24th EUROPEAN ROTORCRAFT FORUM
Marseilles, France – 15th – 17th September 1998

REFERENCE: AE 13

Two-dimensional approximation in the Laplace domain to unsteady aerodynamic of rotary wing

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This paper generalises the Loewy formulation of the aerodynamics of an oscillating rotary wing airflow to the Laplace domain. Loewy postulated a two-dimensional model for the representation of a harmonically oscillating rotary wing airfoil operating at low inflow; forward speed effects were not considered. The transfer function relating the arbitrary airfoil motions to the airloads are derived from the Laplace transforms of the linearised airload expressions for incompressible two-dimensional flow. The transfer function relating the motions to the circulatory part of these loads is recognised as the Loewy's function, extended to complex values of the reduced frequency, and it is termed Loewy's generalised function. A brief review of unsteady aerodynamics in the Laplace domain is given. The ability to calculate airloads for complex values of the reduced frequency allows its application in active control techniques for rotary wings.

1. SYMBOLS

b	blade semichord.
C	generalised Theodorsen's function in the Laplace domain.
C'	modified lift deficiency function for rotors (generalised Loewy's function).
h	vertical distance between successive rows of vorticity.
k	reduced frequency.
n	rotor revolution index.
p	Laplace variable
Q	number of blades in the rotor.
r	blade section radius.
t	time.
u	inflow velocity.
w_a	vertical component of induced flow ("downwash").
V	steady resultant velocity, Ωr.
x	dimensionless ordinate in stream direction along the airfoil.
X	ordinate in stream direction along the airfoil.
z	dimensionless ordinate normal to the stream direction
Z	ordinate normal to the stream direction.
Γ	total circulation around the airfoil.
Γ_O	quasi steady circulation around the airfoil.
γ	line strength of continuously distributed vorticity.
$\eta, \lambda, \xi, \zeta$	Dummy variables.

Subscripts

a	quantity associated with reference airfoil.
n	rotor revolution index
q	rotor blade index.

2. INTRODUCTION

A topic of current interest in the aeronautical community for rotary wings aircraft is the noise and vibration control. Active control of aeromechanical stability is currently investigated since the beginning of this decade. Traditional methods of active control require performing the study in the Laplace domain.

Current unsteady aerodynamic theories for aeroelastic and aeroservoelastic applications, as active flutter control, gust loads alleviation, fatigue reduction or ride controls, are confined to assuming simple harmonic oscillations. This is a severe restriction for the control engineer trying to synthesise active control logic. Nevertheless, active flutter control laws have been formulated using these techniques.

It is shown unnecessary the restriction of unsteady aerodynamic theory to simple harmonic oscillations [1, 2, 3]. Only, one must resort to a consistent formulation of the unsteady aerodynamic in the Laplace domain.

Edwards [1, 2, 3] and Ueda [4] have contributed to Laplace domain aerodynamic in the two-dimensional and three-dimensional area, respectively. The former, using the classical theories of Theodorsen [5] and Garrick y Rubinow [6], obtained the corresponding p-domain solutions for incompressible and supersonic flows. Ueda generalised his doublet point method [7] to the p-domain for three-dimensional subsonic lifting surfaces. García-Fogeda [8] developed a pressure modes method to calculate the unsteady aerodynamic forces for airfoils in arbitrary motions on compressible subsonic flows. The forces are evaluated numerically in the

Laplace domain by a semianalytical method for computing the kernel. First Possio's integral equation was generalised to p-domain, and the resulting kernel was recast to display explicitly its singularities. Then, the kernel was integrated numerically, but the singular terms which were integrated analytically. The pressure modes method is used and the integral equation is evaluated at a set of discrete points. Once pressure distribution is known, generalised aerodynamic forces are calculated by integrating analytically the pressure.

López-Díez and García-Fogeda [9] formulated the sonic unsteady flow in the Laplace domain, and then, estimated the flutter condition evolution from subsonic to supersonic, through the sonic regime. The velocity potential in the Laplace domain was derived following the procedure of Stewartson [10], as it was described by Edwards [1].

The helicopter, or autogiro, rotor is subject to unsteady flow phenomena in many phases of its operation. Unlike fixed-wing aircraft, the rotor experiences oscillatory aerodynamic effects even in steady, gustless forward flight. This is fundamental for rotary wings since variations in relative airspeed occur at a blade section as the blade traverses the azimuth. In addition, flapping and blade pitch angles are made to vary with azimuth in order to tilt the thrust vector in the desired direction.

Loewy [11] studied the unsteady effects in rotors including the returning wake. By way of introduction to this topic, it is useful to consider the physics of the flowfield in the vicinity of a rotor in hover or vertical flight. Such a rotor has an axial component of velocity through the disk plane, accruing from the thrust that the rotor is developing and from the axial translation speed [12].

The predominant feature of the rotor flow field is the strong trailing tip vortex, which is blown downward at the axial velocity to form a contracting helix. In general, the rotor blade develops oscillating loads that, from the basic theory, cause a radial distribution of vorticity to be shed from the trailing edge. This shed vorticity lies on a helical surface. It must be mentioned that the flow description is further complicated.

In order to analyse the flow, some simplifying assumptions could be invoked [11, 12]. Loewy [11] assumed a low inflow rotor, thence, the helical surfaces "stack up" one on another, separated a distance ($2\pi u/Q\Omega$), being u the axial velocity, Q the number of blades and Ω the rotor angular velocity. Historically, the Loewy's analysis was the first successful attempt to incorporate the returning wake into the unsteady aerodynamics problem. Only hovering/axial flight was considered, which enabled a closed form solution.

Loewy's solution has been commonly used in the rotor blades loads analyses and in aeroelastic stability analyses. Johnson [13] related the variables used in unsteady harmonic airfoil theory to the variables used in describing the motion of a rotary wing, which was improved by Kaza [14], pointing out his application to wind turbine blades [15, 16] and non uniform rotor blades in forward flight [17]. Bidimensional models using Theodorsen formulation have been implemented to analyse the airfoil response in a pulsating stream [18].

In the present paper, the velocity potential in the Laplace domain is calculated as in [9], using the formulation of the aerodynamic model proposed by Loewy [11]. From the original postulation of small inflow, the vertical angularity of the wake with respect to the plane of the rotor blade can be neglected. Furthermore, consistent with the idea that only the vorticity near the blade section has an important effect, one may allow planar rows of vorticity to extend to infinity in horizontal direction in order to achieve mathematical simplification.

Therefore, it is assumed that the returning wake can be represented by a countable infinite number of infinite (in length) sheets of vorticity, parallel to the rotor plane as sketched in Fig. 1. Since not all the vorticity has been shed by one blade, the vortex rows must be identified by two indices; n indicates which revolution a given row is associated with, while q reveals which blade has shed the vorticity.

3. AERODYNAMIC MODEL

The linearised equation satisfied by the velocity potential, ϕ, for unsteady incompressible flow is:

$$\nabla^2\Phi(X,Z;t)=0 \tag{1}$$

subject to the two-dimensional boundary condition along the airfiol chord, $2b$:

$$\left.\frac{\partial\Phi}{\partial Z}\right|_{Z=0}=W_a(X;t)=\frac{\partial Z_a}{\partial t}+(\Omega r)\frac{\partial Z_a}{\partial X} \tag{2}$$

Kutta's hypothesis of finite, continuos velocities and pressures at the trailing edge must be imposed. Also continuity of velocities and pressures on the wakes should be satisfied. Velocity potential must be zero far from the airfoil.

Superposition of elementary solutions is normally used to calculate Φ. The potentials due to camber, thickness and steady angle of attack are, therefore, assumed known, and the solution due to the dynamic airfoil motion is sought with respect to the flat plate airfoil. This situation is illustrated in Fig. 1, for a two-degrees-of-freedom typical section in a two-dimensional air stream.

The calculation of unsteady airloads due to simple harmonic motion has traditionally begun with the substitution $\Phi(x,z,t)=\underline{\Phi}(x,z)e^{i\omega t}$, which it is equivalent to applying the Fourier integral transform to the time variable of Eq. (1), and boundary conditions. Dimensionless formulation will be presented, being, b the semichord, the longitude of reference, and Ωr, the velocity. The Laplace variable, p, is dimensionless as it is usual for the reduced frequency k, $k=\omega b/U_\infty$.

In attempting to derive solutions with general time dependence, it is natural to apply the Laplace integral transform. By defining:

$$\phi(x,z;p)=\int_0^\infty \Phi(x,z;t)e^{-pt}dt \qquad (3)$$

the Eq. (1) is transformed into:

$$\nabla^2\phi(x,z;p)=0 \qquad (4)$$

and boundary condition along the airfoil:

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=0}=w_a(x;p)=(p+(\Omega r)\frac{\partial}{\partial x})\mathcal{L}\left[Z_a(x,t)\right] \qquad (5)$$

The potential $\phi(x,z;p)$ can be obtained as sum of one satisfying the initial conditions, and one which satisfies a boundary value problem.

If exact transient response were desired, the term relating the initial conditions should also be calculated. In what follows, since we are interested in the stability problem only, initial conditions, that appear when the Laplace transform is applied, need not to be considered.

4. VELOCITY POTENTIAL

By use of two-dimensional vortex sheets, the following integral equation relating vortex strength and Laplace transform of the airfoil velocity is obtained:

$$w_a(x;p)=\frac{-1}{2\pi}\left[\int_{-1}^{1}\frac{\gamma_a(\xi,p)d\xi}{x-\xi}+\int_{-1}^{1}\frac{\gamma_{00}(\lambda,p)d\lambda}{x-\lambda}+\right.$$
$$+\sum_{q=1}^{Q-1}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\frac{\gamma_{nq}(\zeta,p)(x-\zeta)d\zeta}{(x-\zeta)^2+(nQ+q)^2h^2}+$$
$$\left.+\sum_{n=1}^{\infty}\int_{-\infty}^{+\infty}\frac{\gamma_{n0}(\eta,p)(x-\eta)d\zeta}{(x-\eta)^2+(nQ)^2h^2}\right]$$

The vorticity along the reference airfoil, γ_a, and that in the wake are differentiated from each other by writing wake vorticity as γ_{00} for the attached sheet, and γ_{nq} for the sheet shed by the qth blade in the nth preceding rotor revolution. Note that 0 for the blade index is associated with the reference airfoil.

It should be stated that this is a first order theory, so that such effects as that of the wake upon itself have been neglected. Since the shed vorticity is being accounted for as a perturbation in the main flow, however, this component of induced velocity will be small compared to the free-stream velocity and must react with small, first order, airfoil displacement.

5. VORTEX DISTRIBUTION

The strengths of the trailing and returning wakes are then related to the strength of the bound vorticity, $\Gamma(p)$, by means of:

- The phasing of the motion between the rotor blades, ψ_q.
- The spacing of the returning wakes: $hb=2\pi u/Q\Omega$, being u the axial velocity thorough the disk, depending on the thrust.
- The number of rotor blades, Q.

Since a vortex is shed at the trailing edge of an airfoil with the instantaneous change in the total airfoil circulation, Γ, the following may be expressed for the reference airfoil:

$$\gamma_{00}(1,p)=-p\Gamma \qquad (7)$$

The vortex element at a general point λ was shed in the past at a moment determined by the time interval $(\lambda-1)b/\Omega r$ required for it to reach λ. In the Laplace domain this must be considered as a delay, which means:

$$\gamma_{00}(\lambda,p)=-pe^{p}e^{-\lambda p}\Gamma \qquad (8)$$

The vorticity shed by the qth blade n revolutions ago can be related to that of the reference revolution, n=0, by the time inverted in describing these n revolutions, $\Delta t=2\pi n/\Omega$.

Therefore

$$\gamma_{nq}(\lambda,p)=e^{-2\pi nr'p}\gamma_{0q}(\lambda,p) \qquad (9)$$

being $r'=r/b$.

In addition, a vortex shed from the trailing edge of the qth airfoil is related to the change in circulation of this airfoil by:

$$\gamma_{0q}(1-2\pi r'q/Q,p)=-p\Gamma_q \qquad (10)$$

If the amplitude of the circulation of the qth airfoil is equal to that of the reference airfoil but leading it in time by the phase ψ_q.

$$\gamma_{0q}(\lambda,p)=-pe^{(p-2\pi r'qp/Q+i\Psi_q)}\Gamma \qquad (11)$$

Therefore,

$$\gamma_{0q}(\lambda, p) = -pe^{(p-2\pi r'qp/Q+i\Psi_q)}e^{-\lambda p}\Gamma \quad (12)$$

And, considering the time inverted in describing n revolutions:

$$\gamma_{nq}(\zeta, p) = -pe^{(p-2\pi r'qp/Q-2\pi nr'p+i\Psi_q)}e^{-\zeta p}\Gamma \quad (13)$$

$$\gamma_{n0}(\eta, p) = -pe^{(p-2\pi nr'p)}e^{-\lambda p}\Gamma \quad (14)$$

Substituting the vortex relations in the boundary condition:

$$w_a(x,p) = \frac{-1}{2\pi}\left[\int_{-1}^{1}\frac{\gamma_a(\xi,p)d\xi}{x-\xi} - pe^p\Gamma\int_{-1}^{1}\frac{e^{-p\lambda}d\lambda}{x-\lambda} - pe^p\Gamma \times \left\{\sum_{\substack{n=0 \\ q=1}}^{\substack{\infty \\ Q-1}} e^{-2\pi r'qp/Q-2\pi nr'p+i\Psi_q}e^{-px}\int_{-\infty}^{+\infty}\frac{e^{-p(\zeta-x)}(x-\zeta)d\zeta}{(x-\zeta)^2+(nQ+q)^2h^2} + \sum_{n=1}^{\infty}e^{-2\pi nr'p}e^{-px}\int_{-\infty}^{+\infty}\frac{e^{-p(\eta-x)}(x-\eta)d\zeta}{(x-\eta)^2+(nQ)^2h^2}\right\}\right]$$

This expression relates the boundary condition along the airfoil to the total circulation and the bound vorticity.

Following the procedure in [11], this condition can be expressed as:

$$w_a(x,p) = \frac{-1}{2\pi}\left[\int_{-1}^{1}\frac{\gamma_a(\xi,p)d\xi}{x-\xi} - pe^p\Gamma\int_{-1}^{1}\frac{e^{-p\lambda}d\lambda}{x-\lambda} + \pi pe^pe^{-px}\mathcal{W}\right] \quad (16)$$

Note that the total effect of the infinite number of vortex sheets below the plane of the reference airfoil is contained in the third term on the right hand side of this equation. The function, $\mathcal{W}(p,h,Q,\Psi_q)$ may be thought of as a weighting function for the vorticity shed by preceding blades and/or in previous revolutions. This function was obtained by Loewy [11]. Consistent with the idea that only the vorticity near the blade section has an important effect, in the present case, harmonic vortex distribution has been assumed in the wakes, it is to say, only the imaginary part of p is retained in the last two integrals of equation (15). This hypothesis has been applied only to obtain the weighting function. This is in accordance with the discussion presented in [2] about the convergence of the airfoil motion.

The form of the integral downwash equation (16) may be solved directly by applying Söhngen's inversion formula.

Söhngen showed the solution of an equation of the form:

$$g(x) = -\frac{1}{2\pi}\int_{-1}^{1}\frac{f(\xi)d\xi}{x-\xi} \quad (17)$$

to be

$$f(x) = \frac{\pi}{2}\sqrt{\frac{1-x}{1+x}}\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}\frac{g(\xi)d\xi}{x-\xi} \quad (18)$$

Hence:

$$\gamma_a(x,p) = \frac{2}{\pi}\sqrt{\frac{1-x}{1+x}}\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}\frac{w_a(\xi,p)d\xi}{x-\xi} + \frac{p\Gamma e^p}{\pi}\sqrt{\frac{1-x}{1+x}}\int_{1}^{\infty}\sqrt{\frac{\lambda+1}{\lambda-1}}\frac{e^{-p\lambda}}{x-\lambda}d\lambda + \frac{ip\Gamma e^p\mathcal{W}}{\pi}\sqrt{\frac{1-x}{1+x}}\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}\frac{e^{-p\xi}}{x-\xi}d\xi \quad (19)$$

Performing the second integration on the right hand side of the above equation, and evaluating the circulation over the airfoil, Γ, from:

$$\Gamma(p) = \int_{-1}^{1}\gamma_a(x,p)dx \quad (20)$$

the total bound circulation on the airfoil is:

$$\Gamma(p) = \frac{\Gamma_0(p)}{pe^p\{[K_0(p)+K_1(p)] - i\pi\mathcal{W}[I_0(p)-I_1(p)]\}} \quad (21)$$

being Γ_0 the quasy-steady circulation, defined as:

$$\Gamma_0(p) = -2\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}w_a(\xi,p)d\xi \quad (22)$$

as was defined by Theodorsen[19].

$K_v(z)$ and $I_v(z)$ are the modified Bessel function of second and first kind of imaginary argument respectively [20].

$$K_0(z) = \int_{1}^{\infty}\frac{e^{-zt}}{\sqrt{t^2-1}}dt \quad (23)$$

$$K_1(z) = -\frac{dK_0(z)}{dz} = \int_{1}^{\infty}\frac{te^{-zt}}{\sqrt{t^2-1}}dt \quad (24)$$

$$I_0(z) = \frac{1}{\pi}\int_{-1}^{+1}\frac{e^{-zt}}{\sqrt{1-t^2}}dt \quad (25)$$

$$I_1(z) = \frac{dI_0(z)}{dz} = \int_{-1}^{+1} \frac{-te^{-zt}}{\sqrt{t^2-1}} dt \quad (26)$$

$$K_0(z) + K_1(z) = \int_1^{\infty} \sqrt{\frac{t+1}{t-1}} e^{-zt} dt \quad (27)$$

$$I_0(z) - I_1(z) = \frac{1}{\pi} \int_1^{\infty} \sqrt{\frac{1+t}{1-t}} e^{-zt} dt \quad (28)$$

6. PRESSURE COEFFICIENT

Dimensionless Bernoulli's equation for unsteady motion permits relating the pressure coefficient with the vortex distribution. If Laplace operator is applied to this equation, it results as follows:

$$\Delta c_p(x;p) = -\gamma_a(x;p) - p\int_{-1}^{x} \gamma_a(\xi;p)d\xi \quad (29)$$

This equation can be evaluated in terms of downwash on the airfoil, $w_a(\xi;p)$, by substituting equations (19) and (21) in the appropriate form. It can be shown that the equation for the unsteady pressure distribution becomes:

$$\Delta c_p(x;p) = \frac{2}{\pi}\int_{-1}^{+1}\left[\sqrt{\frac{1-x}{1+x}}\sqrt{\frac{1+\xi}{1-\xi}}\frac{1}{x-\xi} - pA_1(x,\xi)\right]w_a d\xi$$
$$\frac{2}{\pi}[1-C'(p)]\int_{-1}^{+1}\sqrt{\frac{1-x}{1+x}}\sqrt{\frac{1+\xi}{1-\xi}}w_a(x,\xi)d\xi$$

Where

$$A_1(x,\xi) = \frac{1}{2}\ln\left[\frac{1-x\xi+\sqrt{1-\xi^2}\sqrt{1-x^2}}{1-x\xi-\sqrt{1-\xi^2}\sqrt{1-x^2}}\right] \quad (31)$$

and

$$C'(p) = \frac{K_1(p)+i\pi W I_1(p)}{[K_0(p)+K_1(p)]-i\pi W[I_0(p)-I_1(p)]} \quad (32)$$

Equation (30) may be recognised as being in the same form as the corresponding equation for the two-dimensional fixed-wing incompressible flow [2].

The function $C'(p)$ of equation (32) is directly analogous to the generalised Theodorsen's lift deficiency function, $C(p)$ given in [2]. All previous derivation for lift and moment remain unchanged by substituting the Generalised Theodorsen's function for the generalised Loewy's function, $C'(p)$.

7. CONCLUSIONS

Generalised Loewy's lift deficiency function has been presented. For classical airfoil in supersonic unsteady forces in the Laplace domain can be obtained from the harmonic results, replacing the reduced frequency, *ik*, by the Laplace variable, *p*, [2]. This technique is not applicable to the incompressible flow. In this case, the lift deficiency function is similar to the Theodorsen's function replacing the reduced frequency, *ik*, by the Laplace variable, *p*, and the Hankel functions by the corresponding Bessel's functions of second kind, K_ν.

The result obtained here for rotary wing is consistent with this for fixed wings. The lift deficiency function in the Laplace domain is the corresponding Loewy's function replacing the reduced frequency, *ik*, by the Laplace variable, *p*, and the Hankel functions by the corresponding Bessel functions of second kind, K_ν, and J_ν by I_ν.

8. ACKNOLEDGMENTS

This work has been supported by Fundación Juan de la Cierva, under contract number 43700810035.

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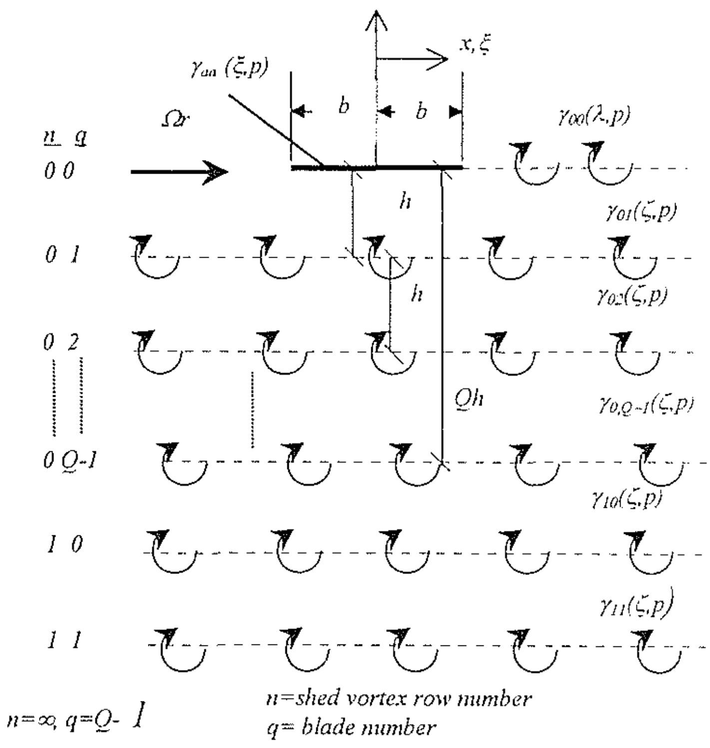

Fig. 1. Sketch of the Loewy problem. Aerodynamic model for a rotor. Notation for mathematical analysis is shown.