

APPLICATION OF MODERN STRUCTURAL OPTIMIZATION
TO VIBRATION REDUCTION IN ROTORCRAFT

BY

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APPLICATION OF MODERN STRUCTURAL OPTIMIZATION
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ABSTRACT

This paper explores a number of techniques which are capable of reducing vibration levels in rotorcraft by redistributing the mass and stiffness properties of the structure. First vibration reduction in the rotor is considered by using formal structural optimization for ensuring optimal frequency placement. Two cases are considered. In the first case aeroelastic constraints are not enforced and the blade is designed for minimum weight. In the second case aeroelastic constraints are enforced and vibration levels are minimized in forward flight. Next vibration reduction in the fuselage is considered and the various methods available for vibration reduction by local structural modification are reviewed. The feasibility of combining local structural modification with modern structural optimization is discussed and some extensions of previous research are suggested.

Nomenclature

a	= two dimensional lift curve slope
b_s	= cross sectional dimension, Fig. 1
\bar{b}	= semi-chord nondimensionalized with respect to R
C_W	= weight coefficient = $W/\pi\Omega^2 R^4$
e_1	= blade root offset
h_s	= cross sectional dimensional, Fig. 1
I_b	= blade flapping inertia
ℓ_{os}	= length defining outboard station, where outboard blade segments start, Fig. 3
ℓ	= length of elastic part of the blade
m_{ns}	= nonstructural mass per unit length of the blade use as tunning, Fig. 3
M_L	= additional mass used inside structural box, Ref. 16
M_{x1}	= hub rolling moment

n_b	= number of blades
P_{z1}	= vertical hub shears
R	= rotor radius
t_h, t_b	= cross sectional dimension, Fig. 1
V	= velocity of forward flight
W	= aircraft weight
x_I	= offset between sectional elastic axis and center of mass
x_m	= offset from counterweight to elastic axis
α_R	= rotor angle of attack
β_p	= precone
γ	= Lock number
ζ_k	= real part of eigenvalue
μ	= advance ratio = $V \cos \alpha_R / \Omega R$
σ	= blade solidity = $n_b 2\bar{b} / \pi$
ω_k	= imaginary part of eigenvalue
Ω	= speed of rotation, radians/sec
$\bar{\omega}_{L1}, \bar{\omega}_{F1}, \bar{\omega}_{T1}$	= rotating fundamental frequency of blade in lag, flap and torsion, nondimensionalized w.r.t. Ω
$\bar{\omega}_{F2}$	= rotating fundamental second flap frequency

1. Introduction

Vibration levels in helicopters have been a problem for many helicopter configurations in the past and it is reasonable to expect that vibration levels and their alleviation will continue to play an important role in the design of the next generation of helicopters. Due to the great practical importance of these problems a considerable amount of research has been aimed at various aspects of vibration reduction and control, as shown in a recent survey by Reichert¹. As indicated in Ref. 1 there is a substantial volume of literature available concerning various devices which can reduce unacceptable vibrations, such as: vibration absorbers, isolators and higher harmonic control devices. Other approaches utilize blade twist and blade tip sweep for reduction of vibrations in the rotor as shown in Ref. 2. Similarly the blade trailing edge tab can be also used to reduce vertical hub shears in the rotor³. The purpose of this paper is to present some recent developments which are based on the use of structural optimization for vibration reduction in rotorcraft. It is

shown that by applying structural optimization one can obtain vibration level reductions which are similar in magnitude to those which can be accomplished by the various other methods described in Refs. 1-3.

The field of structural optimization or structural synthesis has become a practical tool in recent years⁴. This is due to significant advances in high speed computers coupled with considerable research which has led to more efficient methods using a combination of mathematical programming techniques for optimization and the finite element method for structural modeling⁵. Structural optimization has found considerable use in the aerospace industry, although most applications have been in the design of fixed wing aircraft^{6,7}. A detailed review of the application of numerical optimization to helicopter design problems can be found in Ref. 8.

When dealing with the vibration reduction problem in rotorcraft two different potential applications of structural optimization present themselves. The first approach consists of tailoring the rotor mass and stiffness distributions so as to reduce vibration levels in forward flight. This approach attacks the vibration problem at its source, namely the vibratory excitation caused by the main rotor. It is evident that a rotor which produces inherently low hub loads will also produce low vibration levels throughout the airframe and can be expected to offer weight and reliability advantages when compared to other vibration control approaches². Another advantage of this approach is the ability to incorporate the automated optimization procedure in the design process of the rotor, thus yielding a computer aided design capability. The second approach consists of applying structural optimization to locally modify the mass or stiffness of a helicopter fuselage so as to reduce vibration levels at a specific location in the airframe, such as the pilot seat (for example), where excessive vibrations are encountered after the helicopter has been designed and built. This type of local structural modification would play a role similar to introduction of a vibration absorber or a vibration isolator. Both these approaches are discussed in the paper. Furthermore the two approaches can be combined in a single procedure in which the coupled rotor/fuselage system is considered in its entirety. For this case one can use structural optimization to aeroelastically tailor the main rotor for reduced vibrations in forward flight while simultaneously using structural optimization for the airframe structure so as to minimize vibration levels at specified locations. This approach can be incorporated directly in the design procedure of modern helicopters.

It is shown in the paper that modern structural optimization can yield substantial practical benefits in the design process of improved rotor and airframe systems and one hopes that these methods will be adopted by the helicopter industry.

2. Vibration Reduction in the Rotor

2.1 Overview

The idea of reducing vibrations by modifying blade properties so as to reduce vibratory hub shears and moments and thereby reduce the vibration levels experienced in the fuselage is appealing because it reduces the vibrations at the source, namely the rotor. The first studies based on this approach started to appear in the mid fifties⁹⁻¹³ and it was shown

that by addition of tip masses, tuning masses and adjustment of blade torsional frequencies significant amounts of vibration reduction can be obtained.

More recently, two studies have become available^{14,15} aimed at modifying blade properties to reduce vibration levels in forward flight. Taylor¹⁴ has considered the vibration reduction problem of rotor blades in forward flight by using modal shaping. In Ref. 14 vibration levels in forward flight were reduced by modifying the mass distribution, and to a lesser extent the stiffness distribution of the blade, using a so called "modal shaping parameter". The modal shaping parameter can be interpreted as an ad hoc type of optimality criterion, which is used to reduce the response in a particular normal mode to the applied aerodynamic excitation. Strictly speaking Taylor's study is not an optimum design approach because the aeroelastic stability constraints were not imposed, and the procedure was not automated since it was based on repeated analyses and a visual inspection of the results. Another study by Bennett¹⁵ contains a simple example where vertical hub shears, due to blade flapping were minimized, using mathematical programming techniques. Bennett's study neglected the effect of blade dynamics on the airloads and therefore it represents a forced response study. It should be noted that both studies^{14,15} are based on simple linear models for blade vibration employing modal analysis and the principal of superposition. However it is interesting that in both studies vibration reduction in the order of 20-40% in vertical hub shears was achieved.

It was only very recently that studies based upon modern structural optimization methods applied to rotor blade design have become available¹⁶⁻¹⁹. The purpose of this section is to provide a description of the methods used and the results obtained in References 16-19.

2.2 Application of Structural Optimization to Rotor Blade Frequency Placement

The study, described in Ref. 16, is based on the assumption that separation of blade frequencies in flap, lead-lag and torsion from the aerodynamic forcing frequencies, which are occurring at integer multiples of the rotor RPM, will also guarantee vibration reduction in forward flight. The basic optimum structural design problem is one in which the mass and stiffness distributions are selected by an optimization process, such that the uncoupled flap, lag and torsional rotating frequencies are placed in certain predetermined "windows" which are separated from the integer resonances. It should be noted that aeroelastic stability constraints were not considered in Ref. 16, and minimization of hub loads resulting from the aeroelastic response of the blade was not the aim of this study.

The optimization problem, solved in Ref. 16, can be stated in the following form: find the vector of design variables \bar{D} such that

$$\begin{aligned}
 g_q(\bar{D}) &\geq 0; \quad q = 1, 2, \dots, Q \\
 D_i^{(L)} &\leq D_i \leq D_i^{(U)}; \quad i=1, 2, \dots, n_{dv} \\
 \text{and } J(\bar{D}) &\rightarrow \min.
 \end{aligned}
 \tag{1}$$

where $g_q(\bar{D})$ is the q^{th} constraint function in terms of the vector of design variables \bar{D} , D_i is the i^{th} design variable, superscripts *L and U denote lower and upper bounds respectively and $J(\bar{D})$ is the objective function in terms of the design variables.

Design Variables. The blade being optimized consists of typical cross sections shown in Fig. 1. The vibration analysis for the mode shapes and frequencies is based upon a tapered finite element model for the blade¹⁶, using ten spanwise stations. The design variables are the breadth b_s , the height h_s and the thickness t_b and t_h of the thin walled rectangular box section representing the structural member at each spanwise station. The nonstructural mass at each cross section consisted of a tuning mass m_{ns} shown in Fig. 1, together with a second lumped mass M_L was assumed to be inside the structural box and is not shown in Fig. 1. These masses can be placed at each spanwise station.

Constraints. Three types of behavior constraints were imposed. The first and second flapwise bending modes were constrained to be within certain specified limits, i.e.

$$\left. \begin{aligned} \bar{\omega}_{1F}(L) &\leq \bar{\omega}_{1F} \leq \bar{\omega}_{1F}(U) \\ \bar{\omega}_{2F}(L) &\leq \bar{\omega}_{2F} \leq \bar{\omega}_{2F}(U) \end{aligned} \right\} \quad (2)$$

representing "frequency windows" which are separated from the integer resonances of the blade.

A second constraint is imposed on the rotary inertia of the blade so that the rotor has sufficient inertia to autorotate. A third constraint is a constraint on the maximum stresses due to centrifugal loads. In addition, there are side constraints on the design variables t_h , b_s , t_b and h_s to prevent them from reaching impractical values. The last constraint is a physical limitation on the structural box, so that it fits within the aerodynamic airfoil shape.

The Objective Function. Due to the optimizer used two alternate objective functions had to be used¹⁶. In the first stage of the optimization the objective was to minimize the discrepancies between desired frequencies and actual frequencies. The purpose of this stage is to avoid an unfeasible solution, however the true objective function, which is weight, was not used in this stage. Once the frequencies are within the desired window, as represented by Eq. (2), thus guaranteeing a feasible region, the objective function is replaced by the weight of the blade. Subsequently the objective of the optimization is to minimize the weight of the blade.

Solution of the Optimization Problem and Results. The optimization problem is solved using the widely available CONMIN program developed by Vanderplaats²⁰.

The results presented in the paper contain a number of numerical experiments as well as three applications to the design of various rotor blade configurations. In all cases considered the problem solved is the minimization of blade weight such that the first and second flap frequencies

*or subscripts

have certain prescribed values. Three separate configurations are considered: (1) optimization of a wind turbine blade, where a 30.5% reduction in blade weight is obtained, (2) optimization of an articulated rotor blade, where a 26% reduction in blade weight is obtained and (3) optimization of a teetering rotor blade, where no reduction in blade weight is obtained.

It is difficult to determine whether the optimized configurations, obtained in Ref. 16, are meaningful, since aeroelastic constraints are not enforced, nor are any specific dynamic response quantities considered. It should be also noted that somewhat more general studies dealing with the minimum weight optimum design of damped linearly elastic, nonrotating, structural systems, subjected to periodic loading, with behavior constraints on maximum deflection and side constraints on the design variables have become recently available^{21,38}. The methods developed in Refs. 21 and 38 are quite applicable to the problem considered in Ref. 16.

2.3 Application of Structural Optimization to Vibration Reduction in Forward Flight

This research¹⁷⁻¹⁹ was the first, documented, application of optimum structural design to vibration reduction in the rotor, while simultaneously using aeroelastic stability margins as constraints. This optimization problem can be also cast in the mathematical form represented by Eq. (1).

This optimum design problem was solved using mathematical programming methods and approximation concepts^{5,22} were used to improve the cost effectiveness of the mathematical programming methods.

The blade preassigned properties, helicopter performance parameters, design variables, side constraints, behavior constraints and objective function, used in Refs. 17-19, are described next.

The system considered in this study consisted of a four bladed hingeless rotor, attached to a fuselage of infinite mass. Thus the fuselage degrees of freedom were not included.

The following quantities, defining the helicopter blade configuration, were treated as preassigned blade parameters: b - blade semi-chord, β_p - blade precone angle, e_1 - blade root offset from axis of rotations, x_A - blade cross sectional aerodynamic center offset, from elastic axis.

The helicopter performance parameters which define the helicopter flight condition, in trimmed flight are: the advance ratio μ , and the weight coefficient C_W which represents the total weight of the helicopter. These performance parameters were assumed to be specified parameters, characterizing the configuration.

Design Variables. A typical cross section of the rotor is shown in Figure 1.. The design variables were the breadth b_s , the height h_s and the thicknesses t_b and t_h of the thin walled rectangular box section representing the structural member, at each of the seven spanwise stations. Elastic properties of the blade in bending and torsion as well as the structural mass properties were expressed in terms of these design variables. The nonstructural mass of the blade was assumed to consist of two parts.

The first portion was the nonstructural skin and honeycomb core surrounding the structural cell shown in Fig. 1, so as to provide the appropriate aerodynamic shape, which was assumed to be a fixed percentage of the initial blade mass. The second contribution to nonstructural mass was represented by m_{NS} , in Fig. 1, which is a counter weight used as a tuning device for controlling blade frequency placement. The nonstructural masses m_{NS} at three outboard stations of the blade were also used as design variables, while the offsets x_m from the elastic axis were given parameters.

Constraints. The two types of behavior constraints in this optimization study were frequency constraints and constraints on the aeroelastic stability margins.

The frequency constraints were expressed in terms of the square of the nondimensional rotating frequencies $\bar{\omega}^2$ of the blade in flap, lead-lag and torsional degrees of freedom. These uncoupled rotating frequencies were generated from a Galerkin type finite element model of the blade^{23,24}. The fundamental frequencies, of the rotating blade, in flap, lag and torsion were constrained within certain specified upper and lower bounds. The higher frequencies were constrained so as to avoid four per rev resonances in the four bladed hingeless rotor.

A typical frequency constraint in the optimization procedure was expressed in terms of inequality constraints having the mathematical form

$$g_q(\bar{D}) = \frac{\bar{\omega}_i^2}{\omega_{i(L)}^2} - 1.0 \geq 0$$

and

$$g_q(\bar{D}) = 1 - \frac{\bar{\omega}_i^2}{\omega_{i(U)}^2} \geq 0$$

(3)

where $\bar{\omega}_i$ are the fundamental nondimensional rotating frequencies and $\omega_{i(L)}$ and $\omega_{i(U)}$ are the lower and upper bounds imposed on the rotating fundamental frequencies, in flap, lag and torsion respectively.

The aeroelastic constraints are considered next. Aeroelastic stability results presented in Refs. 25 and 26 indicated that forward flight is stabilizing for soft-in-plane hingeless blade configurations. The trend in current hingeless rotor design is to use soft-in-plane blades. Therefore aeroelastic stability margins in hover were assumed an acceptable measure for these margins. The validity of this assumption was verified by subsequent calculations. The aeroelastic constraints represent the requirement that stability margins in hover remain virtually unchanged during the optimization process. The aeroelastic analysis^{25,26}, which served as the basis of the optimization study, uses two uncoupled free vibration modes of the rotating blade to represent the flap, lag and torsional degrees of freedom respectively. The dynamic equations of equilibrium for an isolated rotor blade in hover, and quasisteady aerodynamics, lead to the standard eigenvalue problem. The eigenvalues occur in complex conjugate pairs

$$\lambda_k = \zeta_k \pm i\omega_k \quad (4)$$

The blade is stable when $\zeta_k < 0$, for $k = 1, \dots, 6$. The aeroelastic constraints in the optimization procedure were expressed as follows

$$g_q(\bar{D}) = \frac{\zeta_k}{\zeta_k^{(L)}} - 1 > 0, \quad k = 1, 2, \dots, 6 \quad (5)$$

where $\zeta_k^{(L)}$ is the lower bound on ζ_k . The value of $\zeta_k^{(L)}$ was selected, with a small degree of flexibility, such that aeroelastic stability margins in hover remain practically unchanged during the optimization process.

Side constraints were also placed on the design variables t_h , b_s , h_s , t_b , and m_{ns} , in form of upper and lower bounds in order to prevent the design variables from reaching impractical values during the optimization process.

The Objective Function. To be minimized was a mathematical expression representative of vertical hub shears or hub rolling moments. In Refs. 17-19 the maximum peak-to-peak value of the oscillatory hub vertical shears or the oscillatory hub rolling moments due to the blade flap-wise bending was used as an objective function. The objective functions considered were

$$J(\bar{D}) = P_{z1max} \quad (6)$$

$$J(\bar{D}) = M_{x1max}$$

where P_{z1max} is the maximum peak-to-peak value of the oscillatory hub vertical shears and M_{x1max} is the maximum peak-to-peak value of the oscillatory hub rolling moments due to the flapwise bending.

These objective functions were obtained by using the steady state blade response values in flap, lag and torsion which are generated by the aeroelastic stability and response analysis described in Refs. 25 and 26. A brief description of the relations between the aeroelastic analysis, the loads acting on the blade and the hub shears and moments was given in Refs. 17 and 18, complete details can be found in Ref. 19. The aeroelastic response analysis^{25,26} is based upon two elastic modes for each of the flap, lag and torsional degrees of freedom, respectively (i.e., a total of six elastic modes).

Solution of the Optimization Problem. The optimization problem was treated by using the sequence of unconstrained minimization technique (SUMT) based on an extended interior penalty function and a modified Newton method minimizer²² implemented in a Fortran program called NEWSUMT²⁷. Furthermore approximation concepts^{22,28} were used in the optimization process to reduce computing costs. The organization of the optimization process used in Refs. 17-19 is illustrated in Fig. 2 and described below.

- (1) An initial trial design \bar{D}_0 is chosen by selecting the values of b_s , h_s , t_h , t_b at the seven spanwise stations, and m_{ns} at the three outboard stations.

- (2) The uncoupled rotating modes and frequencies of the blade are obtained using a finite element model. Explicit first order and second order Taylor series approximations to the frequency constraints are calculated in closed form.
- (3) The aeroelastic stability in hover, the response in forward flight, and the vertical hub shears and moment (which constitute the objective function to be minimized) are calculated using the analysis given in Refs. 25 and 26. The gradient information for the explicit approximation of the objective function and aeroelastic constraints is calculated by finite differences.
- (4) The mathematical programming problem represented by Eq. (1) is replaced by an approximate problem where the constraints $g_q(\bar{D})$ and the objective function $J(D)$ are expressed by explicit Taylor series approximations. The approximate problem is solved by the NEWSUMT optimizer to obtain an improved design.
- (5) The entire optimization process is repeated with the improved design as a starting point until the sequence of vectors \bar{D} converges to a solution \bar{D}^* where all inequality constraints are satisfied and $J(D^*)$ is at least a local minimum.

Typical Results and Discussion. Results selected from Ref. 18 are presented here. Numerous additional results can be found in Refs. 17-19. Two slightly different soft-in-plane, four bladed, hingeless rotor configurations were considered. For the first configuration the initial design was a blade with uniform mass and stiffness distribution, and properties similar to the B0-105 rotor which is known to be one of the best hingeless rotors. Two stages of optimization were carried out, without utilizing tuning masses. The first stage of optimization resulted in a 15.9% reduction in the peak-to-peak, oscillatory, vertical hub shears and the second stage yielded an additional reduction of hub shears equal to 1.03%.

The initial design for the second soft-in-plane configuration was also a uniform four bladed hingeless rotor. This initial design had the following properties: $\bar{\omega}_{F1} = 1.125$; $\bar{\omega}_{L1} = 0.732$; $\bar{\omega}_{T1} = 3.16$; $\gamma = 5.5$; $\sigma = 0.07$; $a = 2\pi$; $n_b = 4$; $\bar{b} = 0.0275$; $\Omega = 425$ RPM; $C_W = 0.005$. For this case the nonstructural tuning mass m_{NS} is distributed by the optimizer, along the elastic axis, at the three outboard stations (i.e. the two finite elements close to the tip of the blade), as shown in Fig. 3. Two stages of optimization were carried out. The initial design is denoted D_0 , the design after the first stage of optimization is denoted by D_I , and the design after the second stage of optimization is denoted by D_{II} .

The objective function used in the optimization was the value of the linear peak-to-peak vertical hub shears at $\mu = 0.30$. The reductions in vertical hub shears and rolling moments at $\mu = 0.30$, after two stages of optimization, are presented in Table I. The term linear and nonlinear in Table I refers to the inclusion of geometrically nonlinear effects, due to moderate blade deflections, in the aeroelastic response calculation from which the hub shears and rolling moment are obtained. In the nonlinear case the geometrical nonlinearities are included while in the linear case they are not. For design D_{II} , the linear peak-to-peak vertical hub shear was reduced by 37.9% and the nonlinear hub shear reduced by 35.9%. The corresponding reduction in the hub rolling moments was 24.17% and 25.2%, respectively. An interesting by-product of the optimization is a reduction of total blade mass which is shown at the bottom of Table I. In design D_I

only 0.2% of the blade mass is added as nonstructural mass, whereas for design D_{II} 2.3% of the blade mass is added as nonstructural mass in the same locations. Design D_I produced a 8.7% reduction in total blade mass while design D_{II} resulted in a 19.7% reduction in total blade mass. An examination of the two designs reveals that the reduction in blade mass at the outboard segments of the blade is considerably higher than the reduction experienced by the inboard segments. This indicates that one should be careful about violating constraints associated with energy storage in the rotor which can be important for autorotation.

In Fig. 4 the cross sectional dimensions of the improved designs D_I and D_{II} are compared with those of the initial D₀, which was assumed to be a uniform blade. The spanwise variations of b_s and h_s of the improved design D_{II} are similar to those of the improved design D_I. However, the spanwise variations of the thicknesses t_b and t_h of design D_{II} are considerably different from those of design D_I. Design D_{II} exhibits reduced cross sectional thickness in the inboard 2/3 span, accompanied by nonstructural mass addition, m_{ns} , equal to 2.3% of blade mass, distributed along the elastic axis of the outboard 1/3 span portion of the blade.

Since the objective function used in the optimization was the linear expression of hub shears at $\mu = 0.30$, it was important to determine the variation in hub shears over the whole range of advance ratios considered. The nonlinear vertical hub shears over the advance ratio range $0 < \mu < 0.3$ are shown in Fig. 5 indicating a consistent reduction in hub shears over the whole range of advance ratios. These results demonstrate that for the soft-in-plane configurations, studied in Refs. 17-19, the choice of the linear vertical hub at one particular, moderately high advance ratio ($\mu = 0.30$) as the objective function was sufficient to guarantee a similar amount of reduction in the oscillatory vertical hub shears at the intermediate advance ratios. This statement is also supported by the behavior of the nonlinear hub rolling moments shown in Fig. 6. Again it is evident that improved design D_{II} exhibits a consistent reduction in hub rolling moment compared to design D₀ over the whole range of advance ratios considered.

Two additional relevant quantities are presented in Figs. 7 and 8. Figure 7 presents a comparison of the linear and nonlinear in-plane hub shears associated with design D₀ and D_{II} as a function of advance ratio. Both the linear and nonlinear peak-to-peak values of the in-plane hub shears have decreased for design D_{II} when compared to design D₀ as shown in Fig. 7. This decrease however is small. This reflects upon the well known sensitivity of in-plane hub shears, to higher order nonlinear terms, associated with the lag degree of freedom. The behavior of the root torsional moment, evaluated in the rotating system is shown for designs D₀ and D_{II} in Fig. 8. Again a consistent reduction of root torsional moment is observed over the whole range of advance ratios considered.

The results obtained in Refs. 17-19 have indicated that by applying modern structural optimization to the design of soft-in-plane hingeless rotors, vibratory hub shears in forward flight can be reduced by 15-40%. This reduction is achieved by relatively small modifications of the original design, which yield optimal frequency placement in flap, lag and torsion. It is also interesting to note that as a by product of optimization, the optimized blade configuration is between 9-20% lighter than the initial uniform blade. This result is obtained without using blade weight as the

objective function, in the optimization process. Furthermore, aeroelastic stability margins in hover, are adequate constraints, when dealing with the optimum design problem in forward flight, for soft-in-plane hingeless rotors.

3. Vibration Reduction in the Fuselage

3.1 Overview

The optimum blade design problem, discussed in the previous section, attempts to reduce helicopter vibrations by reducing the vibratory excitation at its source. During the design cycle of a helicopter the need for local vibration reduction, at specific locations in the fuselage or tail boom, frequently arises. Various methods for local vibration reduction have been developed such as: vibration isolation devices, vibration absorbers¹ and the use of local structural modification. The purpose of this section is to describe the available methods for local structural modification and show that they can be combined with structural optimization so as to enhance their effectiveness.

3.2 Vibration Reduction by Local Structural Modification

Local structural modifications are aimed at reducing vibrations by a number of relatively small modifications in mass or stiffness which are computationally efficient to implement and provide the structural dynamicist with some physical insight into both the source and alleviation of the particular vibration problem. These methods can be divided roughly in three separate categories. The first category consists of methods which are based on a basic property of a linear spring mass damper system first noted by Vincent²⁹, which has found application in a number of papers³⁰⁻³³. A second group utilizes the strain energy³³ associated with various components, or modes, so as to determine where the structure should be modified. The third group uses the sensitivity of the response³⁴ or the sensitivity of the mode shapes³⁵ to accomplish the vibration reduction by local structural modification.

The pioneering work in this area was initiated by Done and Hughes^{30,31}. In their first paper³⁰, they extended Vincent's observation (frequently called the Vincent Circle Method) regarding the response of a single degree of freedom damped system to multidegree of freedom systems. The basic property of a linear damped single degree of freedom, noted by Vincent, is as follows: "If a structure is excited by a sinusoidal force while either the mass at a point, or the stiffness between two points (as represented by a spring) is continuously varied, then the response in the complex plane at some other point is seen to trace out part of a circular locus". In Ref. 30 this statement was generalized to multidegree of freedom systems and also to the case when two spring type stiffness terms are changed simultaneously. This method was applied³¹ to the vibration reduction problem of a relatively simple two dimensional model of the Westland Lynx, shown in Fig. 9. The structure consisted of 25 structural elements, having two translational and one rotational degree of freedom at each node. Each beam like element was considered to be a substructure which could be identified with a particular portion of the fuselage. The excitation consisted of an oscillatory couple of frequency 21.7 Hz applied at the hub, additional

possible excitations, shown in Fig. 9, could have been also considered, but were not used for the sake of simplicity. For the same reason only horizontal response at the pilot's seat was considered. The objective of the study was to determine which part of the structure should be modified so as to reduce the rotor induced vibrational response at the pilot seat.

To determine the structural components which should be modified four different criteria which measure the sensitivity of vibration reduction by structural modification are examined, these were:

- (1) Diameters of response circles for stiffness change in each element were computed and presented in a normalized bar graph, both maximum diameters corresponding to each of the nodal degrees of freedom and average values were evaluated.
- (2) Another measure of sensitivity plotted shows the number of times each element appears in a pair of elements which can be varied to give a zero response within a response region.
- (3) The actual minimum response that can be achieved for a single element stiffness change was computed for all possible changes and plotted.
- (4) Response circle diameters for point masses introduced at the structure nodes were plotted in normalized form for all 25 elements.

By a visual inspection of these results the authors conclude that the gearbox stiffnesses play the most important part in controlling vibration in the crew area, and to a lesser extent so do those of the tail cone structure. It also revealed that the fuselage sides represented the next substructure of importance. It is interesting that the same conclusions were reached regardless of the criterion used.

A useful extension of References 30 and 31 can be found in Ref. 32 in which a flight vibration reduction analysis is developed (using concepts presented in Ref. 30 and 31) by determining the effect of impedance changes on the airframe vibration without incorporating the change in the baseline model so that only one dynamic analysis is required. The numerical examples in this paper also deal with the placement of a vibration absorber.

In another very interesting paper, Hanson and Calapodas³³ have compared two different methods of vibration reduction through local structural modification. The two methods considered were the method presented by Done and Hughes^{30,31} and the strain energy method developed by Sciara³⁶. Two different variants of the strain energy method were used.

The first variant uses a conventional expression for strain energy

$$U = \frac{1}{2} \{q\}^T [k] \{q\} \quad (7)$$

where $[k]$ is the element stiffness matrix and $\{q(t)\}$ is the element displacement response vector. For a vibrating structure it is hypothesized that the structural elements having the highest value of strain energy, when vibrating in a particular mode are the best candidates for structural modification.

Another variant of the method uses an alternate expression³³ instead of Eq. (7), which represents the maximum strain energy in an element, during the steady state response of a damped structure to a particular load

application at a particular frequency, and during one period. Again the elements with the highest strain energy levels are indicative of the best candidates for structural modification. This method is denoted the forced response strain energy method, and the examples considered in Ref. 33 indicate that it is superior to the use of Eq. (7).

A considerable number of numerical examples were examined in Ref. 33. First the method described by Donne et al^{30,31} and forced response strain energy method were applied to the elastic line fuselage model of the AH-1G helicopter fuselage, having 70 degrees of freedom, and excited by a 2/rev (10.8 Hz) vertical excitation at the main rotor hub. Fuselage damping was assumed to be 2%, of critical, and the objective was to reduce vertical vibrations at the pilot seat. The results indicated discrepancies between the Vincent circle type method^{30,31} and the forced response strain energy method. The forced response strain energy method points to the tailboom as the area most responsive for dynamic amplification, while the Vincent Circle Method points to the pylon as the area having the most potential for reducing vibrations at the pilot's seat. Next a more sophisticated three dimensional NASTRAN model of the fuselage (with 241 degrees of freedom) was considered and the results obtained with the simple, elastic line structural model were verified. The forced response strain energy method was applied in an iterative manner to modify the stiffness of the tailboom. A stiffness increase of 375% accompanied by a 46% reduction in the strain energy of the tail boom resulted in a near zero response at the pilot seat. The authors concluded that the Vincent Circle property is particularly useful when dealing with local effects in relatively simple structures, however for complex structures such as a helicopter fuselage the forced response strain energy method appears to be preferable.

In another study³⁴ a more general numerical method for the computation of frequency response of a vibrating structure as a function of its structural properties is presented and the results are applied to the problem of vibration reduction. This study represents another extension of Refs. 30 and 31 in which the analogy to the Vincent Circle, is a polar plot of the complex response with the beam type element stiffness (EI) as a parameter along the curve. A sensitivity analysis is used to determine which structural element changes are most effective for vibration reduction. The method is illustrated by applying it to a very idealized, beam type, finite element model of a helicopter.

It is interesting to note that among the various methods discussed in this section only Ref. 33 uses information associated with the free vibration modes of the fuselage. Structural dynamicists frequently use normal modes to gain a better physical understanding of the vibration characteristics of a structural system. Reference 35 utilizes the free vibration modes of the undamped structure to develop an algorithm which estimates the changes in normal modes and natural frequencies of a dynamical system when the system is modified by the addition of mass, stiffness or mass/spring absorber. The only data required are the magnitude of the modification and the modal characteristics of the datum structure. The new modes are expressed as a linear combination of the original datum modes, thus the degree of coupling introduced by the structural change may be found. The methods were applied to three different examples and the results appear to be promising.

3.3 Combination of Formal Structural Optimization and Local Structural Modification

A careful examination of the papers dealing with local structural modification, described in the previous section, reveals that the term optimization is frequently used in either the title or the body of these papers. Unfortunately the use of the word optimization is somewhat misleading, since none of these papers attempt to use formal structural optimization, in the manner in which it was used in Refs. 16-19. In Refs. 30-33 the term optimization is used to indicate that the local structural changes made are the best, based on certain ad hoc considerations, such as reduction of strain energy in a structural member, or reduction of some dynamic response quantity.

Considerable work has been done on structural optimization with dynamic constraints, Refs. 21, 37 and 38 are representative of both past and more recent research. References 21 and 38 in particular are directly applicable to the problem of vibration reduction by local structural modification. To combine local structural modification with formal structural optimization a number of approaches are possible. One could, for example, combine the numerical method and the sensitivity analysis presented in Ref. 34 with an optimization package^{20,27} to obtain an automated procedure. Another more effective approach would be the extension of Refs. 21 and 38 to the helicopter fuselage problem. Using this research one could formulate an approximate optimization problem, in terms of cross sectional properties, and identify the structural members which need to be modified so as to reduce vibration levels at specified locations. In the case when a more complicated structural model of the fuselage is used one could break down the complicated structure into substructures, and use multilevel decomposition³⁹, to deal with other constraints imposed on the substructure level, in addition to constraints on vibration levels.

4. Extensions of Previous Research

In his excellent paper Blackwell² provides practical physical insight by considering the sensitivity of blade vibrations, to useful blade design parameters such as: tip sweep, camber, blade mass and stiffness distribution, chordwise blade center of gravity offset from the aerodynamic center, chordwise blade center of gravity offset from the elastic axis, blade twist and the use of composite materials for tailoring of the vibrational characteristics. Our ongoing research is aimed at extending the research presented in Refs. 17-19, by incorporating some of the effects discussed by Blackwell in a structural optimization study based upon the blade model shown in Fig. 10. The most important effects incorporated are the swept tip and improved unsteady aerodynamic modeling of the excitation. These two items were selected because the swept tip is a powerful means for both modifying the vibratory response as well as optimizing the aerodynamic and acoustic performance of the rotor. Improved unsteady aerodynamics are needed, so as to have a more realistic representation of the vibratory loads. A recently developed two dimensional unsteady aerodynamic theory⁴⁰ is being combined with simple compressibility correction and a simple dynamic stall model so as to yield more realistic airloads. Furthermore the single cell structural model shown in Fig. 1 is replaced by a two cell type structural box, so as to have a capability for modeling more general blade configurations. The need for more complicated aerodynamic modeling, requires

a more flexible formulation of the blade equations of motion. This is accomplished by using an implicit formulation of the aeroelastic problem, as opposed to the explicit formulation used in Refs. 25 and 26. It is expected that this research will enhance the capability for automated design of rotor blades with reduced vibration levels.

A number of other studies^{14,15} mentioned in this paper, are also being currently extended and refined. Taylor's work¹⁴ based on the modal shaping design, is being extended by Davis⁴¹, by coupling it to a formal optimization procedure. The objective of the study is to minimize vibrations in the rotor. Frequency placement, stresses and rotor inertia are used as constraints. However aeroelastic stability margins are not included among the constraints. Bennett's work¹⁵ is also being extended by Sutton and Bennett⁴², so as to include many additional ingredients in the analysis and optimization such as: rotor aerodynamics, rotor dynamics, fuselage dynamics, flight mechanics, aeroelastic analysis, active and passive vibration reduction devices. This program, when completed, will be capable of optimizing tilt rotor, compound and coaxial rotorcraft configurations.

5. Concluding Remarks

It was shown that modern structural optimization offers significant benefits in the structural design of helicopter rotors and fuselages. When applied to the rotor, these methods provide an automated design capability which has the potential for reducing vibration levels in forward flight by 15-40% while simultaneously reducing blade weight by up to 25%. When applied to the fuselage the combination of optimization and local structural modification provides a useful tool for reducing vibration levels at specific locations in the fuselage. Both applications accomplish vibration reduction by redistribution of mass and stiffness in a more optimal manner and hence structural weight is reduced. Therefore these methods deserve to be serious candidates for incorporation in the design process of rotorcraft.

Acknowledgement

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TABLE I: COMPARISON OF VERTICAL HUB SHEARS AND HUB ROLLING MOMENTS AT $\mu=0.30$, AND ADDED NON STRUCTURAL MASSES, AFTER TWO STAGES OF OPTIMIZATION

ITEM		INITIAL DESIGN D_0	IMPROVED DESIGN D_I	REDUCTION 1st STAGE $\frac{(D_0-D_I)}{D_0}$	IMPROVED DESIGN D_{II}	REDUCTION $\frac{(D_0-D_{II})}{D_0}$ %
VERTICAL HUB SHEARS	PEAK-TO-PEAK (LINEAR)	0.0575	0.0408	29.04%	0.0357	37.91%
(P_{z1}^2)	PEAK-TO-PEAK (NON-LINEAR)	0.0602	---	---	0.0386	35.88%
$(\Omega^2 I_b)$						
HUB ROLLING MOMENTS	PEAK-TO-PEAK (LINEAR)	0.0120	0.0104	13.33%	0.0091	24.17%
M_{x1}	PEAK-TO-PEAK (NON-LINEAR)	0.0119	---	---	0.0089	25.21%
$(\Omega^2 I_b)$						
ADDED NON-STRUCTURAL MASS	none		0.17% of blade mass		2.3% of blade mass	

--- not calculated

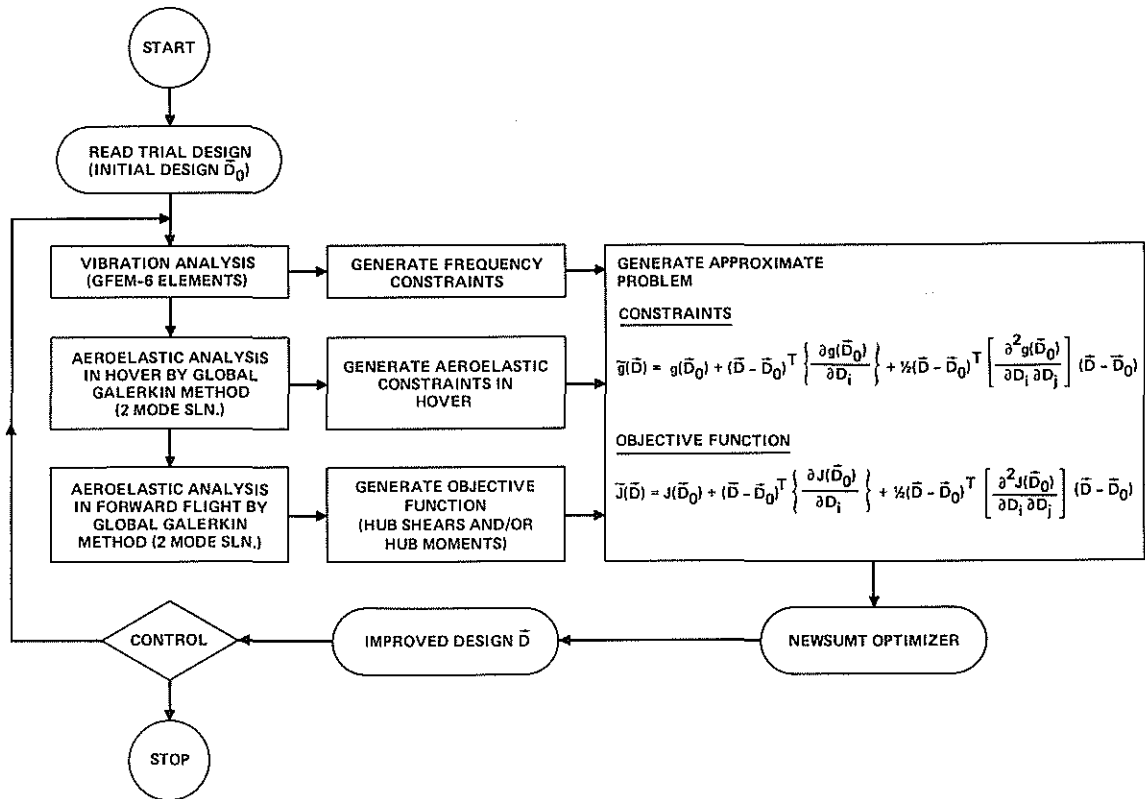


Fig. 2 Basic Organization of the Optimization Process

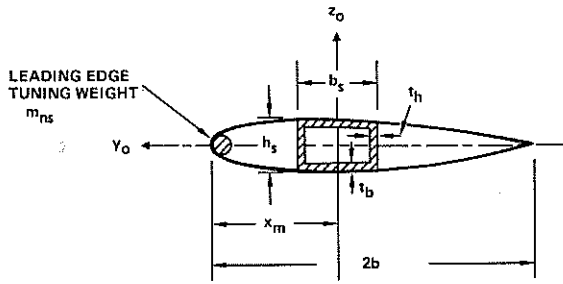


Fig. 1 Typical Blade Cross Section

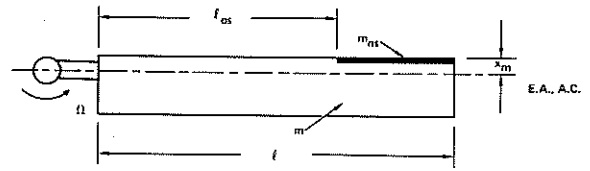


Fig. 3 Spanwise and Chordwise Location of Nonstructural Mass (Figure shows leading edge location, when $x_m = 0$, mass is on elastic axis)

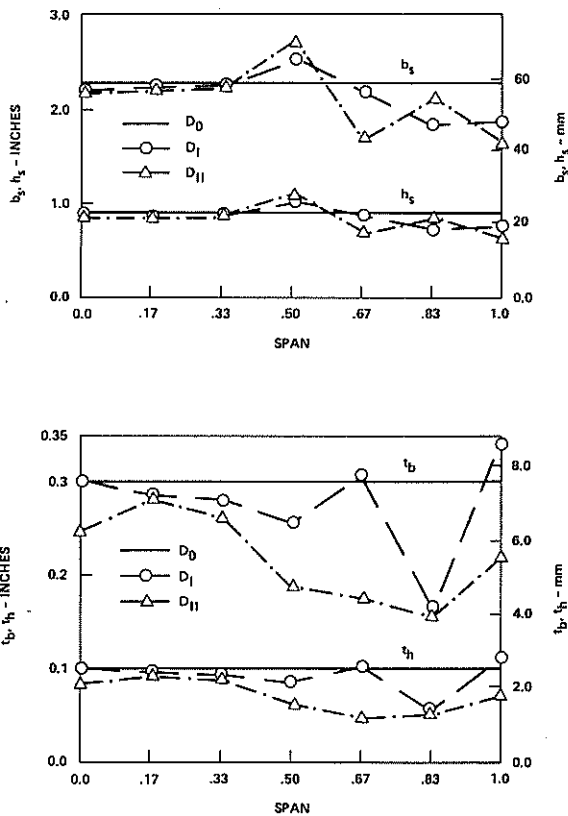


Fig. 4 Cross Sectional Dimensions of Initial and Improved Designs After Two Stages of Optimization

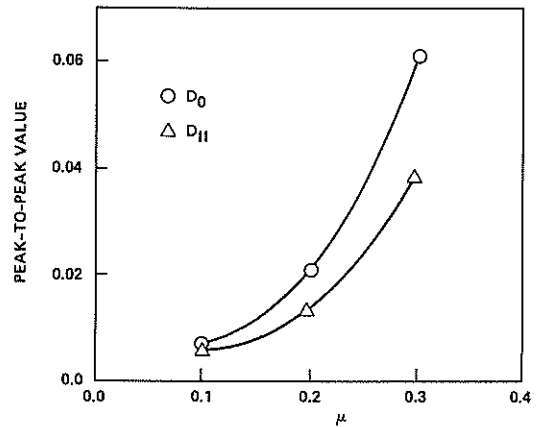


Fig. 5 Vertical Hub Shears, Nonlinear, Peak-to-Peak Values, Nondimensionalized ($P_{z1}l/\Omega^2 I_b$) Vs. μ , Comparison of Initial and Final Designs After Two Stages of Optimization

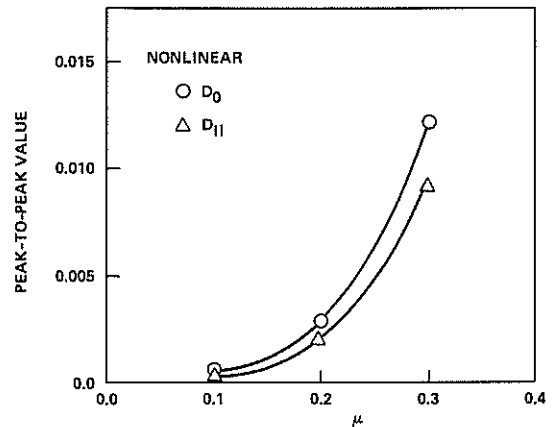


Fig. 6 Hub Rolling Moments, Peak-to-Peak Values, Nondimensionalized ($M_{x1}/\Omega^2 I_b$), Versus μ , Comparison of Initial and Final Designs After Two Stages of Optimization

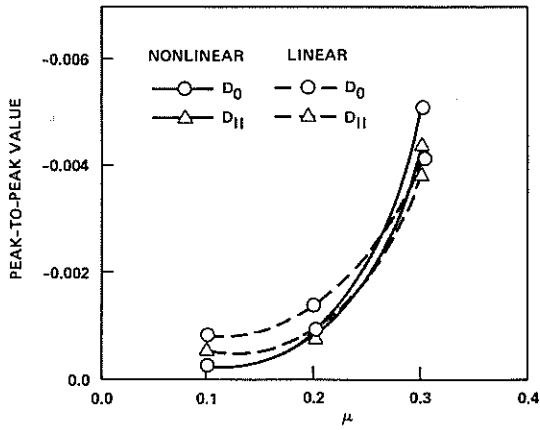


Fig. 7 In-Plane Hub Shears, Peak-to-Peak Values, Nondimensionalized ($P_{y1}l/\Omega^2 I_b$), Versus μ , Comparison of Initial and Final Designs, After Two Stages of Optimization

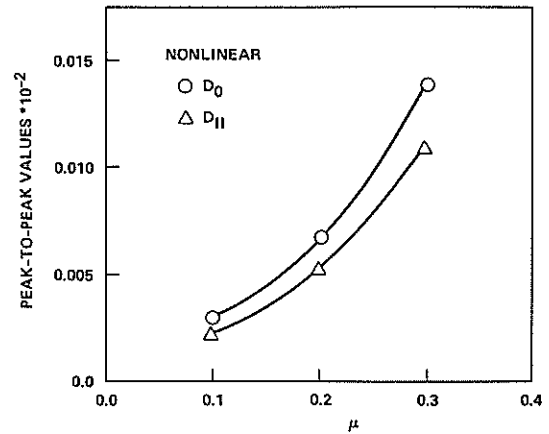


Fig. 8 Blade Root Torsional Moments in Blade Fixed Rotating Reference Frame, Nondimensionalized ($q_{x0}/\Omega^2 I_b$), Versus μ , Comparison of Initial and Final Designs

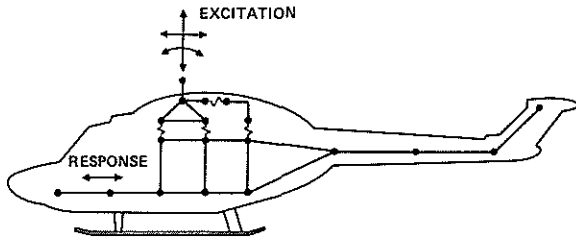


Fig. 9 Beam Type Fuselage Model For Vibration Reduction by Local Structural Modification (Ref. 31)

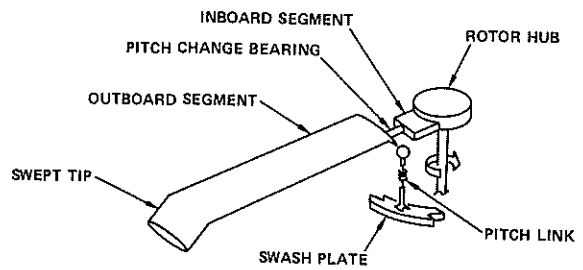


Fig. 10 Swept Tip Hingeless Blade Model