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ROTOR IMPEDANCE MEASUREMENTS AT MODEL SCALE

by

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1 Introduction

The work described in this paper was started at the Royal Aircraft Establishment several years ago[1]. At that time, increasing interest was being taken in what is now known as the 'semi-rigid' rotor, and it was clear that considerable investigation of the dynamic behaviour of such rotors would be necessary, both for the avoidance of dynamic instabilities, and in order to exploit to the full the advantages of semi-rigid design.

The RAE has a history of fixed wing aeroelastic model construction and testing that extends back for more than forty-five years, and it was an obvious step to deploy the existing techniques and expertise on aeroelastic models of rotors. There was an additional incentive to initiate work on scaled rotor models, and this was that relatively little use had been made of rotor models in the UK up to that time, whereas a significant amount of useful research was being undertaken with models in other countries, notably the US. It was felt that, despite the known disadvantages of models, a great deal of fundamental research, particularly in dynamics, could be undertaken at model scale.

It was against this background that a research programme was started at RAE to measure overall rotor dynamic behaviour using accurately-scaled aeroelastic rotor models. The programme was aimed at providing rotor data associated with the investigation of the twin instabilities of 'ground resonance' and 'air resonance in the hover' for semi-rigid rotors. These instabilities can be investigated in a number of ways; one method is to treat the rotor and the fuselage separately, and to obtain, for each, the impedances at the junction point, which is the rotor shaft. An 'impedance-matching' technique can then be applied to determine the overall dynamic behaviour of the composite system of rotor and fuselage. It was decided therefore to measure the rotor impedances at the shaft, so that the measured data could be used either to compare with theoretical values with the object of checking and improving the mathematical model of the rotor, or used with theoretical fuselage impedances to obtain a stability solution by 'impedance matching'.

This paper deals with the technique that has been developed for impedance-measurement, and illustrates some of the results that have been obtained. The programme, like any other research programme, has had its share of successes and failures. With the work not yet completed, it is difficult to attempt a final assessment of its value. What has emerged, however, is that the measurement of impedances is only half the problem: the other half, and possibly the more difficult half, is to interpret and use the measurements. Some of the difficulties are discussed later in the paper.

2 Experimental Requirements

The basic requirements of a test rig can be established by considering oscillatory motions of the rotor shaft that are significant in ground and air resonance instabilities. These motions are: (i) fore and aft translation,

(ii) pitch, (iii) lateral translation, and (iv) roll. In the hover, fore and aft translation and lateral translation are synonymous, as are pitch and roll. The rig requirements reduce, therefore, to the provision of separate oscillatory motions of the rotor in translational and rotational directions, with a capability of measuring, for each motion, the force or moment required to produce that motion, together with the forces and moments required to prevent motions in other directions. Thus, a four-component balance is required at the rotor shaft, together with means of excitation and measurement of shaft motion. The impedances are simply the forces and moments per unit velocity of motion. Each impedance is a complex function of frequency, and it is necessary to measure the real and imaginary parts of the forces and moments relative to the shaft motion over the whole frequency range of interest. This constitutes a severe data-collection problem, for there may be nearly 5000 measurements required for each rotor condition. Obviously, an essential test requirement is some form of rapid testing and analysis.

The model requirements are all-important if the test technique is to have any useful application. Since one is concerned with overall rotor forces, each of which includes components of centrifugal, inertia, aerodynamic and elastic forces, it is desirable to scale the rotor so that these components in the model bear the same relationship to one another as in the full scale rotor. This requirement can only be met with a so-called Froude-scaled model, in which the ratio of model to full-scale tip speed is equal to the square root of the linear scale, and the model is therefore operating at an incorrect Mach number. This is a fact of rotor modelling that is inescapable in atmospheric testing. However, it does not preclude the testing of models scaled for correct Mach number operation for which the gravitational forces will be too small. This may be an acceptable penalty in respect of the rotor forces, and the use of Mach-scaled models in stability investigations certainly cannot be ruled out.

3 Rig Design

At the centre of the rig is a short section of steel shaft; the rotor is attached to the top of the shaft and a torque connector to the bottom.

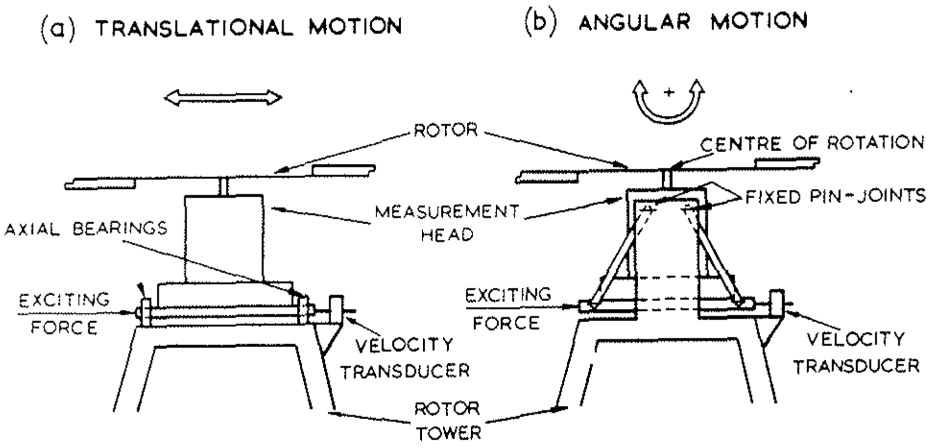

Fig 1 MOTION - CONTROL SYSTEMS

The connector links the shaft to a second, and longer, shaft which is driven by an electric motor, and the purpose of the torque connector is to allow torque to be transmitted to the rotor shaft but to prevent load transmission in any other direction. The rotor shaft is held in position by two bearings near the top of a common housing. The lower part of the housing is a thin cylinder attached at the base to a rigid platform. The walls of the cylinder are strain gauged to measure forces and moments in the required direction. Each force and moment transmitted through the rotor shaft can be obtained as a linear sum of the strain gauge outputs. The required motion of the shaft is obtained by constraining the rigid platform to move in one of two ways (Fig. 1). The plate is either attached to two horizontal parallel bars which can move axially in bearings giving the rotor shaft a translational motion, or it is attached to a non-parallel linkage which gives the shaft an angular motion about the centre of the rotor. With either arrangement the plate is driven by an electromagnetic exciter, and its motion measured by a velocity transducer, both units being anchored to an 'earth' point on the tower on which the rig is mounted (Fig. 2).

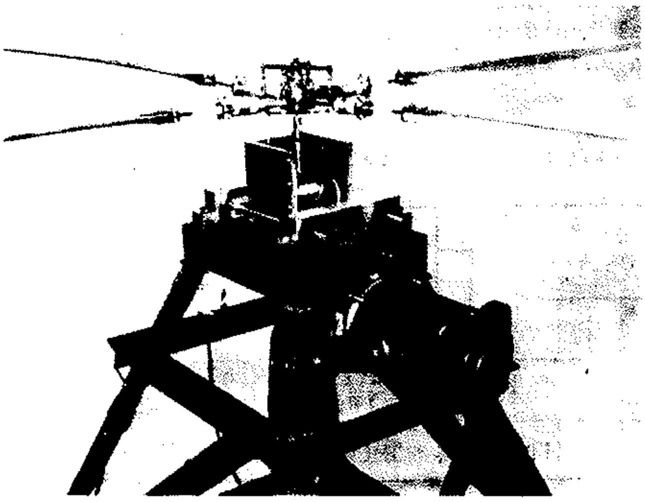

Fig 2. Rig arrangement for translational motion

It is worth noting that by arranging for the force measurements to be made on the rotor side of the motion system, the forces are independent of the dynamics of that system. The advantages of this are that no special precautions have to be taken to eliminate friction in the motion-system, and, if necessary, instabilities of the whole rig can be prevented by adding damping (or mass or stiffness) in the motion-system without affecting the impedance measurements.

Calibration can be effected in a number of ways. Perhaps the most satisfactory is to mount a rigid disc in place of the rotor. With the disc spinning, a pitch motion will give rise to known pitching moments and rolling moments. If the disc is mounted above the pitch axis, an additional fore-and-aft force will also arise. However, whilst it is desirable to calibrate in as near the test conditions as possible (ie with a rotating test piece and with motions imposed), it is difficult to find a simple means of calibrating some of the cross-impedances in this way. For example, only a flexible rotating system will give rise to lateral forces when subjected to pitch or fore-and-aft translatory motions. For such cases, calibration is made by applying known forces to the stationary shaft.

4 Processing of Test Results

The need to minimise test time has been mentioned in section 2. In order to avoid tests at several hundred discrete values of frequencies in covering the frequency range, frequency sweep excitation is applied, the frequency range being swept through in a few seconds. The resulting force and motion time-histories are fed to an on-line Fourier Analyser which takes the Fourier transform of each signal and divides the two to obtain the impedances as functions of frequency. In practice it is necessary to elaborate this procedure slightly (Fig. 3) by measuring the forces or moments (mainly at first rotor order) which may occur when there is no imposed motion of the shaft and subtracting these from the test values; it is also desirable to average the results over several frequency sweeps - generally about ten. A further modification is to subtract the forces and moments due to the inertias of the moving parts of the rig between the rotor and the strain gauges. This is simply done by making a test without a rotor which establishes the relevant forces and moments. The Fourier Analyser is programmed to carry out these procedures automatically, with the result that a graphical presentation of an impedance can be obtained over the whole frequency range in one or two minutes, compared with several hours required to obtain the same results by discrete frequency excitation and separate analysis of each case.

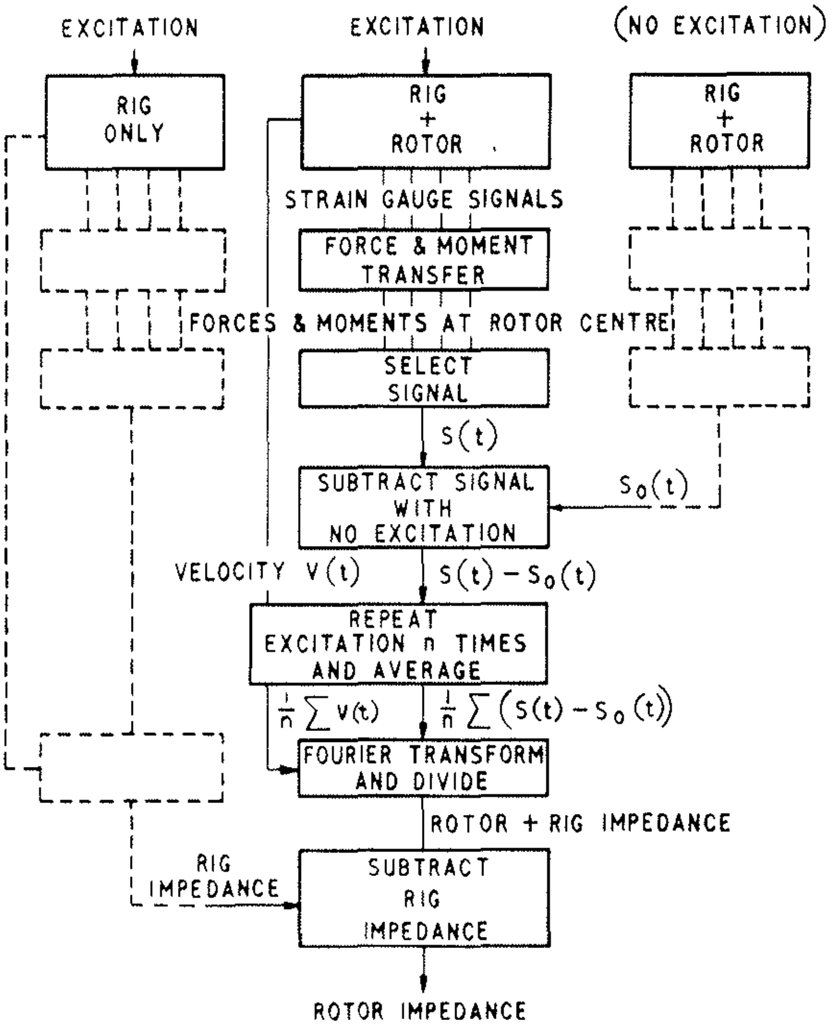

Fig 3. ANALYSIS PROCESS

5 Interpretation of Impedances

Before discussing impedance interpretation, it is as well to be clear about the rotor conditions in which the classical impedance concept, and the technique of 'impedance-matching' are valid. An impedance, in the classical sense, exists if an excitation at a particular frequency results in a response at the same frequency and only at that frequency. For a rotor which is excited in a fixed direction the response in a fixed direction will satisfy this classical impedance definition only in the hovering condition, and provided the rotor has three or more blades. In forward flight (or with a two-blade rotor in hovering flight), an excitation at a particular frequency will give rise to responses both at the original frequency and also at other frequencies which are dependent on the rotational speed. The measurement of rotor impedances is only valid, therefore, in the hover, and this restricts its application to the investigation of hover phenomena such as ground-resonance and hovering air-resonance.

Turning now to impedance interpretation, let us assume that a complete set of impedance measurements has been made on a rotor at a rotor speed and collective pitch appropriate to a full scale condition. Eight impedances have been measured as functions of frequency, and symmetry allows a (4 x 4) impedance matrix to be obtained (Table 1). The question now is how to use the measured data. If the measurements have been made in order to determine the stability behaviour of the rotor when it is mounted on a specific fuselage, then it is necessary to derive, either by measurement or calculation, a (4 x 4) impedance matrix of the fuselage for shaft motions corresponding to those of the rotor measurements. For a rigid fuselage in the hover, for example, the impedance terms will be simple frequency functions of the mass and moments of inertia of the fuselage. The procedure is to add the rotor and fuselage impedance matrices term by term (and, within each term, frequency by frequency). The resulting matrix (Table 2) contains all the data required to calculate the dynamic behaviour of the combined rotor fuselage system. If the elements of the matrix have been derived as analytical expressions, the complex roots of the system could be obtained by equating the determinant of the matrix to zero. However, with the matrix in numerical form, probably the easiest approach is to calculate the response of some point in the system to a sinusoidal force at each frequency for which data have been measured. The resulting vector can then be

Table 1

MATRIX OF ROTOR IMPEDANCES IN HOVER

FORE & AFT TRANSLATION x	LATERAL TRANSLATION y	PITCH θ	ROLL ϕ
f_{xx}	$f_{xy}\ (=-f_{yx})$	$f_{x\theta}$	$f_{x\phi}\ (=f_{y\theta})$
f_{yx}	$f_{yy}\ (=f_{xx})$	$f_{y\theta}$	$f_{y\phi}\ (=-f_{x\theta})$
$m_{\theta x}$	$m_{\theta y}\ (=m_{\phi x})$	$m_{\theta\theta}$	$m_{\theta\phi}\ (=-m_{\phi\theta})$
$m_{\phi x}$	$m_{\phi x}\ (=-m_{\theta x})$	$m_{\phi\theta}$	$m_{\phi\phi}\ (=m_{\theta\theta})$
↑ MEASURED		↑ MEASURED	

Table 2

ANALYSIS OF IMPEDANCE DATA

ROTOR IMPEDANCES FUSELAGE IMPEDANCES

$$\begin{bmatrix} f_{xx} & f_{xy} & f_{x\theta} & f_{x\phi} \\ f_{yx} & f_{yy} & f_{y\theta} & f_{y\phi} \\ m_{\theta x} & m_{\theta y} & m_{\theta\theta} & m_{\theta\phi} \\ m_{\phi x} & m_{\phi y} & m_{\phi\theta} & m_{\phi\phi} \end{bmatrix} + i\omega M \begin{bmatrix} 1 & 0 & \bar{z} & 0 \\ 0 & 1 & 0 & \bar{z} \\ \bar{z} & 0 & \left(p^2-\frac{g\bar{z}}{\omega^2}\right) & 0 \\ 0 & \bar{z} & 0 & \left(r^2-\frac{g\bar{z}}{\omega^2}\right) \end{bmatrix} = \left[\Phi\right]$$

EQUATIONS OF MOTION OF ROTOR + FUSELAGE –

(a) FREE OSCILLATIONS

$$\left[\Phi\right]\begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(b) FORCED OSCILLATIONS

$$\left[\Phi\right]\begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

MODAL FREQUENCIES AND DAMPING RATIOS GIVEN BY –

(a) ROOTS OF .

$$\det\left[\Phi\right] = 0$$

(b) ANALYSIS OF RESPONSE VECTOR

$$V_\alpha = \left[\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\right] \left[\Phi\right]^{-1}\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

analysed by standard numerical or graphical methods to obtain the mode frequencies and damping ratios. This approach has the advantages that it presents the results in a form familiar to the dynamicist, it gives a physical feel for the behaviour of the system, and it indicates an instability by revealing any modes that have negative damping. The latter will be characterised on the vector plot by modal arcs which rotate anti-clockwise with increase of frequency. The disadvantage of the method is that it implicitly involves inverting the impedance matrix, and this is likely to emphasise experimental errors which, even with the most stringent efforts to obtain measurement accuracy, may well be significant at the modal frequencies of the system.

To appreciate why this should be so, it is instructive to examine a simple, if impractical, rotor-fuselage system. Let us postulate a hovering helicopter in which the only motional freedom of the rotor shaft is in a fore-and-aft direction and the only blade mode is a lag mode. To assess the stability of the machine it is only necessary to measure a single impedance - the fore-and-aft force per unit fore-and-aft velocity. The impedance may have the form of Fig. 4. There will be two peaks resulting from excitation of the blade lag mode and the frequencies at which these peaks occur will be when the applied excitation is equal to the shaft-fixed lag mode frequency plus and minus the rotational speed. It is important to note that the impedance peaks occur at frequencies dependent on the shaft-fixed lag mode frequency. If, however, the impedance is inverted, so that the velocity response per unit input force is obtained, the two peaks are then related to the shaft-free lag mode frequency. The stability of the rotor-fuselage system is given by the damping characteristics of the shaft-free modes, and in the case shown in Fig. 4, an analysis of the two resonance peaks of the response curves would indicate the characteristics of the rotor, with no fuselage, if allowed to translate freely in a fore-and-aft direction. The peaks of interest are associated with minimum values of the measured impedances, and this is obvious intuitively because of the inverse relationships between impedance and response. Thus the accuracy with which the desired characteristics can be obtained depends on the accuracy with which minimum values of impedance can be measured. This emphasises the need for a high degree of accuracy in the force-measurement system. In a practical case the position may be rather better than that shown in Fig. 4. If, in the example given, the fuselage mass is realistically represented, and the procedure is followed of adding the impedance of this mass to the rotor impedance

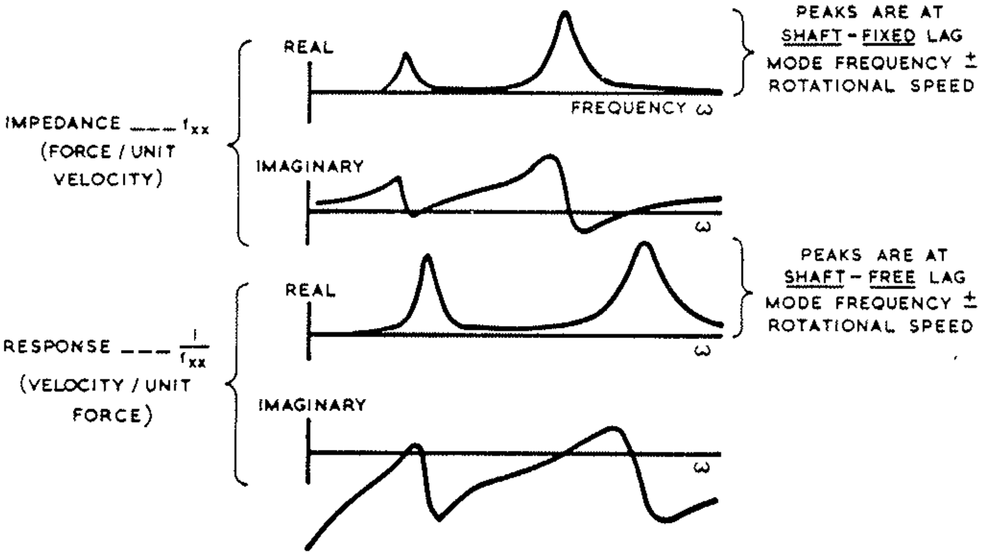

Fig 4 EXAMPLE OF IMPEDANCE INVERSION

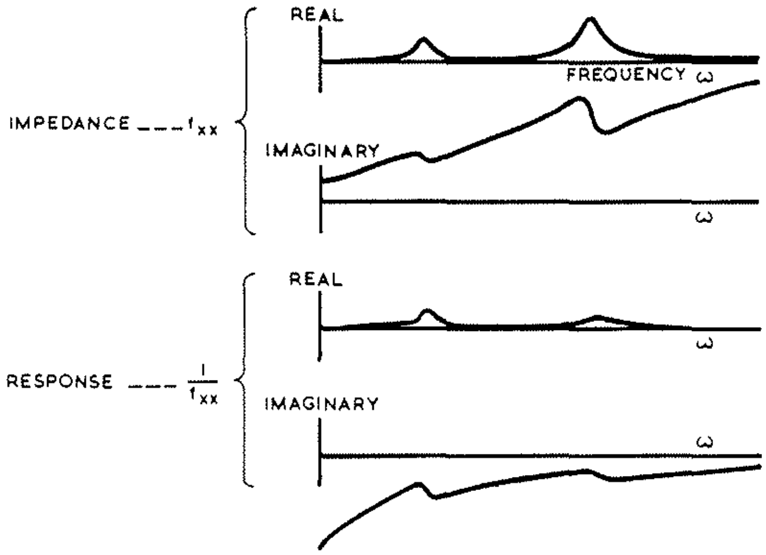

Fig 5 IMPEDANCE AND RESPONSE OF ROTOR + FUSELAGE

and then inverting the sum, the resulting response curves (Fig. 5) have resonance peaks close to the peaks of the original rotor impedance curves. This is so because the addition of fuselage mass reduces the free-shaft frequencies, and, in the limit, with a fuselage of infinite mass the two conditions will be identical. If, therefore, the fuselage impedances are large compared with the rotor impedances, the frequencies of the modes of interest in stability assessments tend to be fairly close to the peaks of the rotor impedances, where a high order of measurement accuracy can be obtained. In the case of ground resonance, however, the frequency of the critical fuselage support mode will be close to that of the rotor lag mode, and the fuselage impedances will be small in this frequency range. Stability assessments will then be very sensitive to measurement inaccuracies.

6 Test Results

We shall now examine one or two typical test results, which demonstrate the characteristics that have been described earlier.

Fig. 6 shows the real and imaginary parts of the pitching moment impedance due to pitch, for a four-blade non-articulated rotor model operating over a range of rotational speeds at zero collective pitch. At zero rotational speed, the two peaks are the blade fundamental and first overtone flap modes, and they can be seen to have quite low damping. As the rotational speed is increased, each mode is excited at two frequencies, and the variation of the actual modal frequency with rotational speed can be traced as the mean of the two frequencies at which the resonance peaks occur. It can be seen that the damping rapidly increases with rotational speed, particularly in the fundamental mode, so that at the higher speeds it is barely possible to detect either mode. In addition to these dominant modal features there can be seen many relatively small features, some of which are due to identifiable modes of the rotor system, whilst others may arise from features of the measurement and analysis technique.

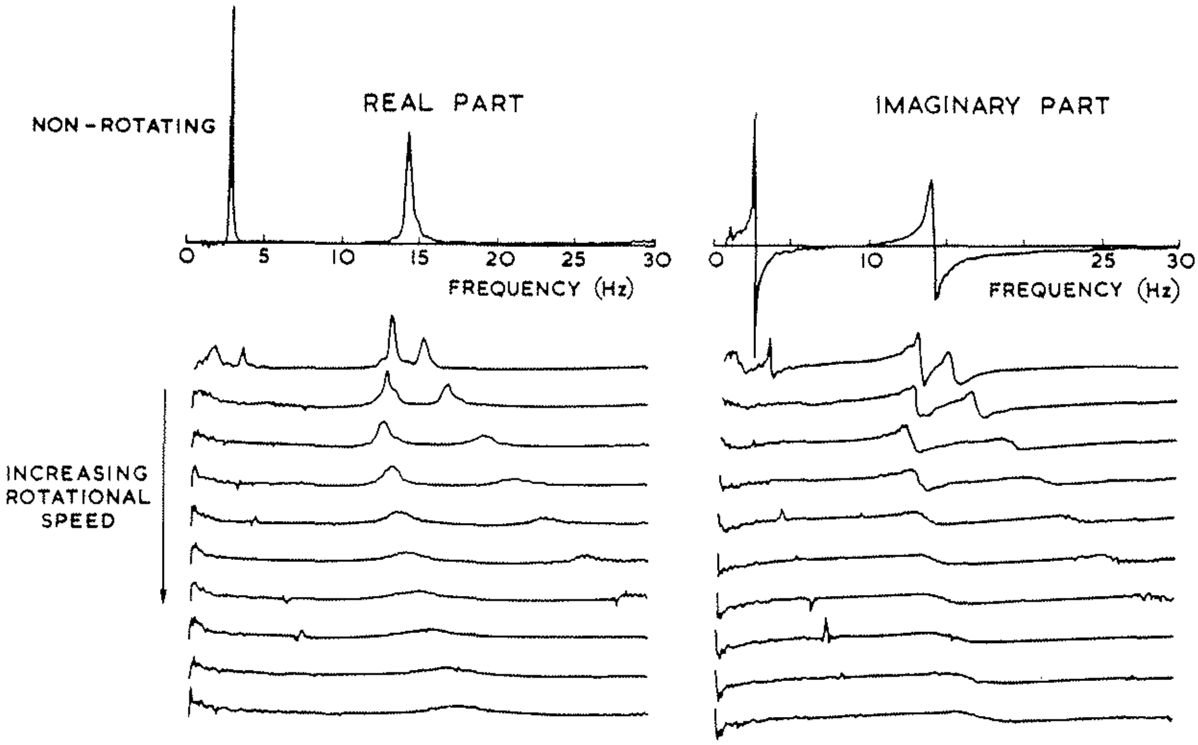

Fig 6 PITCHING MOMENT IMPEDANCE DUE TO PITCH - $m_{\theta\theta}$

In contrast, Fig. 7 shows a corresponding set of fore-and-aft impedances due to fore-and-aft translation. Here one mode, the fundamental blade lag mode, dominates the picture. The damping at zero rotational speed is not high, although the presentation of all the impedance values at the same scale in Fig. 7 tends to conceal this fact. As the rotational speed increases, the

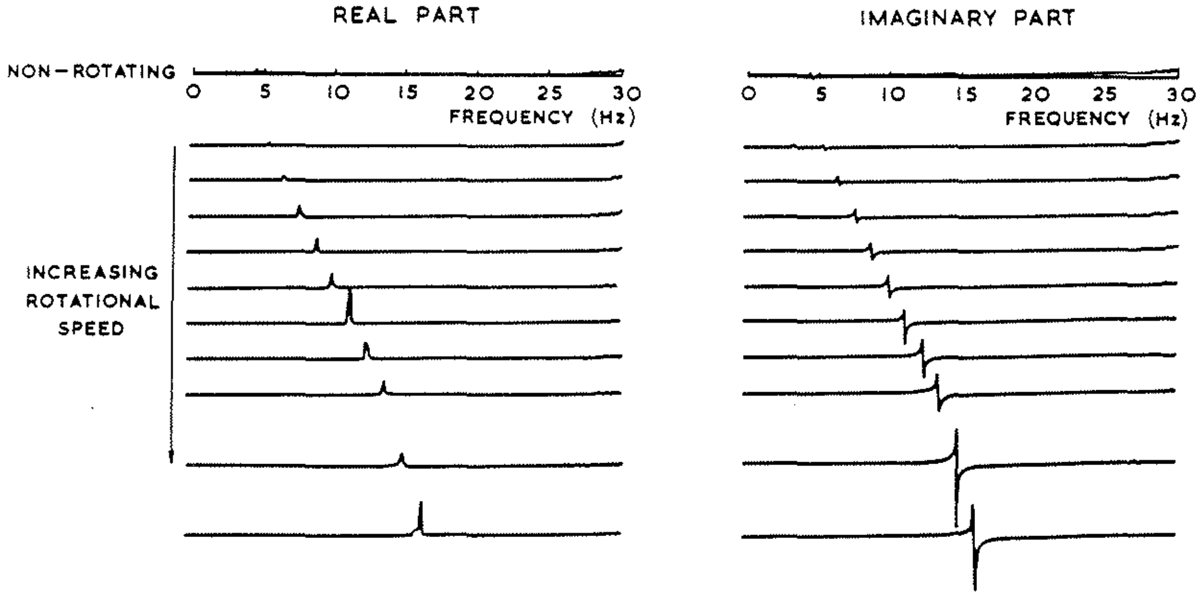

Fig 7 FORE-AND-AFT IMPEDANCE DUE TO FORE-AND-AFT TRANSLATION - f_{xx}

damping falls, and the peak impedance values increase (Fig. 7 does not present a true picture in this respect because the curve plotter does not interpolate a peak between computed values). The excitation at two frequencies can just be detected at low rotational speeds, but at higher speeds, the second peak is difficult to detect.

7 Application of Test Results

It is still far from obvious how to use the test results to maximum advantage, and work is continuing on several lines. It has been found, not entirely unexpectedly, that to throw a complete matrix of impedance measurements into a computer program is unrewarding, to say the least. The resulting vector response data, which should, ideally, enable the frequency and damping characteristics of all the rotor/fuselage modes to be extracted, tend to have a high degree of superimposed scatter, for the reasons that have been mentioned earlier. It is clear that there must be some form of initial data treatment before proceeding to a stability analysis. Two approaches, that are currently under examination, are (i) reducing the number of degrees of freedom of the rotor shaft in the preliminary analysis, and (ii) smoothing the test results in the frequency range of interest. Adopting the first of these approaches in a ground resonance investigation, one might start by considering only the impedances dominated by lag-mode behaviour, and following this by including the other impedances in several stages to determine the importance of their contribution to the analysis. The second approach, of smoothing the test results, is aimed at overcoming some of the shortcomings of measurement accuracy. The risk here, of course, is that the smoothing process might remove some essential feature of the data, and it is for this reason that one needs to have a good engineering feel for the physical interpretation of the measured data. At this stage, comparisons with the calculated impedances can be extremely useful, because they can guide the analyst into making correct decisions on how to 'improve' the raw data.

Another interesting possibility is that of finding analytical functions, the values of which fit the measured impedances in a specified frequency range, and then using these expressions to obtain a stability solution. The shapes of many of the impedance curves suggest that fairly simple functions could be used to obtain an accurate curve fit, and this has been found to be the case. Yet another method of analysis, suggested by W.R. Walker[2], is to obtain the modal characteristics from a vector plot of the inverse of the determinant of the impedance matrix; this has the virtue of avoiding the inversion of the matrix.

It is evident that a good deal of investigation into these and other possible methods of analysis needs to be done before a valid assessment can be made of the value of impedance measurements.

Direct comparison of measured and calculated rotor impedances has indicated that it is extremely difficult to improve a mathematical model so as to eliminate or minimise differences between measured and calculated values. The difficulties arise from the large number of terms in the equations of motion and in choosing which terms to manipulate to produce a desired effect on the impedances. At the moment, therefore, impedance measurements are seen as a useful check of theory but not as a path to the improvement of theory.

8 Concluding Remarks

To summarise the work that has been done at RAE on rotor impedance measurements, it may be said, taking a narrow view, that a technique has been developed which can obtain, within a relatively short time scale, all the measurements on a model that are needed for an assessment of ground and air resonance instabilities, or for comparison with theory. What has not yet been demonstrated is whether the measurement accuracy is sufficiently high to overcome the inherent problems of impedance interpretation, or whether skilful handling of the measurement data can compensate for the experimental weaknesses. Taking a wider view, however, there is no doubt that the work that has been done has very considerably advanced the state-of-the-art, in rotor model testing, in respect of rotor modelling, test techniques and analysis methods. It has also given a new insight into the dynamic behaviour of rotating systems, which many people find difficult to acquire from wholly theoretical considerations. Finally, it has opened the door to several dynamics investigations which might otherwise not have been made. For example, using the rig simply as a shake test facility, it is easy to establish blade mode frequency variations with rotor speed, and, with a small modification to the rig, modal dampings can also be obtained from strain gauges at the blade root. This capability enables one to check such important questions as the amount of damping in the fundamental lag mode that can be obtained as a result of flap-lag couplings. This, and similar investigations, are now being pursued, and will certainly add to our knowledge and understanding of the dynamics of rotors.

REFERENCES

1. R. Cansdale, D.R. Gaukroger and C.W. Skingle, A Technique for measuring impedances of a spinning rotor, RAE Tech Report 71092 (May 1971).

2. W.R. Walker, To assess the stability of a helicopter in air resonance from the impedances of the rotor and fuselage, Unpublished MOD(PE) Report