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A FREQUENCY-DOMAIN APPROACH TO ROTOR STATE ESTIMATION FOR IDENTIFICATION AND CONTROL

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Abstract. A novel technique for accelerometer-based estimation of blade flapping and out-of-plane bending is proposed. The estimators are analog filters, designed in the frequency domain by means of inverse modelling concepts, and taking into account the issues of robustness and modal spillover. A byproduct of the proposed technique is a general rationale for the proper location of accelerometers along the blade. Preliminary results from linear simulations are also presented.

I. INTRODUCTION

In recent years the application of modern optimal control has been investigated (see e.g., [1]-[3]) as a mean to improve the dynamic characteristics of helicopter main rotors. Such control techniques require the availability of measurements or estimates of the state variables of the system to be controlled. Since direct measurement is unlikely to be possible, it is normally necessary to resort to estimation techniques.

The classical solution to the estimation problem for linear and finite-dimensional systems is given by the Kalman filter; unfortunately its application to the rotor is a critical issue. Indeed, the rotor is a time-varying, infinite-dimensional system, so that even the estimation of a subset of its state variables would be unpractical and the estimates would be heavily biased by the presence in the measured signals of components due to (unmodelled) higher order dynamics.

A few years ago a novel technique for accelerometer-based rotor state estimation, called kinematic observer or McKillip filter was proposed (see [5] and [6]). It relied on Kalman filter time-domain concepts, requiring the use of two accelerometers per mode.

In this paper, the state estimation problem is tackled by a frequency-domain approach; the estimation of blade flapping and bending can then be performed by means of simple filters, the design of which is extremely easy. Moreover, only one accelerometer per mode suffices. The designed filters have shown good performance during testing on a simulation program.

II. PROBLEM STATEMENT

Consider as sensor a blade-mounted accelerometer, with a sensitive axis normal to the surface of the blade.
In this paper, in analogy with [5] and [6], we neglect the couplings with in-plane and torsional dynamics.
Thus, the output of the sensor located at $x = \overline{x}$ is given by:

$$\frac{a(\overline{x},t)}{R} = \ddot{z}(\overline{x},t) + \overline{x}\Omega^2 z'(\overline{x},t)$$

where $z(x,t)$ is the out-of-plane bending of the blade. As usual, it can be expressed as an expansion on the basis of the normal bending modes:

$$z(x,t) = \sum_k q_k(t)\eta_k(x).$$

The sensor output can be therefore expressed as:

$$\frac{a(\bar{x},t)}{R} = \sum_k \eta_k(\bar{x})\ddot{q}_k(t) + \bar{x}\eta_k'(\bar{x})\Omega^2 q_k(t) \quad (1)$$

The problem can then be stated as follows: to reconstruct the first K $q_k(t)$'s from K measurements of the $a(\bar{x},t)$ type.
By making reference, e.g., to the problem of estimating flapping and first bending ($K=2$), the solution to this problem proposed in [5] calls for the availability of *two* sensors per mode, so that by inverting the system:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} x_1 - e & x_1\Omega^2 & \eta(x_1) & x_1\eta'(x_1)\Omega^2 \\ x_2 - e & x_2\Omega^2 & \eta(x_2) & x_2\eta'(x_2)\Omega^2 \\ x_3 - e & x_3\Omega^2 & \eta(x_3) & x_3\eta'(x_3)\Omega^2 \\ x_4 - e & x_4\Omega^2 & \eta(x_4) & x_4\eta'(x_4)\Omega^2 \end{bmatrix} \begin{bmatrix} \beta \\ \ddot{\beta} \\ q_2 \\ \ddot{q}_2 \end{bmatrix}$$

the four unknown variables can be estimated from the four acceleration measurements.

Since for this particular problem modal spillover effects are the main source of inaccuracy, any possibility to perform the estimate with just one acceleremoter per mode should be seen as a major improvement, since it would allow to double the number of modes taken into account, given the total number of accelerometers.
To pursue this goal, a frequency domain estimation technique is proposed in the subsequent section.

III. FREQUENCY-DOMAIN APPROACH

III.a The frequency-domain estimation filter

Taking the Laplace transform of eq. (1) one can obtain the following expression for the output of the i-th sensor:

$$\frac{a_i(x_i,s)}{R} = \sum_k \eta_k(x_i)[s^2 + \Omega_{i,k}]q_k(s) = \sum_k D_{i,k}(s)q_k(s) \quad (2)$$

where

$$\Omega_{i,k} = \frac{x_i\eta'(x_i)}{\eta(x_i)}\Omega^2 \quad (3)$$

and

$$D_{i,k}(s) = \eta_k(x_i)[s^2 + \Omega_k].$$

The parameters defined in eq. (3) will be referred to as the *characteristic parameters* of the accelerometers .
By separating the terms related to the first K bending modes from the remaining ones, eq. (2) takes the form:

$$\frac{a_i(x_i,s)}{R} = \sum_{k=1}^{K} D_{i,k}(s)q_k(s) + \varepsilon_i(s) \quad (4.a)$$

where

$$\varepsilon_i(s) = \sum_{l=K+1}^{\infty} D_{i,l}(s) q_l(s) \tag{4.b}$$

represents the spillover term.

Then, in principle, if one had have K (linearly independent) accelerometers, one would have the possibility of estimating the first K bending modes by direct inversion of the transfer function matrix relating the first K bending modes to the K accelerometer outputs. This amounts to inverting the MIMO relationship obtained from eqs. (4) for $i=1,2...,K$ with the spillover terms neglected.

Obviously, this is not feasable since the $D_{i,k}(s)$ present two zeros on the imaginary axis, so that the filter obtained by this direct inversion procedure would not be stable: indeed, it is easily checked that the poles of the MIMO relationship above are symmetrical with respect to the imaginary axis.

A simple remedy consists in resorting to a filter which is a stable approximant of the direct inverse of the MIMO relationship. Of course this introduces further causes of approximation in the estimate of the bending modes, which add to the modal spillover effect.
However, we will show that, since most of the modal response of the blade occurs at integer multiples of the rotation frequency Ω, it is possible to design a filter which achieves a satisfactory trade-off between stability and accuracy, thus providing a good solution for the estimation problem.

III.b The spillover effect
To put on solid grounds the above procedure, it is essential to envisage the effect of the neglected spillover terms. These terms are associated with the modes $K+1, K+2, \ldots$. In practice, we will of course confine to a finite number of spillover modes, from $k+1$ to -say- N, whith N large enough.
Defining then the following vectors and polynomial matrices:

$$Q_K = [q_1 \;\; \cdots \;\; q_K]^T \qquad Q_{sp} = [q_{K+1} \;\; \cdots \;\; q_{K+N}]^T \qquad A_K = [a_1 \;\; \cdots \;\; a_K]^T$$

$$D_K(s) = \begin{bmatrix} D_{1,1}(s) & \cdots & D_{1,K}(s) \\ \cdots & \cdots & \cdots \\ D_{K,1}(s) & \cdots & D_{K,K}(s) \end{bmatrix} \qquad D_{sp}(s) = \begin{bmatrix} D_{K+1,K+1}(s) & \cdots & D_{K+1,K+N}(s) \\ \cdots & \cdots & \cdots \\ D_{K+N,K+1}(s) & \cdots & D_{K+N,K+N}(s) \end{bmatrix}$$

eqs. (4) can be put into the matrix form:

$$A_K = D_K(s) Q_K + D_{sp}(s) Q_{sp} \,. \tag{5}$$

Remark 1
Obviously, det $D_K(s)$ is a polynomial with even powers only. Therefore, the poles of the truncated system are either located on the imaginary axis or are given by pairs symmetric with respect to the imaginary axis.□

Remark 2
The poles of the truncated matrix depend upon the sensor locations, i.e. upon vector $X = [x_1 \;\; \ldots \;\; x_K]$, since such locations determine the parameters $\Omega_{i,k}$ through eq. (3). For instance, if one considers the case of two accelerometers per blade, simple computations show that the poles of $D_K(s)$ are always located on the imaginary axis except when the two sensors are located on a particular portion of blade, in which case all poles have a non-zero real part.□

From eq. (5), it is apparent that the estimate is given by:

$$\hat{Q}_K = D_K^{-1}(s)A_K = Q_K + D_K^{-1}(s)D_{sp}(s)Q_{sp}.$$

Thus the bias in the estimation of the modal coefficient is:

$$\tilde{Q}_K = \hat{Q}_K - Q_K = D_K^{-1}(s)D_{sp}(s)Q_{sp} \qquad (6)$$

Eq. (6) shows what the effect of modal spillover on the estimates would be; according to Remark 2 the characteristics of the polynomial matrices $D_K(s)$ and $D_{sp}(s)$, and therefore those of the modal spillover transfer function matrix, are heavily dependent upon the locations of the K accelerometers. This dependence is investigated in the forthcoming subsection.

III.c Sensor location problem

As seen above, there is a direct relationship between modal spillover and sensor location: as a matter of fact, if X is such that the elements of the transfer function matrix given in eq. (7) are large, then the effect of higher order modes on $\hat{Q}_K$ will be significantly amplified, therefore introducing a large bias in the estimates.

Interestingly enough, the same expression of the bias applies as well to other approaches, such as the direct FFT approach proposed in [6], or notch filtering methods (see e.g., [7]). Therefore, an analysis of the bias given by eq. (6) is important whatever estimation method is used to assess the spillover effect.

To try to keep such effect within acceptable bounds, we study the problem of the accelerometer location so as to maximize accuracy in estimation (namely minimize $\tilde{Q}_K$).
We will illustrate a wise location procedure in the simple case $K=1$:

Example. Suppose to resort to the simplest possible approximation, i.e. let $K=1$:

$$\frac{a}{R} = (x-e)\ddot{\beta} + x\Omega^2\beta \ .$$

Then, under periodic, steady-state conditions, consider the Fourier expansion of acceleration a and flapping angle β, so as to obtain the complex harmonics a_n of the acceleration:

$$\frac{a_n}{R} = \Omega^2\left[x-(x-e)n^2\right]\beta_n.$$

In particular the first harmonic is given by:

$$\beta_1 = \frac{a_1}{R\Omega^2 e} \ .$$

In Table 1, two cases are considered. The first one refers to an accelerometer located near the tip of the blade (90% radius), while the second one refers to a sensor close to the flapping hinge (10% radius).
The first harmonic of flapping has been computed in both cases via a direct FFT technique; a straightforward comparison of the estimates and the true value shows that the tip accelerometer is subject to a very heavy spillover effect.

First harmonic flapping	Est. from root accelerometer	Est. from tip accelerometer
0.6951-0.02339i	0.6904+0.0379i	0.6717-2.3106I

Table 1. Blade flapping estimation under spillover conditions.

The significant error on first harmonic flapping which is present when using measurements from the tip accelerometer is due to spillover from the higher order bending modes.□

In view of such considerations, sensor locations should be cosidered as main design variables in the described optimisation procedure.

Remark 3
The spillover effect is fundamental in order to judge whether an estimation technique can be successfully applied to the rotor. For instance, we have also investigated the possibility of resorting to discrete-time inverse modelling techniques (deconvolution techniques, see e.g., [8]).
However, in order for this technique be applicable, one has to avoid poles on the imaginary axis. Otherwise, the impulse response would not decay, so that deconvolution techniques would fail. As seen in Remark 1, this can be achieved by suitably locating the accelerometers. Unfortunately it turns out that such a positioning is unpracticable from the spillover viewpoint.□

IV. DESIGN OF THE FLAPPING ESTIMATOR

Up to this point, we have always dealt with ideal filters, with poles on the imaginary axis. In the analysis we have therefore considered only the suitable periodic solution of the equations, to avoid raising stability issues.

We now present a simple procedure to work out a stable filter for the estimation of blade flapping $(q_1 = \beta)$ in the simple case $K=1$.
With our approach, only *one* accelerometric measurement is then necessary.
Taking N=2, eqs. (4) read:

$$\frac{a_1}{R} = (x_1 - e)\ddot{\beta} + x_1\Omega^2\beta + \eta_2(x_1)\ddot{q}_2 + x_1\eta_2'(x_1)\Omega^2 q_2 = D_1(s)\beta + D_{sp}(s)q_2$$

Eq. (5) then gives:

$$\hat{\beta} = \frac{1}{D_1(s)}a_1.$$

Having two poles on the imaginary axis, this filter is in fact unreliable. In order to guarantee stability, a suitable stable approximant to $D_1(s)$ will be designed. To this purpose, let's consider the family of polynomials

$$D_{1\alpha}(s,\alpha) = (x_1 - e)\left[s^2 + 2\alpha s + \alpha^2 + \Omega_1^{\ 2}\right].$$

Obviously

$$D_1(s) = D_{1\alpha}(s,0)$$

When the ideal polynomial is replaced by the approximate one, an additional error term is intrduced; the error in the estimation is now constituted by two terms, one $(\tilde{\beta}_{sp}(s))$ due to the spillover effect and one $(\tilde{\beta}_{ap}(s))$ due to the approximation in the above polynomial:

$$\tilde{\beta}(s) = \hat{\beta}(s) - \beta(s) = \tilde{\beta}_{ap}(s) + \tilde{\beta}_{sp}(s)$$

where

$$\widetilde{\beta}_{ap}(s) = \frac{D_1(s,0) - D_1(s,\alpha)}{D_1(s,\alpha)}\beta(s)$$

$$\widetilde{\beta}_{sp}(s) = \frac{D_{sp}(s)}{D_1(s,\alpha)}\beta(s)$$

Therefore, focusing on the filter approximation effect only, the quantities:

$$\widetilde{\beta}_{ap}(\alpha,n) = \frac{|D_{1\alpha}(nj\Omega,\alpha)| - |D_{1\alpha}(nj\Omega,0)|}{|D_{1\alpha}(nj\Omega,\alpha)|} \tag{7}$$

represent the percentile estimation errors at integer multiples of the rotation frequency Ω. Obviously, eq. (7) is large in proximity of the square root of the characteristic parameter Ω_1 of the accelerometer.

In general, the problem of making a fair estimate requires that both $\widetilde{\beta}_{sp}(s)$ and $\widetilde{\beta}_{ap}(s)$ are kept as small as possible. Fortunately, these two effects do not present interactions, and one can face the problem of minimizing $\widetilde{\beta}_{sp}(s)$ by suitably locating the sensors, whereas the tuning knob to reduce $\widetilde{\beta}_{ap}(s)$ is parameter α.
As for the sensor location, we have seen the basic ideas in the preceding section.

For the selection of the value of α, we have to find a compromise between the degree of stability of the filter and the magnitude of errors $\widetilde{\beta}_{ap}(s,n)$. This may be stated as the following optimization problem:

$$\begin{cases} \max(\alpha) \\ \widetilde{\beta}_{ap}(\alpha,\bar{n}-1) < E \\ \widetilde{\beta}_{ap}(\alpha,\bar{n}) < E \end{cases}$$

where $\bar{n}$ is such that $(\bar{n}-1)\Omega < \Omega_1 < \bar{n}\Omega$. Obviously, thia corresponds to imposing a bound, E, to the worst approximation errors, which are those associated with the multiples of Ω closest to the characteristic frequency Ω_1 of the accelerometer.
Once E is fixed, the optimal value of α, i.e., the one ensuring the best possible degree of stability within the prescribed specification for precision, can be easily evaluated, either analytically (as in this simple case) or numerically.

V. ESTIMATION OF HIGHER ORDER MODES

If more than one sensor is available, then the additional information can be used in order to refine estimates for blade flapping. Indeed, once N is fixed, the number of spillover terms in (4.b) can be reduced. Moreover, one can also estimate higher order modal deflections.

Consider, e.g., $K=2$, i.e., the problem for the estimation of flapping and first bending starting from the measurements of two accelerometers. The two sensor outputs are given by:

$$\frac{a_1(x_1,t)}{R} = (x_1 - e)\ddot{\beta}(t) + x_1\Omega^2\beta(t) + \eta(x_1)\ddot{q}(t) + x_1\eta'(x_1)\Omega^2 q(t)$$

$$\frac{a_2(x_2,t)}{R} = (x_2 - e)\ddot{\beta}(t) + x_2\Omega^2\beta(t) + \eta(x_2)\ddot{q}(t) + x_2\eta'(x_2)\Omega^2 q(t)$$

so in the frequency-domain one gets:

$$\begin{bmatrix} \frac{a_1(x_1,s)}{R} \\ \frac{a_2(x_2,s)}{R} \end{bmatrix} = D_2(s)\begin{bmatrix} \beta \\ q \end{bmatrix} = \begin{bmatrix} (x_1-e)(s^2+\Omega_{1,1}^2) & \eta'(x_1)(s^2+\Omega_{1,2}^2) \\ (x_2-e)(s^2+\Omega_{2,1}^2) & \eta'(x_2)(s^2+\Omega_{2,2}^2) \end{bmatrix}\begin{bmatrix} \beta \\ q \end{bmatrix}$$

The filter is then given by:

$$F_2(s,\alpha) = D_{2\alpha}^{-1}(s,\alpha) \tag{8}$$

where $D_{2\alpha}(s,\alpha)$ is a stable approximation of $D_2(s)$. Again, one comes across an optimisation problem with parameter α and the sensor locations as optimization tools.

VI. SIMULATION RESULTS

The testing have been performed on a simulatin model of a five-bladed, articulated rotor.
The simulation program, develeped in a SIMULINK context, enables one to model both rigid and flexible blades. In the latter case, the out-of-plane bending dynamics of the blade is described by its first four modes; the mathematical model is analogous to the classical one described in [9].
The state estimation problem has been considered with $K=1$ and $K=2$. In the first case, only the flapping is estimated; in the second one the first bending mode is estimated too.
In any case $N=4$ has been taken and we have also performed a comparison with the performances achievable with a kinematic observer, proposed in [5].
For each test, two plots are proposed: the first one shows comparisons between time histories of simulated (dashed lines) and estimated (solid lines) flapping; in the second the same comparison is performed in the frequency domain for the steady state periodic regime of the same forward flight condition.
.The simulated model has been given non-zero initial conditions in order to put into focus the bandwidth characteristics of the estimators.

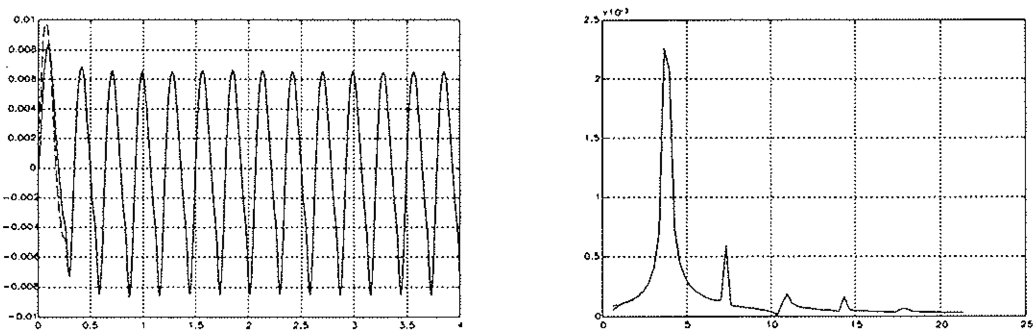

Fig. 1. Kinematic observer, rigid blade with two sensors.

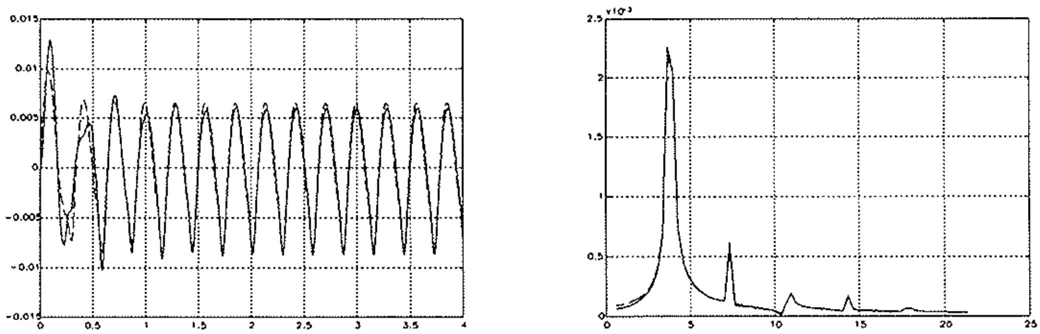

Fig. 2. Inverse model, rigid blade with one sensor.

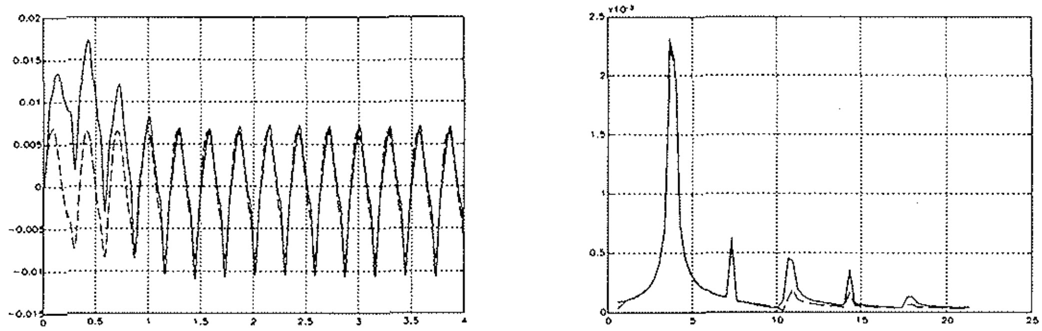

Fig. 3. Kinematic observer, flexible blade with two sensors.

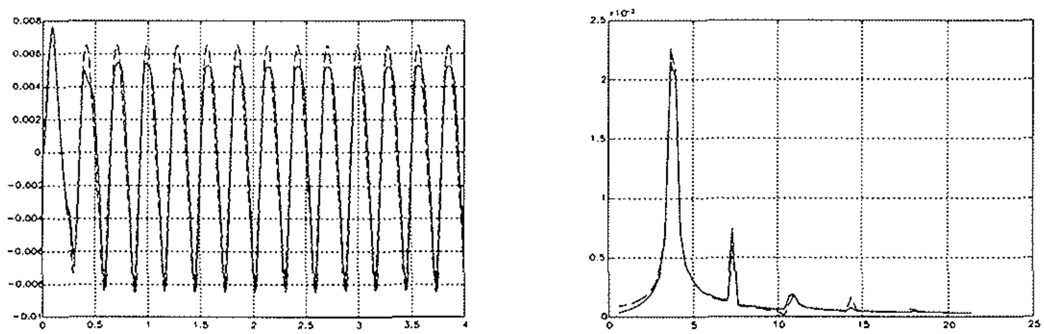

Fig. 4. Inverse model, flexible blade with one sensor (*K=1*).

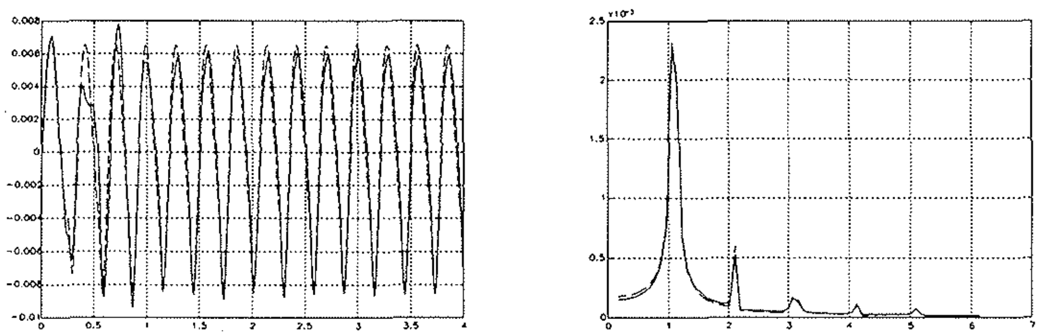

Fig. 5. Inverse model, flexible blade with two sensors (*K=2*).

It appears quite clearly that in absence of any uncertainty (rigid blade case) the Kalman filter viewpoint taken in [5] ensures very good performances; such conditions however are not realistic, given that true acceleration measurements always retain the effect of higher order, unmodelled bending modes.

VII. APPLICATION: IDENTIFICATION OF FLAPPING DYNAMICS

The described estimators have been applied to the problem of identifying a suitable dynamic model of blade flapping dynamics from noisy accelerometer measurements. We assume to have two accelerometers per blade (*K=2*), located according to the rationale described in the previous sections.
The simulated identification experiments, concerning a *hovering rotor*, was performed as follows:

- a multiharmonic periodic perturbation is superimposed to the pitch pilot command.

- the steady state accelerometer measurements are recordered, low-pass filtered to prevent aliasing, and then resampled at a lower rate (175 Hz).

The identification data are constituted by the sequences of pitch angle and acceleration measurement bidimensional vector, sampled at 175 Hz.
The identification procedure consisted of a first filtering phase of the accelerometric data, by means of our inversion based estimator with $K=2$; in other words, the accelerometric data were filtered through system (7). In this way, one obtains in particular the sequence of the (estimated) flapping angle behaviour.

Such estimates have been used as fictitious output measurements for the identification of discrete-time models of the type:.

$$A(q)y(t) = B(q)u(t) + e(t) \tag{9}$$

or:

$$y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \tag{10}$$

where $A(q)$, $B(q)$, $C(q)$, $D(q)$, $F(q)$ are polynomials in the delay operator q, $y(t)$ is the estimated flapping angle, $u(t)$ is the pitch angle and $e(t)$ is a white noise introduced to model disturbances and model inaccuracies.
Models (9) and (10) are usually referred to as ARX and Box-Jenkins models, respectively, see e.g., [11] for more detail.

Three different experimental conditions have been considered, which are characterized by different levels of noise affecting the accelerometrs. Precisely, we have considered the case of noisefree measurement first; then, the case of a Signal to Noise Ratio (SNR) of 20 dB and 10 dB are dealt with. These three experimental conditions will be referred to as A, B, C.

Note: given a signal $s(t)$ and a noise $n(t)$, the signal-to-noise ratio is defined as

$$SNR = 10Log\left[E\left[|s|^2\right] / E\left[|n|^2\right]\right]$$

where $E[]$ is the expected value in the stochastic case, or the corresponding sampled version in the deterministic case (see e.g., [12]). □

Depending on the specific experimental conditions, different models were tuned.
Whenever an ARX model was identified, $A(q)$ polynomials of order four and $B(q)$ polynomials of order two were considered, and the parameters were estimated by the classical Least Squares algorithm.
In the case of Box-Jenkins models , order four for polynomials $F(q)$ and $D(q)$ and order two for polynomials $B(q)$ and $C(q)$ were found to be a good choice; estimation was then performed with Maximum Likelihood techniques.
The results of the identification of ARX models in the three experimental conditions are reported in Table 2. The a_i's and the b_i's are the coefficients of polynomials $A(q)$ and $B(q)$.

	a_1	a_2	a_3	a_4	b_1	b_2	FPE	Cov[β-y]
Exp. A	-3.3153	4.1216	-2.2837	0.4780	0.0078	-0.0072	2.19e-10	4.16e-5
Exp. B	-3.2496	3.9204	-2.0750	0.4048	0.0086	-0.0080	1.32e-8	6.83e-5
Exp. C	-3.0129	3.2027	-1.3385	0.1497	0.0115	-0.0107	1.10e-7	1.10e-3

Table 2. Identification of ARX models.

From the analysis of the Final Prediction Error (FPE) and of the error covariance, it appears that the ARX models provide satisfactory approximations to the actual flapping response in experiments A and B; in other words, in those conditions, the effect of using the estimated flapping instead of the true flapping in the

identification does not affect the model quality in a substantial manner. This is quite remarkable for the case of Model B, which has been obtained under difficult "experimental" conditions.
Only in the experimental conditions C, the ARX modelling is not satisfactory: this is not surprising, since in this (worst) case the SNR has been given an extremely (and possibly unrealistically) small value.

However it is worth noticing that under noisy measurement conditions ARX models are not the most appropriate structure for the kind of I/O relations we are trying to describe; this happens because, due to the filtering applied to the actual accelerometer measurements, the residuals in the signals used in the identification are not white anymore, but have different spectral characteristics.

For all these reasons, the Box-Jenkins model structure of eq. (10) has been considered appropriate for the low SNR case, in that it allows for independent modelling of the I/O relationship and of the residual's dynamics.
In Table 3, the results of the identification of a Box-Jenkins model in experimental conditions C are summarized. Obviously, the f_i's and the b's are the coefficients of polynomials $F(q)$ and $B(q)$.

	f_1	f_2	f_3	f_4	b_1	b_2	FPE	Cov[β-y]
Exp. C	-2.9929	3.4255	-1.8111	0.3817	0.0064	-0.0024	7.99e-8	1.11e-5

Table 3. Identification of Box-Jenkins models.

The Final Prediction Error and the error covariance are, respectively, one and two orders of magnitude smaller than those for Exp. C with ARX modelling, and they are comparable with those obtained using noise-free signals and ARX modelling.

Therefore, it appears that identification of blade flapping response to pitch perturbations can be well performed using estimates of rotor flapping, as this allows for very good results, even under difficult experimental conditions; furthermore ARX models suffice, provided that good to moderate SNR's can be ensured.
If this is not the case, it is always possible to resort to slightly more complex Box-Jenkins models.

Finally, notice that ARX models can be readily extended to account for the periodicity of rotor dynamics under forward flight conditions (PeriodicARX, or PARX, models), see [13]. In principle, one can therefore identify periodic models for forward flight conditions.

VIII. ROBUSTNESS OF THE ESTIMATORS

It is extremely important to ensure that the filters are not oversensitive to (unavoidable) uncertainty in the design process. Two possible sources of inaccuracy have been considered:

- inaccuracy in the assumed mode shapes;
- inaccuracy in the location of the sensors;

In order to check for the effects of uncertainty, the identification of ARX models in the experimental condition A, described in the preceding section, have been repeated under the following inaccuracy assumptions:

- the filters have been re-designed using approximate expressions for the mode-shapes, obtained by Least-Squares fitting a low order set of Duncan polynomials ([10]) to the analytical non-rotating mode-shapes.

- an error in the location of the acceleremeters is included; first, indicating with R the blade length, a location error of order .02R is considered. Then, the error has been reduced to .01R.

For simplicity, we refer to the above conditions as experimental condition A.1 (sensor location error 0.1R) and A.2 (sensor location error 0.2R), respectively.

The results are presented in the following Table 4, in the same form as in Tables 2 and 3:

	a_1	a_2	a_3	a_4	b_1	b_2	FPE	Cov[β-y]
Exp. A.1	-3.5116	4.7043	-2.8666	0.6744	0.0053	-0.0050	5.21e-9	9.02e-4
Exp. A.2	-3.3631	4.2853	-2.4673	0.5458	0.0070	-0.0062	7.41e-10	1.42e-4

Table 4. Identification of ARX models.

A degradation in the accuracy of the identified models is apparent from such results; it should be considered, however, that the inaccuracies introduced in these tests are really worst-case ones (particularly for accelerometer placement inaccuracies). Therefore the performance of the estimators can be considered quite satisfactory.

IX. CONCLUDING REMARKS

An inversion based frequency-domain technique for the estimation of rotor state variables from the measurement of acceleration has been presented and discussed. This technique is easy to implement and requires only one accelerometer for each modal response to be estimated. It is fairly robust to spillover effects and modelling errors. Finally, it is suited for applications like system identification of rotor dynamics.

X. ACKNOWLEDGMENT

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XI. REFERENCES

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