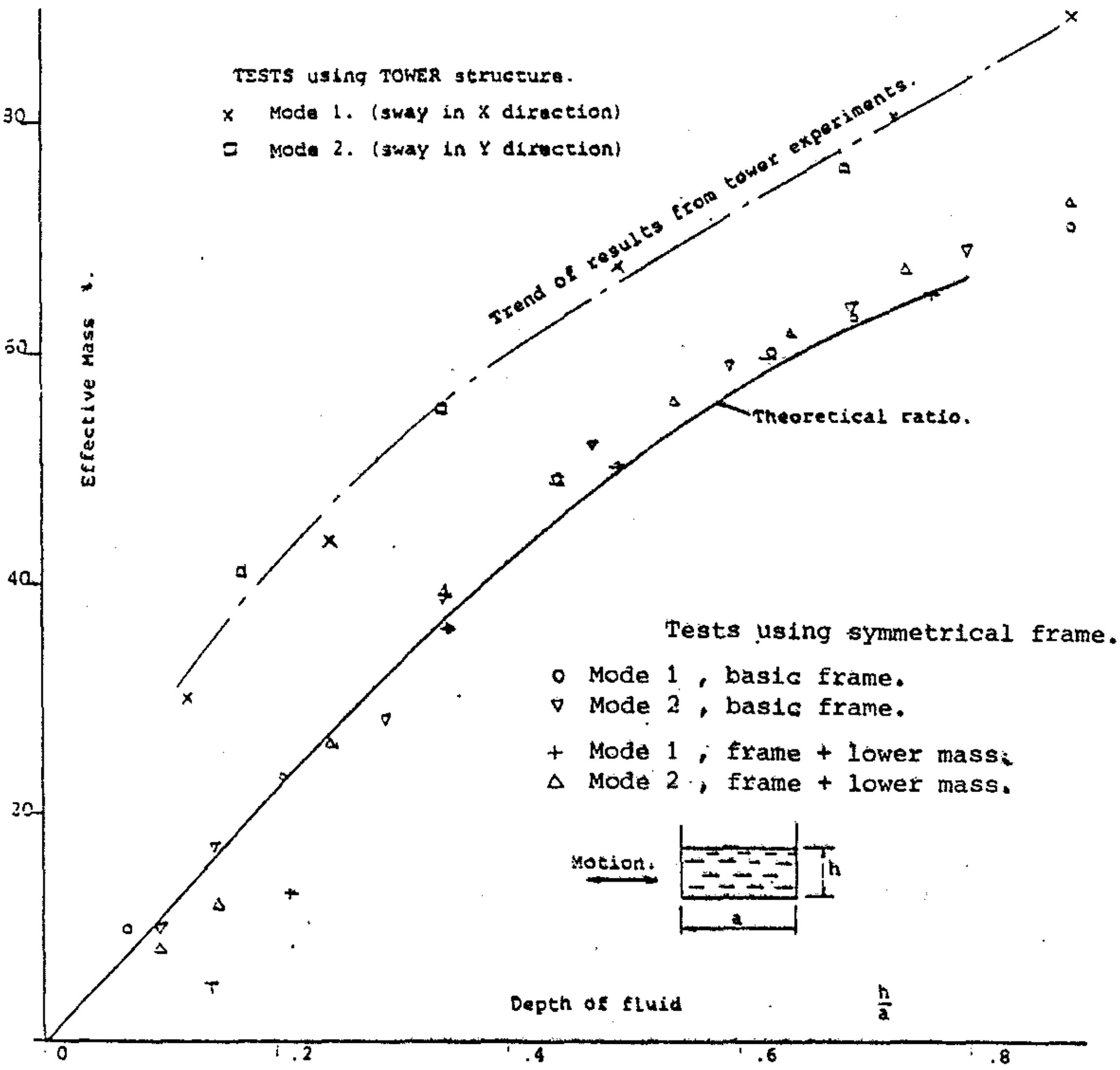
4.5 Effective mass estimates using a simple symmetrical frame

Subsequently, the tank was removed from the tower and mounted onto a symmetrical welded steel frame. Mode shape measurements confirmed that the motion was virtually pure sway for the first and second modes and a symmetrical rotation about the vertical axis of symmetry for the third mode of vibration. The effective mass of fluid was again obtained by finding the mass of iron required to give the same natural frequency. The effective mass for different depths of fluid were determined for each type of motion and the results plotted on Fig 9 for sway modes and Fig 10



EFFECT of varying mass on Natural Frequency.

FIGURE 8.

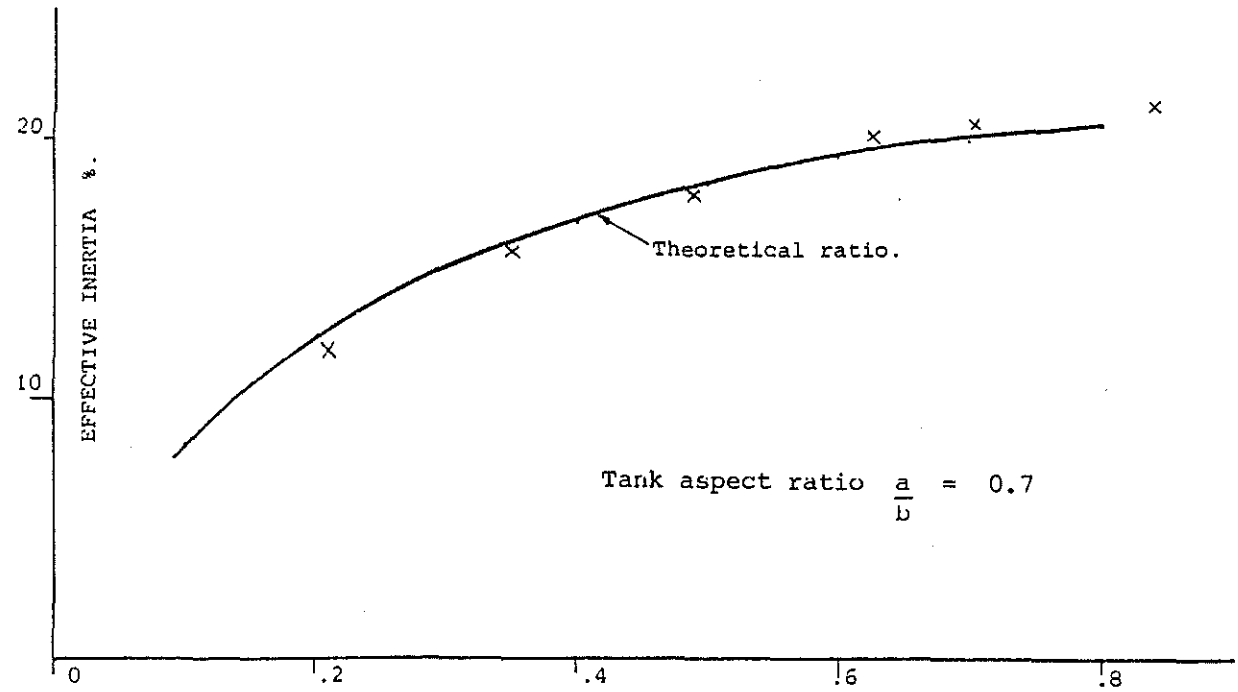


EFFECTIVE MASS of liquid for SWAY Modes.

FIGURE 9.

for the torsion mode. These figures also show the predicted effective mass and it is clear that the predictions are close to the measured values.

These results suggest that the effect of fluid in a structure may be represented in a structural model by an equivalent solid mass, (usually significantly less than the actual mass of the fluid) provided that the proportions of the fluid or the dominant motion are known. For shallow tanks and yaw motion a greater error is introduced if the fluid is modelled as a solid having the same mass than if the fluid is ignored altogether.



EFFECTIVE INERTIA OF LIQUID on symmetrical frame structure.

FIGURE 10

5 CONCLUSIONS

5.1 Modal testing techniques may be applied to structures containing fluids at frequencies above the fundamental sloshing frequency without major difficulty.

5.2 Fluids do not appear to increase the level of non-linear behaviour exhibited by typical fabricated structures, and the frequency response techniques developed to cope with non-linear structures can produce repeatable results.

5.3 When a structure containing some fluid vibrates, a significant proportion of the fluid decouples from the structure, the amount is dependant upon the mode of vibration experienced, and upon the dimensions of the fluid.

5.4 An analytical model of the fluid as a series of spring-mass systems predicts values for the equivalent mass of the fluid for sway motion and for yaw motion which agree closely with the values measured by experiment on a simple test rig. The values obtained on the tower did not agree as well, because of the more complex motion occurring in the tank.

6 ACKNOWLEDGEMENTS

The authors with to acknowledge the financial and technical support provided for this work by Westland Helicopters Ltd.

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