DEMONSTRATION OF CONTROLLER WITH OBSERVER FOR THE ANSAT AIRCRAFT

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Abstract

In this work, the potential use of a controller combined with observer, is assessed for helicopter applications. The idea behind the controller is to combine classic control theory with the benefits offered by an observer to achieve helicopter control with characteristics superior to a typical PID controller. The results show that the controller with observer can filter out disturbances and achieve rapid change of states for all cases tried in this work. The PID controller was tuned using locally-optimal control theory and the Linear Matrix Inequalities (LMI) method taking into account the constraints on control signals and state vector. The estimation of the disturbance vector, corresponding to non-modeled nonlinear helicopter characteristics and wind, is conducted using the observer. The results show that the controller in combination with disturbance observer can filter out disturbances, and allows for rapid changes in the state in all cases considered in this work.

Nomenclature

θ, ϕ, ψ	pitch, roll and yaw angles (deg)	$\lambda_0, \lambda_{eq}, \lambda_t$	main rotor, equivalent rotor, and tail rotor inflow
u, v, w	longitudinal, lateral and normal velocity (m/sec)	$\bar{\upsilon}_i$, $\bar{\upsilon}_{ti}$	average relative induced velocity of main and tail rotor (m/sec)
p, q, r	roll, pitch and yaw rate (deg/sec)	$\delta_{lon}, \delta_{lat}$	lateral and longitudinal cyclic of the swash plate caused by the atmospheric turbulence (deg)
R_m, R_t	main and tail rotor radii (m)	δ_{col}, δ_p	collective deviation of the main and tail rotor caused by the atmospheric turbulence (deg)
θ_0, θ_{0t}	main and tail rotor collective (deg)	V_r	mean wind speed in turbulence model (m/sec)
θ_{1c}, θ_{1s}	longitudinal and lateral cyclics (deg)	σ_w	root mean square value of vertical gust velocity (m/sec)
$a_{10\beta}, b_{10\beta}$	flapping coefficients of equivalent main rotor in stability axis coordinate system coordinate system	w_n	white noise with unit covariance
		ω_{rotor}, ω_t	main and tail rotor rotational speed, (deg/sec)
$b_{1\beta}$	blade flapping coefficient in stability axis coordinate system coordinate system	LMI	the Linear Matrix Inequalities method
		PID	the proportional-integral-derivative controller
		MR, TR	main rotor and tail rotor

1. INTRODUCTION

The development of control systems for helicopters is a complex task due to the large number of degrees of freedom of the vehicle, the highly nonlinear dynamics and the presence of cross-couplings between state variables [1]. Existing dynamic models for helicopters cannot capture all non-linearity and aeromechanical couplings of real vehicles. It is therefore important to develop robust controllers that will be insensitive to the fraction of dynamics not included in the design models, as well as, to external disturbances.

The need for a controller is also dictated by the high pilot workload during maneuvering flight, or flight near the edge of the helicopter envelope, or even due to adverse atmospheric conditions. The control augmentation system should lead to a reduction of pilot stress and workload and improved flight and handling qualities.

Over the past two decades, several approaches were put forward regarding the development of helicopter flight control systems. One of the proposed methods is robust control, which aims to develop controllers insensitive to uncertainties present in a system [2]. For example, Linear Quadratic Regulators (LQR) [3, 4] minimise a quadratic cost function with user-supplied weights. Although this approach guarantees robust stability, it has significant drawbacks. The choice of the necessary weighting matrices is an iterative procedure, which depends on the designer's experience and the accuracy of the model. Also, the standard LQR design does not put any restrictions on the input signal amplitudes and requires full state feedback that is not always realizable. In this case the Linear Quadratic Gaussian (LQG) method can be applied, which uses Kalman filters for the estimation of the state vector. LQG optimality does not automatically ensure good robustness properties [2], and the stability of the closed loop system must be checked separately after the LQG controller has been designed.

Another robust method is the H_∞ technique. It is inherently multivariable and provides robust stability for systems with uncertainties. This technique is often applied to helicopter flight controller design [5] and was successfully tested in-flight [6]. It also suffers from the need of weighting matrices, and the synthesis process is iterative. There are no generic methods for systematically and efficiently selecting the weighting matrices [7], though some authors are exploiting a genetic approach [7], [8] to solve a non-convex problem of finding weighting matrices along with the main optimization problem with more or less success.

The development of the H_∞ method is based on μ-synthesis, which quantifies the uncertainty and un-modeled dynamics present in the target system and models the noise characteristics of the sensor system, but it produces a controller of very high order [9].

In this paper, the LMI-based approach to the controller synthesis is considered, taking into account the constraints on the control and phase coordinates. To reduce the dimension of the problem, the use of a controller structure obtained via locally-optimal control approach [10] is proposed. To improve the accuracy of stabilization, the feedback on the estimation of the vector of reduced external disturbances along with feedback on the state vector is introduced. This estimation is made using an observer [11]. The problem of stabilizing the helicopter in hover, taking into account atmospheric turbulence, is also considered.

2. HELICOPTER MODEL

To simplify the model of the helicopter [12], the equations describing the helicopter motion with main and tail equivalent rotors are used.

Aerodynamic parameters of the main and tail rotors were determined using a mathematical model, which was established on the basis of the classic Glauert and Lock theory of a rotor with hinged blades [13].

In the model the following assumptions were made:

1. the induced velocity is uniformly distributed over the main rotor (MR) disc;
2. the lift slope of the MR blade section is linear;
3. the profile drag coefficient can be replaced by averaged value and identical for all blade sections;
4. blade tip losses are ignored;
5. a hingeless hub is considered (the MR torsion stiffness is taken into account in the model);
6. the dynamics of the hydraulic servo actuators of the main and tail rotors is neglected.

Similar assumptions are made for the tail rotor (TR).

In view of the assumptions, the equations in the fuselage coordinate system, and generalized form are described by nonlinear differential equations, in which the aerodynamic parameters of the main and tail rotors implicitly depend on the coordinates of the state and controls:

(1) $\dot{x} = f\left(x, \mathrm{u}, \varphi(z_0, z_t), \xi\right)$,

(2) $\varphi_0\left(z_0, x, \mathrm{u}\right) = 0$,

(3) $\varphi_t\left(z_t, x, \mathrm{u}\right) = 0$,

where $x = \left(u, v, w, p, q, r, \theta, \phi, \psi\right)^T \in R^n$ is the state vector of the system, $n = 9$; $\mathrm{u} = \left(\theta_{1c}, \theta_{1s}, \theta_0, \theta_{0t}\right)^T \in R^m$ is the vector of the swash plate and tail rotor attitude, $m = 4$; $\varphi(z_0, z_t) \in R^4$ is the vector of aerodynamic forces and moments obtained from the works [13,14]; $z_0 = \left(a_{10\beta}, b_{10\beta}, \lambda_0, \lambda_{eq}, b_{1\beta}, \bar{\upsilon}_i\right)^T \in R^6$ are the aerodynamic parameters of the MR, $z_t = \left(\lambda_t, \bar{\upsilon}_{ti}\right)^T \in R^2$ are aerodynamic parameters vector of the TR; $\xi \in R^4$ is the vector of atmospheric turbulence, which, according to [15], is modeled in the form of additional impacts causing change in the position of the swash plate controls.

Modelling of the helicopter dynamics using equations (1)-(3) is performed with a constant integration step. For each point in time the values z_0, z_t that satisfy equations (2), (3) are obtained from the known values x, u using Newton's method.

From equations (1)-(3) with $\dot{x} = 0$, $\xi = 0$ the trim values $x^*, \mathrm{u}^*, z_0^*, z_t^*$ are obtained with a given accuracy. Then from the equation (1) the simplified equation of deviations $\Delta x = x - x^*$, $\Delta \mathrm{u} = \mathrm{u} - \mathrm{u}^*$ from trim are:

(4) $\Delta\dot{x} = A\Delta x + B\Delta \mathrm{u} + D\,\mathrm{w}\left(x, \mathrm{u}, \xi\right)$,

where $B = [\tilde{B}^T \; 0_{m\times(n-s)}]^T$, $D = \left[I_s \; 0_{s\times(n-s)}\right]^T$, $\mathrm{w}\left(x, \mathrm{u}, \xi\right) = \tilde{B}\xi + \Delta\,\mathrm{w}\left(x, \mathrm{u}, \xi\right)$, $\Delta\,\mathrm{w} \in R^s$.

It is assumed that the initial deviation of the system and external disturbances are constrained:

(5) $\Delta x(t_0)\Delta x^T(t_0) \le Q_{\Delta x}$,

(6) $\mathrm{w}\mathrm{w}^T \le Q_{\mathrm{w}}$, $\dot{\mathrm{w}}\dot{\mathrm{w}}^T \le Q_{\dot{\mathrm{w}}}$,

where $Q_{\Delta x}$, Q_{w} are positive definite matrices of appropriate dimensions. Note that these restrictions are equivalent to the corresponding ellipsoid membership of the vectors (for example, $\Delta x(t_0) Q_{\Delta x}^{-1} \Delta x^T(t_0) \le 1$).

In addition, during flight, the deviation of controls from the autopilot is not more than 20% of a possible deviation, because the main function of control rests on the pilot. Therefore, further in the synthesis of the autopilot control law, the following restriction is taken into account:

(7) $\Delta \mathrm{u} \Delta \mathrm{u}^T \le Q_{\Delta \mathrm{u}}$

for some range of values Δx .

3. ATMOSPHERIC TURBULENCE MODEL

In this paper, atmospheric turbulence is modeled according to the approach, described in [15], [16]. Turbulence effects are obtained as additional control inputs by passing a white noise through an appropriate transfer functions, parameterized by main rotor diameter, angular velocity of the rotor, turbulence intensity, and mean wind speed. The transfer functions obtained for one helicopter model can be scaled for other using the technique in [15]. The turbulence model for the UH-60 rotorcraft was scaled for the Ansat aircraft of the Kazan Helicopter Plant at the following conditions: the mean wind speed was $V_r = 5{,}144$ m/sec, and the turbulence intensity was $\sigma_{\mathrm{w}} = 1{,}03$ m/sec. The transfer functions for the control inputs of the UH-60 were also scaled for the Ansat helicopter:

(8) $$\frac{\delta_{lat}}{\mathrm{w}_n} = 0{,}837\sigma_{\mathrm{w}}^{-0{,}6265}\sqrt{\frac{\sigma_{\mathrm{w}}^2 V_r}{\pi R_m}} \times \left[\frac{1}{s+(2V_r/R_m)}\right]\frac{\omega_{rotor}|_{UH-60}}{\omega_{rotor}|_{Ansat}}H,$$

(9) $$\frac{\delta_{lon}}{\mathrm{w}_n} = 1{,}702\sigma_{\mathrm{w}}^{-0{,}6265}\sqrt{\frac{\sigma_{\mathrm{w}}^2 V_r}{\pi R_m}} \times \left[\frac{1}{s+(2V_r/R_m)}\right]\frac{\omega_{rotor}|_{UH-60}}{\omega_{rotor}|_{Ansat}}H,$$

(10) $$\frac{\delta_{col}}{\mathrm{w}_n} = 0{,}1486\sigma_{\mathrm{w}}^{-0{,}7069}\sqrt{\frac{3\sigma_{\mathrm{w}}^2 V_r}{\pi R_m}} \times \left[\frac{(s+33{,}91(V_r/R_m))}{\left[s+1{,}46(V_r/R_m)\right]\left[s+9{,}45(V_r/R_m)\right]}\right] \times \frac{R_m\omega_{rotor}|_{UH-60}}{R_m\omega_{rotor}|_{Ansat}}H,$$

(11) $$\frac{\delta_p}{\mathrm{w}_n} = 1{,}573\sigma_{\mathrm{w}}^{-0{,}6493}\sqrt{\frac{\sigma_{\mathrm{w}}^2 V_r}{\pi R_t}} \times \left[\frac{1}{s+(2V_r/R_t)}\right]\left[\frac{R_t\omega_t|_{UH-60}}{R_t\omega_t|_{Ansat}}\right]^2,$$

where the transfer function

$$H = \frac{\left.\dfrac{(\pi V_r/8R_m)}{s+(\pi V_r/8R_m)}\right|_{Ansat}}{\left.\dfrac{(\pi V_r/8R_m)}{s+(\pi V_r/8R_m)}\right|_{UH-60}}.$$

This atmospheric turbulence model resulted in vector of external disturbances $\xi = [\delta_{lon}\ \delta_{lat}\ \delta_{col}\ \delta_p]^T$.

4. PROBLEM STATEMENT

Considering hovering flight, the velocities u, v, w are not measured. The vector of measured output $y \in R^l$, where $l = 6$:

(12) $$y = C\Delta x,$$

with $C = \left[0_{l\times(n-l)}\ I_l\right]$. In the case of axial flight of the helicopter, the velocity v is measured.

The problem of the autopilot control law synthesis is based on the results of measurements (12) subject to the restrictions set out above and atmospheric turbulence. The objective is to improve the accuracy of stabilizing the helicopter and provide the specified control time.

5. CONTROLLER DEVELOPMENT

To improve the accuracy of the helicopter stabilization we will employ feedback using the estimate $\Delta\hat{x}$ of the state vector Δx, as well as the estimates $\hat{\mathrm{w}}$ of the unknown disturbances vector w. This method improves the accuracy of stabilization without significantly increasing the gains, that would lead to a restriction of the autopilot control signals, and loss of the helicopter control quality.

The block diagram of the helicopter control system is shown in Figure 1. This control law is given as

(13) $$\mathrm{u} = \mathrm{u}_e + \mathrm{u}_{\Delta\hat{x}} + \mathrm{u}_{\hat{\mathrm{w}}} + \mathrm{u}^*$$

where u_e is formed at the output of the PID - controller from the vector $e = r - \bar{y}$ of the error between the reference signal r and the controlled variables $\bar{y} = \bar{C}y = (v, \Delta\theta, \Delta\phi, \Delta\psi)^T$; $u_{\Delta\hat{x}}$ and $u_{\hat{w}}$ are formed according to the estimates of the vector $\Delta\hat{x} = \hat{x} - x^*$ and vector $\hat{w}$ of irregular disturbances, respectively. The trim values x^*, u^* are stored in the controller and may vary in accordance with a specified motion of the vehicle.

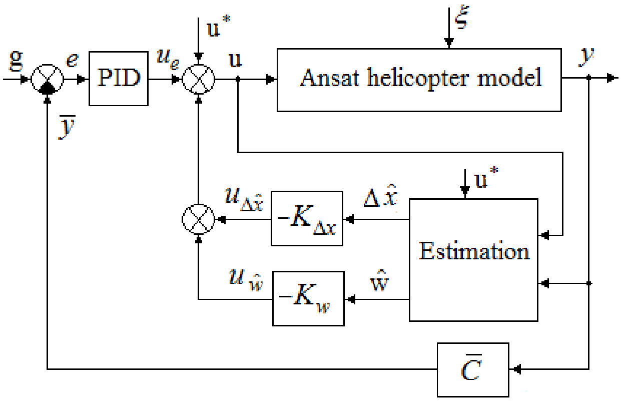

Figure1: Block diagram of the proposed controller

For tracking the reference signal vector r, the control u_e is used, and the control $u_{\Delta\hat{x}}$ is disabled. To improve the accuracy of hover stabilization, the control $u_{\Delta\hat{x}}$ is used, while u_e is disabled. In order to suppress the external disturbances for all flight modes, the control $u_{\hat{w}}$ is applied.
As a basic control law the PID controller is used. In order to improve the quality of the dynamics and reliability of the control law, the feedback $u_{\Delta\hat{x}}$ is introduced. And to improve the accuracy of the stabilization, the signal $u_{\hat{w}}$ is used to compensate the irregular disturbances, limited in size and velocity.

The problem of control synthesis is carried out for hover, considering that the coordinates $p, q, r, \Delta\theta, \Delta\phi, \Delta\psi$ are measured. In case of collective pitch changing, the coordinate v is also measured.

6. TUNING THE PID CONTROLLER

Subsystems with vectors $\bar{x}_1 = v$ and $\bar{x}_2 = (p, q, r, \Delta\theta, \Delta\phi, \Delta\psi)^T$ for the linear helicopter model (4) are then considered separately.

The control law for the first subsystem with $\bar{y}_1 = v$ has the form

(14) $$u_{e1} = k_1(g_1 - \bar{y}_1) + k_2\int_0^t (g_1 - \bar{y}_1)d\tau .$$

For the second subsystem:

(15) $$\dot{\bar{x}}_2 = A_2\bar{x}_2 + B_2 u_{e2} .$$

The control law u_{e2} with $\bar{y}_2 = (\Delta\theta, \Delta\phi, \Delta\psi)^T$ can be represented as

(16) $$u_{e2} = K_1\int_0^t (g_2 - \bar{y}_2)d\tau + K_2(g_2 - \bar{y}_2) + $$
$$+ K_3(\dot{g}_2 - \dot{\bar{y}}_2)$$

or with $g_2 = 0$ we obtain

(17) $$\dot{u}_{e2} = -K_1\bar{y}_2 - K_2\dot{\bar{y}}_2 - K_3\ddot{\bar{y}}_2 .$$

Then for the augmented system (15), (17) with the state vector $\bar{x} = (\bar{x}_2^T\ u_{e2}^T)^T$ the control law is initially determined by

(18) $$\dot{u}_{e2} = -K\bar{x} ,$$

whereby, in view of the transformation $\bar{x} = N^{-1}\left[\bar{y}_2^T\ \dot{\bar{y}}_2^T\ \ddot{\bar{y}}_2^T\right]^T$, where

$$N = \left[\begin{array}{c|c} C_2 & 0_3 \\ \hline C_2 A_2 & C_2 B_2 \\ \hline C_2 A_2^2 & C_2 A_2 B_2 \end{array}\right], \; C_2 = \left[0_3 \; I_3\right],$$

the control law matrix (16) is determined by

$$\left[K_1 \; K_2 \; K_3\right] = K N^{-1}.$$

To implement the control law (16), the vector $\dot{\bar{y}}_2$ is calculated using the signals p, q, r or their estimates, obtained by the observer with measured signals $\Delta\theta, \Delta\phi, \Delta\psi$.

7. THE OBSERVER SYNTHESIS

To construct the observer for the disturbance estimation (w) the approach reported in [11] is used.

From equation (4) the expression for the disturbance is:

$$\mathrm{w} = D^{+}\left(\Delta\dot{x} - A\Delta x - B\Delta \mathrm{u}\right), \tag{19}$$

where $D^{+} = \left(D^T D\right)^{-1} D^T$. Equation (19) can be seen as an implicit model of the disturbances, and the observer takes the form:

$$\Delta\dot{\hat{x}} = A\Delta\hat{x} + B\Delta \mathrm{u} + D\,\hat{\mathrm{w}} + L_1\left(y - C\Delta\hat{x}\right), \tag{20}$$

where $\Delta\hat{x}(t)$ is the state vector estimate, $\hat{\mathrm{w}}$ is the disturbance vector estimate, defined by:

$$\begin{aligned} \mu\dot{\hat{\mathrm{w}}} + \hat{\mathrm{w}} = D^{+}\left(\Delta\dot{\hat{x}} - A\Delta\hat{x} - B\Delta \mathrm{u}\right) + \\ + L_2\left(y - C\Delta\hat{x}\right). \end{aligned} \tag{21}$$

Here, μ is a small parameter, L_1, L_2 are coefficient matrices to be determined.

Taking into account (20), equation (21) can be rewritten as

$$\dot{\hat{\mathrm{w}}} = \mu^{-1}\left(D^{+}L_1 + L_2\right)\left(y - C\Delta\hat{x}\right). \tag{22}$$

It is obvious, that for an arbitrary disturbance and for (22) to be valid it is necessary that $rank\left(D^{+}L_1 + L_2\right) = s$, $l \geq s$.

Introducing the extended vector $\hat{x}_{ext} = \left[\Delta\hat{x}^T \; \hat{\mathrm{w}}^T\right]^T$, we can then write the equation of the observer:

$$\begin{aligned} \dot{\hat{x}}_{ext} = A_{ext}\hat{x}_{ext} + B_{ext}\Delta \mathrm{u} + \\ + H_{ext}L_{ext}\left(y - C_{ext}\hat{x}_{ext}\right), \end{aligned} \tag{23}$$

where $C_{ext} = \left[C \; 0_{l\times s}\right]$,

$$A_{ext} = \begin{bmatrix} A & D \\ 0_{s\times n} & 0_{s\times s} \end{bmatrix}, \; B_{ext} = \begin{bmatrix} B \\ 0_{s\times m} \end{bmatrix},$$
$$L_{ext} = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \; H_{ext} = \begin{bmatrix} I_n & 0_{n\times s} \\ \mu^{-1}D^{+} & \mu^{-1}I_s \end{bmatrix}.$$

Then the deviations $\Delta x_{ext} = x_{ext} - \hat{x}_{ext}$, where $x_{ext} = \left[\Delta x^T \; \mathrm{w}^T\right]^T$, will have the form:

$$\Delta\dot{x}_{ext} = P_{ext}\Delta x_{ext} + D_{ext}\,\dot{\mathrm{w}}, \tag{24}$$

where $P_{ext} = A_{ext} - H_{ext}L_{ext}C_{ext}$, $D_{ext} = \left[0_{s\times n} \; I_s\right]^T$.

Equations (20), (24) imply that for the observer (23) to work, the matrices $A - L_1 C$ and P_{ext} must be stable. At the same time taking into account the constraints (6), the solution $\Delta x_{ext}(t)$ will be limited.

A key feature of the observer (23) is the presence of the matrix H_{ext}, which, depending on the setting of the parameter μ, affects the accuracy of the external disturbance estimation.

8. SYNTHESIS OF LOCALLY-OPTIMAL CONTROL LAWS

Using the estimates $\Delta\hat{x}$ and $\hat{\mathrm{w}}$, we obtain:

(25) $\mathrm{u}_{\Delta\hat{x}} = -K_{\Delta x}\Delta\hat{x}$.

To determine the coefficient matrices K of (18) and $K_{\Delta\hat{x}}$ of (25), a procedure for the control synthesis in the presence of uncertain external disturbances is used. For the control law of (25) with measurement of the vector Δx this reads:

(26) $\mathrm{u}_{\Delta x} = -K_{\Delta x}\Delta x$

within the constraints $\Delta x(t_0)\Delta x^T(t_0) \le Q_{\Delta x}$, $\mathrm{w}\,\mathrm{w}^T \le Q_{\mathrm{w}}$. Using the method of locally-optimal control synthesis [10], the matrix $K_{\Delta x}$ is defined as:

(27) $K_{\Delta x} = \lambda B^T P^{-1}$,

where the positive definite matrix P is the solution of the matrix equation or inequality:

(28) $AP + PA^T - 2\lambda BB^T + \alpha P + \alpha^{-1}Q_{\mathrm{w}} \le 0$.

Here, the parameters $\alpha > 0$, $\lambda > 0$ are chosen to provide specific dynamic properties of the closed-loop system. With a stable matrix $A - \lambda BB^T P^{-1} + \frac{\alpha}{2} I_n$, the parameter α affects the response time, and the parameter λ affects the magnitude of control inputs (25). In addition, for the solution P of equation (28) the estimate $\Delta x \Delta x^T \le P$. The constraint $\mathrm{u}_{\Delta x}\,\mathrm{u}_{\Delta x}^T \le Q_{\Delta \mathrm{u}}$ on the control $\mathrm{u}_{\Delta x}$ is satisfied, if the following inequality is true:

$$K_{\Delta x} P K_{\Delta x}^T = \lambda^2 B^T P^{-1} B \le Q_{\Delta \mathrm{u}}.$$

According to Schur's lemma [17], this inequality is equivalent to

(29) $\begin{pmatrix} Q_{\Delta \mathrm{u}} & \lambda B^T \\ \lambda B & P \end{pmatrix} \ge 0, \quad P > 0$.

We also introduce the restrictions on the allowed change of coordinates $|\Delta x_7| = |\theta - \theta^*| \le \Delta\theta^*$, $|\Delta x_8| = |\phi - \phi^*| \le \Delta\phi^*$, this can be rewritten in the form:

(30) $P_{77} \le \Delta\theta^{*2}$, $P_{88} \le \Delta\phi^{*2}$.

In such a way, to determine the feedback matrix (25), it is necessary for the fixed parameter α to find the parameter λ and matrix P satisfying the system of (28) - (30). Obviously, such a solution is not unique, a solution can be found, for example, by minimizing the trace of the matrix $tr\{RP\}$, where $R > 0$ is defined diagonal matrix weights, using the LMI method [17].

Note that in contrast to known synthesis algorithms using the LMI method, here, because of the given feedback matrix structure (27), the number of unknown parameters is much smaller.

Using the considered algorithm of control synthesis, the coefficient matrix K can be found. This method can also be used to determine the coefficient matrix L_{ext}.

9. COMPENSATION OF EXTERNAL DISTURBANCES

To compensate for the external disturbances the following equation is considered:

(31) $\mathrm{u}_{\hat{\mathrm{w}}} = -K_{\mathrm{w}}\,\hat{\mathrm{w}}$,

where the coefficient matrix K_{w} is determined from the condition of the best disturbance w suppression, which includes atmospheric turbulence and also nonlinearities and parametric perturbations not included in the linearized model (4). There are various ways of determining the matrix K_{w}. The helicopter dynamics modelling has shown that for the control (25) and $u_e = 0$ the best disturbance suppression is attained when

(32) $$K_w = \left(B^T B\right)^{-1} B^T D .$$

In the PID-controller (14), (16) is used with $u_{\Delta\hat{x}} = 0$, for compensation of external disturbances the following control is applied:

(33) $$\left[u_{\hat{w}1}\, u_{\hat{w}2}\, u_{\hat{w}4}\right]^T = \tilde{K}_2 \left[0_3\; I_3\right] \hat{w}, \quad u_{\hat{w}3} = 0,$$

where $\tilde{K}_2 = \left(\tilde{B}^T \tilde{B}\right)^{-1} \tilde{B}^T \tilde{D}$, $\tilde{D} = \left[I_3\; 0_3\right]^T$, the matrix $\tilde{B} = \left[b_1^{(2)}\; b_2^{(2)}\; b_4^{(2)}\right]$ is composed of the columns of the matrix $B^{(2)}$ which is of 6×4 dimensions with a representation of $B = \left[B^{(1)T}\; B^{(2)T}\right]^T$.

10. MODELLING THE DYNAMICS OF THE HELICOPTER STABILIZATION

As a test case, the ANSAT helicopter in hover is considered at a height of 10 m. In this mode, the helicopter is the most susceptible to wind effects, which can significantly alter the quality of control processes.

From equations (1)-(3) the trim values are obtained:

$$u^* = v^* = w^* = 0,\; p^* = q^* = r^* = 0,$$
$$\theta^* = 4{,}36 \text{ deg},\; \phi^* = 3{,}45 \text{ deg},\; \psi^* = 0,$$
$$\theta_{1c}^* = -0{,}05 \text{ deg},\; \theta_{1s}^* = -0{,}34 \text{ deg},$$
$$\theta_0^* = 10{,}22 \text{ deg},\; \theta_{0t} = 12{,}49 \text{ deg}.$$

The matrix $K_{\Delta x}$ is determined using the constraints $Q_{\Delta u} = \Delta u^{*2} I_m$, $\Delta u^* = 2$ deg; $Q_w = w^{*2} I_n$, $w^* = 0{,}1$; $\Delta\theta^* = 5$ deg, $\Delta\phi^* = 5$ deg, and the weight matrix $R = diag\{I_6, 10^3 I_3\}$.

Similar constraints are used to determine the matrix K of controller (14), (16), which was evaluated against the ADS-33E handling qualities specification [18]. This document specifies the desired handling qualities of military rotorcraft and is frequently used for the control law analysis.

The linear model of the ANSAT helicopter was used to obtain frequency responses. A nonlinear model was also used to obtain time-domain characteristics. The assessment criteria consists of small amplitude, moderate amplitude, large amplitude responses, and cross-coupling.

The small-amplitude response comprises short-term and mid-term responses. The short-term response is defined by a bandwidth (frequency giving $45°$ phase margin) and a phase delay. The phase delay is defined as:

(34) $$\tau_p = \frac{\Delta\Phi_{2\omega_{180}}}{57.3(2\omega_{180})},$$

where $\Delta\Phi_{2\omega_{180}}$ is difference in phase between ω_{180} and $2\omega_{180}$ frequencies. The mid-term response rates the damping ratio at all frequencies below the obtained bandwidth frequency. The minimum damping ratio is $\zeta = 0.35$. This characteristic describes the ability of the controller to filter out high-frequency disturbances.

The moderate-amplitude response (attitude quickness) is defined as the ratio of peak rate to change in attitude for different magnitude of attitude changes.

The large-amplitude response defines achievable angular rate of attitude change from trim. It is important for assessing the ability of the helicopter to retain high levels of handling at attitudes where the non-linearities are most severe [14]. For aggressive agility maneuvers, the minimum requirements for hover and low speed flight are $\pm 30°/\text{sec}$ for pitch and $\pm 50°/\text{sec}$ for roll.

The inter-axis coupling criteria require that control inputs to achieve a response in one axis shall not result in objectionable responses in one or more of the other axes. For Level 1 handling quality the ratio of roll due to pitch and pitch due to roll should be smaller than 0.25.

The results obtained for the developed controller are summarized in Tables 1 and 2.

Table 1

ADS-33 criteria	Pitch	Roll	Yaw
Short term response to control input	Level 1	Level 1	Level 1
Mid-term response to control input	Level 1	Level 1	Level 1
Moderate-amplitude attitude changes	Level 1	Level 1	Level 2
Large-amplitude attitude changes	Level 1	Level 1	Level 1

Table 2

ADS-33 criteria	Level of handling quality
Pitch due to roll coupling	Level 1
Roll due to pitch coupling	Level 1

The obtained results show that the developed controller is robust and meets the Level 1 requirements (only moderate-amplitude response of the yaw has Level 2 handling quality). This is also confirmed by the transient responses, presented in figures (5)-(7).

In the synthesis of the observer (23) in the hover, the parameters u, v, w were not measured. However, in determining the matrix L_{ext} full measurement of the state vector Δx was assumed. As the helicopter dynamics simulation shows, the use of such an observer with zeroing (lack of) measurement signal of the parameters u, v, w makes it easier to compensate the disturbances w than with an observer, built for the subsystem (15).

To test the observer of equation (23) for the estimate of the external disturbance vector w, the atmospheric turbulence model (8)-(11) was used with $V_r = 5{,}144\ \text{m/sec}$, $\sigma_{\mathrm{w}} = 1{,}03\ \text{m/sec}$. For the ANSAT parameters $R_m = 5{,}75\ \text{m}$, $R_t = 1{,}05\ \text{m}$, $\omega_{rotor} = 38{,}22\ \text{rad/sec}$, $\omega_t = 209{,}44\ \text{rad/sec}$, Figure 2 presents the process of the estimation of the external disturbance vector w of equation (4) without measurement of the velocities u, v, w. In this case the PID-controller was used without input signals and with $u_{\Delta\hat{x}} = 0$, $u_{\hat{\mathrm{w}}} = 0$. When the measurement of u, v, w is taken into account, all the elements of the vectors $\hat{\mathrm{w}}$ and w were practically identical.

To check the efficiency of external disturbance suppression on Figure 3, the results of modelling the helicopter dynamics in hover with the control (25), (31), (32) with $\mathrm{u}_e = 0$ are shown, where $V = \sqrt{u^2 + v^2 + w^2}$. The accuracy of stabilization is preserved in the presence of the external disturbance compensation.

Figures 4-7 show the results of the system modelling with the control law of equations (14), (16), (33) with $\mathrm{u}_{\Delta\hat{x}} = 0$ for different input signals $g_i = g_i^* \operatorname{sign}(\sin 0{,}5t)$, $g_j = 0$, $j \neq i$; $i, j = \overline{1,4}$. The results with and without compensation of external disturbances are also shown.

11. CONCLUSION

The results presented in this paper show that the controller in combination with an observer of external influences can filter out disturbances, and allows for rapid changes in the state for all cases considered in this work. In the future this work will be directed towards further validation of the functionality of the controller using test data.

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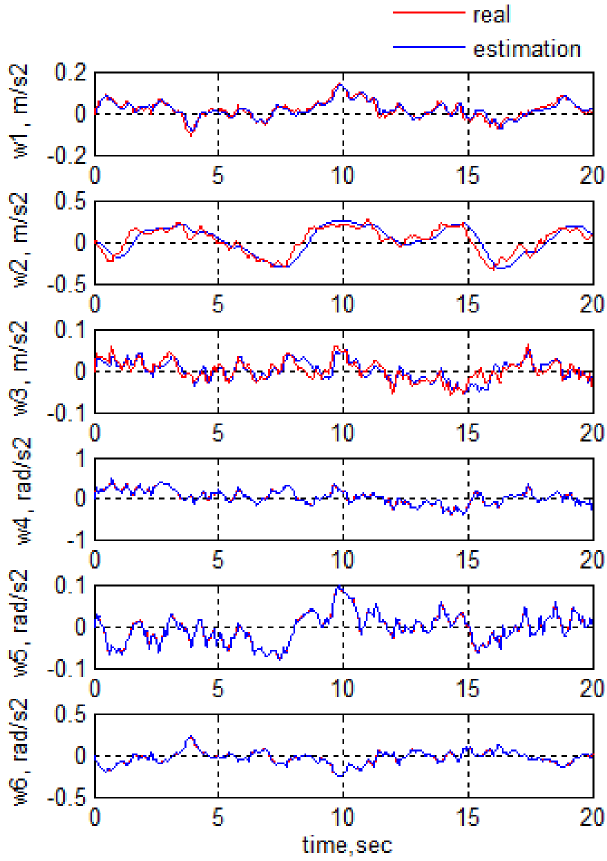

Figure 2: Estimation of disturbance w without measurement of u, v, w.

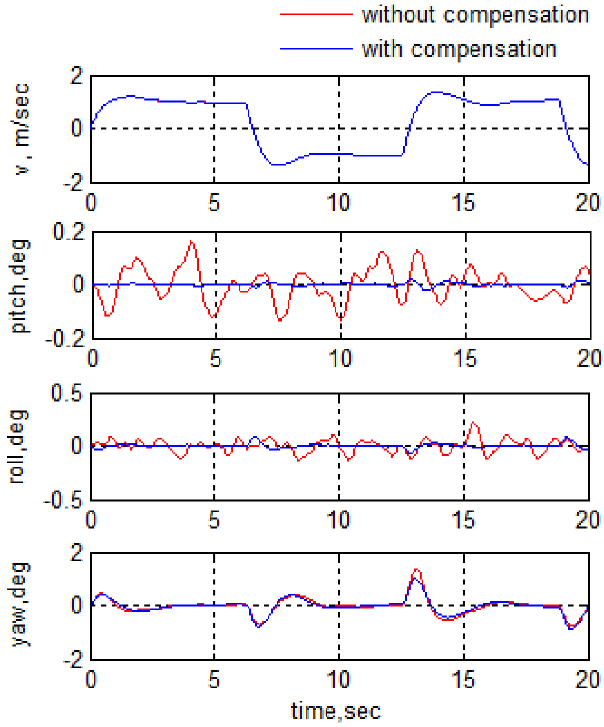

Figure 4: Helicopter stabilization with control input (14), (16), (33) at $g_1^* = 1\text{m/sec}$

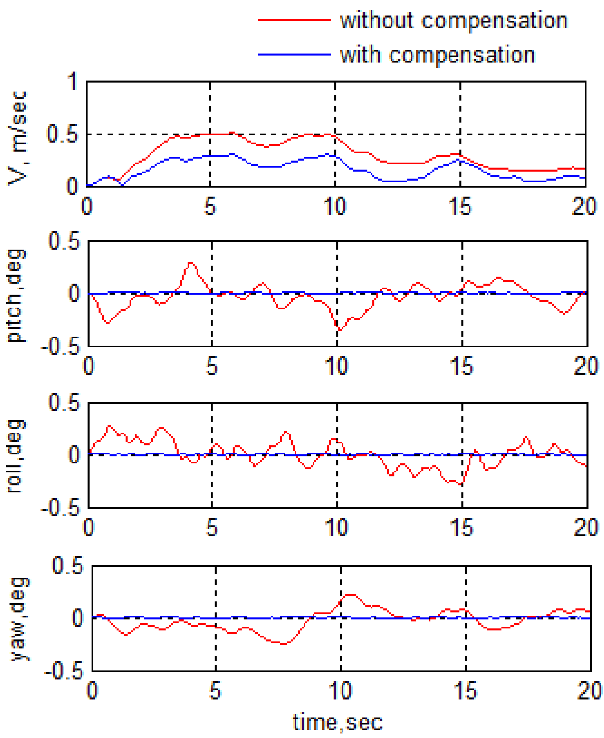

Figure 3: Helicopter stabilization with control input (25), (31), (32)

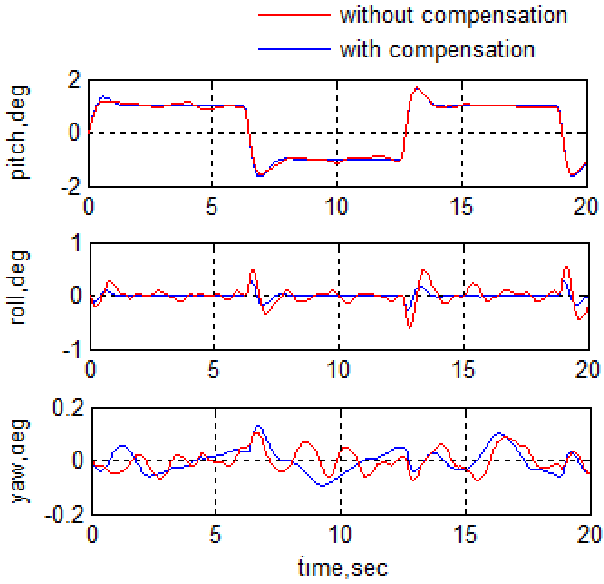

Figure 5: Helicopter stabilization with control input (14), (16), (33) at $g_2^* = 1\text{deg}$

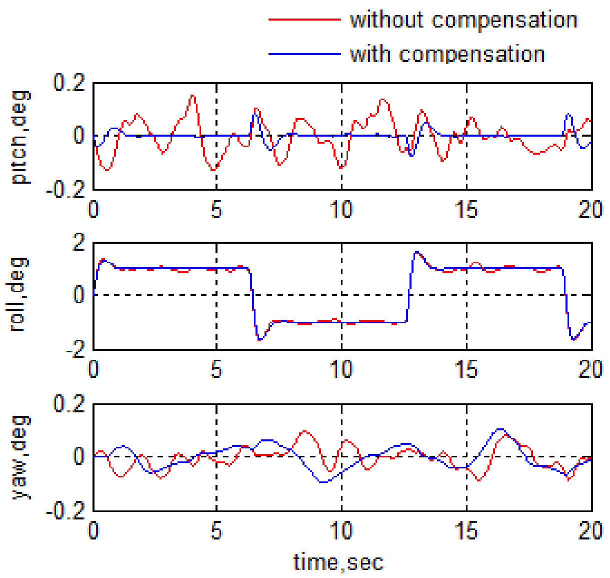

Figure 6: Helicopter stabilization with control input (14), (16), (33) at $g_3^* = 1\text{deg}$

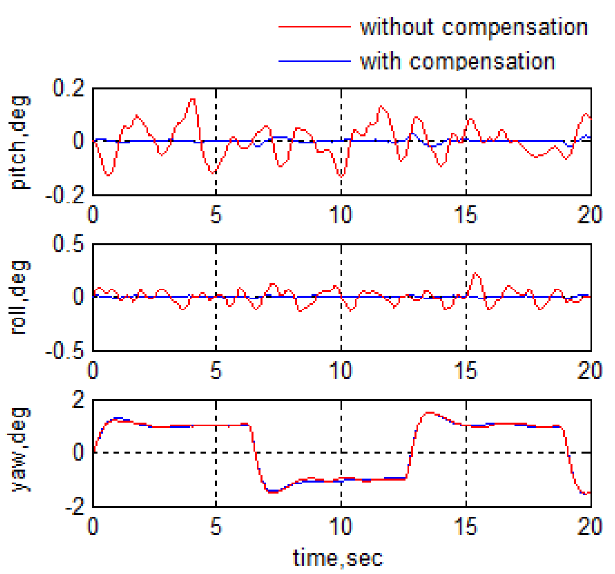

Figure 7: Helicopter stabilization with control input (14), (16), (33) at $g_4^* = 1\text{deg}$