SIXTH EUROPEAN ROTORCRAFT AND POWERED LIFT AIRCRAFT FORUM

Paper No. 57

ON THE USE OF APPROXIMATE MODELS IN HELICOPTER FLIGHT MECHANICS

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September 16-19, 1980
Bristol, England

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SUMMARY

This paper addresses several aspects of the prediction of helicopter flight behaviour and emphasises the need for low order approximations to aid physical interpretation of important flying qualities. The centre spring, rigid blade rotor model is used for predicting the integrated loads from hingeless and articulated rotors. Stability derivatives, derived with this model, are then used in the search for simplified approximations to the short term pitch attitude response to cyclic pitch control, throughout the speed range. The method of weakly coupled systems provides a mathematical framework for the analysis which is applied to the prediction of flight path trajectories during transient manoeuvres. The use of truncated dynamic models for combined pitch and roll manoeuvres is also discussed.

1 GENERAL INTRODUCTION

As computational methods and hardware develop so also does the attraction of including all possible effects when creating a theoretical model for the solution of a complicated technological problem. In this approach, the danger of masking the primary cause of some resulting phenomenon should be apparent. This is not to say that experiments with large scale theoretical models will not be valuable during a design process, far from it, but where possible, truncated and simplified elements are often invaluable in understanding underlying physical mechanisms. This argument is particularly germane to the prediction of helicopter flight behaviour where an adequate simplified model is not altogether obvious. The many facets of helicopter modelling are extensively discussed in the literature, particularly those concerned with rotor dynamics and aerodynamics, to the extent that it is the very selection and adequacy of the degree of approximation, when considering the requirements of the task under investigation, that needs careful consideration. In such cases the most simple adequate model is the most effective. These and related topics are reviewed in Reference 1 where, in particular, evidence is presented for the validity of the centre spring rotor model for simulating the behaviour of both articulated and hingeless rotors. Along with the additional assumption of quasi-steady blade dynamics, this type of modelling forms the basis on which the linearised derivatives used in this paper were calculated and as such is most suitable for parametric studies, where trends, rather than absolute accuracy, are required. This paper presents such a study and the telescope of approximation is used to bring into focus the physical mechanisms characterising helicopter short term pitching motions.

Although it is recognised that many current helicopters need, and future ones may continue to need, some form of stability and control augmentation, the present study concentrates mainly on the natural aircraft behaviour, for three reasons:

(a) a knowledge of the inherent flying deficiencies of the helicopter forms a valuable basis for the design of an augmentation system;

(b) the more satisfactory the natural characteristics, the lower the gain and authority required of the augmentation system and the easier it is to endow a 'fly home' capability after a failure of the augmentation;

(c) for a helicopter with a limited authority augmentation system, the handling characteristics during rapid manoeuvre are more likely to be akin to the natural characteristics if and when saturation of the augmentation occurs.

The first two reasons are considered self-evident but the third is not so obvious and may not be widely appreciated.

The rather elementary study of helicopter short term pitching characteristics presented in this paper was stimulated by the comparative dearth of rational approximations that highlight important handling parameters and also the disturbingly frequent occurrence of misleading remarks on this topic in the literature.

2 THE PREDICTION OF LONGITUDINAL STABILITY AND CONTROL CHARACTERISTICS

2.1 Introduction

Considering the symmetric motions of a helicopter in isolation for the moment, the linearised form of the equations of fuselage motion can be written in the matrix differential form:

$$\frac{d}{dt}\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} - \begin{bmatrix} X_u & X_w & X_q - W_e & -g\cos\theta_e \\ Z_u & Z_w & Z_q + U_e & -g\sin\theta_e \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} = \begin{bmatrix} X_{\theta_{1s}} & X_{\theta_0} \\ Z_{\theta_{1s}} & Z_{\theta_0} \\ M_{\theta_{1s}} & M_{\theta_0} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \theta_{1s} \\ \theta_0 \end{bmatrix} \tag{1}$$

where u, w, q and θ are deviations in forward and normal speeds, pitch rate and attitude respectively; X_u, Z_w etc, are semi-normalised stability derivatives and θ_{1s}, θ_0 the main rotor longitudinal cyclic and collective pitch respectively. Drawing from the results in the Appendix we shall be concerned with the validity of approximations to the short term response to cyclic pitch: in particular the following two questions will be addressed:

(1) what are the important characteristics of the longitudinal response that can affect handling?

(2) can these characteristics be predicted with adequate accuracy by simple models, perhaps leading to a better understanding of the effect of design parameters on handling qualities?

The main concern here will be the range of application of the usual approximation for the conventional aeroplane short period mode, where forward speed changes are neglected (*cf* equation (A-11)),

$$\frac{d}{dt}\begin{bmatrix} w \\ q \end{bmatrix} - \begin{bmatrix} Z_w & Z_q + U_e \\ M_w & M_q \end{bmatrix}\begin{bmatrix} w \\ q \end{bmatrix} = \begin{bmatrix} Z_{\theta_{1s}} \\ M_{\theta_{1s}} \end{bmatrix}\theta_{1s} \quad . \tag{2}$$

The system (2) has the characteristic equation,

$$\lambda^2 - (Z_w + M_q)\lambda + Z_w M_q - M_w(Z_q + U_e) = 0 \quad . \tag{3}$$

The use of this type of approximation for helicopter short term pitching motion has been discussed on several occasions in the literature. In Reference 2, Bramwell shows how, as in fixed wing aircraft, the constant term in (3) can be related to the manoeuvre margin and the attendant handling qualities. Also, in Reference 3, the authors apply the Bairstow factorisation to the stability quartic and assess the validity thereof for VTOL aircraft. The coverage in this second reference is comprehensive and very useful but the analysis is mainly carried out in the frequency domain. The status of (2) as a control response approximation needs further consideration and it is hoped that this paper will partly meet this need and hence aid applications with the reduced model.

The approximate method for weakly coupled systems[4] outlined in the Appendix provides the basis for the use of (2) and can be referred to, in an application, for the conditions of validity. These conditions are similar to, but more precise than those in Reference 3 and include as the main condition the separation of the system eigenvalues into two or more widely separated sets. When (3) serves as a good approximation to the high modulus eigenvalues of the system matrix in (1), then the low modulus eigenvalues can often be approximated by a second order system in the dependent variable u and w_0 (where $w_0 = w - U_e\theta$, the vertical velocity).

The details are given in Reference 4 and the approximation can be written as the quadratic,

$$\lambda^2 + \left\{ - X_u + \frac{\left(X_w - \dfrac{g\cos\theta_e}{U_e}\right)(Z_u M_q - M_u(Z_q + U_e)) + (X_q - W_e)(Z_w M_u - M_w Z_u)}{M_q Z_w - M_w(Z_q + U_e)} \right\} \lambda$$

$$- \frac{g\cos\theta_e}{U_e}\left(Z_u - \frac{Z_w(Z_u M_q - M_u(Z_q + U_e))}{M_q Z_w - M_w(Z_q + U_e)}\right) = 0 \quad . \tag{4}$$

In the following sections the foregoing results are applied to the prediction of short term cyclic response of an articulated and hingeless rotor helicopter: the various shortcomings are reviewed and the use of the approximation for continuous manoeuvres is assessed.

2.2 Natural configuration characteristics

The two configurations chosen for the present study differ only in the magnitude of main rotor blade flapping frequency ratio λ_β , and hence rotor flapping stiffness. Configuration A (with $\lambda_\beta^2 = 1.05$), represents a small offset articulated rotor and configuration H (with $\lambda_\beta^2 = 1.225$), a typical hingeless rotor. All other parameter values defining the configurations are common to both and are given in Table 1. Stability and control derivatives, calculated by a familiar, though somewhat novel technique[5], are illustrated in Figures 1

Table 1

Basic helicopter data

MAIN ROTOR		
Blade lift curve slope	(per rad)	5.8
Rotor hub height above CG	(ft, m)	4.77, 1.45
Blade flap moment of inertia	(slug ft^2, kg m^2)	500, 679
Blade radius	(ft, m)	21, 6.4
Rotor solidity	-	0.078
Rotor shaft tilt forward	(deg)	4.0
Flap frequency ratio^2 - λ_β^2		
Configuration A	-	1.05
Configuration H	-	1.225
Rotor speed	(rad/s)	33.0
TAILPLANE		
Lift curve slope	(per rad)	3.5
Distance aft of shaft	(ft, m)	25.2, 7.68
Plan area	(ft^2, m^2)	11, 1.02
Incidence setting (positive up)	(deg)	-2.0
GENERAL		
Aircraft mass	(lb, kg)	9500, 4308
Pitch moment of inertia	(slug ft^2, kg m^2)	10250, 13903
CG location forward of shaft base	(ft, m)	0.33, 0.1

and 2, as a function of forward speed, for the two configurations in straight and level flight. It can be seen that the force derivatives remain virtually unchanged for the two configurations, as expected, and that the moment derivatives are significantly increased for configuration H. In particular, the angle of attack derivative has changed sign, becoming destabilising, for configuration H. The fuselage trim attitudes are shown in Figure 3 to complete the data.

The eigenvalues for the two configurations are displayed in Figure 4, illustrating how the damping and frequency of the natural modes vary with flight speed. The slow oscillatory mode, sometimes referred to as the pendulum mode in the hover, persists but changes its character with flight speed in a different manner for the two configurations. For the articulated rotor a second oscillation forms with increasing speed so that at high forward speed a situation develops that is reminiscent of conventional aeroplane phugoid and short period characteristics. For the hingeless rotor the second pair of roots remain real.

A perusal of Figure 4 suggests that the only obvious candidate that satisfies the primary condition for weak coupling (widely separated roots) is the articulated rotor at mid speed and above. This is indeed the case but before examining these aspects further it is worth reviewing established hover results. It is well known that heave motions can be treated independently, and without recourse to variable transformations, a satisfactory approximation to the other modes requires all three degrees of freedom and is given by the two equations,

$$\dot{U} + g \cos \theta_e \theta = 0 \qquad \dot{q} - M_u u - M_q q = M_{\theta_{1s}} \theta_{1s} \quad . \tag{5}$$

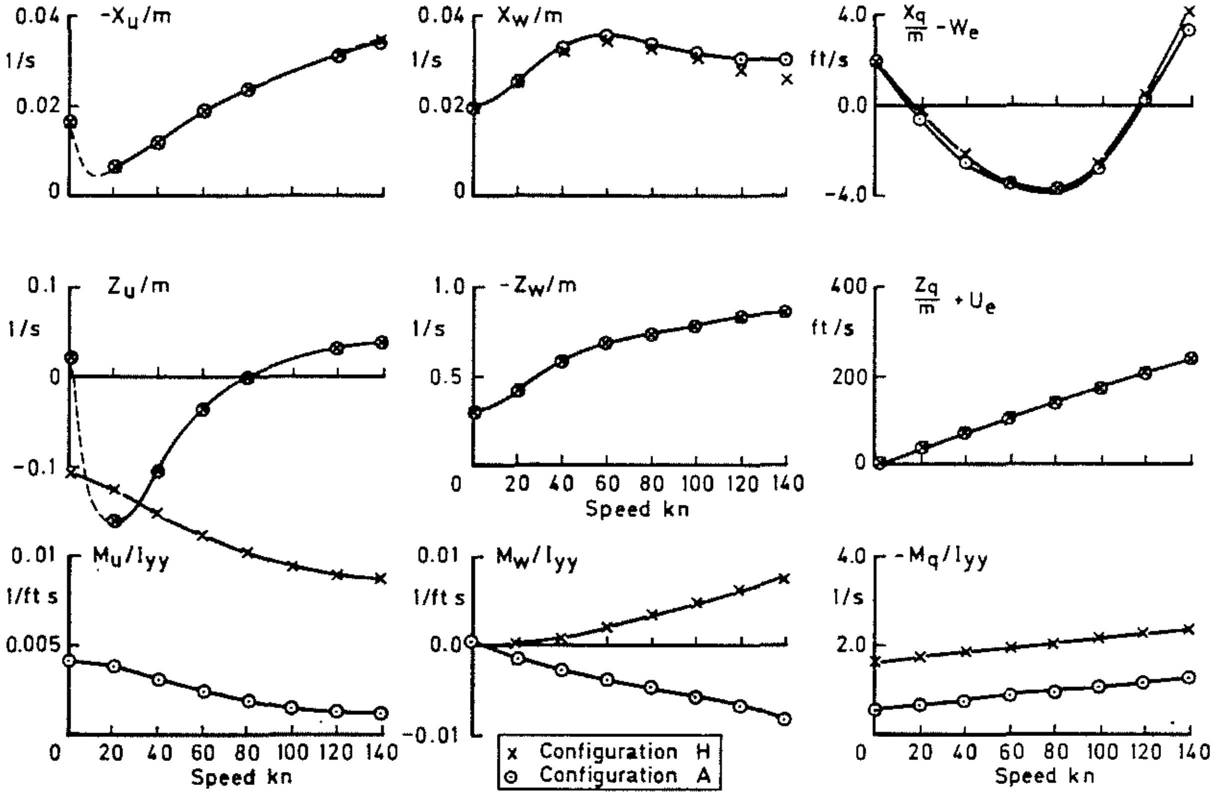

Figure 1 Variation of longitudinal stability derivatives with speed for articulated (A) and hingeless (H) rotor helicopters

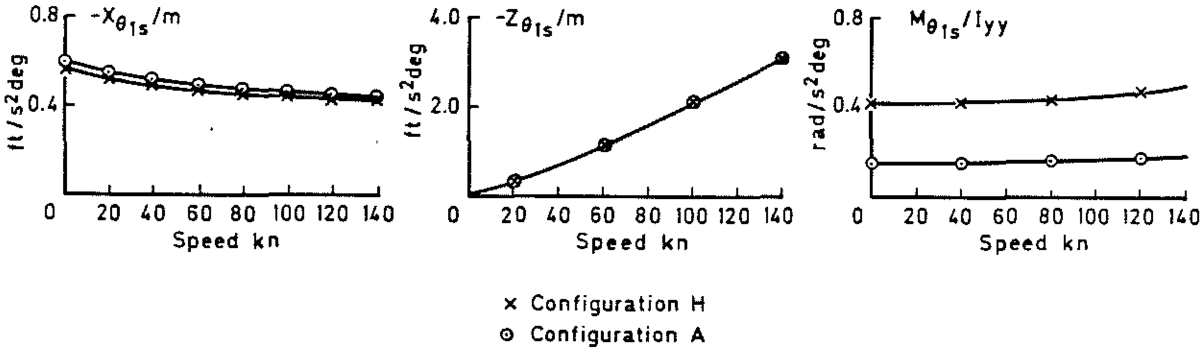

Figure 2 Variation of longitudinal control derivatives with speed

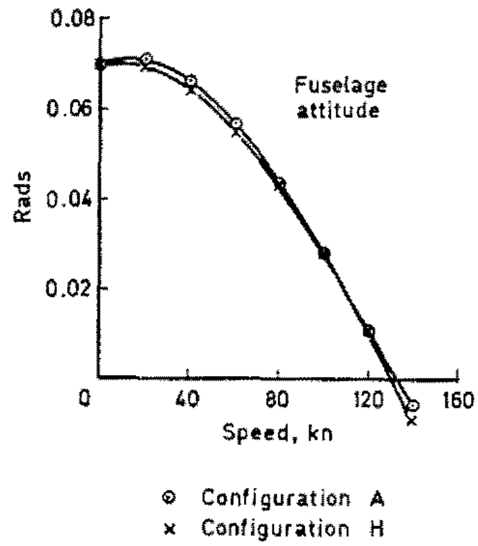

Figure 3 Variation of fuselage attitude with speed

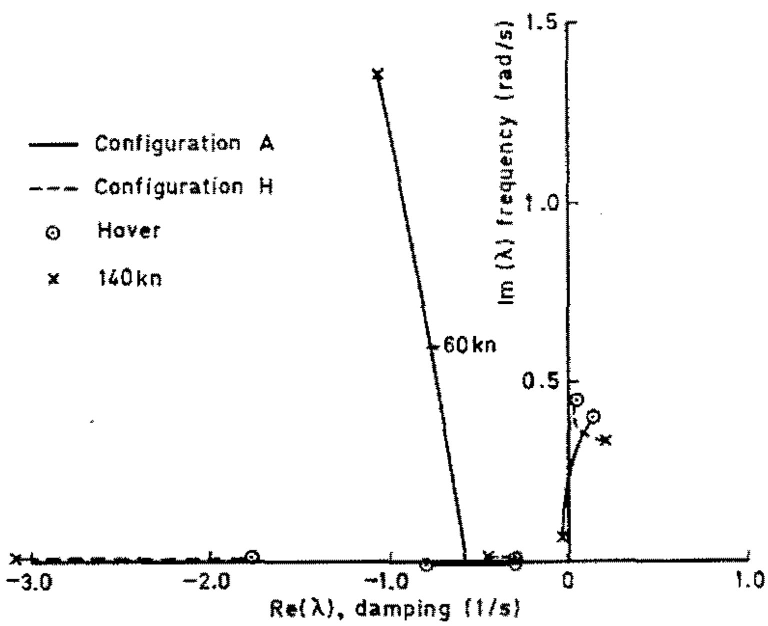

Figure 4 Root loci with increasing flight speed

Speed variations are thus intimately linked with pitching motion in the hover, an effect that, as will be shown, reduces in magnitude with small-offset articulated rotors but increases with hingeless rotors, as forward speed is increased.

A comparison of exact and approximate values (equation (3)) of the higher modulus roots (λ_3, λ_4), as a function of speed, is given in Figures 5 and 6.

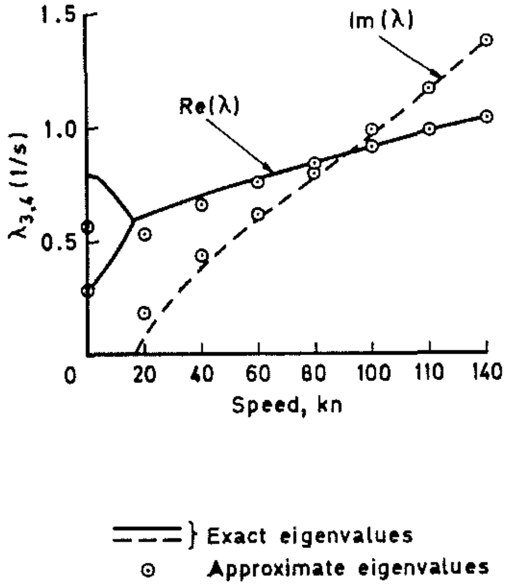

Figure 5 Comparison of exact and approximate high modulus eigenvalues with speed - configuration A

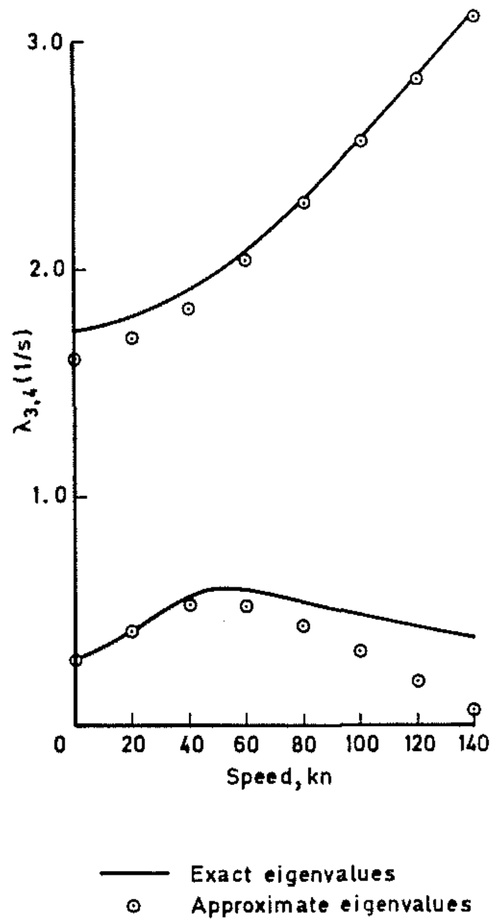

Figure 6 Comparison of exact and approximate high modulus eigenvalues with speed - configuration H

There is excellent agreement for the articulated rotor above about 60 kn and surprisingly, the agreement for the hingeless rotor breaks down only at the higher speeds. On Figure 7 the root corresponding to the slower oscillatory mode (λ_1, λ_2) is plotted at an increased scale. The approximation according to (4), shown for the articulated rotor, is seen to be converging on the exact locus at high speed. The corresponding approximation for the hingeless rotor, however, has not improved with speed and is too far to the left to be plotted on Figure 7.

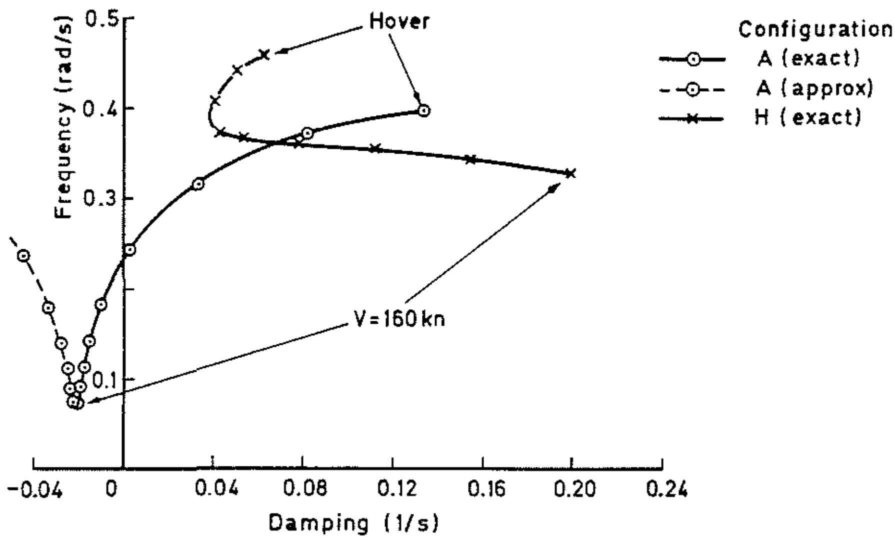

Figure 7 Root loci for slow longitudinal oscillatory mode with forward speed (0-160 kn (20))

The failure of this 'phugoid' type of approximation for the long period mode clearly precludes its use in establishing whether or not a configuration satisfies specified criteria for this mode. The principal reason for the breakdown in the high modulus approximation for the hingeless rotor is the contribution of the forward speed perturbation u to the make-up of this mode. This is illustrated in Figure 8 where the eigenvector ratio |u/w| is plotted as a function of speed for the high modulus complex eigenvalue of configuration A and the smaller real root of configuration H. Clearly, as speed increases so the mode changes character and neglect of forward speed variations for configuration H

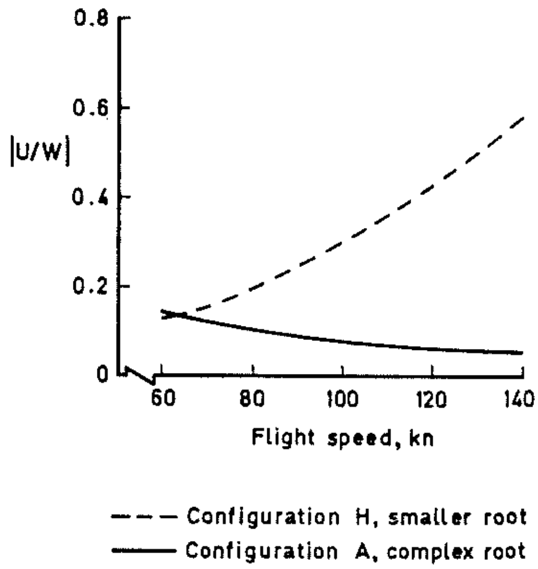

Figure 8 Variation of eigenvector ratio |u/w| with speed for high modulus roots of configurations A and H

are invalid. It appears that for hingeless rotors of the type discussed here the approximation to short term cyclic pitch response given by (2) will have little utility. However, it can be shown that at high speed the approximation improves as the aircraft centre of gravity is moved aft and the unstable oscillation degenerates into two pure divergences. The eigenvalues approximated by the high modulus system are then the largest positive and largest negative ones.

2.3 Short term cyclic control response

The pitch rate response, q , following a step input in longitudinal cyclic pitch, θ_{1s} , is illustrated in Figures 9 and 10, for configurations A and H

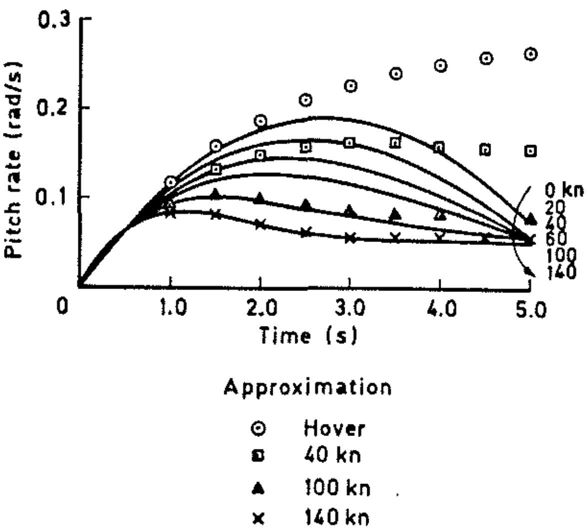

Figure 9 Pitch response to step in longitudinal cyclic pitch (1^o) - configuration A

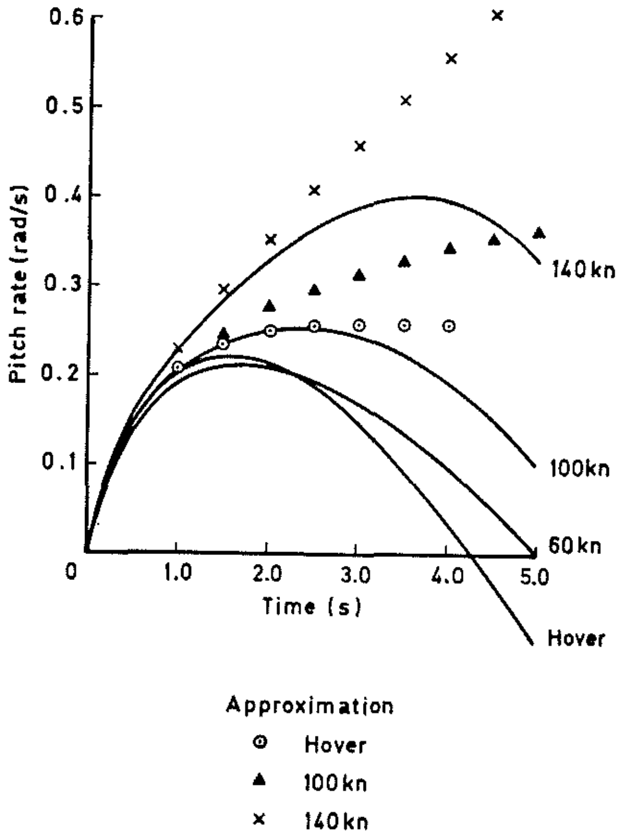

Figure 10 Pitch response to step in longitudinal cyclic pitch (1^o) - configuration H

respectively. Reponses at several speeds are compared with the approximation given by (2) which can be written in the alternative form,

$$\ddot{q} - (Z_w + M_q)\dot{q} + Z_w M_q - M_w(Z_q + U_e) = M_{\theta_{1s}}\dot{\theta}_{1s} - (Z_w M_{\theta_{1s}} - M_w Z_{\theta_{1s}})\theta_{1s} . \quad (6)$$

As forward speed increases the 'exact' linearised theory (bold lines) predicts a striking difference between the two configurations. For configuration A both the apparent time constant (time to 63% peak rate) and rate sensitivity (peak rate) decrease with increasing forward speed, whereas they increase for configuration H. Again, for configuration A, the approximate solutions follow this pattern though pointwise accuracy is not achieved beyond 2 seconds until flight speeds exceed 60 kn. With marked contrast the approximation for configuration H is of little value beyond 2 seconds at any speed. It follows from the above discussion that the often used handling qualities diagram where pitch damping is plotted against control sensitivity is therefore quite inappropriate for short term pitching motions of helicopters in forward flight. For roll motion the damping and control sensitivity often reflect the corresponding time constant and rate sensitivity in forward flight. For pitching motions this correspondence is certainly not one-to-one and Figure 11 illustrates this point. Here the

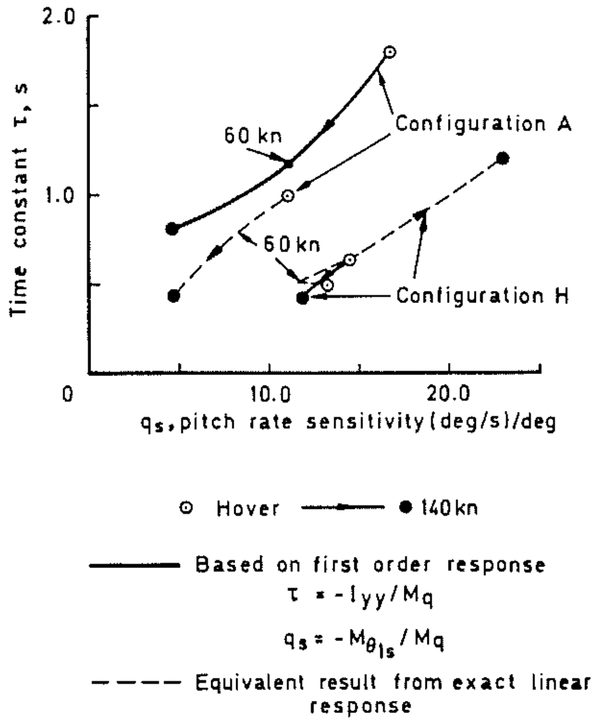

Figure 11 Time constant - rate sensitivity diagram

apparent time constant is plotted against the apparent rate sensitivity taken from the responses in Figures 9 and 10, compared with the result predicted by a first order response analysis. The latter can obviously be very misleading, particularly for hingeless rotors and the situation is further compounded for these rotors by nonlinear effects present during large pitching manoeuvres, the origin again being in the relatively large speed excursions[6].

For articulated rotors of similar type to configuration A the key parameters that reflect short term pitch handling qualities at mid to high speed are therefore embodied in (6). These are the frequency and damping of the oscillation together with the control sensitivity $M_{\theta_{1s}}$. It is usually valid to assume that

$|Z_w M_{\theta_{1s}}| \gg |M_w Z_{\theta_{1s}}|$. Flight measurements on a Gazelle helicopter of the response to a step in longitudinal cyclic stick, shown in Figure 12, qualitatively confirm the results predicted by the present method for configuration A. In particular,

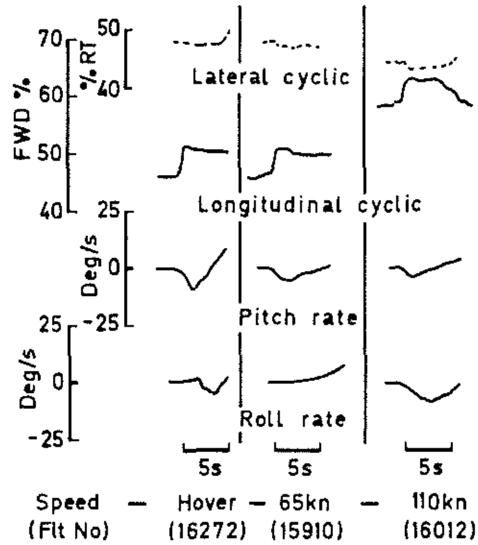

Figure 12 Gazelle XW846 - rate response to longitudinal cyclic step at three speeds (lateral control notionally constant)

for nominally the same longitudinal cyclic inputs, the peak pitch rate, and time to achieve this, both reduce with speed in a similar fashion to that predicted for configuration A.

The handling qualities parameters discussed above, so familiar in fixed wing stability and control, are clearly inadequate to portray the characteristics of unaugmented hingeless rotors typified by configuration H. As already discussed the effect of speed variation has to be incorporated before ground can be made.

Intuitively, the reason for the bending over of the pitch rate response after a few seconds in Figure 10 stems mainly from the contribution of the term $M_u u$ to the pitching moment equation (1), resulting in the apparent oscillatory character to the response. This effect has been discussed qualitatively in Reference 7. Retaining the speed terms leads to the equation,

$$\ddot{q} - (Z_w + M_q)\dot{q} + (M_q Z_w - M_w(Z_q + U_e))q = M_u \dot{u} - (Z_w M_u - M_w Z_u)u$$

$$+ M_{\theta_{1s}} \dot{\theta}_{1s} - (Z_w M_{\theta_{1s}} - M_w Z_{\theta_{1s}})\theta_{1s} \, . \quad (7)$$

For configuration H, $|Z_w M_u| \gg |M_w Z_u|$, so the only significant additional parameter defining the motion is the derivative M_u . The sensitivity of the short term behaviour to M_u can be examined by incorporating a feedback loop in (7) from speed variations to cyclic pitch θ_{1s} . Proportional control with a gain of -0.015 deg/ft s^{-1} serves to eliminate the M_u terms and the derivatives Z_u, X_u are only slightly affected. For the augmented configuration the cyclic response is shown in Figure 13 together with the unaugmented result for comparison. It can be seen that the short term approximation now gives an excellent fit. In fact the high modulus roots are little affected by the augmentation whereas the slower mode is stabilized.

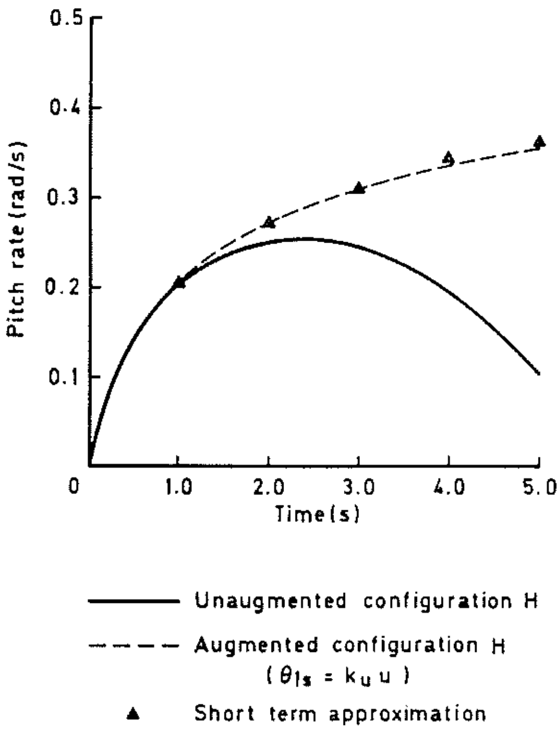

Figure 13 Comparison of pitch rate response with and without M_u - configuration H

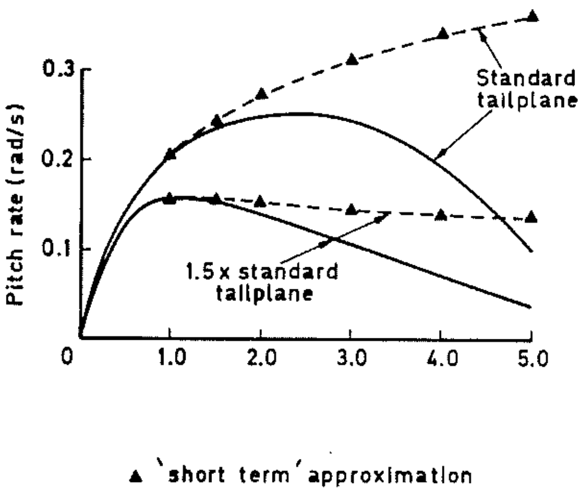

Figure 14 Effect of tailplane area increase on short term pitch response - configuration H

It is probably fair to reflect that neither response in Figure 13 would be suitable for tasks demanding crisp attitude or 'g' control. Although it is not the purpose of this paper to discuss handling qualities themselves, it is interesting to see how the approximate method predicts the changing character of the response with tailplane size, shown in Figure 14. With a 50% increase in tailplane size the crisper but reduced response, resulting from the increased damping (M_q) and static stability ($-M_w$), is broadly predicted by (6), though inferred speed effects are still apparent.

In the next section we shall apply the short term approximation to an applied flying task to discover how well flight path trajectories are predicted.

2.4 Hurdle hopping

In this section the value of the short term approximation for predicting flight path trajectories will be demonstrated. This value lies in the increased understanding of the effect of handling parameters on flight path control and also in the potential application to small scale simulations.

The manoeuvre considered involves clearing an obstacle and returning to the original height, described as hurdle hopping. This kind of task was used during a recent piloted simulation at Bedford[6,8] as a method of highlighting configurational effects on helicopter agility. Typical time histories of longitudinal variables for an articulated rotor (λ_β^2 = 1.05) are shown in Figure 15. From these records a model input was synthesised that characterises the main features of the pilot's input that would roughly reproduce the flight path trajectory. The general form of the input is shown in Figure 16, along with a typical simulation record. For the present

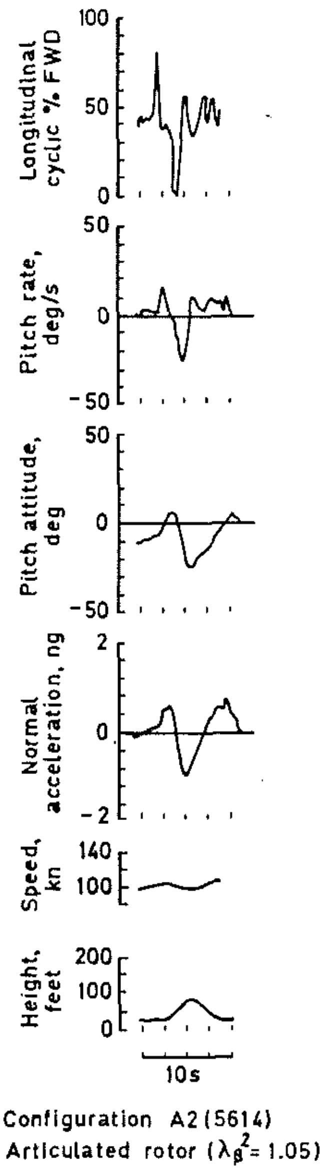

Figure 15 Piloted simulation results for a hurdle-hopping manoeuvre

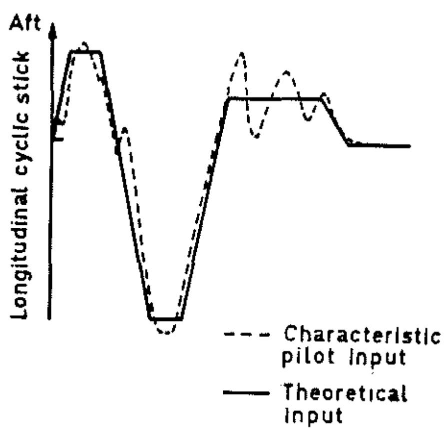

Figure 16 General form of longitudinal cyclic stick movement for hurdle-hopping manoeuvre

exercise the cyclic input was simplified further by assuming step rather than ramp growth. The input is made up of a +1 degree step for 1.5 seconds followed by a −2 degree step for 1.5 seconds and followed finally by a +1 degree step for a further 2.5 seconds. A comparison between the exact linear solution to (1) and the approximate result given by the solution of (2) is shown in Figure 17 for a 100kn entry speed. The primary response variables pitch rate, attitude and normal acceleration are virtually indistinguishable over the 6 second period whereas the height approximation shows a slight departure from the exact solution for the last second or so. In this type of manoeuvre and others where speed excursions are relatively small, the approximation is clearly satisfactory.

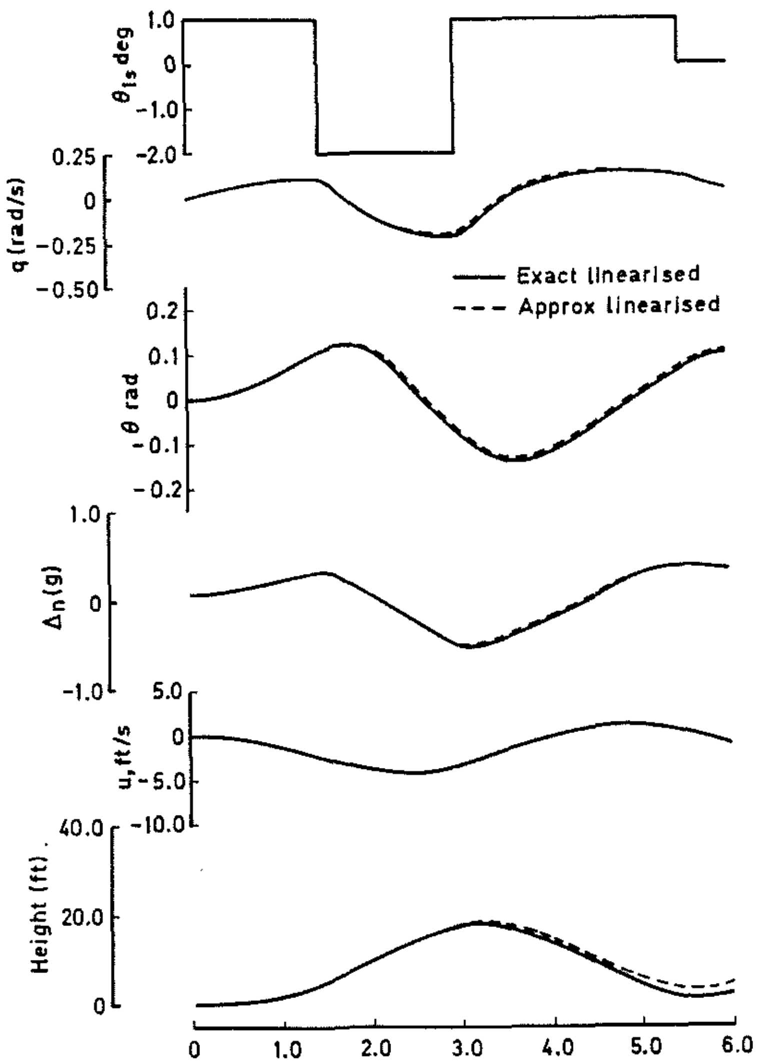

Figure 17 Hurdle-hopping manoeuvre at 100 kn - comparison of exact and approximate solutions - configuration A

In conclusion we shall consider briefly approximations relating to combined pitch and roll manoeuvres, *eg* pitch up and roll over, roll reversals at constant height.

2.5 Approximations for combined pitch and roll manoeuvres

For combined manoeuvres in the vertical and horizontal plane the short term approximation for pitching motion cannot be expected to portray faithfully all the important handling qualities or predict the flight path trajectory adequately in all cases. Control and rate couplings as well as sideslip effects will manifest themselves, but, if we assume that the short term roll response is of the rate type so that sideslip and oscillatory mode excursions can be neglected then a useful approximation can again be constructed. Combining the roll subsidence mode with the longitudinal approximation given by (2) leads to the coupled three-degree of freedom system

$$\frac{d}{dt}\begin{bmatrix} w \\ q \\ \hline p \end{bmatrix} - \left[\begin{array}{cc|c} Z_w & Z_q + U_e & Z_p \\ M_w & M_q & M_p \\ \hline L_w & L_q & L_p \end{array}\right]\begin{bmatrix} w \\ q \\ \hline p \end{bmatrix} = \begin{bmatrix} Z_{\theta_{1s}} & Z_{\theta_{1c}} \\ M_{\theta_{1s}} & M_{\theta_{1c}} \\ \hline L_{\theta_{1s}} & L_{\theta_{1c}} \end{bmatrix}\begin{bmatrix} \theta_{1s} \\ \theta_{1c} \end{bmatrix} \tag{8}$$

Included in (8) are the coupling derivatives M_p, L_q, etc and the derivatives with respect to lateral cyclic pitch θ_{1c}. Once again, the solution to (8) is straight-forward (see (A-2)), even on a hand programmable calculator, but we can seek a further approximation to highlight handling parameters. Assuming weak coupling[4] between the lateral and longitudinal modes, the partitioning shown in (8) seems appropriate (see Appendix). If we assume that Z_p is small enough to be neglected then the approximating polynomial for the eigenvalues of the system take the form (*cf* (A-5) and (A-6)):

$$\lambda^2 - \left(Z_w + M_q - \frac{M_pL_q}{L_p}\right)\lambda + Z_w\left(M_q - \frac{M_pL_q}{L_p}\right) - \left(M_w - \frac{M_pL_w}{L_p}\right)(Z_q + U_e) = 0 \tag{9}$$

$$\lambda - L_p = 0 \; . \tag{10}$$

In addition to the root modulus separation condition, which should apply when the roll inertia of the helicopter is very much lower than the pitch inertia, the validity of (9) and (10) as approximations depends upon the magnitude of the coupling terms. In more precise terms, these conditions can be written as,

$$\left|\frac{M_pL_q}{L_p}\right| \ll M_q \quad \text{and} \quad \left|\frac{M_pL_w}{L_p}\right| \ll M_w \; . \tag{11}$$

When these conditions apply then (9) should serve to predict the sensitivity of the frequency and damping of the longitudinal short period oscillatory mode to the coupling derivatives. For example with $M_p > 0$, $L_q < 0$, $L_p < 0$ and $L_w > 0$, then the effective pitch damping will increase but the static stability derivative M_w will be destabilised.

A numerical example should serve to illustrate the above points. For configuration A, at 100 kn, the system matrix in (8) has numerical elements given by

$$\left[\begin{array}{cc|c} -0.78 & 168.33 & 0.0 \\ -0.0058 & -1.044 & 0.275 \\ \hline 0.0226 & 1.294 & -2.8 \end{array}\right] .$$

When comparing the magnitude of the rolling moment derivatives in the third row, with the pitching moment derivatives in the second row, it should be remembered that these are divided by the appropriate aircraft moments of inertia; the pitch inertia is approximately five times greater than the roll inertia for the present example. The eigenvalues of the above matrix are,

$$\lambda_{1,2} = 1.28 \pm 0.572i$$

$$\lambda_3 = -2.06 \ ,$$

whereas the approximations given by (9) and (10) are,

$$\lambda_{1,2} \simeq -0.98 \pm 0.7i$$

$$\lambda_3 \simeq -2.8 \ .$$

The increase in damping and reduction in frequency trends for the oscillatory mode are seen to be predicted by the approximation though magnitudes are somewhat reduced. The 25% reduction in the roll damping root could not, of course, have been predicted by the approximation.

Regarding the conditions of validity for (9) and (10), on reflection, the increase in the pitch and heave damping with speed results in the condition of widely separated roots being unacceptable. The coupling terms in (11) are given by

$$\frac{M_p L_q}{L_p} = 0.127 \ , \qquad \frac{M_p L_w}{L_p} = -0.00261 \ .$$

The first condition in (11) is therefore valid but the relatively high value of L_w makes the second condition unacceptable. The derivative L_w originates from the lateral flapping induced by the change in coning angle produced by the perturbation in normal velocity w, which is of comparable magnitude to the longitudinal flapping induced directly by the change in w.

For response calculations when speed and sideslip excursions are small, the three degrees of freedom in (8) may well need to be retained in many cases. When sideslip excursions are not small, as is often the case, then it is likely that the full set of lateral/directional equations will need to be coupled with the longitudinal short period mode. In such cases a weakly coupled system approximation may still be applicable, in a wider sense, and prove to be a useful investigative tool in establishing design trends. The author hopes to be able to expand on these ideas in a future paper.

3 DISCUSSION AND CONCLUSIONS

A treatment on the formulation and use of approximations for the short term stability and response characteristics of helicopters has been presented. The method of approximation proposed, not in itself restricted to a small interval of time, is based on rational analysis where conditions of validity in particular applications are readily derived. The author is aware, however, that application of the method of weakly coupled systems often requires a transformation of variables in order to yield a successful partitioning, and that experience based on a knowledge of the behaviour of similar or derived systems can be invaluable. The subject of the present application is of course not new, but it is hoped that clearer physical understanding of this old ground will develop from presenting it in a new light and a sound mathematical framework.

The character, and the quality of prediction by approximate models, of short term longitudinal handling characteristics for helicopters, have been shown to vary significantly with configuration. For an articulated rotor configuration the usual 'short period' approximation works well over the mid to high speed range whereas for the hingeless rotor configuration studied that same approximation breaks down as a result of the relatively large speed excursions present during the short time response. However, for hingeless rotors, the elimination of the M_u effects by feedback from forward speed perturbations to longitudinal cyclic pitch renders the approximations again valid.

The results obtained with the articulated rotor were applied to the simulation of a hurdle hopping manoeuvre where the accuracy of flight path prediction was satisfactory. Besides the ability to predict basic attitude dynamics the value of such approximations is also believed to lie in their ability to predict flight path trajectories, particularly for low level helicopter applications where continuous terrain and obstacle avoidance is required and where the outer loop position variables are of primary interest to the pilot. The relationship between outer loop (flight path) control and inner loop (attitude) control for totally visual flying tasks, where points of reference can move rapidly relative to the helicopter and other objects, has probably not received the consideration it deserves in terms of the safety implications. Once again it is hoped that accurate but simple vehicle models can lead to increased understanding of this relationship and highlight the main features of a preferred control strategy.

Nothing has been said on the short term longitudinal response to collective pitch and since the associated control derivatives differ from those due to cyclic pitch we must expect that vehicle modes will be excited in a different manner. Some calculations performed for collective inputs indicate similar levels of accuracy from the approximation though these clearly relate to additional handling characteristics. However, the ability of a helicopter pilot to change flight path and aircraft attitude independently through the use of collective and cyclic allows him to adopt a flying technique, for evasive manoeuvring for example, whereby he elects to maintain speed or pitch attitude constant. The same effect can be brought about by augmentation and the relevance to ride smoothing systems should also be apparent. For such cases the use of simple approximations may still be valid even for large amplitude manoeuvres in pitch and roll, though it may be necessary to include inertial and aerodynamic nonlinearities in the unconstrained degrees of freedom. The use of the weakly coupled approximation for constrained aircraft motion is further developed for fixed wing aircraft in Reference 9. Some considerations on the same theme, using approximate transfer function relationships for helicopters, have been given by Heffley in Reference 10.

Appendix

MATHEMATICAL OUTLINE OF THE APPROXIMATE METHOD

The method, described more fully in Reference 4, can be applied to linear stationary systems described by the equation,

$$\frac{d\mathbf{y}}{dt} - \mathbf{C}\mathbf{y} = \mathbf{f}(t) \tag{A-1}$$

where $\mathbf{C}$ is a real constant matrix, $\mathbf{y}(t)$ the ℓ-dimensional state vector and $\mathbf{f}(t)$ is a given vector function of time.

The solution of (A-1) can be written in terms of the eigenvalues λ_i, $i = 1,2,\ldots,\ell$ and eigenvectors $\mathbf{u}_i$, $i = 1,2,\ldots,\ell$ of the matrix $\mathbf{C}$ as,

$$\mathbf{y}(t) = \mathbf{Y}(t)\mathbf{y}(0) + \int_0^t \mathbf{Y}(t-\tau)\mathbf{f}(\tau)\,d\tau \tag{A-2}$$

where $\mathbf{y}(0)$ is the initial value of $\mathbf{y}(t)$ and the principal matrix solution $\mathbf{Y}(t)$ is given by

$$\left.\begin{aligned} \mathbf{Y}(t) &= 0 & t < 0 \\ \mathbf{Y}(t) &= \mathbf{U}\,\mathrm{diag}[\exp \lambda_i t]\,\mathbf{U}^{-1} & t \geqslant 0. \end{aligned}\right\} \tag{A-3}$$

Here, the $(\ell \times \ell)$ matrix $\mathbf{U}$ is made up of the columns of eigenvectors $\mathbf{u}_i$.

Since, according to (A-2), a knowledge of $\mathbf{Y}(t)$ will yield the full solution of (A-1) then approximate methods can be confined to the search for approximations to λ_i and $\mathbf{u}_i$. However, unlike numerous methods developed for aircraft stability and control where approximations are based on factorisation of the characteristic polynomial for the system, the present method is based on the complete dynamical system (A-1) and its effective replacement by the direct sum of simpler subsystems. In many cases approximations to the modes of motion of a system will suggest themselves naturally from the most primitive form of the equations of motion; in other cases a transformation of variables may be required to re-cast the equations into an appropriate form. Some prior knowledge of the expected behaviour of the system is therefore of value. The present method is based on the assumption that the complete dynamic system is made up of a set of weakly coupled subsystems from which lower order approximations to the behaviour of the system can be derived.

Let the complete system (A-1) be partitioned into the two-level form,

$$\frac{d}{dt}\left[\begin{array}{c} \mathbf{y}_1 \\ \hline \mathbf{y}_2 \end{array}\right] - \left[\begin{array}{c|c} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \hline \mathbf{C}_{21} & \mathbf{C}_{22} \end{array}\right]\left[\begin{array}{c} \mathbf{y}_1 \\ \hline \mathbf{y}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{f}_1(t) \\ \hline \mathbf{f}_2(t) \end{array}\right] . \tag{A-4}$$

Here, $\mathbf{y}_1$ and $\mathbf{y}_2$ are m- and n-element vectors respectively; $\mathbf{C}_{11}$ is an $(m \times m)$ matrix and $\mathbf{C}_{22}$ an $(n \times n)$ matrix, where $\ell = m + n$. The partitioned system, described by (A-4), is referred to as 'weakly coupled' if the eigenvalue sets of $\mathbf{C}_{11}$ and $\mathbf{C}_{22}$ are widely separated in modulus and the coupling matrices are, in some sense, small. These conditions are made more precise in Reference 4, to which the reader is referred for a full and proper understanding of the method. For the present purposes it is sufficient to state the consequences of the weak coupling. These are as follows,

(a) The eigenvalues of $\mathbf{C}$ also form two sets widely separated in modulus that can be determined from the lower order characteristic equations,

$$\det\left[\lambda I - \mathbf{C}_{11} + \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}\right] = 0 \tag{A-5}$$

$$\det\left[\lambda I - \mathbf{C}_{22}\right] = 0 \tag{A-6}$$

where the solutions to (A-5) are, without loss of generality, assumed to be of lower modulus.

(b) The eigenvectors of $\mathbf{C}$ can be approximated by the matrix

$$\mathbf{U} \simeq \begin{bmatrix} \mathbf{U}_1 & \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{U}_2 \\ -\mathbf{C}_{22}^{-1}\mathbf{C}_{21}\mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \tag{A-7}$$

where $\mathbf{U}_1$ and $\mathbf{U}_2$ are the eigenvector matrices of the submatrices in (A-5) and (A-6) respectively.

(c) The principal matrix solution $\mathbf{Y}(t)$ can be approximated by the sum,

$$\mathbf{Y}(t) = \begin{bmatrix} \mathbf{Y}_1 & -\mathbf{Y}_1\mathbf{C}_{12}\mathbf{C}_{22}^{-1} \\ -\mathbf{C}_{22}^{-1}\mathbf{C}_{21}\mathbf{Y}_1 & \mathbf{C}_{22}^{-1}\mathbf{C}_{21}\mathbf{Y}_1\mathbf{C}_{12}\mathbf{C}_{22}^{-1} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{Y}_2\mathbf{C}_{22}^{-1}\mathbf{C}_{21} & \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{Y}_2 \\ \mathbf{Y}_2\mathbf{C}_{22}^{-1}\mathbf{C}_{21} & \mathbf{Y}_2 \end{bmatrix} \tag{A-8}$$

where $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are, respectively, the principal matrix solutions of the m-dimensional, low modulus, homogeneous system,

$$\frac{d\mathbf{y}_1}{dt} - \left[\mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}\right]\mathbf{y}_1 = 0 \tag{A-9}$$

and the n-dimensional, high modulus system,

$$\frac{d\mathbf{y}_2}{dt} - \mathbf{C}_{22}\mathbf{y}_2 = 0 \ . \tag{A-10}$$

For the application of the above technique in the present paper we shall be most concerned with the short term response of the system and in particular with the validity of the further approximation for the response of the high modulus system, given by,

$$\frac{d\mathbf{y}_2}{dt} - \mathbf{C}_{22}\mathbf{y}_2 = \mathbf{f}_2(t) \quad . \tag{A-11}$$

The technique outlined above has been applied successfully to finding approximations for the short period and phugoid modes of conventional aeroplanes[4] and for the lateral modes of slender aircraft at high angle of attack[11]. Also, the method is extended to automatically controlled aircraft dynamics in Reference 9, where further guidance on the practical application is given.

NOTATION

$\mathbf{C},\mathbf{C}_{12}$ etc	system matrices
$\mathbf{f}(t),\mathbf{f}_1(t)$ etc	forcing vectors
g	gravitational acceleration (ft/s^2, m/s^2)
L,M	roll and pitch moments normalised by respective fuselage moments of inertia (rad/s^2)
M_u,L_p etc	moment stability derivatives
$M_{\theta_0},L_{\theta_{1s}}$ etc	moment control derivatives
ℓ,m,n	vector space dimensions
p,q	fuselage roll and pitch rates (rad/s)
t	time (s)
$\mathbf{U},\mathbf{U}_1$ etc	eigenvector matrices
U_e,W_e	aircraft trim velocity components (ft/s, m/s)
u,w	aircraft perturbation velocities along fuselage x and z axes directions (ft/s, m/s)
u_i	eigenvector
$w_0 = w - U_e\theta$	vertical velocity component (ft/s, m/s)
X,Z	mass normalised forces along fuselage x and z directions (ft/s^2, m/s^2)
X_u,Z_w etc	force stability derivatives
$X_{\theta_{1s}},Z_{\theta_0}$	force control derivatives
x,z	body fixed axes directions centred at CG; z direction down and parallel to hub-normal
$\mathbf{Y}(t),\mathbf{Y}_1(t)$	principal matrix solutions
$\mathbf{y}(t),\mathbf{y}_1(t)$	state vectors
θ	fuselage pitch attitude (rad)
$\theta_0,\theta_{1s},\theta_{1c}$	main rotor collective, longitudinal and lateral cyclic pitch respectively
λ,λ_i	eigenvalues
λ_β	natural rotating flap frequency ratio of rotor blade

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