"A Frequency Domain Theory for Structural Identification"

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ABSTRACT

Structural identification refers to those methodologies which seek to correct or improve analytical dynamics models by the rational application of dynamic test data. A new theory for structural identification is under formulation, and has been shown to address critical issues in the airframe vibration prediction problem. The theory allows a finite element model to be corrected such that its frequency response predictions will precisely match the corresponding test data, at all frequencies of interest. The theory is based on *frequency domain structural synthesis*, a structural modification and substructure coupling methodology which provides a natural analytic bridge between the finite element frequency response model of a linear structural dynamic system, and its experimentally derived counterpart. The *structural synthesis transformation* quantifies the difference between the two models by extracting from their matrix difference a complex and frequency-dependent frequency response error matrix. The associated error impedance matrix can then be decomposed into the constituent missing mass, damping, and stiffness matrices, each calculated as a function of frequency. The theory demonstrates that an exact solution for the corrective structural matrices is available directly from spatially complete frequency response data. Furthermore, the theory reveals that structural identification using spatially incomplete data (e.g., applying dynamic reduction to the finite element model) imposes a frequency-dependency in the identified missing parameters. Operating spatially, the theory can provide precise information as to the location of modeling errors, a capability compromised only by the requirement of dynamic reduction of the finite element description. The theory inherently provides corrections for errors due to the discretization of a continuous structure, the further discretization errors imposed by dynamic reduction of the finite element model, and accommodates any frequency-dependent mechanisms inherent in the structure.

OVERVIEW

A new theory for structural identification is under development, and its capabilities developed to date are reported in what follows. The formulation is based on *frequency domain structural synthesis* [1], a theory for structural modification and sub-structure coupling. Given a frequency response matrix for a *baseline* or pre-synthesis structural system, a *structural synthesis transformation* is constructed from the matrix descriptions of the structural changes to be installed into the baseline system, (either additional elements or appended sub-structures), and is used to transform the baseline frequency response model into the *reference* or synthesized frequency response model. The synthesis transformation is performed at each frequency of interest, and provides an exact solution for the synthesized system dynamics. The changes that may be installed into the baseline system may be of any size, and may be comprised of any number or distribution of linear structural elements, including both lumped elements, such as simple springs and dashpots, and finite (distributed) elements.

A reinterpretation of the structural synthesis transformation reveals its suitability as the basis for a new approach to the identification of finite element modeling errors. The synthesis transformation provides a natural analytic bridge between two frequency domain matrix descriptions of a given linear structural dynamic system. Finite element modeling provides the *baseline* or predicted frequency response description; a vibration test provides the *reference* or true frequency response description. The inverse synthesis transformation extracts from the matrix difference of the two descriptions an error impedance matrix. This error impedance matrix can then be decomposed into associated error matrices of mass, damping and stiffness. These error matrices can be calculated at any frequency of interest, thereby allowing frequency dependent structural mechanisms to be identified, and subsequently included in the finite element model. The structural synthesis transformation has the unique ability to spatially locate modeling errors. Once the location determination has been made, the structural synthesis transformation will then subsequently provide a unique solution for the missing structural mechanisms, in terms of the error

matrices of mass, stiffness and damping. Note that the structural synthesis transformation demonstrates that an exact solution for the missing *physical* parameters is available directly from frequency response data, no modal analysis or modal parameter identification need be performed.

The finite element model is necessarily comprised of a greater number of degrees of freedom than can be measured in a test. This disparity in information is a fundamental obstacle in any structural identification methodology. The ability of the new theory to function effectively with spatially incomplete data is demonstrated. Identification with spatially incomplete data is shown to be equivalent to the dynamic reduction of a discrete model, with respect to the frequency dependent distortion necessarily and unavoidably imposed on the structural parameters to be identified. Numerical examples of the identification theory are presented.

INTRODUCTION

Structural identification refers to those methodologies which seek to correct or improve analytical dynamics models by the rational application of dynamic test data. Aerospace structures are commonly analyzed by large, linear finite element (FE) models. The structural representation which results from the FE discretization is a second order, linear, constant coefficient ordinary differential equation, whose solution is typically based on the solution of the associated eigensystem. Structural identification methodologies provide the means to correct the structural parameters contained in the FE model, namely the (symmetric) matrices of mass, stiffness, and damping (if an analytical damping model has been established), such that improved estimates of the modal parameters and/or frequency response are obtained from the FE model. The corrections are determined from the relationship established by the identification methodology between what is measured from the structure and the corresponding analytically derived quantities.

The majority of existing identification methodologies seek to identify the required corrections to the FE model by quantifying the difference between the modal parameters extracted from test data and those calculated from the FE analysis. Although many innovative and effective schemes have been developed by numerous investigators, the limitations and uncertainties peculiar to modal parameter estimation and identification have motivated the development of a general approach to structural identification which avoids modal analysis altogether. Those characteristics which are common to modal identification methodologies, and to be avoided in the present formulation, will be briefly described; the interested reader is referred to the literature for a wide variety of objective and indepth analyses.

A fundamental source of "error" which impacts on all identification schemes, and one to be accommodated in the present theory, is as follows. A distributed system has, in theory, an infinity of degrees of freedom, while a finite element model possesses a limited number of degrees of freedom. The representation of a real (distributed) structure by a discrete, finite-order (finite element) model therefore constitutes a reduced order description. The reduction procedures often applied to finite element models, which result in the elimination of a selected set of degrees of freedom, also yield a reduced order description. Any discrete representation of a distributed system, and, equivalently, the reduction of a finite element discretization to a model order whose size is consistent with the limited amount of available test data, necessarily and unavoidably introduces nonlinear dependencies of the structural parameters of mass and damping, with the frequency, Ω [2]. Both 'reductions' are inherent in all structural identification schemes. In order for any finite element representation to accurately predict the structure's response throughout an unrestricted frequency band, the finite element model parameters of mass and damping must necessarily vary non-linearly with frequency. This requirement is revealed in the development of improved dynamic reduction methods, whereby higher order corrections to the missing inertia forces on eliminated degrees of freedom appear as ascending powers of Ω^2 [3,4]. Modal analysis is strictly associated with constant coefficient differential equation descriptions, and therefore modal methods of identification must limit the frequency range in which the corrections so identified are valid. The frequency domain theory to be described not only clearly demonstrates this phenomenon, but inherently accommodates the computational frequency dependencies imposed on the structural parameters to be identified.

While the measurement of natural frequencies is routinely made with confidence, the measurement and estimation of the higher mode shapes presents difficulties, and makes severe demands on the number of transducers required for spatial definition [5,6]. The rational dissemination of the wealth of modal information generated by a FE analysis and from the test is itself not a straightforward task. Typically, a large finite element model will contain more modes than the actual structure, within a given bandwidth [2]. It is often not clear which analytical modes correspond with which test modes. Modal identification methods have required a one to one correspondence between test modes and analysis modes [7,8]. If this correspondence is violated in the course of the identification procedure, the resulting parameter corrections so determined will be unduly large, and could introduce previously non-existent load paths. Additional computation, in at least one method, was provided to insure that test-analysis mode correlation, once established, is maintained [9]. This is further complicated by the fact that test modes are, in general, complex, and analytical modes are real valued. This not only impairs the correlation of test modes with the analytical modes, but can introduce significant errors in an identification procedure which requires that the test modes be orthogonal to the FE mass or stiffness matrix. The presence of complex modes is unavoidable, given the significant levels of non-proportional damping which exists in large and complex aerospace structures. Furthermore, the estimation of the mode shapes can be complicated in the presence of high modal density and the rapid change in phase angles near resonances [10].

One area of structural identification much in need of improvement is the experimental determination of structural damping. Structural damping, currently beyond our predictive capabilities, is a source of a variety of frequency (and amplitude) dependent effects. The inclusion of an accurate damping model in a finite element analysis requires its measurement. Modal methods, however, do not provide a robust approach to the identification of damping in aerospace structures. The helicopter airframe has consistently defied efforts to accurately predict higher mode and inter-resonant response, and this has been attributed, to a significant extent, to the failure to accurately include non-proportional damping in the finite element model [11]. What is particularly troublesome about these modeling limitations is that the helicopter must be operated in the inter-resonant regions to insure acceptable levels of noise and vibration, yet the characteristics of these regions can depend significantly on the nature of the damping. The significant levels of higher frequency excitation to which the helicopter is subjected mandates that the higher mode spectral regions be predicted accurately. The various dissipative mechanisms in a structure exhibit frequency and amplitude dependencies which cannot be addressed by a constant coefficient model, i.e., the modal parameters. The spatial, frequency domain formulation to be presented provides a natural means of quantifying structural damping.

THEORY

The derivation of the structural synthesis transformation will be repeated here for completeness; for a thorough treatment of the subject the reader is referred to [1,12]. Vector quantities are denoted by boldface symbols. Second order linear structural systems can be described in the frequency domain by:

$$[\ \mathbf{K} - \Omega^2\ \mathbf{M} + j\ \mathbf{C}(\Omega)]\ \{\mathbf{q}\} = \{\mathbf{f}\} \tag{1a}$$

Here, the vector $\{\mathbf{q}\}$ contains generalized responses for a set of *motion coordinates* (to be defined shortly) about which this information is desired. The vector $\{\mathbf{f}\}$ contains generalized forces. The matrices **K** and **M** are order n, symmetric, and real valued stiffness and mass matrices, respectively, as might be calculated from a finite element analysis. Damping is introduced through the frequency-dependent matrix, $\mathbf{C}(\Omega)$. In compact notation, Eq. (1a) is written:

$$[\mathbf{Z}(\Omega)]\ \{\mathbf{q}\} = \{\mathbf{f}\} \tag{1b}$$

The matrix $\mathbf{Z}(\Omega)$ is referred to as the system *impedance* matrix, and is both complex-valued and frequency dependent. Matrix inversion of Eq. (1) provides the frquency response relation:

$$[\mathbf{Y}(\Omega)]\ \{\mathbf{f}\} = \{\mathbf{q}\} \tag{2}$$

The matrix $Y(\Omega)$ is known alternatively as a frequency response function matrix, a multiple input/multiple output transfer function, or a mobility matrix. Any element y_{ij} of this matrix is defined as the dynamic response of motion coordinate "i" due to a unit oscillatory generalized force acting at the motion coordinate "j", or:

$$y_{ij} = \partial q_i / \partial f_j$$

The matrix $Y(\Omega)$ is usually calculated from $Z(\Omega)$, in analytical studies, or measured directly, in an experiment.

The popularity of modal methods in the calculations of response, modification, and identification stems not only from their accepted analytical power, but also from the natural language they bring to bear on problems in structural dynamics; a physically intuitive description consistent with the way engineers work. Frequency domain models that operate spatially cannot take advantage of these many benefits, and it is therefore advantageous to seek another way of approaching these models.

With this goal in mind, it is constructive to distinguish between a degree of freedom and a motion coordinate. The partial differential equations from which structural models originate admit an infinity of both degrees of freedom and motion coordinates, a direct result of their distributed nature. A discretization such as a finite element model typically employs a large number of computational degrees of freedom, or *motion coordinates,* which exceeds the number of useful degrees of freedom obtained, the natural modes. A similar discrepancy exists in experimentally derived analytical models. The number of measurement locations (*motion coordinates*) is typically greater than the number of useful degrees of freedom identified, the modes of response.

Structural synthesis operations can be viewed as alterations to the existing *connectivity* (which motion coordinate is connected to which) of a baseline structural system. The mobility and impedance matrices required for synthesis contain all the pertinent connectivity information about the baseline structural system and the structural change, respectively. Two structural system representations are involved in what follows. The pre-synthesis structural system is referred to as the *baseline* model, and represents the finite element prediction. The post-synthesis system representation is referred to as the *reference* model, in anticipation of its role as the test, or true system. The reference model differs from the baseline model in its *connectivity*. Any discrete structural system can be distinguished by its connectivity; a set of motion coordinates joined by a set of complex and frequency-dependent load paths. The structural synthesis transformation effects an alteration in the connectivity of the baseline model, producing the reference model. The transformation is performed at all frequencies of interest.

Consider the following partitioned structural system representation:

$$\begin{Bmatrix} q_i \\ q_c \end{Bmatrix} = \begin{bmatrix} Y_{ii} & Y_{ic} \\ Y_{ci} & Y_{cc} \end{bmatrix} \begin{Bmatrix} f_i \\ f_c \end{Bmatrix} \tag{3}$$

The various quantities are defined as follows:

q_c, f_c: A set of generalized responses and excitations, respectively, at (connection) motion coordinates directly involved in the synthesis. These motion coordinates may be associated with a single structure, or with several structures.

q_i, f_i: A set of generalized responses and excitations, respectively, at (internal) motion coordinates not directly involved in the synthesis. Again, these motion coordinates may be associated with a single structure, or with several structures.

Y_{ii},Y_{ic}, etc.: Frequency response function (mobility) matrices relating the above defined quantities.

In general, the connection motion coordinates may experience coupling forces (to be established through synthesis) and external forces, i.e.,

$$f_c = f_c^{ext} + f_c^{cpl} \tag{4a}$$

By definition of the subscript "i", we may have only

$$f_i = f_i^{ext} \tag{4b}$$

Introducing Eqs. (4) into Eq. (3) allows the expansion of Eq. (3):

$$\begin{Bmatrix} q_i \\ q_c \\ q_c \end{Bmatrix} = \begin{bmatrix} Y_{ii} & Y_{ic} & Y_{ic} \\ Y_{ci} & Y_{cc} & Y_{cc} \\ Y_{ci} & Y_{cc} & Y_{cc} \end{bmatrix} \begin{Bmatrix} f_i^{ext} \\ f_c^{ext} \\ f_c^{cpl} \end{Bmatrix} \tag{5}$$

Here, the superscripts "ext" and "cpl" mean externally applied forces, and coupling forces, respectively. Note that with the introduction of Eqs. (4a) and (4b) into Eq. (3), a redundant equation, - the third row of Eq. (5), has been appended. A new, condensed partitioning of Eq. (5) is now possible, suggested by the "ext" and "cpl" superscripts in Eq. (5), and is based on the union of the two sets of motion coordinates, internal with connection. Hence, the partitioning is now characterized by two new operative sets of motion coordinates which are distinguished by the presence of either externally applied forces, with subscript "e", or coupling forces, with subscript "c", acting on them. The set union is denoted as:

$$e = i \cup c$$

This equation (5) is then more compactly written as:

$$\begin{Bmatrix} q_e \\ q_c \end{Bmatrix} = \begin{bmatrix} Y_{ee} & Y_{ec} \\ Y_{ce} & Y_{cc} \end{bmatrix} \begin{Bmatrix} f_e \\ f_c \end{Bmatrix} \tag{6}$$

where $\{f_e\} = \{\, f_i^{ext} \;\; f_c^{ext} \,\}^T$ and $\{f_c\} = \{f_c^{cpl}\}$.

Eq. (6) is general in that it may describe any baseline linear structural system. The system is described by its motion coordinates of interest, and the impedance paths between them, i.e. the estimate of the connectivity. The synthesis operation involves the construction of new and/or redundant load paths between the connection motion coordinates of Eq. (6). To this end, we now seek to construct a transformation, which when applied to Eq. (6), will provide the operative equation of synthesis. The transformation is constructed from consideration of the structural change to be made. The synthesis transformation allows the structural change to be comprised of any number and type of linear structural element, lumped or distributed (finite). The only requirement is that the element be amenable to description by the following equation:

$$\{f_c\} = - \{ [K_c] - \Omega^2 [M_c] + j [C_c(\Omega)] \} \{q_c\} \tag{7a}$$

or, in compact notation,

$$\{ f_c\} = - [Z_c] \{q_c\} \tag{7b}$$

The subscript "c", with respect to an impedance quantity, denotes the impedance change being considered. The minus sign in Eq. (7) indicates that we are considering reaction forces, imposed by the structural change on the baseline structural system. The transformation which operates on Eq. (6) is now given. The transformation, in effect, installs the coupling forces from the impedance change being made at the connection motion coordinates.

$$\begin{Bmatrix} f_e \\ f_c \end{Bmatrix} = \begin{bmatrix} I & 0 \\ 0 & -Z_c \end{bmatrix} \begin{Bmatrix} f_e \\ q_c \end{Bmatrix} \tag{8a}$$

$$\begin{Bmatrix} q_e \\ f_c \end{Bmatrix} = \begin{bmatrix} I & 0 \\ 0 & -Z_c \end{bmatrix} \begin{Bmatrix} q_e \\ q_c \end{Bmatrix} \tag{8b}$$

Substitution of Eqs. 8 into Eq. 6, and some algebra provides:

$$Y_{ee}^* = Y_{ee} - Y_{ec} [Z_c^{-1} + Y_{cc}]^{-1} Y_{ce} \tag{12a}$$

or in the original, full notation:

$$\begin{bmatrix} Y_{ii} Y_{ic} \\ Y_{ci} Y_{cc} \end{bmatrix}^* = \begin{bmatrix} Y_{ii} Y_{ic} \\ Y_{ci} Y_{cc} \end{bmatrix} - \begin{bmatrix} Y_{ic} \\ Y_{cc} \end{bmatrix} [Z_c^{-1} + Y_{cc}]^{-1} \begin{bmatrix} Y_{ic} \\ Y_{cc} \end{bmatrix}^T \tag{12b}$$

This is the fundamental operative equation of structural synthesis. The superscript "*" indicates a reference or synthesized quantity, i.e., the counterpart to the mobility matrix of Eq. (3), but with alterations made to the baseline connectivity due to the installation of the structural change. All terms on the right hand side, except for Z_c, are calculated for the original, baseline structural system description. The matrix Z_c is the impedance matrix for the structural change itself.

As a qualitative example of the theory functioning as a structural modification methodology, consider the helicopter, depicted in Figure 1. A finite element analysis of the complete aircraft produces a large and detailed frequency response model, of order n.

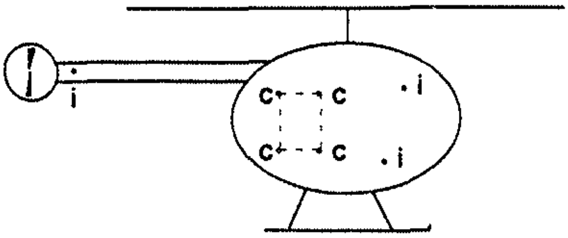


Figure 1: Pre-Modification Helicopter

FE modeling provides:

$q = Y(\Omega)\ f$

where $Y(\Omega) \equiv Z(\Omega)^{-1}$

and $Z(\Omega) = K - \Omega^2 M + ...$

K, M symmetric, order n.

Furthermore, dynamic response predictions reveal unacceptable response levels at the pilot seat, tail rotor, and at an avionics rack below. It is postulated that stiffening the fuselage locally through the installation of an additional plate element will mitigate these problems.

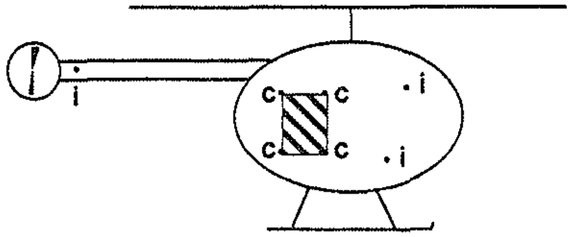


Figure 2: Post-Modification Helicopter

Therefore, motion coordinates at the the pilot's seat, tail rotor, and avionics rack are taken as internal motion coordinates, and the four motion coordinates at the rear fuselage, to which the new plate element (see Figure 2) will be attached, are taken as connection motion coordinates. The appropriate terms of the full order FE frequency response matrix, which correspond to the connection and internal motion coordinates, are extracted to form Y_{ee}, the structural change, which consists of the plate element, is described by its impedance matrix, Z_c. The matrices Y_{ii}, Y_{ic}, and Y_{cc} are themselves extracted from Y_{ee}.

Performing the matrix operations defined by the right hand side of Eq. 12 will result in the synthesized frequency response matrix, Y^*_{ee}. The structural synthesis provides an exact solution for the synthesized system dynamics, and is also an economical solution, due to the fact that only the motion coordinates of interest need be included in the synthesis, and no re-assembly of the original, large order FE model need be performed to incorporate the changes.

Note that the theory is equally effective in problems of substructure coupling. In fact, the structural synthesis transformation makes plain the equivalence of structural modification and substructure coupling, with respect to both the derivation and application of the theory. Frequency domain structural synthesis provides an effective design and analysis tool, allowing proposed structural changes to be evaluated rapidly and economically. These characteristics are unique to the structural synthesis methodology, and they are a result of casting the problem spatially in the frequency domain. A complete discussion of the theory is given in [1,12]. As already mentioned, it is these characteristics which are exploited in the application of structural synthesis to the identification problem. Modal methods, such as the numerous variants of modal synthesis [for example, 13,14], do not offer comparable economy, simplicity, or an exact solution.

STRUCTURAL IDENTIFICATION

A reinterpretation of the structural synthesis equation, Eq. 12, reveals its intrinsic ability to quantify the difference between two frequency response models of a given structural system. The finite element model provides an estimate of the structural system (symmetric) mass, stiffness and damping in terms of the associated baseline impedance matrix, comprised of **M**, **K**, and **C**. Inversion of the finite element impedance matrix provides the baseline frequency response matrix, Y. The vibration test provides the reference frequency response matrix, Y^*, directly, no modal parameter estimation is required. The reinterpretation of the synthesis equation. Eq. 12, is as follows. Retaining the partitioning from before, a *frequency response error matrix* is defined as:

$$\Delta Y = Y - Y^* = \begin{bmatrix} Y_{ii} Y_{ic} \\ Y_{ci} Y_{cc} \end{bmatrix} - \begin{bmatrix} Y_{ii} Y_{ic} \\ Y_{ci} Y_{cc} \end{bmatrix}^* \tag{13}$$

The balance of the synthesis equation is written as:

$$Y_{ec}\ D^{-1}\ Y_{ce} = \begin{bmatrix} Y_{ic} \\ Y_{cc} \end{bmatrix} [\ Z_c^{-1} + Y_{cc}\ ]^T \begin{bmatrix} Y_{ic} \\ Y_{cc} \end{bmatrix}^T \tag{16}$$

The structural synthesis transformation is compactly rewritten as:

$$\Delta Y = Y_{ec}\ D^{-1}\ Y_{ec} \tag{17}$$

It is clear from Eq. 17 that the rank of the error frequency response matrix, ΔY can be no more than the rank of **D**. The rank of **D** is determined by the number of independent elasto-dynamic mechanisms (unique load paths) which comprise the difference between the baseline and reference systems. These mechanisms are the errors we seek to identify, and are contained in the impedance error matrix, Z_c. We are now in the position to detail the identification process using the structural synthesis transformation. The discussion will initially assume the availability of spatially complete test data, all motion coordinates of the FE model having experimentally measured counterparts. The practical situation of spatially incomplete data will be addressed below, along with numerical examples of both spatially complete and incomplete identification, demonstrating the various features of the new theory.

ERROR LOCATION DETERMINATION

One of the benefits of the theory is its ability to disclose the locations of errors, a unique characteristic which arises from the spatial formulation. We initially assume that all motion coordinates (of the full order FE model) are potential connection motion coordinates, i.e., are associated with load paths in error. The structural synthesis transformation will filter out all motion coordinates not associated with load paths in error, leaving definitive information as to which FE motion coordinates are truly connection motion coordinates. The spatial formulation provides an exact determination of the set of connection motion coordinates which should be extracted from the full order frequency response matrix. The *location matrix* L is defined as

$$L = Z\ \Delta Y\ Z^T \tag{18}$$

where Z = full order (e∗e) symmetric finite element impedance matrix, defined by Eq. 1.
ΔY = frequency response error matrix, defined by Eq. 13.

Substituting the structural synthesis transformation, Eq. 17, into Eq. 18,

$$L = Z \; Y_{ec} \; D^{-1} \; Y_{ce} \; Z^{T}$$

This equation can be written at any frequency, and will always produce:

$$L = \begin{array}{cc} \begin{array}{cc} i & c \end{array} & \\ \left[\begin{array}{cc} [\,0\,] & [\,0\,] \\ [\,0\,] & [\,X\,] \end{array} \right] & \begin{array}{c} i \\ c \end{array} \end{array}$$

The L matrix contains zeros in the rows and columns corresponding to motion coordinates of the FE model not associated with load paths in error. The L matrix calculation therefore reveals these motion coordinates as internal motion coordinates. There will always be non-zero values in the rows and columns corresponding to motion coordinates of the FE model associated with load paths in error. The L matrix calculation therefore reveals these motion coordinates as connection motion coordinates. Note that although Eq. 18 can be written at any frequency desired, this property of the L matrix is invariant with frequency (for frequency independent errors), and is based on the identity,

$$Z \, Y = I$$

The determination of the location of errors as provided by the L matrix is functionally equivalent to the determination of the maximally-sized connection partition matrix of full rank. This insures that the subsequent calculation of the error impedance will not fail due to attempted inversion of a singular matrix. Of course, one is free to choose a subset of the maximally-sized set so identified, but this will impose the reduction errors in the calculation of the error impedance, as mentioned in the introduction, and to be further discussed below.

THE ERROR IMPEDANCE

Having made the determination of which motion coordinates are connection motion coordinates, we can now proceed to solve for the error impedance, Z_c. The L matrix calculation provides the information necessary to identify the relevant elements to be extracted from the FE model and the test model. Extracting the connection-connection partition from Eq. 12. (or equivalently, from Eq. 17) and solving for the error impedance,

$$Z_c^{-1} = [Y_{cc}^{-1} \ (\Delta Y_{cc}) \ Y_{cc}^{-1}]^{-1} - Y_{cc} \tag{19}$$

The overdetermined solution, obtained by including internal motion coordinate information, can be used when it is desired that the impedance errors to be identified should be restricted to a subset of the connection motion coordinates identified by the **L** matrix calculation. This might correspond to the situation where certain measured motion coordinates are known to be in regions where the structure has been modeled with high confidence, and therefore should be excluded. The overdetermined solution is given by,

$$Z_c^{-1} = [Y_{ec}^{\dagger} \ (\Delta Y) \ Y_{ce}^{\dagger}]^{-1} - Y_{cc} \tag{20}$$

where the superscript ($\dagger$) indicates the pseudo-inverse. The solution for the error impedance. Eq. 19 or Eq. 20, is performed at all frequencies of interest. This characteristic allows the identification of frequency dependent structural mechanisms. Having done this, it is then possible to decompose the error impedance into its constituent mass, stiffness and damping matrices.

$$\begin{array}{l}
Z(\Omega_1) = K(\Omega_1) - \Omega^2 \, M(\Omega_1) + j \, C(\Omega_1) \\
Z(\Omega_2) = K(\Omega_2) - \Omega^2 \, M(\Omega_2) + j \, C(\Omega_2) \\
Z(\vdots) = K(\vdots) - \Omega^2 \, M(\vdots) + j \, C(\vdots) \\
Z(\Omega_n) = K(\Omega_n) - \Omega^2 \, M(\Omega_n) + j \, C(\Omega_n)
\end{array}$$

This set of equations may be manipulated to solve for the error matrices of mass. stiffness. and damping. As a minimum. two equations are required to solve for the mass and stiffness. If this is done sequentially. where a minimum of two equations are solved for **K** and **M** at a time. frequency dependencies in these matrices will be identified. Of course. (weighted) least squares solutions for **K** and **M** in extended bandwidths may be calculated.

NUMERICAL SIMULATION

The capabilities of the theory will be demonstrated by the following numerical examples. The simulation will at first assume spatially complete data, to provide an unhindered examination of the theoretical capabilities of the formulation. The case of spatially incomplete data will then be considered. The simulation makes use of the following dynamic systems, shown in Figure 3. A clamped-free beam is modeled using five undamped Euler-Bernoulli elements, and this system represents the finite element prediction, or the *baseline system*. The 'test' system, denoted the *reference system*, is identical in configuration, except that two of the five elements possess significant differences in mass, stiffness and damping ($j=\sqrt{-1}$) from their counterparts in the baseline system. These "impedance errors," are implanted in the baseline system as redundant elements. The reference system is the synthesis of the baseline system and the impedance errors.

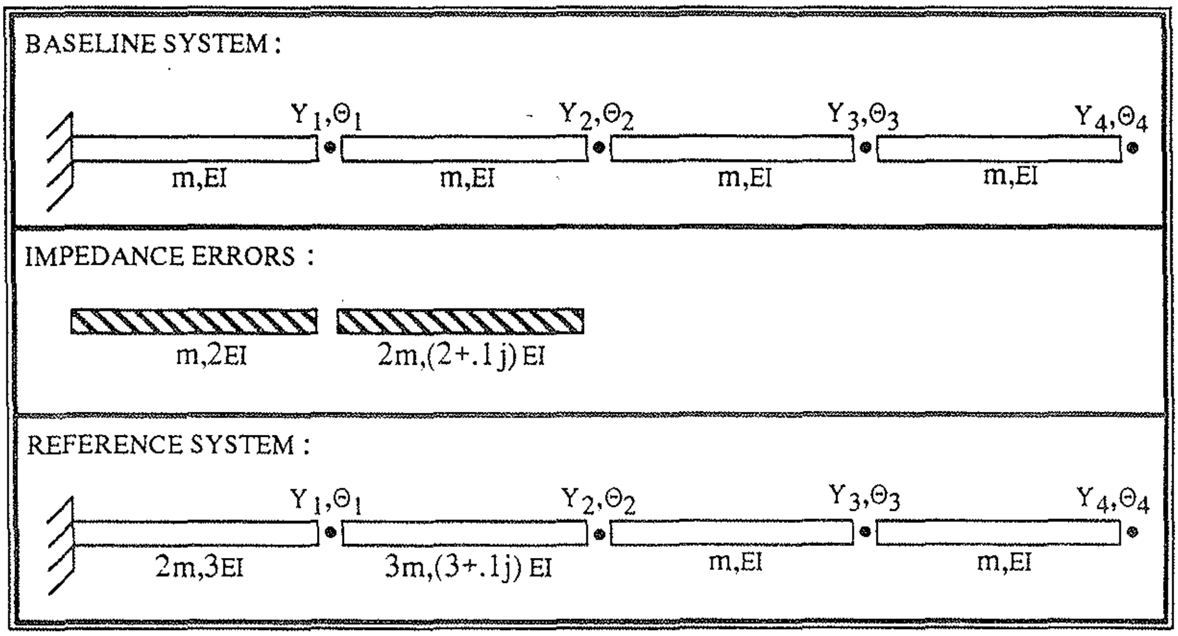


Figure 3: Dynamic Systems for Simulation

The impedance and frequency response for the baseline and reference systems are now given. Figures 4a and 4b show a typical term of the impedance and frequency response for the baseline system, and for the reference system. Note that significant differences exist between the baseline frequency response, and the reference frequency response. The differences include significant frequency spacing errors, and the fact that the baseline (FE prediction) system is undamped. The actual errors implanted in stiffness and mass are of the order of one hundred percent, for illustrative purposes. The identification theory seeks to identify the corrections to the baseline impedances, such that the (corrected) baseline frequency response will precisely match the reference frequency response. Therefore, the error impedance, as identified by the theory, quantifies the difference between the baseline impedance and the reference impedance.

Before the error impedance can be calculated, the location of the errors is required, such that a maximally ranked partition of the relevant matrices will be manipulated. If this is not done, the resulting attempt to invert a singular matrix will cause the procedure to fail. The **L** matrix calculation is shown in Figure 5. The frequency response matrices are depicted by surface plots, the two horizontal axes corresponding to the indices of the matrices, the

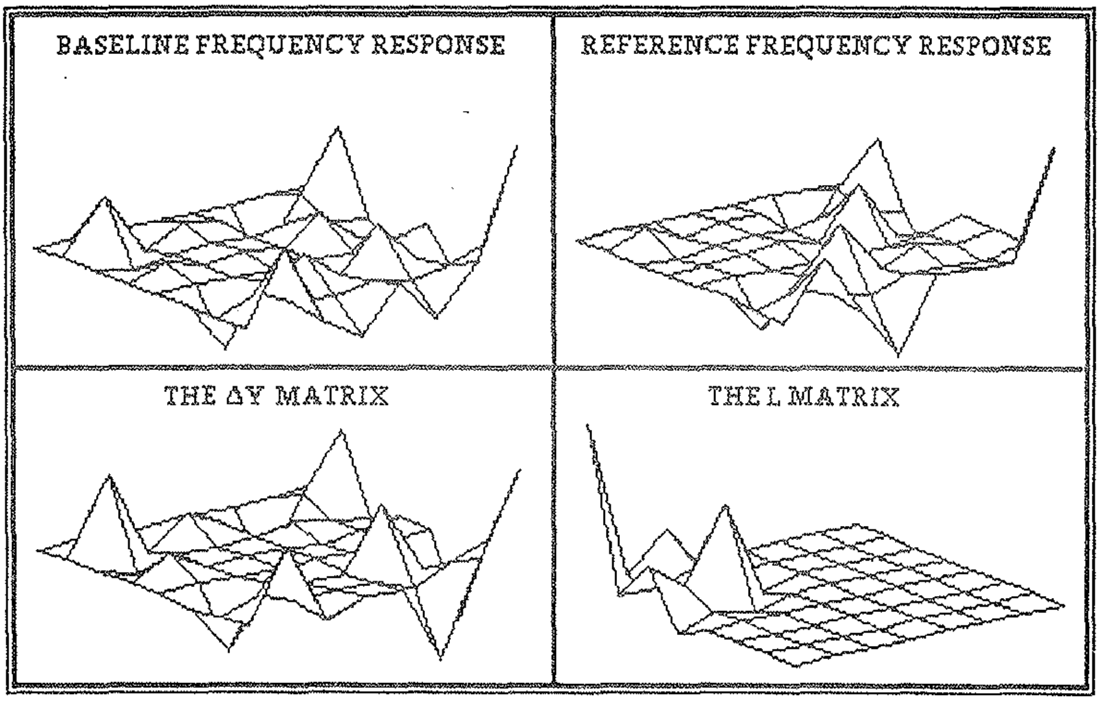
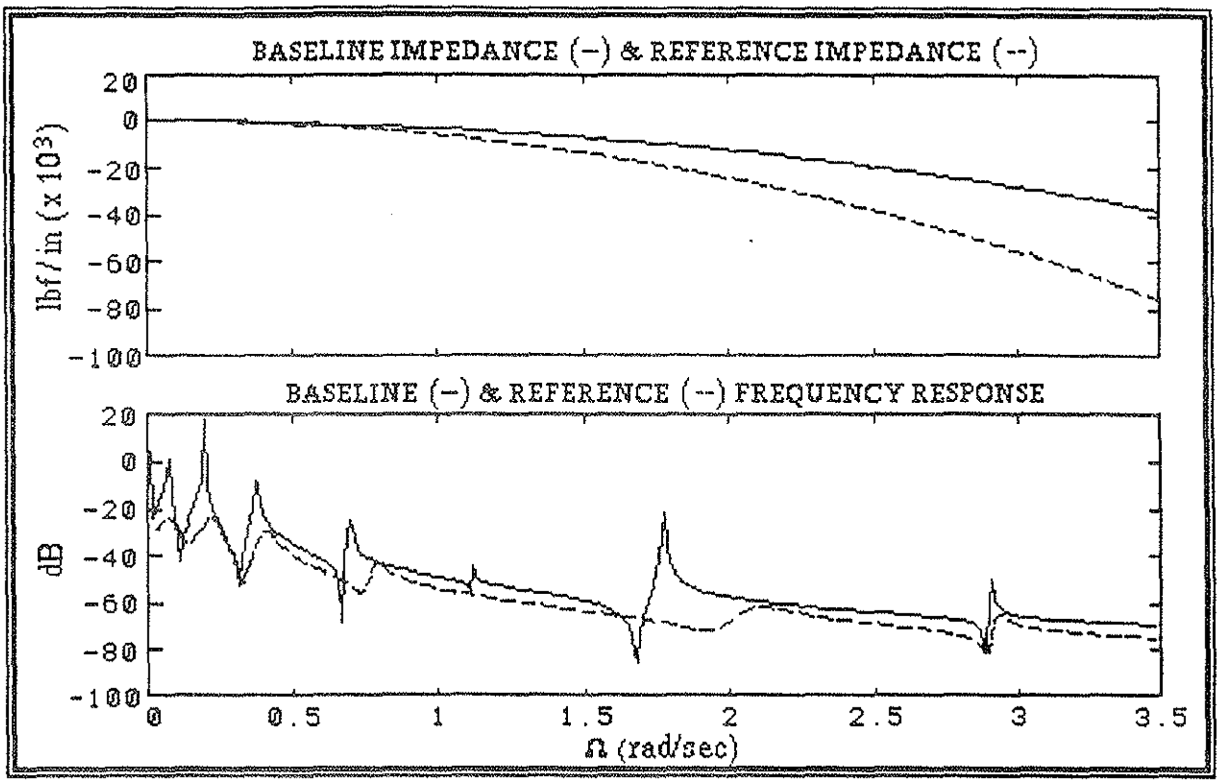


Figure 5: The L Matrix Calculation

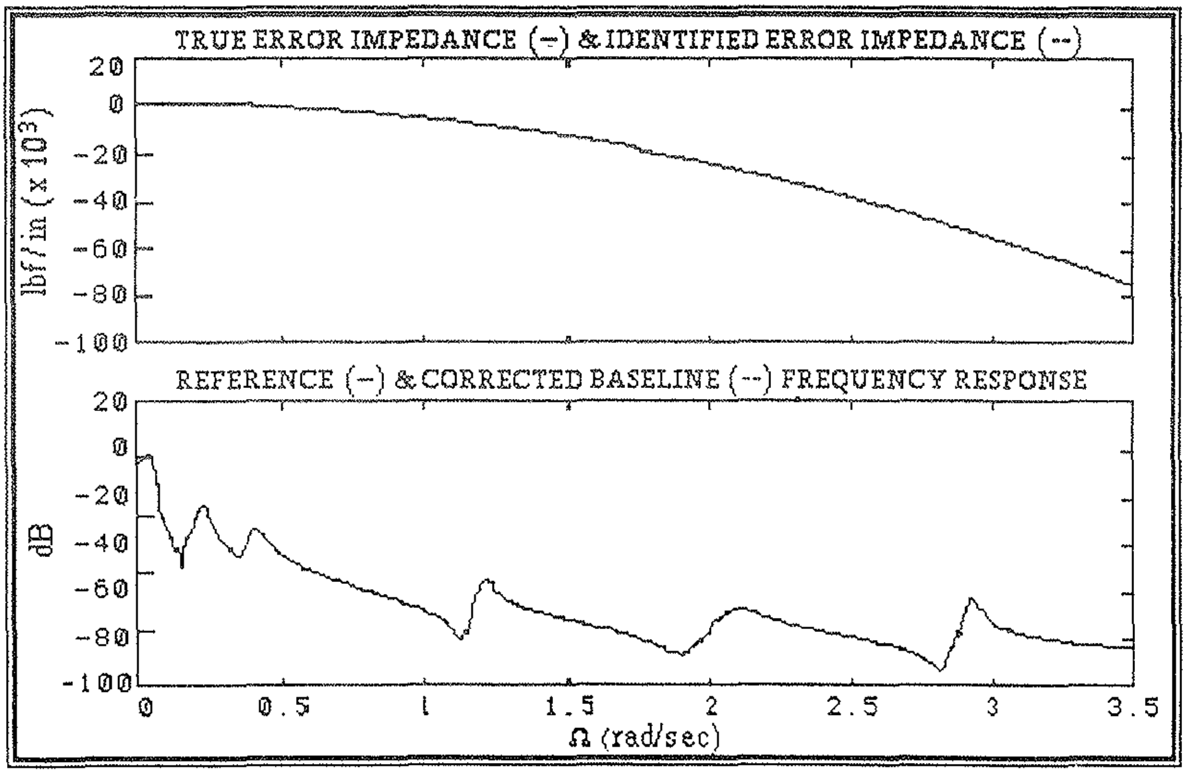


Figures 4a (upper) and 4b (lower)

vertical axis corresponding to the value of the individual elements. The structural synthesis transformation is unique in that it provides an exact relationship for the location of the differences between the baseline and reference systems. The **L** matrix itself, shown in Figure 5, reveals those motion coordinates which are associated with elements in error.

The solution for the error impedance is now given, as shown in Figure 6a. the top trace. Two quantities are plotted in Figure 6a. The first curve is a typical term of the true error impedance, the algebraic difference between the reference impedance and the baseline impedance. The second curve in Figure 5a is the error impedance as calculated by the theory. The two curves match exactly, making clear that the theory will identify the exact impedance errors, given spatially complete data. The lower plot, Figure 6b, also contains two curves. The first curve is the reference frequency response, which corresponds to the test data. Also plotted in Figure 6b is the corrected baseline frequency response, which corresponds to the finite element prediction. after the identified impedance error corrections have been made. Again. the two curves match exactly, making clear that the method will identify those corrections to the finite element model such that the resulting FE frequency response will *precisely* match the test data. at *all* frequencies of interest.

An important point to note here is that the structural synthesis transformation identifies those corrections to the baseline *impedance* such that when these corrections are installed in the baseline model, the resulting baseline *frequency response* will precisely match the reference frequency response. From Figure 6a, it is seen that the corrected baseline impedance also precisely matches the reference impedance. This result is true *only* when the identification is performed using spatially complete data, where every finite element motion coordinate has a corresponding measured motion coordinate, a practical impossibility for any structural identification effort. The situation of spatially incomplete test data, where the number of measured motion coordinates is less than the number of finite element motion coordinates is discussed below, with implications for all identification methodologies.



Figures 6a (upper) & 6b (lower)

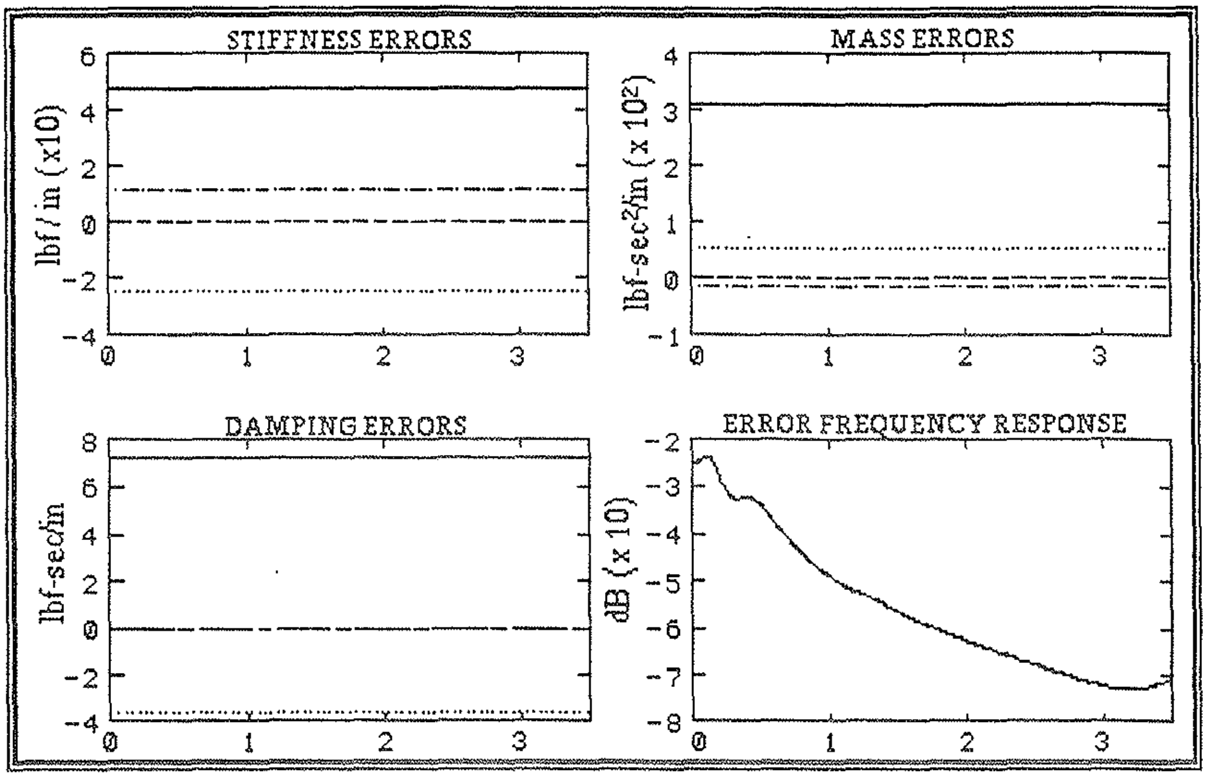


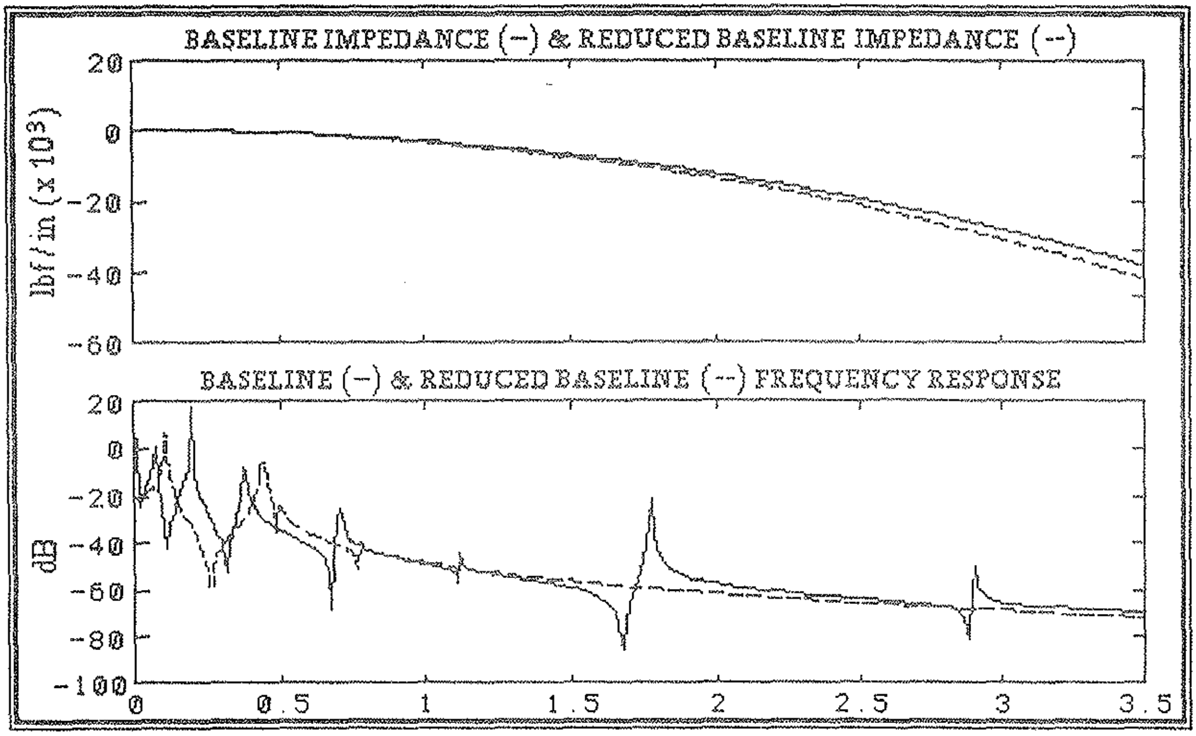
Figure 7: Identified Errors in Structural Parameters

The identified error impedance can be decomposed into its constituent mass, stiffness, and damping matrices, as described earlier. The results of such a calculation are shown in Figure 7. The recovered stiffness, mass, and damping values are the exact values corresponding to the true error impedance.

SPATIALLY INCOMPLETE DATA

A fundamental obstacle to structural identification, and one which will not be eliminated in the foreseeable future, is that a vibration test cannot provide as much information as is necessarily generated by the finite element analysis. The finite element model must possess a spatial resolution, as defined by the number of model degree-of-freedom, much finer than that which is practically generated in a test. This disparity in the amount of information generated is exactly that which prevents us from uniquely identifying particular elements in error. The fact that we seek to correct many more finite element parameters than the number of measured parameters available requires such "best-fit" solutions such as a minimum norm (direct) solution, the use of optimization, or the reduction of the finite element model to the order of the test model. This consideration is common to *all* identification schemes, and is a reflection of the constraint of the conservation of information.

The example to follow provides a direct, fully determined solution for corrections to a Guyan (statically) reduced [15] finite element model. The same beam models will be used as before, with the difference that the baseline system has had all rotational degrees of freedom condensed out, to correspond with the common situation of no rotational measurements. Figure 8a shows a typical term from the the full baseline (FE) impedance, and one from the reduced baseline impedance. As expected, the error incurred grows with frequency, due to the missing modal content. Figure 8b shows the corresponding comparison of frequency response, where the deficiency of modal content and the distortion of the retained modes in the reduced model is quite plain. This is the expected result of a static condensation.

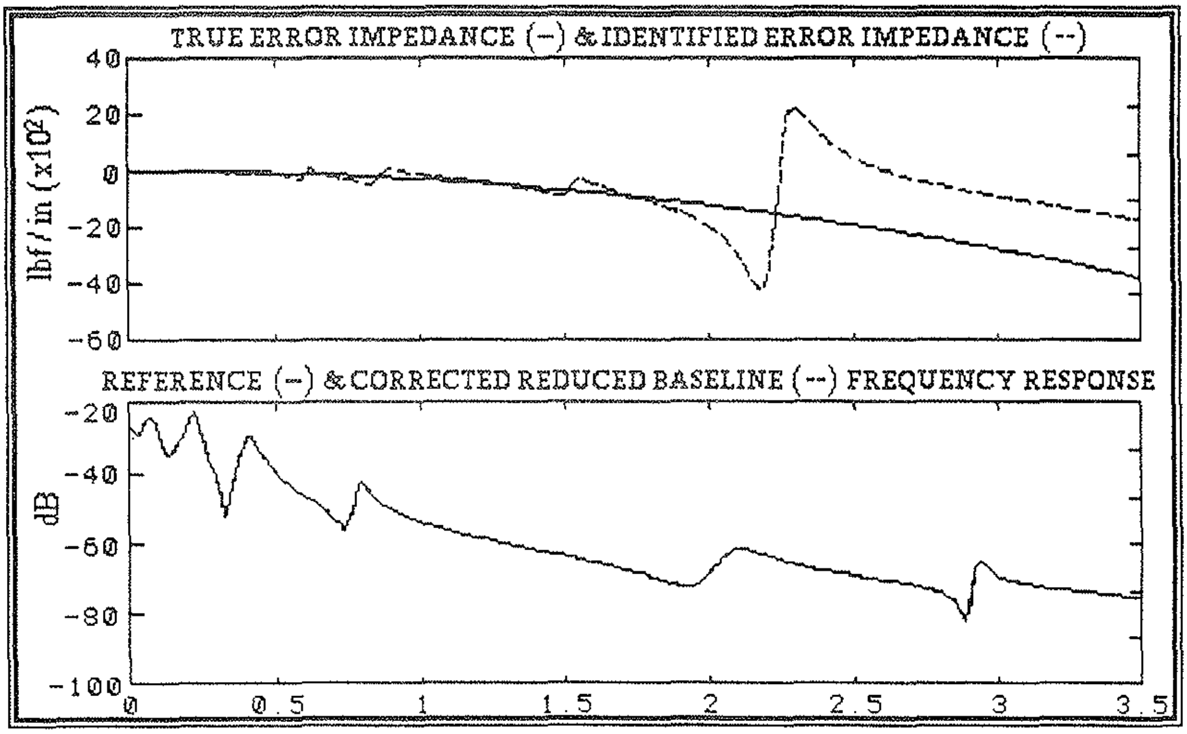

BASELINE IMPEDANCE (–) & REDUCED BASELINE IMPEDANCE (--)

BASELINE (–) & REDUCED BASELINE (--) FREQUENCY RESPONSE

Figures 8a (upper) & 8b (lower)

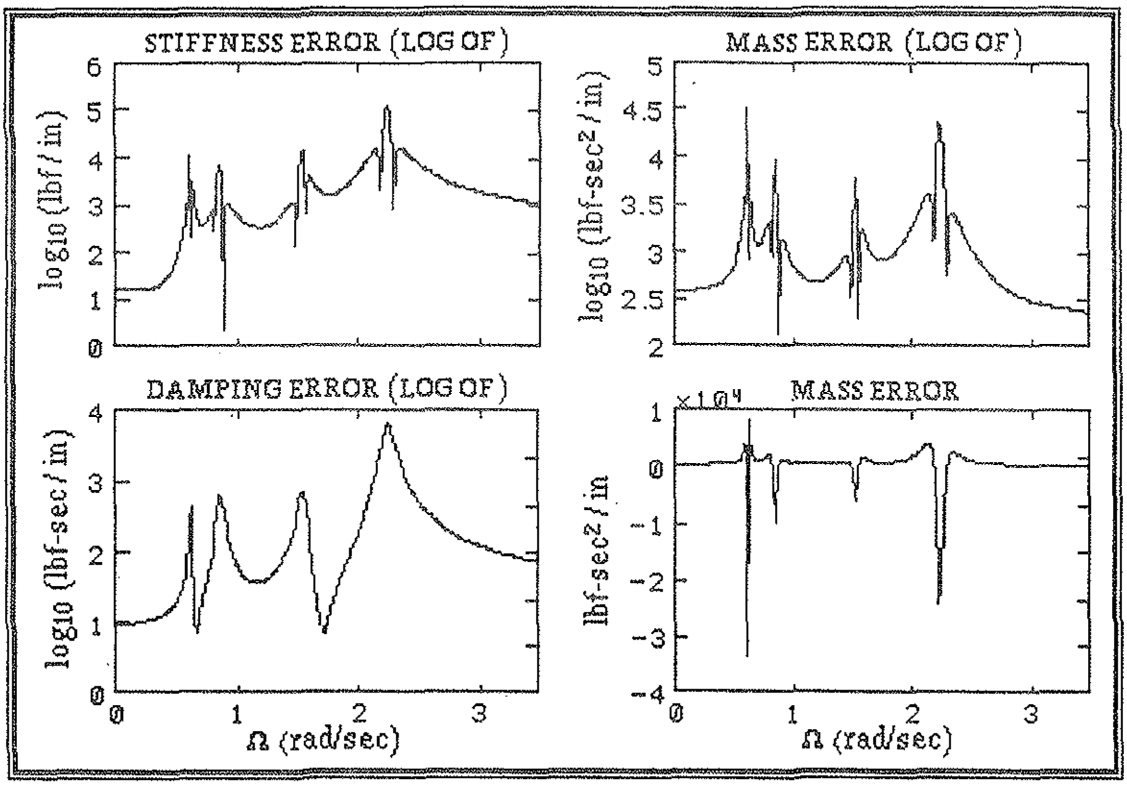
The use of spatially incomplete data naturally compromises the determination of the location of the errors in the baseline model. The location matrix calculation based on a reduced baseline impedance matrix cannot render an exact solution for the location of errors do to the smearing which results from the model reduction transformation. Although the capability of interrogating the L matrix for spatial information is compromised, the L matrix still provides the critical information as to which baseline model motion coordinates are associated with load paths which can provide a full rank transformation, as required by Eq. 19. The sizing of the connection partition must be based on the results of the L matrix calculation. If this is not done, as in the case of spatially complete data, the identification calculation will fail due to the resulting attempt at inverting a rank deficient matrix.

The reduced model replaces the full model in the identification procedure. The true error impedance, as calculated with the full order FE model, and the identified error impedance, are shown in Figure 9a. The corrections determined are with respect to the reduced model, and are such that the corrected reduced baseline frequency response will precisely match the test data as shown in Figure 9b.



Figures 9a (upper) & 9b (lower)

Figure 9a reveals that the error impedance identified by the structural synthesis transformation does not match the 'true' or spatially complete error impedance, yet the corrected baseline (FE) model frequency response precisely matches the reference (test) frequency response, as shown in Figure 9b. As discussed above, any reduction in model order necessarily imposes on the reduced set of structural parameters a nonlinear dependence with frequency. Any structural identification procedure with spatially incomplete data represents such a reduction, and hence complies with this requirement. The structural synthesis transformation provides an exact solution for the actual or true errors in the finite element structural parameters, *when possible.* The necessity of dealing with spatially incomplete data precludes this possibility. From Figures 6a and 6b it is seen that the error impedance, and its constituent structural parameters of mass, stiffness, and damping, as identified with *spatially complete* data, not only precisely match the true error impedance, but when installed in the baseline model as corrections, precisely reproduce the frequency response of the reference or test system. The structural synthesis transformation, in fact, reveals that the identification of the true error impedance is possible *only* with spatially complete information, and demonstrates that when correcting a reduced baseline model, the true errors (corresponding to spatially complete identification), when installed into the reduced model, *are not those required for the precise reproduction of the actual test frequency response data.* The structural parameter corrections for mass, stiffness, and damping, identified with spatially incomplete data, cannot match their 'true' values, if they are to provide the precise reproduction of the reference or test frequency response. This is shown in Figures 9a and 9b. Figure 9a shows that the identified error impedance required to reproduce the frequency response as measured in a test does not correspond with the actual error impedance. If it is desired to precisely reproduce the test frequency response, which the structural synthesis identification guarantees (Figure 9b), then the identified structural parameter errors (corrections) must exhibit the non-linear dependence with frequency, as shown in Figures 10a-10d. This is a fundamental requirement imposed by the conservation of information. The structural synthesis transformation is unique in that it not only plainly reveals these requirements imposed on any spatially incomplete identification, but also inherently accommodates the imposed computational frequency dependencies, so as to provide precise response prediction capability, regardless of the finite element reduction applied.



Figures 10a-d

SUMMARY AND CONCLUSIONS

A new theory for structural identification has been developed which exploits its spatial formulation in the frequency domain. The structural synthesis transformation, a structural modification and substructure coupling methodology, allows the installation of any number and distribution (within topological constraints) of both lumped/finite structural elements, and appended substructures. The formulation inherently treats any form of damping that can be expressed as a frequency dependent matrix. These operations are seen to be equivalent to effecting an alteration to the connectivity of the baseline system, producing the reference system. The theory, operating as a substructure coupling and structural modification methodology is thoroughly described in [1,12].

The structural synthesis transformation is shown to inherently provide a natural analytic bridge between two frequency domain descriptions of a structural system. The transformation quantifies the difference between the two descriptions in terms of an error impedance matrix, extracted at all frequencies of interest. This allows not only the *physical* frequency dependent mechanisms in the structure to be captured, which may be of any size and type, but also accommodates the *computational* frequency dependencies imposed as a natural and unavoidable outcome of the reduced order modeling inherent in identification. The error impedances so produced can be decomposed into their constituent error matrices of stiffness, mass, and damping. The method provides the corrections necessary such that the corrected finite element model dynamic response will *precisely* reproduce the corresponding test data, at *all* frequencies of interest, a unique capability critical to such applications as the helicopter fuselage vibration prediction problem. The structural synthesis transformation, cast as the location matrix, L, provides an exact determination of the location of modeling errors, a capability compromised only by the limitations on the amount of available test data. The location matrix calculation always allows the determination of the maximally sized error impedance matrix that may be calculated.

The frequency domain formulation presented avoids modal analysis altogether, with the distinct benefit of bypassing the concerns of test-model modal correlation, modal density and residual mode errors. The identification procedure inherently corrects for the missing modal content of the finite element model, at all frequencies of interest. The formulation allows the direct and spatial identification of frequency dependent structural damping. The flexibility and simplicity of the formulation allows its straightforward implementation in a finite element analysis environment. Furthermore, the method can easily be employed as an objective function module for optimal solutions of the identification problem. This is explored briefly in [12], in the framework of frequency domain structural synthesis, and in [17], where the fundamental concerns of spatial identification are not addressed, and the implementation restricted in generality due to a rudimentary formulation without the structural synthesis transformation.

Various issues pertaining to the theory are under further investigation, and experimental benchmarking has not yet been performed. The theory of (linear) Structural Identification, as a whole, is characterized by the inversion of the matrix relation, however formulated, between the experimentally determined response model, and the structural parameters, or their errors, to be identified. The inversion of a matrix is an operation quite sensitive to measurement error, and this sensitivity increases with the size of the matrix [6,9,18]. While this has not yet been experimentally investigated, the structural synthesis transformation has the distinct advantage of allowing a minimally sized error impedance matrix to be identified and used as the basis for the model improvement.

The method has demonstrated unique and effective capabilities in structural identification, and provides a direct and efficient basis for approaching a variety of problems, most notably, the helicopter fuselage vibration problem.

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