DETECTION AND CONTROL OF GROUND RESONANCE USING PHASE OF FUSELAGE ATTITUDE RATES

Salini S. Nair
snairsalini@gmail.com
Research Scholar
IIT Madras, Chennai,
India-600036

Ranjith Mohan
ranjith.m@iitm.ac.in
Assistant Professor
IIT Madras, Chennai,
India-600036

Abstract

Ground resonance is a type of aeromechanical instability that occurs when the helicopter is in contact with the ground. It may occur due to coalescence between frequencies of two modes of the system if damping is insufficient. In this paper, we analyze the phase relations between the fuselage states corresponding to the least damped mode (regressive lag mode) and its correlation with frequency coalescence. The phase of fuselage states (attitude or attitude rates), which are easily measurable, is observed to exhibit certain trends with variation in parameters like rotor speed and landing gear stiffness. The phase data can aid in the design of a stability augmentation system for ground resonance. It can serve as a parameter to detect the possibility of instability and in systems with uncertainties in parameters, like landing gear stiffness, it can aid in selecting the appropriate feedback gain for stabilization. The model we have primarily considered has isotropic rotor and anisotropic hub, hence multiblade coordinate transformation is used and the stability analysis is done in fixed frame. The analysis is further extended to incorporate dynamic inflow effects and anisotropy in rotor blades, where Floquet method is used for stability analysis. Air resonance instability is also investigated on similar lines and the proposed method is found to be good for its detection.

1. NOMENCLATURE

$A(t)$	System matrix
$F(t)$	Periodic eigenvector
J	Diagonal matrix of the eigenvalues of the system
K	Steady state gain
K_ϕ	Fuselage roll stiffness
K_θ	Fuselage pitch stiffness
$[L]$	Influence Coefficient Matrix
$[M]$	Mass matrix
M	Number of harmonics
$[V]$	Mass flow matrix
Y_o	Response of second order system
Y_i	Input to second order system
a_n^m	Inflow states
t	Time
v_z	Inflow
x, y	Lateral and longitudinal displacement of the fuselage
Ω	Rotor rpm
Φ	Phase of the frequency response
Φ_{rp}	Phase difference between the fuselage attitude or attitude rates
β	Blade flap motion
γ	Blade lock number
ω	Natural frequency of the degrees of freedom
ω_n	Natural frequency of a second order system
ϕ	Fuselage roll angle
ϕ_n^m	Pressure potential
ψ	Blade azimuth angle
τ_n^m	Pressure expansion coefficients
θ	Fuselage pitch angle
ζ	Blade lag motion, Damping ratio
ζ_c, ζ_s	Cyclic lag displacement
c, s	Cosine and sine terms
fx, fy	Forces in the lateral and longitudinal directions

2. INTRODUCTION

Aeromechanical instabilities are self-excited oscillations in helicopters, which may occur when the frequencies of the fuselage mode and a rotor mode of the system coalesce. However, frequency coalescence need not lead to instability provided there is enough damping present in the system. Ground resonance is a type of aeromechanical instability that occurs when the helicopter is in contact with the ground. It has been demonstrated both analytically and experimentally in literature[1–3] that ground resonance occurs due to coalescence between the regressive lag mode of the rotor and the fuselage modes. The similar kind of phenomena experienced while the helicopter is in flight is air resonance[3,4] . This occurs due to coupling between the blade flap motion, lead-lag motion and the body modes. The focus in this paper is on ground resonance although air resonance will also be discussed very briefly.

The degrees of freedom of the coupled rotor-fuselage system considered in ground resonance analysis are the flap, lead-lag degrees of freedom of the rotor blades and the roll, pitch degrees of freedom of the fuselage. The equations of motion are inherently periodic with frequency equal to the rotor rotational frequency. However, if the blades are isotropic, it is possible to convert the system equations to constant coefficient form by using Multiblade Coordinate Transformation[5,6] . Anisotropy in blades can arise due to wear and tear and this results in a periodic coefficient system. In such cases, the equations are retained in periodic form and the analysis is done using Floquet method[7,8] . In either case, the modal damping and modal frequency plots are obtained by sweeping across a range of rotor frequencies. Instability is indicated by the modal damping being greater than zero and by the coalescence between the modal frequencies of the participating modes.

Several techniques including passive, semi-active, and active methods[9] have been proposed in literature for control of ground resonance. Active and semi-active control strategies require some triggering mechanism, usually amplitude of the fuselage signals, to activate the control system in the event of instability. The signals are composed of many modes and are noisy, hence the triggering of the control system based on amplitude alone can lead to errors.

In this paper, we analyze the phase relations between the fuselage states corresponding to regressive lag mode and its correlation with frequency coalescence. The difference in phase of the fuselage states (attitude or attitude rates), which are easily measurable, is observed to exhibit certain trends, which can aid in the designing of a stability augmentation system for ground resonance. The phase data can serve as a parameter to detect the possibility of instability and in systems with uncertainties in parameters, like landing gear stiffness, it can supplement in the feedback gain selection.

It has been verified in literature that the inflow through the rotor has a significant effect on the stability[10–12] . The inflow through the rotor can be modeled in different ways, uniform or dynamic. In the initial analysis in this paper, the inflow through the rotor is assumed uniform. However, the inflow in real scenario varies in both radial and azimuthal directions. Peters-He dynamic inflow model[13] is a finite state dynamic inflow model developed from the basic potential flow equations and is used in this paper to analyze the effects of dynamic inflow on phase relations. The model is in the form of a set of ordinary differential equations (Eqn. 1), which can be easily incorporated into the stability analysis model.

(1a) $$[M]\left\{\dot{a}_n^{mc}\right\} + [L]^{-1}[V]\left\{a_n^{mc}\right\} = \left\{\tau_n^{mc}/2\right\}$$

(1b) $$[M]\left\{\dot{a}_n^{ms}\right\} + [L]^{-1}[V]\left\{a_n^{ms}\right\} = \left\{\tau_n^{ms}/2\right\}$$

(2) $$v_z = \sum_{m=0}^{\infty}\sum_{n=m+1}^{\infty}\left(a_n^{mc}\phi_n^{mc} + a_n^{ms}\phi_n^{ms}\right)$$

Here all the matrices have closed form expressions, c, s represent the cosine and sine terms, a_n^m the inflow states and τ_n^m the pressure expansion coefficients computed from the lift. The inflow is computed using Eqn. 2, where the variation of m denote the number of harmonics and n the number of radial shape functions. ϕ_n^m denote the pressure potential incorporating the variations in the radial and azimuthal directions. Convergence in inflow can be obtained with a limited number of harmonics (M). If $M = 1$, for an isotropic rotor system, the system equations will be in constant coefficient form. However $M > 1$ leads to a periodic coefficient system and hence Floquet method has to be used for stability analysis.

The paper is structured as follows. The methodology used for analysis is explained first followed by results and discussion. First an isotropic rotor model for ground resonance is analyzed using Mutiblade Coordinate Transformation assuming uniform inflow. Second, phase change with parametric variations are considered and the applicability to stability augmentation is discussed. Third, the dynamic inflow model is applied to the system dynamics and the effect on phase relations and stability is analyzed. Fourth, anisotropy is introduced into the rotor and the analysis is done using Floquet method. In the last part the analysis is extended to case of air resonance.

3. METHODOLOGY

The analytical model used in this study for ground resonance analysis is based on the experimental model in Ref. [2]. The model has roll, pitch degrees of freedom for the fuselage and lag, flap degrees of freedom for the rotor blades and inflow is assumed uniform. Multiblade Coordinate Transformation is used to convert the system equations to constant coefficient form and eigenvalue analysis is used for computing

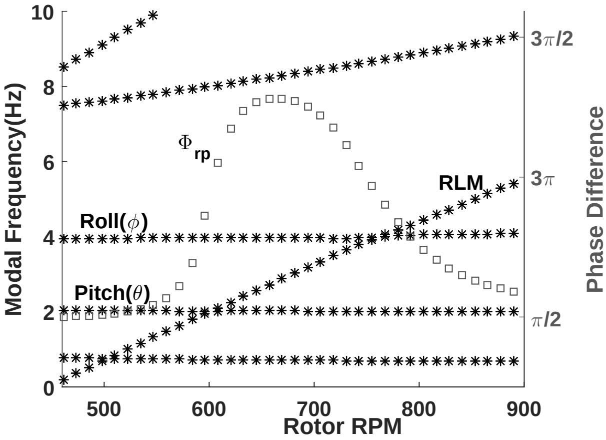

Figure 1: Variation of modal frequency and phase difference between fuselage attitude rates with rotor speed

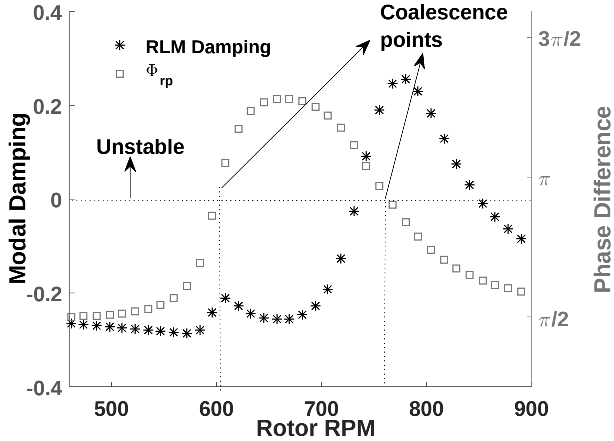

Figure 2: Variation of RLM damping and phase difference between fuselage attitude rates with rotor speed

the modal damping and frequency across a range of rotor rotational frequencies (Ω). The soft-in-plane region of the rotor, which is prone to ground resonance instability, alone, is considered in the current analysis. The mode corresponding to the minimum damping is identified as the regressive lag mode (RLM). The phase difference between the fuselage states corresponding to the RLM at different rotor speeds is investigated. The phase difference between the fuselage attitudes or attitude rates corresponding to RLM will be referred to as Φ_{rp} in the subsequent sections.

The effect of parametric variations on system stability and phase(Φ_{rp}) characteristic is analyzed by varying the fuselage roll and pitch stiffness. To use the phase characteristic as a supplement in stability augmentation in real scenario, it should be computed from the time response of the system obtained from sensors. To demonstrate this, the phase difference is computed from the time response of the nonlinear system using appropriate method.

To analyze the effect of wake, Peters-He dynamic inflow model is applied to the system dynamics and the number of harmonics (M) is restricted to three. Floquet method is used for stability analysis and the periodic eigenvector ($F(t)$) is computed using Eqn. 3, where J is the diagonal matrix of the eigenvalues of the system, $A(t)$ is the system matrix and $F(0)$ is the eigenvector of the Floquet transition matrix[14] . The eigenvectors are converted from blade coordinates to multiblade coordinates and the phase difference between the fuselage attitude rates for the regressive lag mode is considered.

$$\frac{d}{dt}F(t) = A(t)F(t) - F(t)J \tag{3}$$

For anisotropic rotor, Floquet method is used for analysis. Anisotropy is introduced by varying the stiffness of the blades. The least damped mode is considered and the phase difference between the fuselage attitude rates for this mode is considered in the analysis.

The analytical model in Ref. [3] is considered in this study for air resonance analysis. The model has the same degrees of freedom as the ground resonance model but with the landing gear stiffness equal to zero. The analysis is done in multiblade coordinates and the phase difference between the fuselage attitude rates for the regressive lag mode is considered.

4. RESULTS AND DISCUSSION

4.1. RPM Sweep

The soft-in-plane region of the rotor for the model considered[2] corresponds to $\Omega > 400\,rpm$. The modal damping, frequency and the phase difference between the fuselage attitude rates (Φ_{rp}) or equivalently the states are computed and plotted. Figure 1 shows the variation of Φ_{rp} as a function of rotor speed. The modal frequency plots are also shown in the same figure. Since the analysis is done in fixed frame, the frequency of the roll and pitch modes remain almost constant with rpm variation. The regressive lag mode frequency variation is denoted in the figure as RLM. It can be observed that the coalescence between the pitch mode frequency and RLM frequency occurs around $\Omega = 600\,rpm$ and between roll mode frequen-

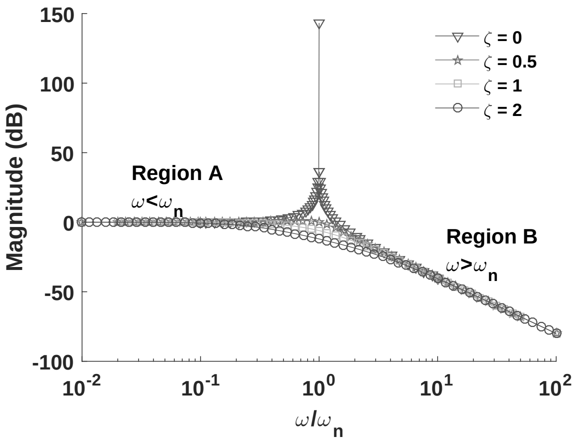

Figure 3: Second order system frequency response (Magnitude)

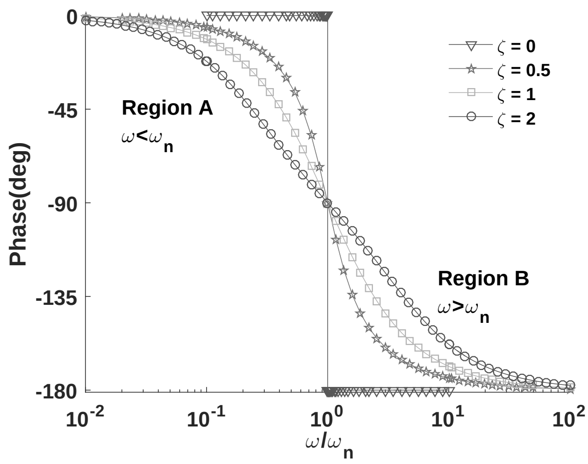

Figure 4: Second order system frequency response (Phase)

cy and RLM frequency around $\Omega = 775\ rpm$.

From the Φ_{rp} variation, it can be noted that the phase deviates from $\pi/2$ to about $3\pi/2$ passing through π at the coalescence between pitch frequency and RLM frequency and from $3\pi/2$ to $\pi/2$ passing through π at the coalescence between roll frequency and RLM frequency. Figure 2 shows the variation of RLM damping with rotor rpm along with the Φ_{rp} plot. Comparing the two figures, it can be observed that reduction in damping occurs corresponding to coalescence between the fuselage mode frequencies and the RLM frequency as expected. In addition, it can be inferred that the minimum damping region for the two coalescence is accompanied by a phase difference (Φ_{rp}) change from $\pi/2$ or $3\pi/2$ and that the stable regions are characterized by a phase difference (Φ_{rp}) of about $\pi/2$ or $3\pi/2$.

4.1.1. Second Order System Analogy

Ground resonance occurs when the frequency of the whirling motion of the center of gravity of rotor equals the frequency of the fuselage motion on its landing gear[15–17] . The instability can be explained by considering the lead-lag motion (whirling motion) to be a forcing to a second order system with pitch or roll degrees of freedom. This implies the phase relations between the lead-lag motion and roll or pitch displacements (or velocities) corresponding to the regressive lag mode will follow the typical phase relationship between forcing and response of a second order system near the coalescence regions.

Second order system characteristic: A typical second order system frequency response characteristic is given by Eqn. 5, where Y_o and Y_i denote the output and input respectively, ω, the input frequency, ω_n the natural frequency of the system, ζ the damping ratio and K the steady state gain.

$$\frac{Y_o}{Y_i}(i\omega) = \frac{K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \frac{4\zeta^2\omega^2}{\omega_n^2}}} \angle - \tan^{-1}\frac{2\zeta}{\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)} \tag{4}$$

The condition $\omega = \omega_n$ is known as the resonance condition, and if sufficient damping is not present (low ζ), the system response will rise to very high magnitudes. For a particular value of ζ, the phase characteristic can be described as below,

$$\Phi = \angle - \tan^{-1}\frac{2\zeta}{\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)} = \angle - \tan^{-1}\frac{2\zeta\omega\omega_n}{(\omega^2 - \omega_n^2)} \tag{5}$$

When $\omega < \omega_n$, $\Phi > -\pi/2$. It varies from 0 at $\omega/\omega_n = 0$ to $-\pi/2$ at $\omega/\omega_n = 1$. Similarly when $\omega > \omega_n$, $\Phi < -\pi/2$. It varies from $-\pi/2$ to $-\pi$ as ω/ω_n tends to ∞. A representative second order system frequency response (magnitude and phase) is shown in Figs. 3 and 4. It can be observed from Fig. 4 that as ζ increases, the width of the transition region increases. Hence for high ζ, the entire transition region will not exactly correspond to the coalescence region ($\omega/\omega_n = 1$). However, for reasonably moderate (low) values of ζ as is observed in mechanical systems, the trend (variation from 0 to $-\pi$ through $-\pi/2$) can be observed in correspondence with the frequency coalescence region.

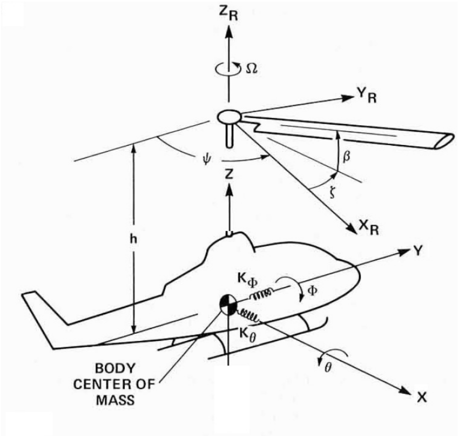

Figure 5: The reference coordinate system used in the analysis[3]

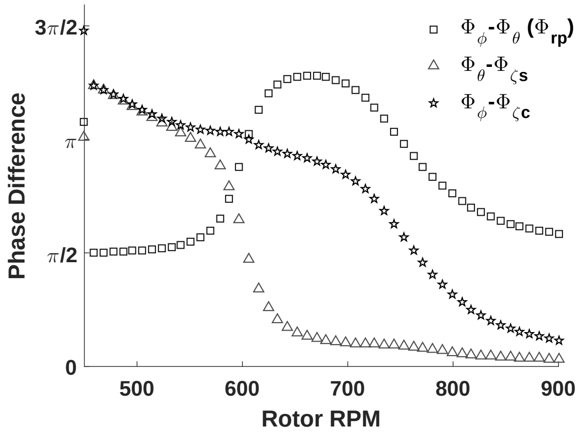

Figure 6: Second order system analogy - phase relations between response(fuselage motion) and force (lead-lag motion)

Table 1: Computation of phase difference between fuselage attitude rates using the second order system analogy

	$\Phi_x - \Phi_{fx}$	$\Phi_\phi - \Phi_{fx}$	$\Phi_\phi - \Phi_{\zeta c}$	$\Phi_y - \Phi_{fy}$	$\Phi_\theta - \Phi_{fy}$	$\Phi_\theta - \Phi_{\zeta s}$	$\Phi_\phi - \Phi_\theta$ (Φ_{rp})
Region 1 - Forcing frequency $<$ Pitch frequency $\Omega - \omega_\zeta < \omega_\theta$	0	0	π	0	π	π	$\pi/2$
Region 2 - Pitch Coalescence $\Omega - \omega_\zeta = \omega_\theta$	0	0	π	$-\pi/2$	$\pi/2$	$\pi/2$	π
Region 3 - Pitch frequency $<$ Forcing frequency $<$ Roll frequency $\omega_\theta < \Omega - \omega_\zeta < \omega_\phi$	0	0	π	$-\pi$	0	0	$3\pi/2$
Region 4 - Roll coalescence $\Omega - \omega_\zeta = \omega_\phi$	$-\pi/2$	$-\pi/2$	$\pi/2$	$-\pi$	0	0	π
Region 5 - Forcing frequency $>$ Roll frequency $\Omega - \omega_\zeta > \omega_\phi$	$-\pi$	$-\pi$	0	$-\pi$	0	0	$\pi/2$

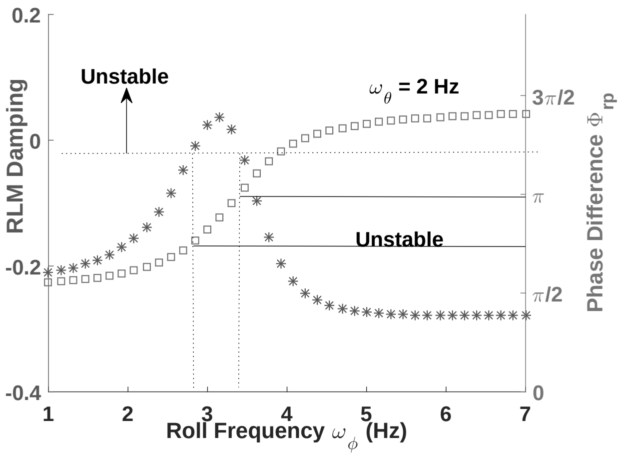

Figure 7: Effect of variation of ω_ϕ on RLM damping and Φ_{rp} for rpm $= 700$

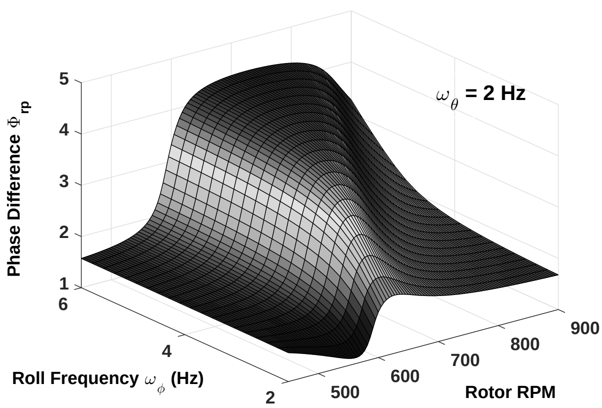

Figure 8: Effect of variation of ω_ϕ on RLM damping for a range of rpm

We will now show how the observed trend in Φ_{rp} (Fig. 1) can be explained using the above simplified analogy. The reference coordinate system (Fig. 5) used in the analysis is based on Ref. [3] with only a minor modification. From the figure, it can be observed that a positive displacement in y-direction corresponds to negative pitch motion and a positive displacement in x-direction corresponds to positive roll motion.

In the derivation, Φ represents the phase of the frequency response function, Ω the rotor rpm and ω the natural frequency of the degrees of freedom. The least damped mode that is the regressive lag mode is actually a low frequency progressive lag mode, implying $\Phi_{\zeta c} - \Phi_{\zeta s} = \pi/2$. Also $\Phi_{\zeta s} = \Phi_{fy}$ and $\Phi_{\zeta c} = \Phi_{fx} \pm \pi$, since blade-lead motion is considered positive (Fig. 5). Here, Φ_{fx} and Φ_{fy} represent the phase of the forces in the lateral and longitudinal directions. As mentioned before the lead-lag motion is considered to be the forcing and fuselage displacement (ϕ, θ) the response of the system. For the model considered, fuselage pitch natural frequency is less than the roll natural frequency ($\omega_\theta < \omega_\phi$). The analysis is done by dividing the rotor frequency sweep considered into five regions, the regions before and after the coalescence points. The forcing frequency is the RLM frequency $\Omega - \omega_\zeta$ and the response frequencies are ω_ϕ and ω_θ .

In Table 1, the derivation for the phase difference is done for all the five regions. A sample derivation is done here for region 1. Here $\Omega - \omega_\zeta < \omega_\theta$. As per the second order system resonance model (Fig. 4 - Region A), here the phase difference between the response and forcing is zero, implying $\Phi_x - \Phi_{fx} = 0$ and $\Phi_y - \Phi_{fy} = 0$. As already mentioned, positive x-displacement correspond to positive roll motion and positive y-displacement correspond to negative pitch motion. This implies, $\Phi_\phi = \Phi_x$ and $\Phi_\theta = \Phi_y + \pi$, hence, $\Phi_\phi - \Phi_{fx} = 0$ and $\Phi_\theta - \Phi_{fy} = \pi$. Again using the relations connecting the phase of the forces in the lateral and longitudinal directions and the lead-lag motion, $\Phi_\phi - \Phi_{\zeta c} = \pi$ and $\Phi_\theta - \Phi_{\zeta s} = \pi$. $\Phi_{\zeta c} - \Phi_{\zeta s} = \pi/2$, implying $\Phi_\phi - \Phi_\theta = \pi/2$ for region 1.

Table 1 thus clearly explains the trend observed in Fig. 1. The phase variation between the lead lag displacement (ζ_c, ζ_s) and the fuselage motions (ϕ, θ) is also shown in Fig. 6, which correspond to columns 4 and 7 in Table 1. For the model considered $\omega_\theta < \omega_\phi$, hence the Φ_{rp} deviates from $\pi/2$ to $3\pi/2$ at the first coalescence and back to $\pi/2$ at the second coalescence. For a reversed configuration, $\omega_\phi < \omega_\theta$, Φ_{rp} will shift from $3\pi/2$ to $\pi/2$ at the first coalescence and back to $3\pi/2$ at the second coalescence.

Due to non-zero damping present in the system, the transitions between the regions indicated in Table 1, will not be sharp but gradual. This is the reason for the smooth curve obtained in the phase difference plot (Fig. 1). When the roll and pitch frequencies are close, the system dynamics become more coupled. Hence in such cases, the second order system analogy cannot be applied and the above derivations (Table 1) will not hold.

4.2. Parameter Variations

Ground resonance occurs when the helicopter is in contact with the ground. Improper maintenance of landing gear components like struts, dampers or tyres, or based on the nature of the ground the helicopter is landing on, slopes, uneven rough ground, the landing gear stiffness may vary from the actual expected values. This may result in frequency coalescence occurring in the operating rpm range and if sufficient damping is not present can lead to ground resonance. To study the effect of landing gear parameter variation on stability and phase as the helicopter lands, the landing gear roll stiffness (frequency ω_ϕ) is varied across a range for a fixed rpm $\Omega = 700$. The results are as shown in Fig. 7. It is observed that the stable regions (modal damping < 0) are characterized by a phase difference (Φ_{rp}) of about $\pi/2$ or $3\pi/2$.

For further clarification, the parametric study of landing gear stiffness is performed in conjunction with rotor rpm variation. Figure 8 shows the change in phase with roll frequency variation for a range of rpm. The roll frequency is varied from the isotropic case where $\omega_\theta = \omega_\phi$ to a range of increasing values of ω_ϕ with $\omega_\phi > \omega_\theta$. Figure 1 is a baseline case for this analysis, for a specific value of ω_ϕ. Similar to the baseline case, there will be two coalescence (roll and pitch mode with RLM) in all these cases and with increasing ω_ϕ, the coalescence point between lag and roll mode shifts towards higher rpm. In addition, similar to Fig. 1, here we can see that there is an upward and downward shifts in the phase (Φ_{rp}). It is observed that as the difference between the two fuselage frequencies increase (increasing ω_ϕ), the downward shift of the phase difference plot also shifts towards the right while the upward shift occurs around the same point (rpm Ω) in all cases. This further emphasizes the fact that the upward shift (from $\pi/2$) occurs around the coalescence between the lag and pitch modes and the downward shift (from $3\pi/2$) occurs around the coalescence between the lag and roll modes.

4.2.1. Control

From the above discussion, we can conclude that the stable regions are characterized by a phase difference value of $\pi/2$ or $3\pi/2$. A deviation from this value indicates frequency coalescence and can be used to trigger the stability augmentation system and in choosing the required feedback gain. The disadvantage in using increasing amplitude as the trigger for control system is that the sensor, measurement and component noise effects amplitude much more than phase. Hence, phase difference estimation can be used in conjunction with amplitude to trigger the control system.

The system can be stabilized using pole placement or LQR control strategy. The conventional method requires analysis of the system matrix and determination of feedback gain based on this. To estimate the control gain in a real scenario, without going through the entire system analysis, we can create a reference chart for gain to be selected based on phase difference. The difficulty in this regard is that the phase characteristic is dependent only on modal frequency coalescence, implying it does not provide any information with regard to the intensity of the instability. The feedback gain required is however directly related to the intensity of the instability.

To overcome this disadvantage, an analysis to determine the RLM damping that correspond to each Φ_{rp} for a range of roll and pitch frequencies is determined assuming the mechanical damping (roll, pitch and lag damping) present in the system is very low. The roll and pitch frequency ranges are determined such that the equivalent roll and pitch stiffness varies from very low values (around $\frac{1}{10}^{th}$ the actual values) to around 1.5 times the actual values. For the model considered, the actual fuselage roll frequency is $4Hz$ and fuselage pitch frequency is $2Hz$. Figure 9 shows the variation of phase difference with roll and pitch frequencies and Fig. 10 shows the corresponding variation in RLM damping. For each Φ_{rp}, we can determine the minimum damping (worst case) and the corresponding gain required for stabilization.

Keeping the worst case data (Φ_{rp}, minimum damping and gain) as reference, we can determine the gain required in any case from the phase difference and damping values. The worst case damping corresponding to each Φ_{rp} is plotted in Fig. 11. The plot also enables us to determine the unstable region in terms of Φ_{rp}. The region around $\pi/2$ and $3\pi/2$ as seen from the figure are stable regions. The control algorithm we have employed computes the phase difference Φ_{rp}. If the value is different from $\pi/2$ or $3\pi/2$, the control system is triggered. If the damping of the least damped mode can be determined the feedback gain is chosen taking this into consideration, otherwise the worst case feedback gain is selected. Full state feedback was assumed and simple Linear Quadratic Regulator with cyclic pitch as control input is applied to stabilize the system. The fuselage pitch response for the system without and with feedback control is shown in Fig. 12.

The system is able to stabilize under most of the possible variations in the configuration in terms of landing gear stiffness and mechanical damping present in the system. As mentioned before, if the two fuselage frequencies are close, the phase (Φ_{rp}) derivations are not valid and the method will need some modifications. There are certain other drawbacks, one of them being the assumption of full-state feedback, which is not practically possible. The feasibility of the control strategy and the control input magnitudes should also be considered. In addition a better approach towards selection of feedback gain should be developed. These drawbacks will be considered in future work and is not discussed in this paper.

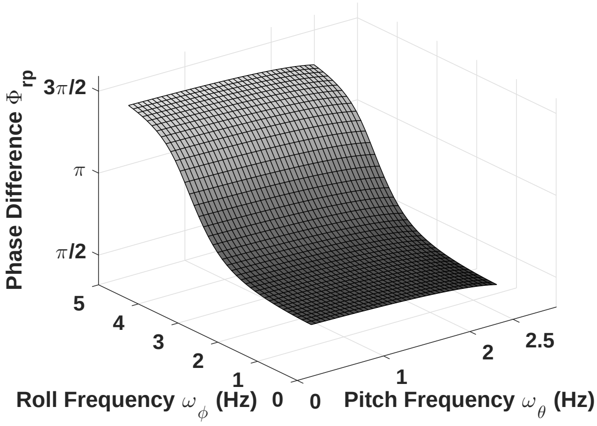

Figure 9: Variation of phase difference between fuselage attitude rates with $\omega_\theta, \omega_\phi$

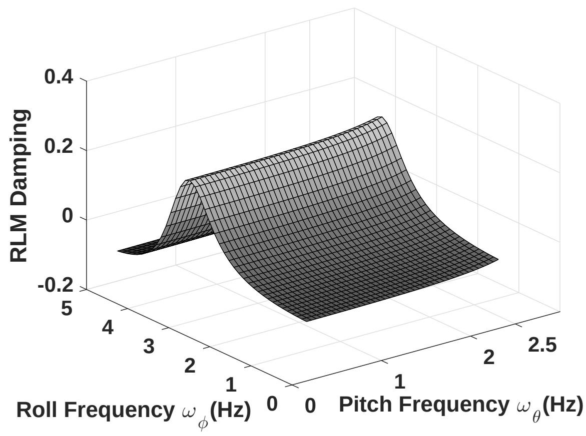

Figure 10: Variation of RLM damping with $\omega_\theta, \omega_\phi$

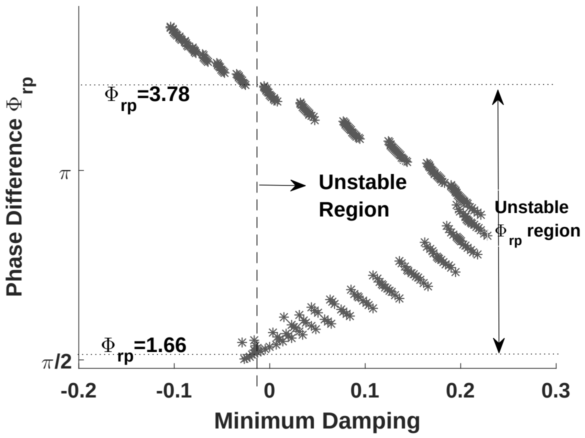

Figure 11: Plot showing the worst case damping for different values of Φ_{rp} used for preparation of the chart for gain selection

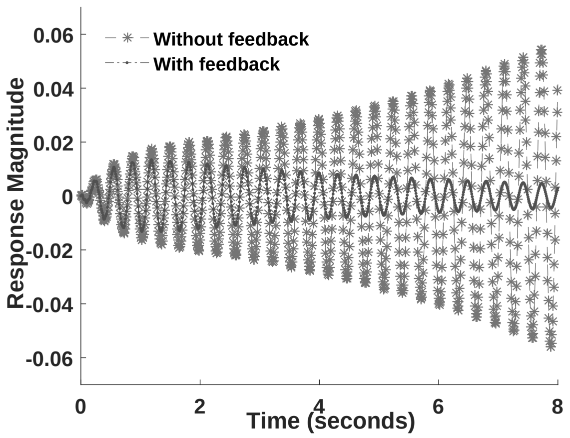

Figure 12: Time response (fuselage pitch) of the system without and with feedback control

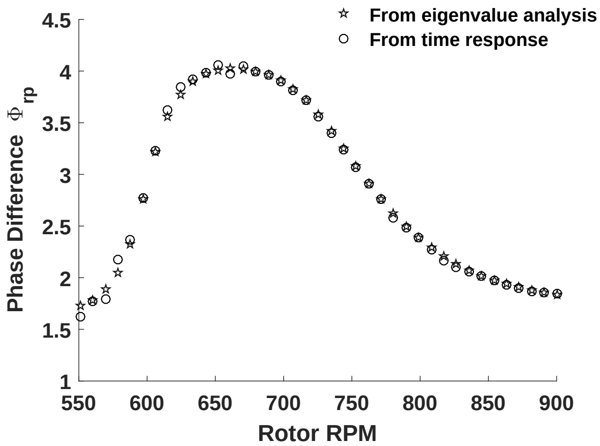

Figure 13: Phase difference computed using FFT from time response compared with the eigenvalue analysis results

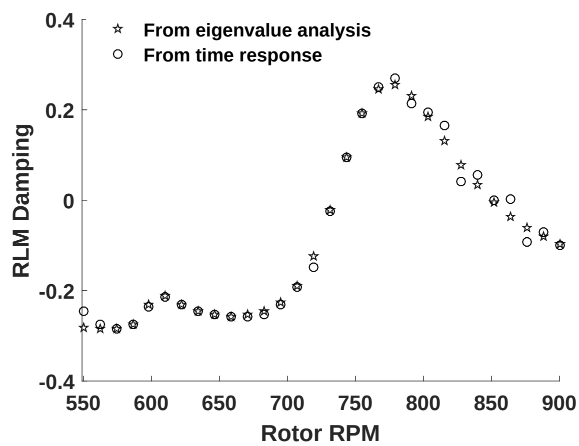

Figure 14: Modal damping of the least damped mode computed using FFT from time response compared with eigenvalue analysis results

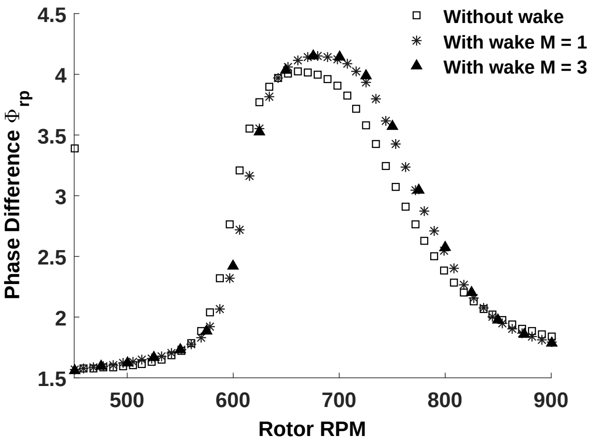

Figure 15: Phase difference variation with rpm, with and without wake effects

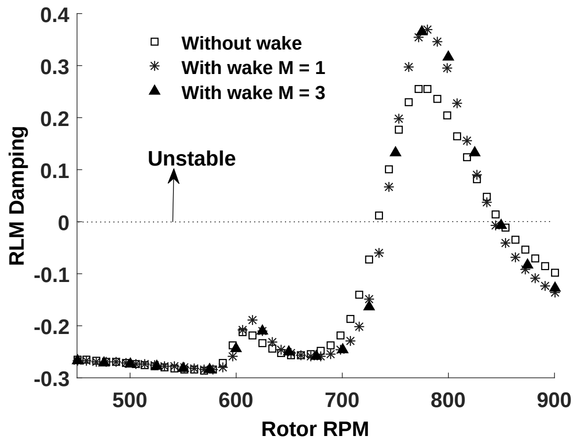

Figure 16: RLM damping variation with rpm, with and without wake effects

4.3. Using time response to compute phase difference

For real time analysis, the phase difference need to be computed from the system response, in this case the fuselage attitude or attitude rate measurements. This is comparable to the response of the nonlinear system obtained using simulation. The least damped mode or the RLM should be obtained from the signals using appropriate modal analysis technique and the phase difference should be evaluated. This can be done using different methods, Fourier analysis, Prony's method, Hilbert transform.

Here, the time response of the non-linear system was computed and Fast Fourier Transform was used to evaluate the phase difference between the fuselage attitude rates. Reasonably good correlation is observed between the results (damping, frequency and Φ_{rp}) obtained using the eigenvectors of the linearized model and the non-linear model time response. The results are shown in Figs. 13 and 14.

4.4. Effect of wake

In the above computations, the inflow through the rotor was assumed to be uniform. To analyze the effect of wake, Peters-He dynamic inflow model[13] is applied to the system dynamics. For $M = 1$, the system is constant coefficient and the analysis is done using constant coefficient eigenvalue analysis. For $M = 3$, the analysis is done using Floquet method and periodic eigenvectors are computed using Eqn. 3. The phase difference and modal damping results obtained with the inclusion of wake are plotted in Figs. 15 and 16 respectively. The results obtained without the inclusion of wake are also shown for comparison. The results show that the trend observed in phase

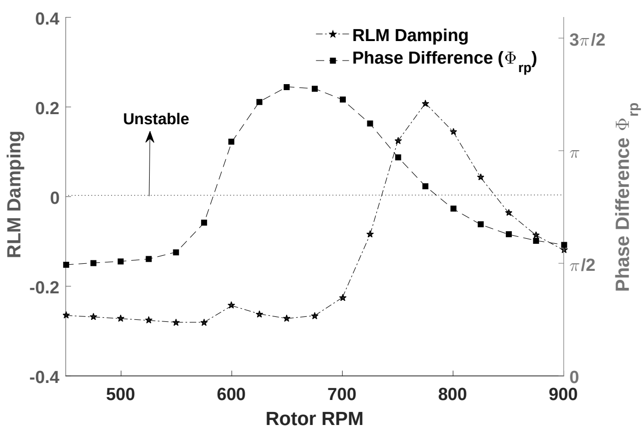

Figure 17: Variation of phase difference and RLM damping with rotor rpm for anisotropic rotor configuration

Φ_{rp} is preserved with the inclusion of wake.

4.5. Anisotropic rotor analysis

The individual rotor blade properties will differ due to manufacturing errors or wear and tear. Here, anisotropy is introduced into the model[2] by varying the blade stiffness in a range of $2-5\%$ from each other. Since the anisotropy introduced and expected in real scenario is less, a constant coefficient analysis will give approximately accurate results. However, for a detailed analysis, Floquet method is used here. The periodic eigenvectors are computed using Eqn. 3 and the phase difference is computed for a range of rpm. Figure 17 shows the results obtained and it can be observed that the phase relations between fuselage attitude rates hold for this configuration.

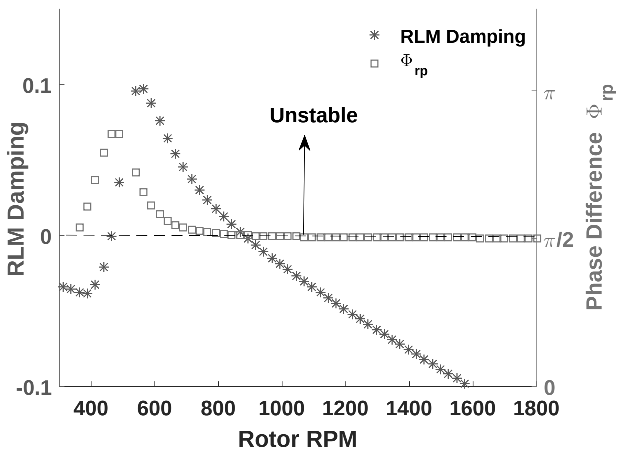

Figure 18: Variation of phase difference and modal damping (least damped mode) with rotor rpm for air resonance model

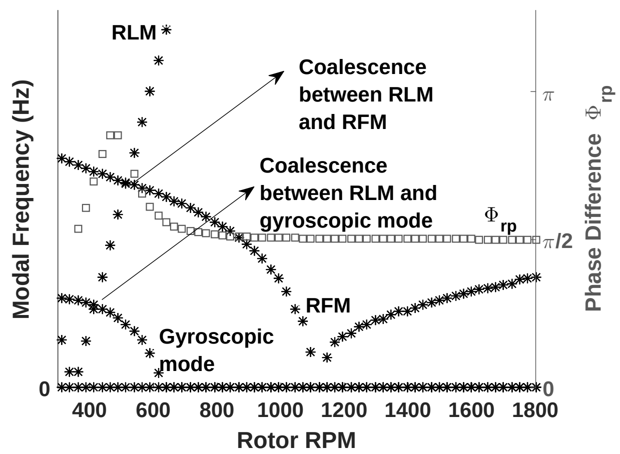

Figure 19: Variation of phase difference and modal frequency with rotor rpm for air resonance model

4.6. Air Resonance

Air resonance occurs while the helicopter is in flight and is highly influenced by aerodynamics. It can occur due to coalescence between the regressive lag mode and the gyroscopic mode frequencies, or due to coalescence between the regressive lag mode and the regressive flap mode(RFM) frequencies[3,4]. However an eigenvector analysis of both the gyroscopic mode as well as the regressive flap mode show significant participation from the flap states as well as the fuselage states, thereby relating the phenomena to ground resonance. It can be observed that with increasing aerodynamic forces, the air resonance instability is reduced significantly. This has been demonstrated through the effect of increasing lock number (γ) on stability[3]. The instability due to coalescence between RLM frequency and gyroscopic mode frequency is seen to occur only if the aerodynamic conditions resemble near vacuum ($\gamma = 0$)[3].

Here we have considered the model from Ref. [3] with $\gamma = 5$ and the results are as shown in Figs. 18 and 19. The frequency plot shows coalescence between gyroscopic mode and RLM and between RFM and RLM. The phase deviation from $\pi/2$ occurs corresponding to the coalescence between gyroscopic mode and RLM. However since the coalescence between the RLM and RFM occurs at a nearby rotor rpm, the phase difference shifts back to $\pi/2$ before reaching $3\pi/2$. However, similar to ground resonance case, we can see two shifts corresponding to the two coalescence. As mentioned before since lock number (or equivalently aerodynamic damping) is high, here instability occurs corresponding to the coalescence between RFM and RLM alone as seen from the modal damping plot. In spite of this, we can conclude that the stable regions are characterized by a phase difference of $\pi/2$.

5. CONCLUSION

- Ground resonance instability occurs due to frequency coalescence between the lead-lag motion of the rotor blade and the fuselage motion on its landing gear. A second order system analogy considering the lead-lag motion as the excitation force and the fuselage motion as the response is used to explain the phase difference between fuselage attitudes (or attitude rates) in region of frequency coalescence.
- It is observed that the stable regions correspond to a phase difference value of $\pi/2$ or $3\pi/2$ and the unstable (frequency coalescence) regions are characterized by a deviation from these values.
- A parametric study (landing gear stiffness variation) on the phase difference variation is also performed and this information can be useful for stability augmentation.
- To demonstrate practical implementation, the phase difference is computed from time response and compared with eigenvector analysis results.
- The effect of wake and rotor anisotropy is investigated. Floquet method is used if system is periodic coefficient and similar phase difference characteristics as with constant coefficient analysis is observed.

- The analysis is extended to air resonance cases, where flap motion and aerodynamics play a prominent role. Similar features as in ground resonance is observed in this case also which shows the feasibility of the method for air resonance detection.

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