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IMPROVED PILOT MODEL FOR APPLICATION TO A COMPUTER-FLIGHT TESTING PROGRAM FOR HELICOPTERS

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ABSTRACT

A Computer-Flight Testing (CFT) program for helicopters is developed at NLR for evaluation of helicopter dynamics and handling and control qualities. A control (pilot) model to "fly" the mathematical helicopter model is designed to provide the necessary control inputs for maneuvering flight. This model has previously been discussed (paper nr. 38, 7th European Forum). Further development of the pilot model was necessary to improve controllability so as to be able to keep some state variable deviations (e.g. side-slip angle) small during a maneuver. This required augmentation of the cost function (to be optimized) by specifying weighting terms not only on control rate but also on the control and the state as well. Terminal conditions now have to be met approximately, rather than exactly so as to eliminate the problem of the pilot's feedback gains becoming infinitely large. Typical results of an application are shown for a landing flare with an S-61-class helicopter.

Results show that the control model performs well and offers a framework within which a wide variety of maneuvering, controlled flight can be handled.

LIST OF SYMBOLS

A	system dynamics matrix, eq. (A.1)
A_{1_s}	lateral cyclic pitch control, eq. (2.1.4)
B	control distribution matrix, eq. (A.1)
B_{1_s}	longitudinal cyclic pitch control, eq. (2.1.4)
$\underline{d}$	time function input vector, eqs. (2.1.1), (A.1)
E	input distribution matrix, eq. (A.1)
E_d	helicopter input distribution matrix, eq. (2.1.1)
F	helicopter dynamics matrix, eq. (2.1.1)
G	helicopter control distribution matrix, eq. (2.1.1)
$\underline{h}$	optimal control solution vector, eq. (A.10)
h	aircraft height, eq. (2.1.3)
J	cost function, eqs (2.1.5), (A.2)
J_{int}	integral cost function, eq. (2.1.5)
J_{term}	terminal cost function, eq. (2.1.5)
K	Ricatti matrix, eq. (A.10)
K_{xx}, K_{xu}, K_{uu}	partitions of K, eq. (A.18)
L	optimal feedback gain, eq. (A.16)
L_x	feedback gain on the helicopter state $\underline{x}_p$, eqs. (2.2.1) and (A.19)
L_u	feedback gain on the helicopter control, eqs. (A.19) and (A.21)
$n_{(--)}$	dimension of vector (--)
p	aircraft roll rate, eq. (2.1.3)
$Q_{(--)}$	weighting matrix on (--)

LIST OF SYMBOLS (cont'd)

q	aircraft pitch rate, eq. (2.1.3)
$\underline{r}(t)$	tracking function
$\underline{r}(t_f)$	soft terminal condition
r	aircraft yaw rate, eq. (2.1.3)
$S_{(--)}$	terminal weighting matrix on (--)
T_N	neuro-muscular lag, eq. (2.2.1)
t	time
t_i	discrete time instant
t_o	starting time, eq. (2.1.6)
t_f	final time, eq. (2.1.6)
$\underline{u}$	control vector
u	longitudinal velocity, eq. (2.1.3)
$\underline{v}$	open-loop control, eq. (2.2.1)
v	lateral velocity, eq. (2.1.4)
w	vertical velocity, eq. (2.1.3)
$\underline{u}$	state vector
x	longitudinal distance
y	lateral distance
δ	perturbation from trimmed path
θ	aircraft pitch angle, eq. (2.1.3)
$\theta_{o_{mr}}$	collective pitch of main rotor, eq. (2.1.4)
$\theta_{o_{tr}}$	collective pitch of tail rotor, eq. (2.1.4)
λ	Lagrange multiplier vector, eq. (A.7)(a)
Q	aircraft bank angle, eq. (2.1.3)
ψ	aircraft yaw angle, eq. (2.1.3)
Ω	main rotor rpm

LIST OF SYMBOLS (cont'd)

Subscripts

d	flight path generation
p	helicopter pilot model
s	stabilization

Superscripts

T	transpose
-1	inverse

$(\dot{--}) = d(--)/dt$ time derivative

1 INTRODUCTION

A Computer-Flight Testing (CFT)-program has been developed at NLR, with which relatively short-term maneuvers can be calculated for a helicopter, as result of control inputs etc. In order to be able to perform maneuvers, starting from a specified initial trim condition (which may include turning flight) and ending at another terminal condition, a control (pilot) model had been developed (Haverdings, 1981). Because of the optimization function used to design this model (see chapter 2), it became apparent that there were some shortcomings in its performance during specific maneuvers. When testing the pilot model in making a ½S-turn for example, it appeared that the airplane skidded around trying to make the necessary heading changes while not keeping side-slip to a minimum. This was the direct result of the fact that only the control rate was weighed in the cost function for the terminal controller or pilot model (Haverdings, 1981). Expansion to include state variables in this cost function is the only approach to control such excursions of state variables from their trim value, in this case to keep the side-slip angle approximately zero. The consequence of this is an increased complexity in calculating the optimal control solution, as will be pointed out in chapter 2. Instead of a rather simple, analytical solution one now has to resort to numerical techniques to solve the two-point boundary value problem (TPBVP).

The control model resulting from the newly devised cost function allows for an even greater flexibility to cope with various situations and missions. A more elaborate description of the controller is given in chapter 2. In terms of human factor analysis using the framework of optimal control theory, the present pilot model is assumed to have all information available, noise free and without perceptual delay.

2 CONTROLLER DESCRIPTION

2.1 General description

The sole task of the controller part of the computer program is to generate control inputs into the nonlinear Computer Flight Testing (CFT)-model (or the helicopter dynamical model). The theoretical framework which is used to develop the relevant control laws originates from optimal control theory using state-space techniques. The control laws are derived using a linear(ized) dynamical system representation with a quadratic optimization function. The controller bears resemblance to the human controller only in the sense that control deflections cannot be made instantaneously, i.e. there is a certain (neuro-muscular) time delay in generating the required control input.

The human pilot model, or controller, is to control the following linear dynamical system, which is a linearization of the nonlinear CFT-model:

$$\delta\dot{\underline{x}}_p(t) = F(t)\delta\underline{x}_p(t) + G(t)\delta\underline{u}_p(t) + E_d(t)\delta\underline{d}(t) \tag{2.1.1}$$

with initial conditions at time t_o:

$$\delta\underline{x}_p(t_o) \qquad (2.1.2)(a)$$

$$\delta\underline{u}_p(t_o) = \underline{0} \quad . \qquad (b)$$

Here F(t) represents the linear helicopter dynamics matrix of dimension $n_{\underline{x}_p} \times n_{\underline{x}_p}$, G(t) is the $n_{\underline{x}_p} \times n_{\underline{u}_p}$-dimensional control distribution matrix, and $E_d(t)$ is the $n_{\underline{u}_p} \times n_{\underline{d}}$-helicopter input distribution matrix.

The $n_{\underline{x}_p}$-dimensional vector $\delta\underline{x}_p(t)$ is the perturbation of the helicopter state vector $\underline{x}_p$ from trimmed flight:

$$\delta\underline{x}_p^T(t) = [\delta u \; \delta v \; \delta w \; \delta p \; \delta q \; \delta r \; \delta\theta \; \delta\varphi \; \delta\psi \; \delta h \; \delta x \; \delta y] \quad . \tag{2.1.3}$$

In case of autorotational flight $\delta\underline{x}_p$ is augmented to include the perturbation from trim of the main rotor rpm, $\delta\Omega$. For powered flight (normal condition) $\delta\Omega = 0$ through action of an engine governor (i.e. constant rpm). Furthermore steady-state dynamical rotor behaviour is assumed which is an adequate assumption, so long as no high frequency turbulence is injected into the system or no large abrupt control inputs are generated by the pilot. Because of control rate weighting, leading to neuro-muscular lag, the pilot dynamics is of relatively low bandwidth.

The $n_{\underline{u}_p}$-dimensional vector $\delta\underline{u}_p(t)$ is the perturbation from trim of the helicopter control vector $\underline{u}_p$ as generated by the controller:

$$\delta\underline{u}_p^T(t) = [\delta\theta_{o_{mr}} \; \delta A_{1_s} \; \delta B_{1_s} \; \delta\theta_{o_{tr}}] \; . \tag{2.1.4}$$

When control system gearing is available, also the cockpit controls can be used instead to define the control vector.

The $n_{\underline{d}}$-dimensional input vector $\delta\underline{d}(t)$ is a pure time function, prescribed for all times t on the interval (t_o, t_f). It can be used to model specific inputs which are only a function of time, e.g. step or ramp-like control inputs which are only a function of time (e.g. to test helicopter dynamic stability). It may also represent other inputs, the influence of which on the dynamics of the helicopter is known by the pilot (e.g. engine power failure effects). By including $\delta\underline{d}(t)$ in equation (2.1.1) the resulting optimal (human pilot) control will automatically compensate for this input. In case $\delta\underline{d}(t)$ contains elements of the control vector, these same elements should not also be used in the control vector of the system, eq. (2.1.4), in order to prevent the optimal controller from counteracting its own outputs.

The matrices F(t), G(t) and $E_d(t)$ are, although time-varying, piecewise continuous linearizations of the non-linear CFT-model. Initially they are derived by linearizing about the initial condition. Details of the linearization process to obtain F(t) and G(t) are given by Haverdings (1981). Presently the time-varying matrices remain fixed at their initial values. In the future an updating scheme will be devised to better match the linearized model, eq. (2.1.1), to the nonlinear CFT-model.

The helicopter has to meet a set of terminal conditions at (fixed) terminal time t_f. These are so-called "soft" constraints, i.e. the set of terminal conditions does not have to be met exactly but approximately. This differs from the previous pilot model framework, where terminal conditions had to be met exactly (Haverdings, 1981). Weighting factors are used to weigh deviations from the terminally required values more or less heavily, and a "cost" value is assigned to it, which has to be minimized. This (quadratic) cost term is expressed in the terminal cost term in the cost function J, to be discussed later-on. By proper choice of terminal value and corresponding weighting factor, the program user can "play" with these soft constraints as seems appropriate for the maneuver at hand.

The human pilot controller is to transfer the helicopter from its initial state to a terminal state at time t_f where the "soft" constraints have to be met. There are still an infinite number of ways to do this; however, that one is chosen which minimizes a certain cost function, or performance index, J, consisting of a terminal cost term J_{term} (previously discussed) and an integral term J_{int}, thus:

$$J = J_{term} + J_{int} \quad . \tag{2.1.5}$$

Concurrent with optimal control theory developed by Bryson and Ho (1969) both cost terms are quadratic in terms of the perturbation vectors $\delta \underline{x}_p$ and $\delta \underline{u}_p$. This assures that the minimum value of J is unique and should reach the value zero as an absolute minimum. In addition Bryson and Ho (1969) show, when using a quadratic cost function (or optimization criterion) that the resulting optimal control law is linear in $\delta \underline{x}_p$ and nonsingular.

For the given case the cost function is defined as follows:

$$J = \tfrac{1}{2}[\delta \underline{x}_p(t_f) - \underline{r}_p(t_f)]^T S_{\underline{x}_p} [\delta \underline{x}_p(t_f) - \underline{r}_p(t_f)] + \tfrac{1}{2}\delta \underline{u}_p^T(t_f)\, S_{\underline{u}_p}\, \delta \underline{u}_p(t_f)$$

$$+ \tfrac{1}{2} \int_{t_o}^{t_f} \{ [\delta \underline{x}_p(t) - \underline{r}_p(t)]^T Q_{\underline{x}_p} [\delta \underline{x}_p(t) - \underline{r}_p(t)] + \delta \underline{u}_p^T(t) Q_{\underline{u}_p} \delta \underline{u}_p(t) + \delta \dot{\underline{u}}_p^T(t) Q_{\dot{\underline{u}}_p} \delta \dot{\underline{u}}_p(t) \} dt \tag{2.1.6}$$

where the first two terms are identified with J_{term}, and J_{int} is clearly identified with the term $\tfrac{1}{2} \int_{t_o}^{t_f} \{---\} dt$.

Three comments can be made. First, the integral term includes a term with $\delta \dot{\underline{u}}_p$. This term, which will increase the total cost J whenever $\delta \dot{\underline{u}}_p(t)$ is not equal to zero ($\delta \dot{\underline{u}}_p^T(t) Q_{\dot{\underline{u}}_p} \delta \dot{\underline{u}}_p(t)$ is positive semi-definite, i.e. > 0), will prevent the controller from using infinitely large control rates. When increasing $Q_{\dot{\underline{u}}_p}$ the controller will behave "more cautiously" by using smaller control rates to effect control deflections for maneuver inputs. The program user, who has to choose all weightings ($S_{\underline{x}_p}$, $S_{\underline{u}_p}$, $Q_{\underline{x}_p}$, $Q_{\underline{u}_p}$ and $Q_{\dot{\underline{u}}_p}$) himself, thus can trade control rate against control or against state by increasing one weighting factor while decreasing another. Since this above-mentioned choice is rather difficult to make off-hand, a suggestion how to choose these weighting matrices will be given later-on. It is shown in Appendix A that the weighting on control rate leads to a time delay in the control response, which can be considered to be the neuro-muscular lag of the pilot, or his reluctancy to make rapid control inputs.

The second comment which can be made about the cost function, eq. (2.1.6), concerns the time-varying vector $\underline{r}_p(t)$ in J_{int}, with its terminal value $\underline{r}_p(t_f)$ in J_{term}. The $n_{\underline{x}_p}$-dimensional vector $\underline{r}_p(t)$ is a time function, specified by the program user. It may contain elements which are constant or even zero. The weighting in eq. (2.1.6) is such that deviations of the perturbation vector $\delta\underline{x}_p(t)$ from $\underline{r}_p(t)$ are weighed, thus forcing the helicopter state $\delta\underline{x}_p(t)$ to follow, or to "track", this vector. The user is free to specify this function, but it should obviously not lead to impossible demands on aircraft performance. Another way of looking at $\underline{r}_p(t)$ is that it represents some flight path which the human considers internally to be ideal in some sense. He has made up some preconception of a smooth flight profile which he wants to follow, although not at all cost. One example of this preconception is the pitch-up maneuver when flaring, where the pilot has some predetermined, or learned, idea about how to flare the helicopter. As long as no feedback signals are used, also flight director (open-loop) signals can be represented by $\underline{r}_p(t)$. In the majority of cases, however, $\underline{r}_p(t)$ (or some of its elements) will simply be a constant vector, e.q. $\underline{r}_p(t) = \delta\underline{x}_p(t_o)$, and in that case only the perturbations $\delta\underline{x}_p(t)$ from its (normally zero) initial value $\delta\underline{x}_p(t_o)$ are minimized. By increasing the terminal weighting matrix $S_{\underline{x}_p}$, the contribution to the terminal cost due to $\delta\underline{x}_p(t_f)-\underline{r}_p(t_f)$ will increase, and the resulting optimal control will even "try harder" to bring $\delta\underline{x}_p(t_f)$ as closely to $\underline{r}_p(t_f)$ as possible. Here $\underline{r}_p(t_f)$ constitutes the set of terminal (soft) constraints. By "playing" the weighting matrices $S_{\underline{x}_p}$ and $Q_{\underline{x}_p}$, the program user can set them such that the "terminal miss", i.e. $||\delta\underline{x}_p(t_f)-\underline{r}_p(t_f)||$, becomes satisfactory.

The third comment to be made concerns the weighting of the helicopter state in the integral term, through the term $[\delta\underline{x}_p(t)-\underline{r}_p(t)]^T Q_{\underline{x}_p} [\delta\underline{x}_p(t)-\underline{r}_p(t)]$. It can be used to force $\delta\underline{x}_p(t)$ to follow $\underline{r}_p(t)$ approximately <u>during</u> the maneuver, independent of any terminal condition. For example, when side-slip angle has to be kept approximately zero <u>during</u> the maneuver (i.e. co-ordinated flight), the corresponding element of $\underline{r}_p(t)$ will be zero, and the corresponding element of $Q_{\underline{x}_p}$ has some non-zero, positive value. The larger this value is chosen, the closer the side-slip angle will remain to zero. By including the <u>state</u> weighting in the cost function it is now possible to overcome the problem with a non-zero, large side-slip angle (i.e. an element of the state vector $\delta\underline{x}_p(t)$) which did occur with the previous pilot model. More on this will be said in chapter 2.3.

The time function vector $\underline{r}_p(t)$ may show discontinuities. It may therefore be possible to specify $\underline{r}_p(t) = \underline{0}$ during the maneuver, from $t = t_o$ to $t = t_f$, and specify a non-zero $\underline{r}_p(t_f)$ at $t = t_f$. In this way one can more or less force the state vector $\delta\underline{x}_p(t)$ to remain close to $\underline{0}$ during the maneuver, and still have a "soft" constraint at $t = t_f$. By increasing $Q_{\underline{u}_p}$ relative to $S_{\underline{x}_p}$ (or vice versa) the state vector $\delta\underline{x}_p(t)$ will be closer to $\underline{r}_p(t)$ than to $\underline{r}_p(t_f)$ (or vice versa), thus it becomes possible to trade one versus the other by "playing" the matrices $Q_{\underline{x}_p}$ and $S_{\underline{x}_p}$ to yield desirable results.

2.2 Improved control laws

The control $\delta\underline{u}_{p_{opt}}(t)$ (generated by the pilot) which minimizes the cost function, eq. (2.1.6), of the linear(ized) helicopter model, eq. (2.1.1), and brings it so as to meet the set of terminal constraints as closely as possible, is derived in Appendix A, and is as follows:

$$T_N(t)\delta\dot{\underline{u}}_{p_{opt}}(t)+\delta\underline{u}_{p_{opt}}(t) = -T_N(t)L_{\underline{x}_p}(t)\delta\underline{u}_p(t)+T_N(t)\underline{v}(t) \quad . \qquad (2.2.1)$$

Here $T_N(t)$ is the (time-varying) neuro-muscular lag of the controller, $L_{\underline{x}_p}(t)$ is the feedback gain on the state $\delta\underline{x}_p$, and $T_N(t)\ \underline{v}(t)$ is the open-loop, or feed-forward, part of the control. Typically the values for the time delay $T_N(t)$ are in the order of 0.1-0.2 seconds.

The time-varying feedback gain $L_{\underline{x}_p}(t)$, neuro-motor lag $T_N(t)$ and open-loop part $\underline{v}(t)$ are determined by integrating a series of vector and matrix differential equations backwards in time from known terminal condition at $t = t_f$, and storing the results as function of time. Details are given in Appendix A.

As can be seen from eq. (2.2.1), the human pilot for one part behaves in a closed-loop sense, and for the other part in an open-loop sense, as shown in figure 1. The controller has a priori knowledge of the helicopter dynamics, task requirements and terminal conditions, which are all incorporated in generating the open-loop, or feed-forward, control $\underline{v}(t)$. Additionally, all perturbations of the helicopter state $\delta\underline{x}_p(t)$ are used in a feedback loop, such that any deviation from trim generates corrective control inputs $-T_N(t)L_{\underline{x}_p}(t)\delta\underline{x}_p(t)$, where the "amplification factor", or feedback gain, $T_N(t)L_{\underline{x}_p}(t)$ depends on the helicopter dynamics, the "soft" terminal conditions and on the requirements during the maneuver, in terms of maximum allowable deviations of the state $\delta\underline{x}_p(t)$ from its required path ($\underline{r}_p(t)$), maximum allowable (or available) control inputs $\delta\underline{u}_p(t)$ and the pilot's reluctancy to make rapid control inputs.

2.3 Comparison with old control model

A few comparative notes have already been given. Actually the closed-loop part and the open-loop part of the controller were generated separately by the old model. The flight path generation (FPG)-model (Haverdings, 1981) generated the open-loop control $\delta\underline{u}_d(t)$ and flight path $\delta\underline{x}_d(t)$, while the stabilization (STAB)-model generated the closed-loop control $\delta\underline{u}_s(t)$ (see Haverdings, 1981) to keep deviations of the perturbation state $\delta\underline{x}_s(t)$ from $\delta\underline{x}_d(t)$ small. There are three major differences between the new and the old control model. First, the cost function used in generating $\delta\underline{u}_d(t)$ did not contain any state $\delta\underline{x}_d$ and control $\delta\underline{u}_d$ weighting but only $\delta\dot{\underline{u}}_d(t)$ weighting. The advantage was an entirely closed-form solution to the optimal control $\delta\underline{u}_d(t)$. The disadvantage was that unwanted perturbations $\delta\underline{x}_d(t)$ could not be checked.

Secondly, the definition of the state $\delta\underline{x}_d$ and control $\delta\underline{u}_d$ was such that $\delta\underline{x}_d$ was of reduced order, as was $\delta\underline{u}_d$. For example, when only a longitudinal maneuver (e.g. pull up or flare) was required, $\delta\underline{x}_d$ contained only the longitudinal variables (δu, δw, δq, $\delta\theta$, δh, δx) and the control vector $\delta\underline{u}_d$ only contained "longitudinal control" parameters ($\delta\theta_{o_{mr}}$ and/or δB_{1_s}). The advantage of this approach was a significantly reduced computing effort. The disadvantage was that sometimes large modeling errors existed with respect to the CFT-model, aggravated by cross-coupling effects which were not taken into account.

Thirdly, the feedback control $\delta\underline{u}_s$ stabilized perturbations of the total helicopter state such that the generated flight path would be followed. If that flight path contained unwanted perturbations (see remarks above), the STAB-model would regard this as something to be followed. The feedback gains were time-invariant. Soft constraints could not be considered in the analysis. Great advantage of the old control model, however, was its ease of implementation and associated calculational effort. However, the development of unwanted side-slip angles during turning flight indicated the inadequacy of this model.

2.4 Choice of model parameters

Because of the complexity of the human controller it is a tedious task to choose the various parameters which are required in order to be able to perform the calculations. One major task is the determination of the weighting matrices which appear in the cost function, eq. (2.1.6).

Firstly, all weighting matrices are diagonal, i.e. all off-diagonal elements are zero. Secondly, the weighting is done in a relative way, i.e. they are scaled such that the maximum allowable perturbation is incorporated. Since some weighting matrices are in the terminal cost term while others are in the integral term, the (inverse of the) weighting matrices are chosen according to the suggestion given by Bryson and Ho (1969):

$$S_{\underline{x}_p}^{-1} = (n_{\underline{x}_p} + n_{\underline{u}_p}) \text{x maximum acceptable value of}$$

$$\text{diag}\left[[\delta\underline{x}_p(t_f) - \underline{r}_p(t_f)][\delta\underline{x}_p(t_f) - \underline{r}_p(t_f)]^T\right]$$

$$S_{\underline{u}_p}^{-1} = (n_{\underline{x}_p} + n_{\underline{u}_p}) \text{ x maximum acceptable value of diag } [\delta\underline{u}_p(t_f)\delta\underline{u}_p(t_f)]$$

$$Q_{\underline{x}_p}^{-1} = (n_{\underline{x}_p} + n_{\underline{u}_p})(t_f - t_o) \text{ x maximum acceptable value of diag}$$

$$\left[[\delta\underline{x}_p(t) - \underline{r}_p(t)][\delta\underline{u}_p(t) - \underline{r}_p(t)]^T\right]$$

$$Q_{\underline{u}_p}^{-1} = (n_{\underline{x}_p} + n_{\underline{u}_p})(t_f - t_o) \text{ x maximum acceptable value of}$$

$$\text{diag}\,[\delta\underline{u}_p(t)\delta\underline{u}_p^T(t)]$$

$$Q_{\dot{\underline{u}}_p}^{-1} = n_{\underline{u}_p}(t_f - t_o) \text{ x maximum acceptable value of}$$

$$\text{diag}\,[\delta\dot{\underline{u}}_p(t)\delta\dot{\underline{u}}_p^T(t)] \;. \qquad (2.4.1)$$

Because of the additional requirement that the neuro-muscular lag should be about 0.2 s, the relation between $Q_{\underline{u}_p}$ and $Q_{\dot{\underline{u}}_p}$ should be:

$$Q_{\underline{u}_p} \approx 0.04\, Q_{\underline{\dot{u}}_p} \quad . \qquad (2.4.2)$$

Therefore, when defining $\delta\dot{\underline{u}}_{p_{max}}$ also $\delta\underline{u}_{p_{max}}$ is determined, or vice-versa. Quite often, however, the choice of the maximum acceptable value of $\delta\underline{u}_p$ is made more easily because $\delta\underline{u}_{p_{max}}$ is available from the control system.

3 EXAMPLE

3.1 System set-up and definition

As an example to show the operation of the improved controller two landing flare maneuvers are calculated for an S-61 class helicopter. Initial conditions are a speed of 60 kts (30 m/s), on a 6 degrees glide slope, with a helicopter mass of 8620 kg and initial height of 15 m. The terminal time t_f = 6 s, and the following state vector is defined at t_f (first flare):

$$\underline{x}_p^T(t_f) = [30,\ 0,\ 2.62,\ 0,\ 0,\ 0,\ .0873,\ 0,\ 0,\ 0,\ x(t_f),\ 0]$$

where $x(t_f)$ is not prescribed. The terminal speed is about 60 kts, and final pitch angle is prescribed at .0873 rad (5 degrees). For the second flare the only change is a reduced speed, viz. u = 21 m/s, w = 1.79 m/s, thus a stronger deceleration has to be performed (final speed ~ 40 kts). To provide for a smooth transition to the terminal condition, the tracking function vector $\underline{r}_p(t)$ was not kept zero, but was phased from zero to its terminal value $\underline{r}_p(t_f)$ using a 1-cos² function as follows:

$$\underline{r}_p(t) = \underline{r}_p(t_f)\left(1-\cos^2\left(\frac{\pi}{2}\,\frac{(t-t_o)}{(t_f-t_o)}\right)\right) . \qquad (3.1)$$

This function may be representative of the smoothness which the human would apply (see section 2.1), and can be a learned, or some other, mental process. Further research in this area is required.

The results of the calculations for the normal flare (solid line) and the decelerating flare (dashed line) are shown in figure 2 for the state variables v, w, θ and h, and the controls $\theta_{o_{mr}}$ and B_{1_s}. The time-varying nature of the feedback gains of B_{1_s} on some of the elements is shown in figure 3, together with its open-loop time history. Obviously the decelerating flare requires a stronger pitch up to bring the deceleration about. All time histories of states, controls and control rates are smooth and exhibit normal tendencies. For future exercises the behaviour of the controller will be examined for other, more complex maneuvers. No time was available to test again the ½-S turn which led to the further development of the pilot model, as described here.

4 CONCLUDING REMARKS

Further development of the control model for a computer-flight testing program for helicopters, by augmenting the optimization criterion, increased the controllability of the man-machine system. The present model offers a framework within which a wide variety of maneuvers can be performed. The controller has many options and possibilities which have not yet been explored. The structure of the optimal control law allows the program user to be very flexible in the definition of the various system parameters pertaining to a specific problem, by trading closed-loop (or feedback) control against open-loop control through proper choice of control parameters. The adequate performance of the controller has been exemplified by a typical flare maneuver.

5 REFERENCES

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APPENDIX A

Derivation of optimal control laws

In standard textbooks on optimal control theory with quadratic cost functions, the following optimization problem is solved: given the dynamical system:

$$\delta\underline{\dot{x}}(t) = A(t)\delta\underline{x}(t) + B(t)\delta\underline{u}(t) + E(t)\delta\underline{d}(t) \tag{A.1}$$

$\delta\underline{x}(t_o)$ given

find the $n_{\underline{u}}$-dimensional optimal control vector $\delta\underline{u}_{opt}(t)$ which brings the $n_{\underline{u}}$-dimensional state vector $\delta\underline{u}(t)$ to a terminal time, while minimizing (i.e. optimizing) the quadratic function:

$$J = \tfrac{1}{2}[\delta\underline{x}(t_f)-\underline{r}(t_f)]^T S_{\underline{x}}[\delta\underline{x}(t_f)-\underline{r}(t_f)]$$
$$+ \tfrac{1}{2}\int_{t_o}^{t_f}\{[\delta\underline{x}(t)-\underline{r}(t)]^T Q_{\underline{r}}[\delta\underline{x}(t)-\underline{r}(t)]+\delta\underline{u}^T(t)\ Q_{\underline{u}}\delta\underline{u}(t)\}\ dt\ . \tag{A.2}$$

By comparing (A.1) and (A.2) with resp. eqs (2.1.1) and (2.1.6), some redefinitions are necessary in order to adapt the human controller optimization problem, eq. (2.1.6), to the optimization problem here. The main difference is that $\delta\underline{\dot{u}}(t)$ is not included in eq. (A.2). The following definitions are made:

$$\delta\underline{u}(t) \triangleq \delta\underline{\dot{u}}_p(t) \qquad \text{(A.3)(a)}$$

$$\delta\underline{x}^T(t) \triangleq [\delta\underline{x}_p^T(t),\ \delta\underline{u}_p^T(t)] \qquad \text{(i.e. augmented)} \qquad \text{(b)}$$

$$\underline{r}^T \triangleq [\underline{r}_p^T,\ \underline{0}] \qquad \text{(c)}$$

Then eqs (2.1.1) and (2.1.6), can be written as:

$$\begin{Bmatrix} \delta\dot{\underline{x}}_p(t) \\ \delta\dot{\underline{u}}_p(t) \end{Bmatrix} = \begin{bmatrix} F(t), & G(t) \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta\underline{x}_p(t) \\ \delta\underline{u}_p(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \delta\dot{\underline{u}}_p(t) + \begin{bmatrix} E_d(t) \\ 0 \end{bmatrix} \delta\underline{d}(t) \qquad (A.4)$$

$$J = \tfrac{1}{2}\left[\left[\begin{Bmatrix} \delta\underline{x}_p(t_f) \\ \delta\underline{u}_p(t_f) \end{Bmatrix} - \begin{Bmatrix} \underline{r}_p(t_f) \\ \underline{0} \end{Bmatrix}\right]^T \begin{bmatrix} S_{\underline{r}_p}, & 0 \\ 0, & S_{\underline{u}_p} \end{bmatrix} \left[\begin{Bmatrix} \delta\underline{x}_p(t_f) \\ \delta\underline{u}_p(t_f) \end{Bmatrix} - \begin{Bmatrix} \underline{r}_p(t_f) \\ \underline{0} \end{Bmatrix}\right]\right] +$$

$$+ \tfrac{1}{2}\int_{t_o}^{t_f} \left|\left[\begin{Bmatrix} \delta\underline{x}_p(t) \\ \delta\underline{u}_p(t) \end{Bmatrix} - \begin{Bmatrix} \underline{r}_p(t) \\ \underline{0} \end{Bmatrix}\right]^T \begin{bmatrix} Q_{\underline{x}_p}, & 0 \\ 0, & Q_{\underline{u}_p} \end{bmatrix} \left[\begin{Bmatrix} \delta\underline{x}_p(t) \\ \delta\underline{u}_p(t) \end{Bmatrix} - \begin{Bmatrix} \underline{r}_p(t) \\ \underline{0} \end{Bmatrix}\right]\right. +$$

$$\left. \delta\dot{\underline{u}}_p^T(t)\, Q_{\dot{\underline{u}}_p}\, \delta\dot{\underline{u}}_p(t) \quad dt \right| . \qquad (A.5)$$

By comparing eq. (A.4) with (A.1) and (A.5) with (A.2), bearing in mind the definitions (A.3), it becomes clear that the human optimal control system is identical to the optimal control problem, eqs (A.1)-(A.2), when in addition the following definitions hold:

$$A(t) \triangleq \begin{bmatrix} F(t), & G(t) \\ 0, & 0 \end{bmatrix} \qquad \text{(i.e. an augmented matrix)} \qquad (A.6)(a)$$

$$B(t) \triangleq \begin{bmatrix} 0 \\ I \end{bmatrix} \qquad (b)$$

$$E(t) \triangleq \begin{bmatrix} E_d(t) \\ 0 \end{bmatrix} \qquad (c)$$

$$S_{\underline{x}} \triangleq \begin{bmatrix} S_{\underline{x}_p}, & 0 \\ 0, & S_{\underline{u}_p} \end{bmatrix} \qquad (d)$$

$$Q_{\underline{x}} \triangleq \begin{bmatrix} Q_{\underline{x}_p}, & 0 \\ 0, & Q_{\underline{u}_p} \end{bmatrix}$$

$$Q_{\underline{u}} \triangleq Q_{\dot{\underline{u}}_p} .$$

The system, eq. (A.1) is generally called the augmented system because the state vector $\delta\underline{x}(t)$ equals the state vector $\delta\underline{x}_p(t)$ augmented with the control vector $\delta\underline{u}_p(t)$. Now the human controller optimization problem has been specified in terms of the augmented system optimization problem, eqs (A.1) through (A.2), by defining the augmented vectors and matrices in terms of helicopter system vectors and matrices as given by eq. (A.3) and (A.6).

The optimality conditions for the (augmented) optimization problem, using calculus of variation, are derived by Bryson and Ho (1969). The results, applied to the problem here, are given without proof:

$$\delta\dot{\underline{x}}(t) = A(t)\delta\underline{x}(t) + B(t)\delta\underline{u}(t) + E(t)\delta\underline{d}(t) \quad \underline{x}(t_D) \text{ given} \qquad (A.1)$$

$$\dot{\underline{\lambda}}(t) = -A^T(t)\,\underline{\lambda}(t) - Q_{\underline{x}}\,[\delta\underline{x}(t) - \underline{r}(t)] \qquad (A.7)(a)$$

with terminal condition: $\underline{\lambda}(t_f) = S_{\underline{x}}[\delta\underline{x}(t_f) - \underline{r}(t_f)]$ (A.7)(b)

and optimal control law:

$$\delta\underline{u}_{opt}(t) = -Q_{\underline{u}}^{-1}\, B^T(t)\,\underline{\lambda}(t) . \qquad (A.8)$$

The boundary conditions for $\delta\underline{x}(t)$ and the Lagrange multiplier $\underline{\lambda}(t)$ are split: for $\delta\underline{x}(t)$ the <u>initial</u> condition at time t_o is given, whereas for $\underline{\lambda}(t)$ they are given at the <u>final</u> time t_f.

By substituting the optimal control law, eq. (A.8), into eq. (A.1), and combining it with eq. (A.7)(a), we have the following set of equations:

$$\begin{Bmatrix} \delta\dot{\underline{x}}(t) \\ \dot{\underline{\lambda}}(t) \end{Bmatrix} = \begin{bmatrix} A(t), & -B(t)\,Q_{\underline{u}}^{-1}\,B^T(t) \\ -Q_{\underline{x}}, & -A^T(t) \end{bmatrix} \begin{Bmatrix} \delta\underline{x}(t) \\ \underline{\lambda}(t) \end{Bmatrix} + \begin{bmatrix} E(t) \\ 0 \end{bmatrix} \delta\underline{d}(t) + \begin{bmatrix} 0 \\ Q_{\underline{x}} \end{bmatrix} \underline{r}(t). \qquad (A.9)$$

The boundary conditions for this system, eqs. (A.9), are mixed, i.e. at different times. This is the wellknown two-point boundary value problem (TPBVP) (Bryson & Ho, 1969).

Instead of trying to determine the state transition matrix of eq. (A.9) and solving for the unknowns, use is made of the "sweep method" as used by Bryson and Ho (1969). From the linearity of the nonhomogeneous equations (A.9) the following solution is postulated:

$$\underline{\lambda}(t) = K(t)\delta\underline{x}(t) - \underline{h}(t) \quad . \tag{A.10}$$

By differentiating this equation with respect to time one gets, using also eq. (A.9):

$$\begin{aligned} \dot{\underline{\lambda}} &= \dot{K}\delta\underline{x} + K\delta\dot{\underline{x}} - \dot{\underline{h}} \\ &= \dot{K}\delta\underline{x} + K[A\delta\underline{x} - BQ_{\underline{u}}^{-1}B^{T}\lambda + E\delta d] - \dot{\underline{h}} \\ &= \dot{K}\delta\underline{x} + KA\delta\underline{x} - KBQ_{\underline{u}}^{-1}B^{T}[K\delta\underline{x} - \underline{h}] + KE\delta\underline{d} - \dot{\underline{h}} \quad . \end{aligned} \tag{A.11}$$

But, from eq. (A.9):

$$\dot{\underline{\lambda}} = -\,Q_{\underline{x}}\,\delta\underline{x} - A^{T}[K\delta\underline{x} - \underline{h}] + Q_{\underline{x}}\underline{r} \quad . \tag{A.12}$$

Equating eq. (A.11) with (A.12) and collecting like terms, one gets:

$$\begin{aligned} &[\dot{K} + KA + A^{T}K - KBQ_{\underline{u}}^{-1}B^{T}K + Q_{\underline{x}}]\ \delta\underline{x}\ + \\ &+ [KBQ_{\underline{u}}^{-1}B^{T}\underline{h} - \dot{\underline{h}} - A^{T}\underline{h} + KE\delta\underline{d} - Q_{\underline{x}}\underline{r}] = 0 \ . \end{aligned} \tag{A.13}$$

Equation (A.13) can only be identical to zero for any arbitrary value of $\delta\underline{x}$, when the terms between square brackets each vanish to zero. Thus when:

$$\dot{K}(t) = -K(t)A(t) - A^{T}(t)K(t) + K(t)B(t)Q_{\underline{u}}^{-1}B^{T}(t)K(t) - Q_{\underline{x}} \tag{A.14)(a}$$

$$\dot{\underline{h}}(t) = -[A(t) - B(t)Q_{\underline{u}}^{-1}B^{T}(t)K(t)]^{T}\underline{h}(t) + K(t)E(t)\delta\underline{d}(t) - Q_{\underline{x}}\underline{r}(t) \ . \tag{b}$$

The boundary conditions for the above equations can be found by substituting $t = t_f$ in eq. (A.10) and comparing that with eq. (A.7)(b).

This yields:

$$K(t_f) = S_{\underline{x}} \qquad (A.15)(a)$$

$$\underline{h}(t_f) = S_{\underline{x}}\underline{r}(t_f) \ . \qquad (b)$$

When substituting eq. (A.10) into eq. (A.8), the optimal control law becomes:

$$\delta\underline{u}_{opt}(t) = L_{\underline{x}}(t)\delta\underline{x}(t)+\underline{v}(t) \qquad (A.16)$$

where:

$$L_{\underline{x}}(t) = -Q_{\underline{u}}^{-1}\, B^T(t)\, K(t) \qquad (A.17)(a)$$

$$\underline{v}(t) = Q_{\underline{u}}^{-1}\, B^T(t)\, \underline{h}(t) \ . \qquad (b)$$

Transformation to the helicopter pilot model system

The optimal control law, eq. (A.16), is still given in terms of parameters of the general system, eq. (A.1). By involving the definitions, eqs. (A.3) and (A.6), the results can be translated into quantities compatible with the helicopter pilot model notation. This is done by noting that the solution of the Ricatti equation (eq. (A.14)(a)), with boundary condition eq. (A.15)(a), yields a Ricatti matrix K(t) which is symmetric (Bryson and Ho, 1969), and can then be split up as follows:

$$K(t) = \begin{bmatrix} K_{xx}(t), & K_{xu}(t) \\ K_{xu}^T(t), & K_{uu}(t) \end{bmatrix} \ . \qquad (A.18)$$

By writing:

$$L_{\underline{x}}(t) = [L_{\underline{x}_p}(t),\ L_{\underline{u}_p}(t)] \qquad (A.19)$$

where $L_{\underline{x}_p}(t)$ is a $(n_{\underline{u}_p} \times n_{\underline{x}_p})$-dimensional matrix, $L_{\underline{u}_p}(t)$ is a $(n_{\underline{u}_p} \times n_{\underline{u}_p})$-square matrix, then the control law, eq. (A.16), can be written as:

$$\delta\underline{\dot{u}}_{p_{opt}}(t) = [L_{\underline{x}_p}(t),\ L_{\underline{u}_p}(t)] \begin{Bmatrix} \delta\underline{x}_p(t) \\ \delta\underline{u}_p(t) \end{Bmatrix} + \underline{v}(t)$$

or:

$$\delta\underline{\dot{u}}_{p_{opt}}(t) = L_{\underline{x}_p}(t)\delta\underline{x}_p(t) + L_{\underline{u}_p}(t)\delta\underline{u}_p(t) + \underline{v}(t) \qquad (A.20)$$

and where, by proper substitution in eq. (A.17)(a) of the definitions, one can derive:

$$L_{\underline{x}_p}(t) = -\ Q_{\dot{\underline{u}}_p}^{-1}\ K_{xu}^T(t) \qquad (A.21)(a)$$

$$L_{\underline{u}_p}(t) = -\ Q_{\dot{\underline{u}}_p}^{-1}\ K_{uu}(t)\quad . \qquad (b)$$

Then the optimal control law for the helicopter pilot model can be written as:

$$\delta\underline{\dot{u}}_p(t) = -\ Q_{\dot{\underline{u}}_p}^{-1}\ K_{xu}^T(t)\delta\underline{x}_p(t) - Q_{\dot{\underline{u}}_p}^{-1}\ K_{uu}(t)\delta\underline{u}_p(t) + \underline{v}(t)\ .$$

Premultiplying by $Q_{\dot{\underline{u}}_p}(t)$ and then premultiplying by the inverse of $K_{uu}(t)$ eventually yields:

$$T_N(t)\delta\underline{\dot{u}}_p(t)+\delta\underline{u}_p(t) = -\ T_N(t)\ L_{\underline{x}_p}(t)\delta\underline{x}_p(t) + T_N(t)\ \underline{v}(t)\ . \qquad (A,22)$$

Here T_N represents the neuro-motor lag resulting from weighting control rate. It can easily be derived that:

$$T_N(t) = K_{uu}^{-1}(t)\ Q_{\dot{\underline{u}}_p}\ . \qquad (A.23)$$

This equation shows that when the feedback gains on the control increase (K_{uu} becomes "larger") then the neuro-motor lag T_N becomes "smaller". Heavier weighting of control rate ($Q_{\dot{\underline{u}}_p}$ "larger") "increases" the neuro-motor lag and results in a more sluggish pilot response. According to general practice in human factor analysis work, the weighting on the control $\delta\underline{u}_p(t)$ and control rate $\delta\dot{u}_p(t)$ is chosen such that the resulting neuro-motor lag is about 0.1-0.2 seconds.

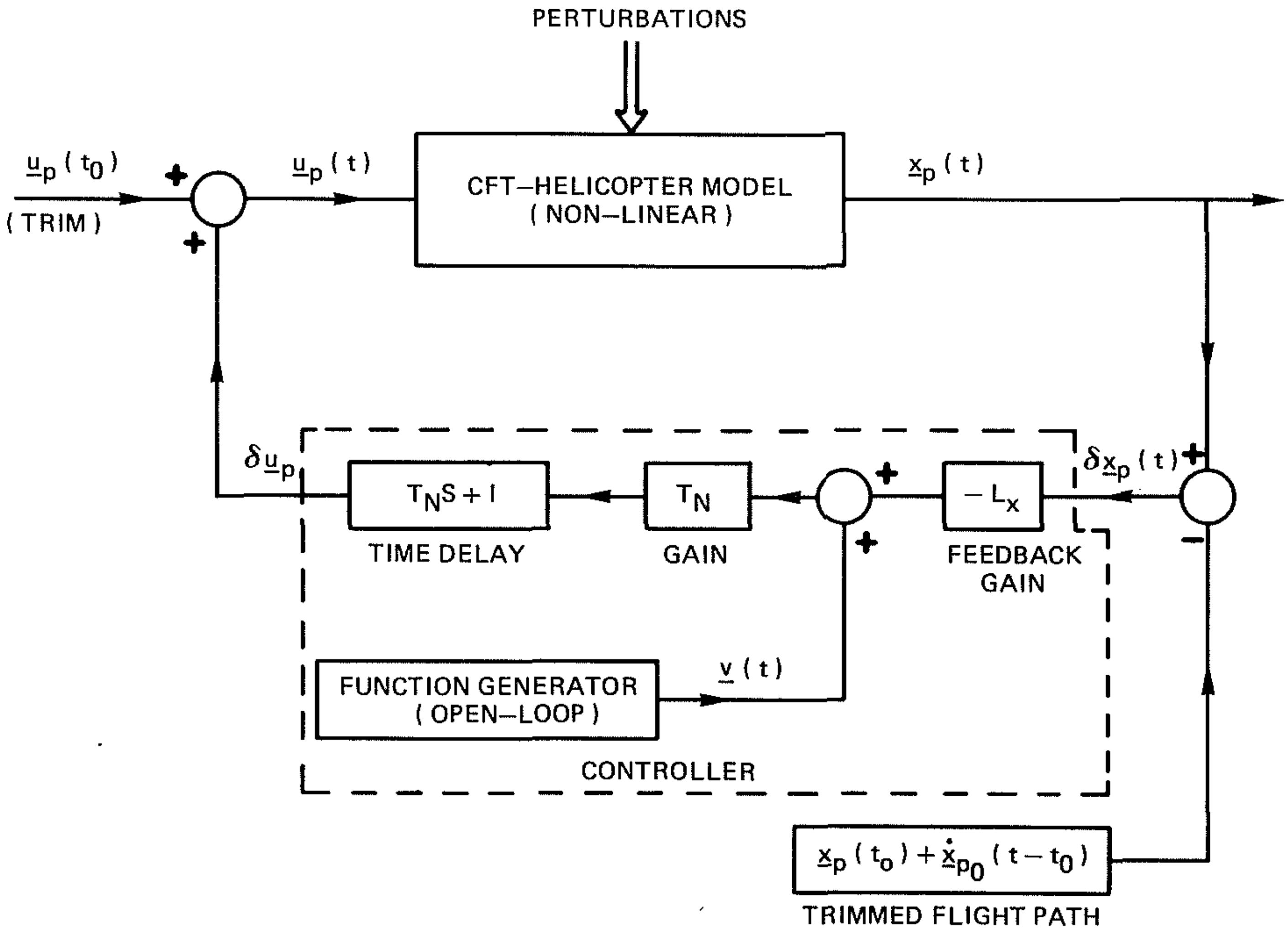

Fig. 1 Operational scheme of the controller, combined with the nonlinear CFT-model

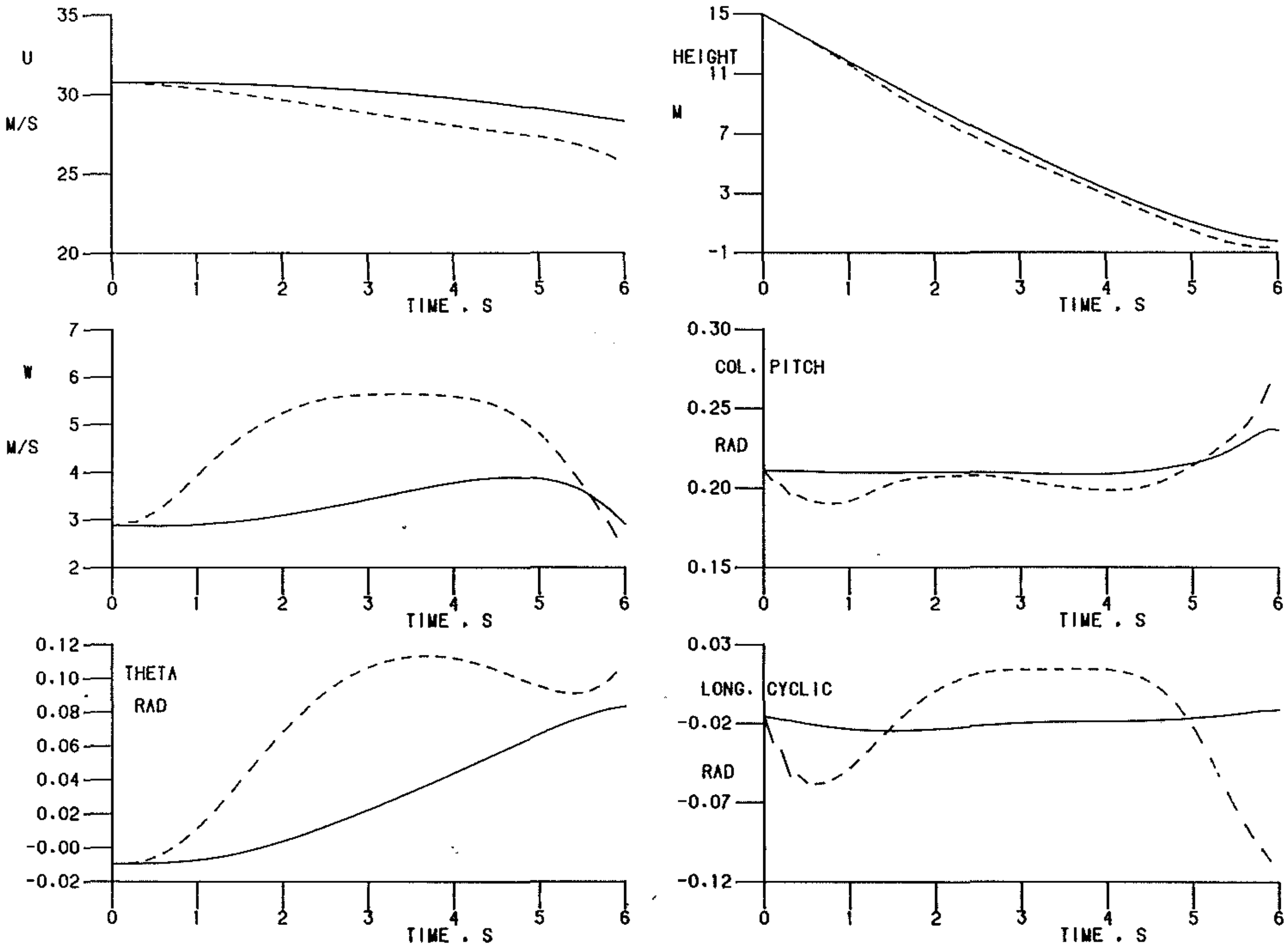

Fig. 2 Time histories of state and control elements during normal flare (solid line) and decelerating flare (dashed line)

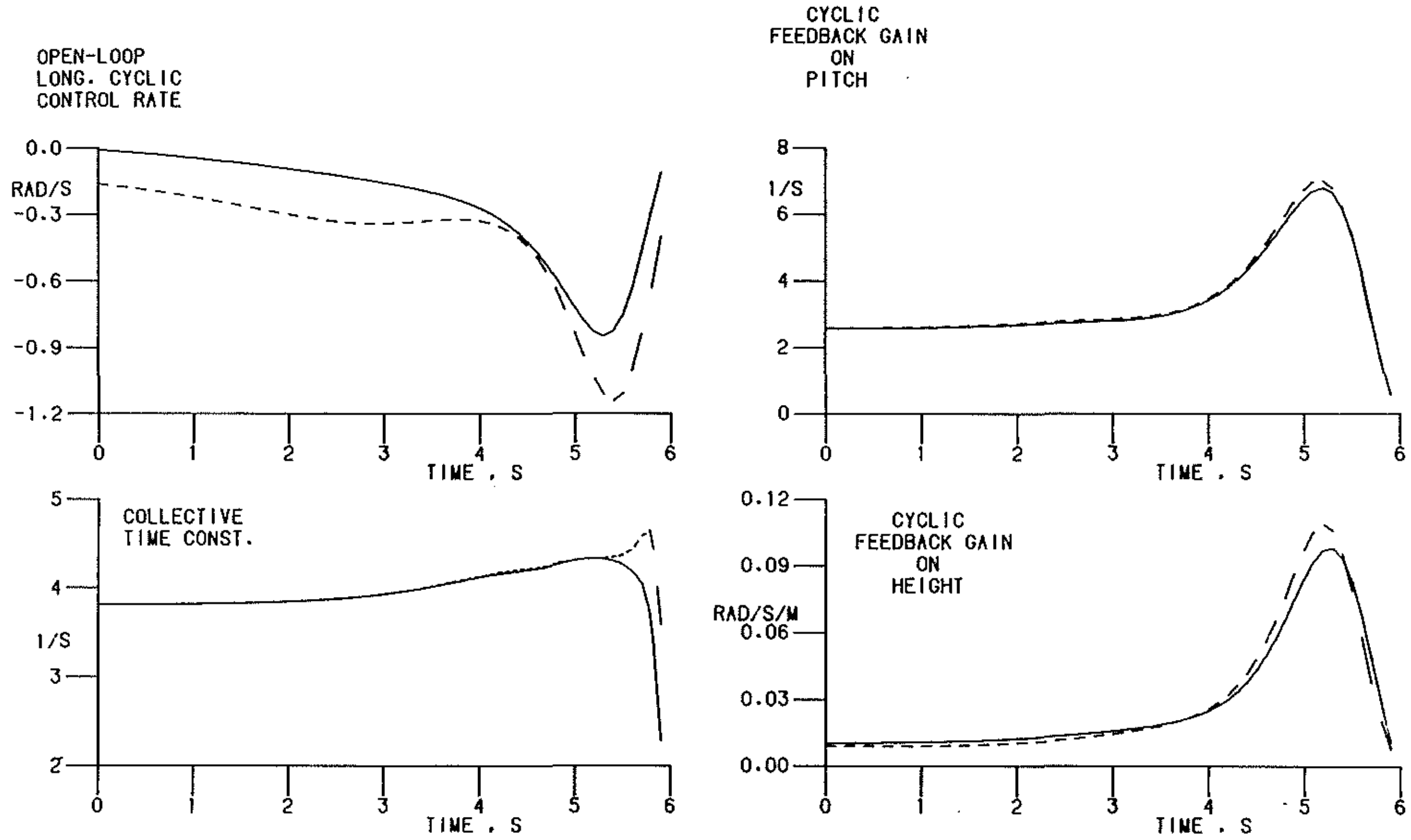

Fig. 3 Time histories of open-loop control rate and several feedback gains for longitudinal cyclic