Nonlinear Rotorcraft Pilot Coupling Analysis and Anti-windup Design

Sébastien Kolb
CreA
French Air Force Research Centre
BA 701
13661 SALON AIR
FRANCE

Pierre-Marie Basset
ONERA
French National Aerospace Research Centre
BA 701
13661 SALON AIR
FRANCE

Nomenclature

ADOCS	Advanced Digital Optical Control System
FCS	Flight Control System
LMI	Linear Matrix Inequalities
PIO	Pilot Induced Oscillations

Abstract

Rotorcraft flight dynamics is not only governed by the properties of the bare airframe but also by the signicant ones of the pilot and of the flight control system. The way they acquire and assimilate the information, the delay and the limitations which restrict their response are key elements in the overall behaviour. The presence of a nonlinear element in the flight control system such as a rate limiter can lead to both problematic phenomenons of pilot induced oscillations and of wind-up.

Pilot induced oscillations due to rate saturation are analysed thanks to the describing function method. As perspective, an anti-windup scheme is designed by optimisation of domains of attraction. The dynamical system theory formalism (bifurcation theory, etc.) is used to examine and to describe the abrupt changes.

Finally, the mathematical tools used in this study reveal to be interesting when performing investigations and concrete calculations about the nonlinear rotorcraft pilot couplings and the design of anti-windup schemes.

Presented at the 34th European Rotorcraft Forum, Liverpool, UK, September 16-19, 2008.

I. Introduction

This paper deals with the analysis and improvement of a rotorcraft flight control system including nonlinear elements such as a rate limiter. In practice the presence of a rate limiter in the command channel may give rise to problematic phenomena such as pilot induced oscillations (PIO) and wind-up (large overshoots and possible loss of stability due to saturation). The interest is here mainly focused on the methodological aspects. A mathematical approach using bifurcation theory is proposed for addressing such an issue in a practical way.

The pilot induced oscillations correspond to a critical dynamic coupling between the pilot controls and the aircraft responses which leads to a global destabilisation of the aircraft-pilot closed-loop system. The terminology of “nonlinear PIOs” is employed when the trigger is a nonlinear element. Some practical cases of such PIOs will be examined “through the prism of” the describing function method which considers the evolution of the first-harmonic properties of the signal. The results will be expressed by the formalism of bifurcation theory and interpreted from the viewpoint of flight dynamics.

The saturated actuators (in position or rate) which are physically or numerically restricted remain besides the source of disturbances. The phenomenon of wind-up caused by (integrator) saturation can be handled and smoothed through several approaches. Here the calculations are based on a LMI (Linear Matrix Inequalities) formulation which determines the gains of an eventually added anti-windup compensator. It also provides a “guaranteed domain of attraction" of the system near the equilibrium point.

The selected concrete case is the (near-) hover flight of the ADOCS (Advanced Digital Optical Control System) helicopter prototype and the physical data comes from flight tests and identification results of M. B. Tischler [1,2].

Different configurations giving rise to pilot induced oscillations will be studied. Tasks with fixed target or with sinusoidal inputs will be examined. The pilot nervousness for which some PIO occur will be assessed. Next a first try to apply anti-windup techniques based on LMI formulation will be accomplished.

II. Pilot induced oscillations analysis using the describing function method

The rotorcraft flight dynamics is governed not only by the equations modelling the air vehicle behaviour (airframe plus rotors), but also by a key contributor which must not be forgotten, (even if it often is) : the pilot. The closed-loop combination of the pilot and the aircraft can be at the origin of the so-called Pilot Induced Oscillations. Besides, in order that pilots seem less guilty for the appearance of such undesirable behaviour, other names are employed such as pilot-in-the-loop.

Several categories of PIO can be distinguished [3]:

- Category I: Linear PIOs resulting from excessive time delays generated by sensors, signal filters, actuators, incorrect control/response sensitivity, …
- Category II: Quasi-linear PIOs whose trigger is an identifiable nonlinear element. They are mainly caused by position saturations or rate limiters present in actuators.
- Category III: Nonlinear PIOs with transients corresponding to abrupt changes of the controlled element or pilot behaviour. There are the most unpredictable and dangerous ones, but their occurrences e. g. mode switching in the Flight Control System (FCS) are less commonly seen.

As nonlinear behaviour is the heart of this research contribution, the analysis will concern the PIOs of category II which consist of the simplest nonlinear occurrences. As far as rotorcraft field is concerned, category I PIOs are still under study whereas for fixed-wing aircraft, the ones of categories I and II were examined and research is actually conducted on the category III PIOs.

For the examination of a helicopter FCS containing a nonlinear element, the describing function method is applied. This last one is based on considering the evolution of the first-harmonic properties. It is chosen for its simplicity of use among other nonlinear-PIO analysis tools such as limit cycle continuation (bifurcation theory), mu-tools or robust stability methods (cf. European project GARTEUR FM/AG 12 or [4]).

This part about PIO is built on several steps. After presenting some theory about the underlying mathematics of the describing function method and about PIOs, several concrete cases are analysed. Firstly, PIOs are considered in the case of a fixed target or control reference. Then, PIOs occurring with a sinusoidal piloting demand and a closed-loop system are treated. Finally, an open-loop system with sinusoidal inputs is examined.

A. Theory about describing function method and PIO

The describing function method [5] is based on considering the evolution of the first-harmonic properties (amplitude and pulsation) of a signal in a closed-loop system. Several constraints must be verified in order to apply this approach successfully. Firstly, the system must be separable in a linear part and a nonlinear one (i.e. an identifiable quasi-linear element). Secondly, the linear part must behave like a low-pass filter. If these two assumptions are verified, this method allows predicting the existence of limit cycles and their characteristics.

The procedure corner stone consists of solving the "harmonic balance equation" whose other name is "self-oscillation equation". Let A and ω be respectively the amplitude and pulsation of potential oscillations, $L(j\omega)$ be the linear part (of whole system : aircraft, control system and pilot) and $N(A,j\omega)$ the nonlinear part, the necessary condition for the existence of oscillations is then written:

$$\det\left(1+L\left(j\omega\right)N\left(A,\omega\right)\right)=0$$

It is necessary to find the values of the variables (A,ω) and of every other unknowns of the system (intervening in the expressions of $L(j\omega)$ and $N(A,\omega)$).

For a one-dimensional system, the equation can be rewritten:

$$L\left(j\omega\right)=-\frac{1}{N\left(A,j\omega\right)}$$

In this case, solving can be accomplished graphically by tracing the Nichols or Nyquist diagrams of $L(j\omega)$ and $-1/N(A,j\omega)$. The existence of an intersection point reveals the existence of a limit cycle. Moreover the values of A and ω associated to the intersection point are estimations of its amplitude and pulsation.

Furthermore the stability of the limit cycle is given by another theorem which is called the *Loeb criterion* [5]. When the describing function is only amplitude-dependent, it stipulates that:

The oscillation is stable if the intersection of L(jω) and -1/N(A) is such that by going along the Nyquist locus of L(jω) with the increasing-frequency order, it leaves on the left the locus of -1/N(A) with rising A. When using the Nichols diagram, "left" must be replaced by "right".

There exists besides a generalised version (with the describing function $N(A,j\omega)$ depending both of amplitude and pulsation).

After dealing with the describing function method, further elements need to be exposed in order to initiate the analysis: some information about the identification of a pilot model and some clues about a rate limiter.

During this research, the model used to represent the pilot is selected as simple as possible i.e. the pure gain. It indicates that the pilot corrects immediately the gap between the reference and the observed state with a proportional response. Of course, there are other possible modelling ways. A delay in the pilot response may be taken into account ($K_p e^{-sT}$). More complete models are also available such as the Neal-Smith one (taking into account closed-loop resonance property, gradients of variation and bandwidth $K_p(T_{p1}\,s+1)/(T_{p2}\,s+1)\,e^{-sT}$) [6] or the structural one (including descriptions of nervous and neuromuscular systems) [6].

In order to identify the pilot model, several theories have been developed and are based on different kinds of assumptions. Usually, the pilot control over the vehicle may be compensatory or precognitive synchronous.

In compensatory control, the pilot seeks to reduce the observed error by adapting the controls. Such a philosophy makes the pilot-vehicle system behaviour tends to the so-called crossover model [3]:

When the gain of the open-loop pilot-vehicle system is equal to 1 (0 dB), (researchers remarked that) the global system transfer function looks generally like $K/(j\omega)\,e^{-\tau j\omega}$ *which corresponds to an integrator plus a delay.*

In precognitive synchronous control, the pilot commands the vehicle thanks to learning process and experience. When the PIO is fully developed, it may be successfully assumed that the pilot reacts like a completely synchronous pure gain.

Another point to note in this study concerns the rate limiter. It is the trigger of many category II PIOs of fly-by-wire aircraft like the well-known ones of the Saab Gripen or YF-22 [4,7].

Actually, it is important to be aware of the fact that the describing function of that kind of nonlinear element depends only on the ratio ($A\omega/R$) where R is the variation velocity bound. For low amplitudes and pulsations, the rate limiter is not activated and its equivalent complex gain is $N(A\omega/R)=1$, but when the signal characteristics are too high, the rate limiter is completely saturated and its corresponding formula is $N(A\omega/R)=4/(\pi R)\exp(-j\arccos(\pi/2\;1/R))$. Between this two states, there is a continuous variation of $N(A\omega/R)$.

B. Case of a fixed reference signal

1. Presentation of the helicopter model and of the flight conditions

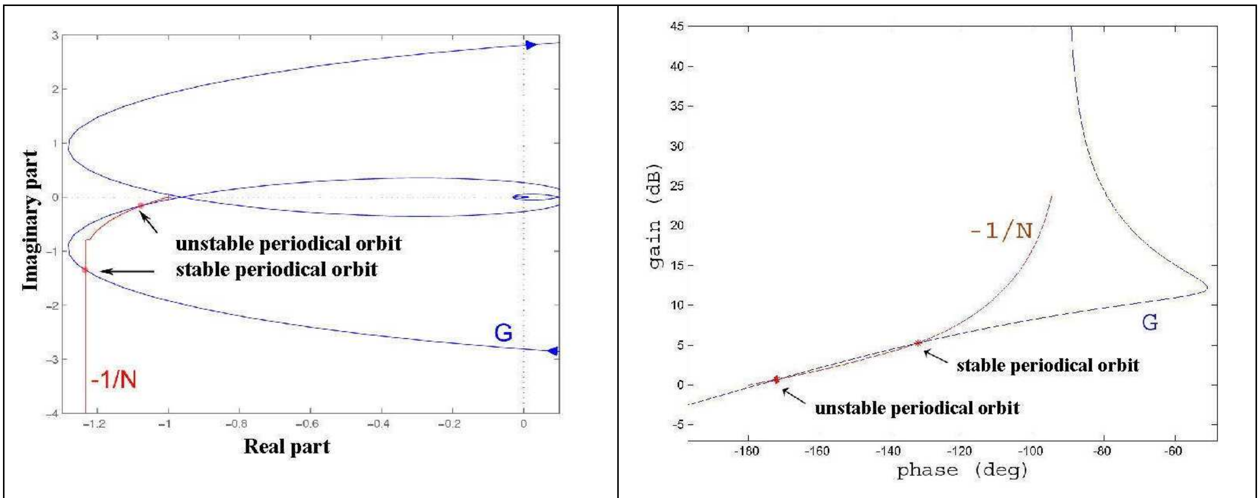
The ADOCS concept is a helicopter prototype whose name is an acronym of “Advanced Digital Optical Control System”. It was a program involving the NASA research centre and its Canadian counterpart which were aiming at developing an advanced flight control system for combat rotorcraft. Flight tests and concrete realisations were made by equipping an UH-60 Black Hawk in the eighties [1,2].

The flight case is a near hover flight and the pilot tries to maintain the pitch angle at the desired attitude e.g. for target tracking. In the first configuration examined hereafter, in order to avoid that abrupt stick inputs might be transmitted to the remaining command channel, a rate limiter is included. Unfortunately, such an element can provoke PIO phenomenon.

The response of the pitch angle θ in function of the stick position δ_s is a second-order system (with a delay): $\dfrac{5.26(0.2)e^{-0.244s}}{s[0.964, 2.35]}$ with the conventional notations *(a)* for *(s+a)* and *[ζ,ω]* for $(s^2 + 2\zeta\omega + \omega^2)$ [2]. Besides, the actuator variation rate is restricted to *15deg/s*.

2. Fixed nervousness of the pilot

The pilot gain K_{pil} is chosen to be equal to *3.9* (the reason of this choice will appear more clearly in the next part). The Nyquist and Nichols diagrams of the linear part (bare airframe, pilot) and of the negative inverse describing function of the nonlinear element (rate limited actuator) are presented on Fig. 1.

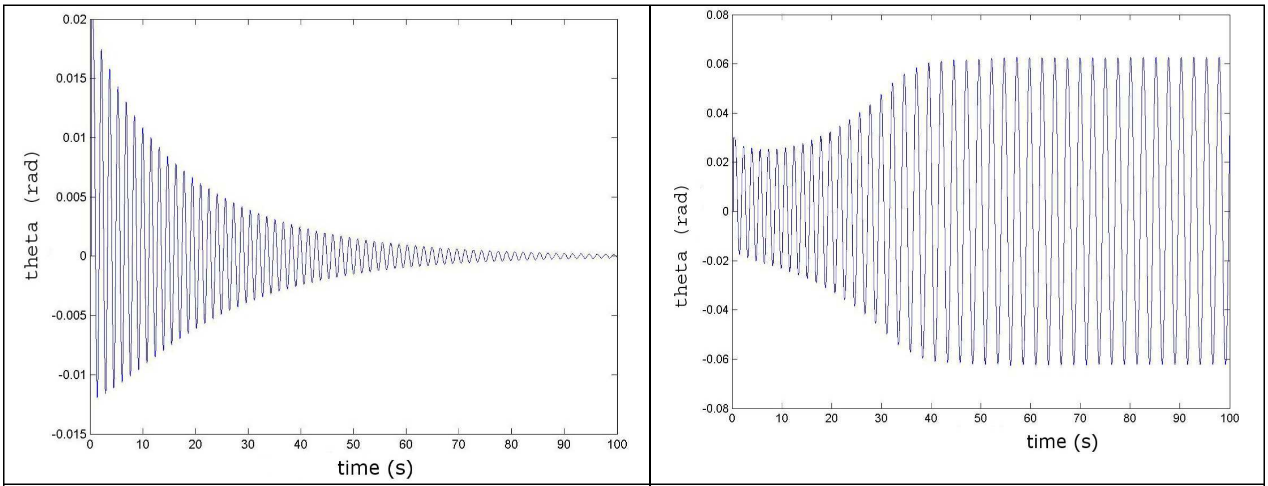


Nyquist	*Nichols*

Fig. 1: Determinination of the limit cycles with the Nyquist and Nichols diagrams.

There are two intersection points. The corresponding pulsations are *3.8rad/s* and *2.5rad/s* whereas the ratios *X=Aω/R* are *1.3* and *2.3*. As a consequence, at the entry of the rate limiter, the amplitudes of the limit cycles are *A=X R/ω* i.e. *0.1rad* and *0.25rad* respectively. Moreover, the Loeb criterion allows predicting their stability. In short, there are one unstable limit cycle of pulsation *ω=3.8rad/s* and amplitude *A=0.1rad* and one stable limit cycle of pulsation *ω=2.5rad/s* and amplitude *A=0.25rad/s*. The unstable periodical orbit is inside the stable one.

Time simulations with initial conditions inside and outside the unstable limit cycle illustrate the prediction (Fig. 2).



$\theta(0)=0.02$	$\theta(0)=0.03$

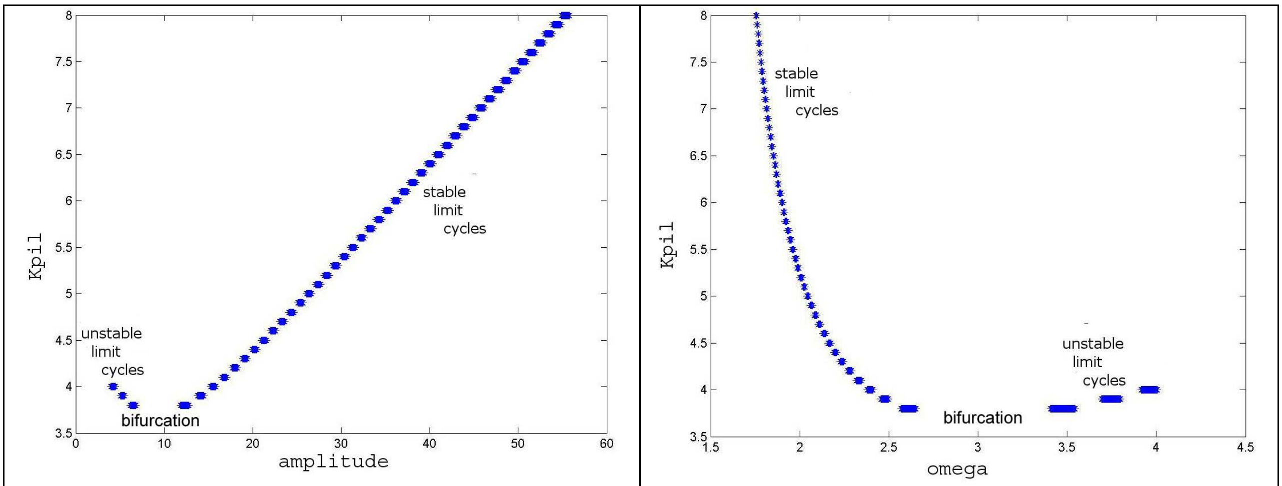
Fig. 2: Time simulations showing the system behaviour with respect to these limit cycles.

The first simulation converges to zero (i.e. at the equilibrium point for which the model has been linearised) whereas the other one converges to the stable periodical orbit. Thus this configuration may give rise to pilot-vehicle couplings and oscillations depending on the input amplitude. On the right diagram of figure 2, the situation exposed is such that the pilot doesn't succeed in obtaining the desired longitudinal attitude and the pilot-aircraft system oscillates.

3. Varying nervousness of the pilot

The appearance of pilot induced oscillations depends often on the pilot nervousness. A quiet pilot is less likely to meet such negative phenomenon than a nervous one. The overall dynamics and the critical values for which changes occur are important to diagnose.

In order to predict the global aircraft behaviour, it is necessary to solve the harmonic balance equation which involves the transfer function *H* of the linear aircraft response the pilot gain *Kpil* and the first harmonic *N(A,ω)* of the nonlinear element: $K_{pil}H\left(j\omega\right)N\left(A,\omega\right)+1=0$.



amplitude	*pulsation*

Fig. 3: Effect of the pilot gain on the PIO occurences.

According to the graph, three ranges of pilot gain values K_{pil} can be distinguished and correspond to different numbers of limit cycles:

- For $K_{pil}<3.72$, there is no limit cycle.
- For $3.72<K_{pil}<4.1$, there are two limit cycles : the one with the lowest amplitude and biggest pulsation is unstable whereas the other one is stable.
- For $K_{pil}>4.1$, there is one unique stable limit cycle.

The time simulations corroborate the predicted behaviours even if some oscillations appear for a little bit higher pilot gain than the predicted critical one.

From the mathematical point of view, there is a saddle-node bifurcation of periodical orbits i.e. creation of two branches of periodical orbits: one stable and one unstable. As partial conclusion, in the ADOCS prototype whose equations were adapted here some PIOs may occur even if the pilot is only a little nervous.

C. Closed-loop system with sinusoidal target

The second longitudinal command channel under consideration here contains more details and is more realistic. The command block, the actuator system, the rotor tip path plane dynamics, the rigid body airframe and the feedback stabilisation are precisely described. The upper boost actuator which represents the swashplates is mechanically limited in rate variation with a bound of *10 inch/s*.

In this situation, the rate limiter is included in the feedback loop, which implies a bigger instability. That is why the feedback gains are reduced to $K_\theta=14.52$, $K_q=9.19$. The task that the pilot must achieve consists of following a sinusoidal reference (pitch attitude) e.g. during obstacle avoidance maneuvers. The pilot model is precognitive synchronous and identified by the crossover model theory. The pilot is chosen such that the “crossover phase angle” is *-130°* and such that the pilot is quite nervous, concretely *Kp=1* [6,8].

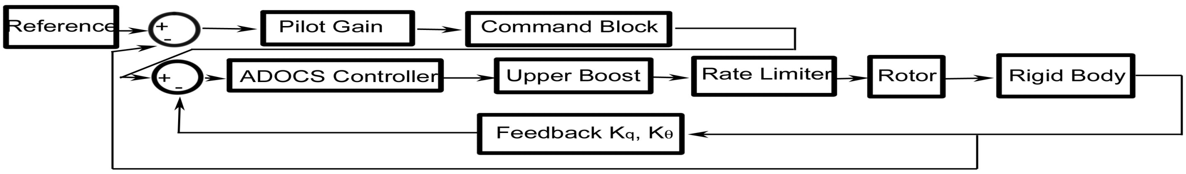


Fig. 4: Block diagram of the closed-loop system.

In order to determine the amplitude *A* and the phase ϕ of the entry state of the rate limiter, the following equation must be solved. It depends on the reference amplitude θ_c and exhibits the relationship between the different components:

$$\left(1+Rotor\cdot RigidBody\cdot N\left(A,\phi\right)\cdot Actuator\cdot\left(K_p\cdot Command+Feedback\right)\right)\cdot A\exp\left(j\phi\right)$$
$$=Actuator\cdot Command\cdot\theta_c$$

The continuation algorithm is used to find the solution of the (implicit) problem whose equations are the real and imaginary parts of this expression and whose variables are A,ϕ,θ_c (one equation less than the number of variables).

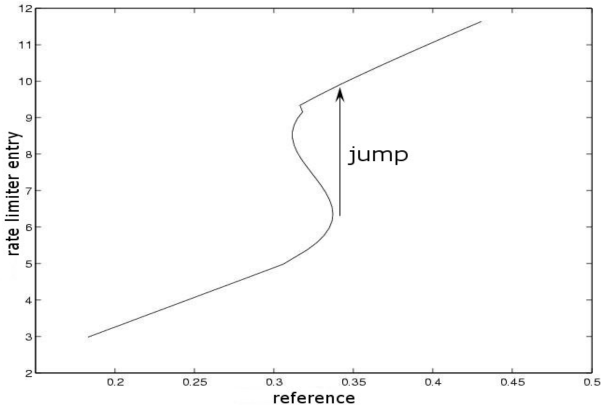


Fig. 5: Equilibrium curve with a fold (two bifurcations).

An amplitude jump is observed when the reference signal pulsation is increased from *0.33rad* to *0.34rad*. This bifurcation leads to the sudden variation of the entry state of the rate limiter from *6* to *10*.

D. Open-loop system with sinusoidal inputs

In this open-loop configuration, the pilot gives direct sinusoidal inputs to the rotorcraft command channel without taking into account the pitch angle value. The input signal amplitude is *20deg=20π/180rad* and the pulsation is varied.

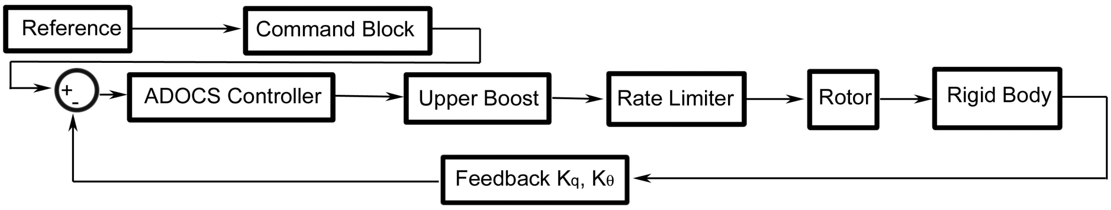


Fig. 6: Open-loop command channel.

The Nichols diagrams of the linear (blue) and nonlinear (green) command channels are compared. In fact the parameters of the nonlinear configuration are determined for the closed-loop system. Then formulas permit to calculate the values of the equivalent open-loop system. These last ones are presented on the diagram.

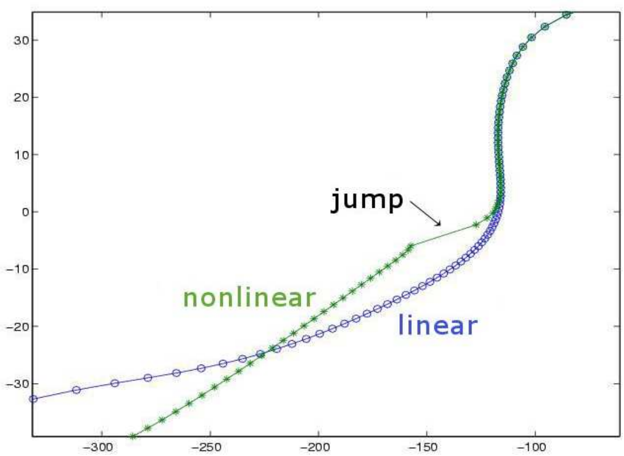
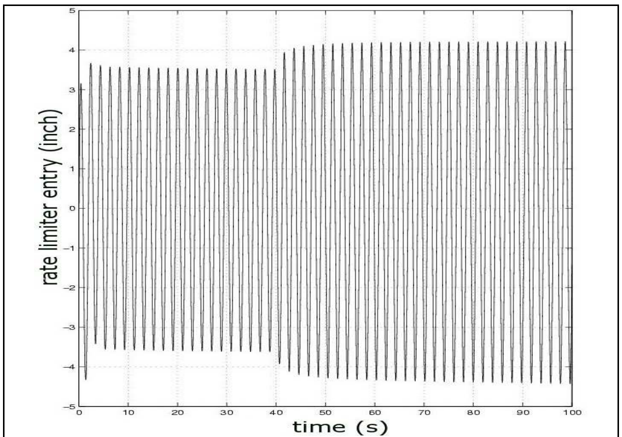


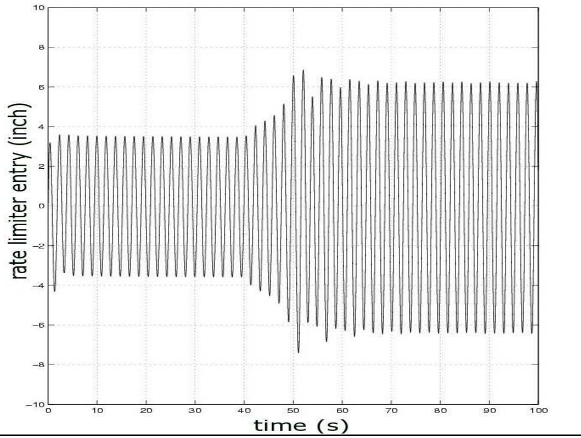
Fig. 7: Nichols diagram of the linear and nonlinear command channels.

A jump is observed at a critical pulsation. For bigger input pulsations than this one, the properties of the linear and nonlinear configurations differ absolutely.

Nevertheless at the beginning of the simulation, the steady regime is not established and rapid transient changes can occur. It produces persistent saturation of the rate limiter for numerical reason and the simulation cannot be exploited. That's why the rate limiter is activated here only after *40s*.



ω=3.2rad



ω=3.3rad

Fig. 8: Time simulations before and after the critical pulsation.

Time simulations showing the entry of the rate limiter illustrates that the oscillation amplitude increases abruptly at this critical parameter value. A little increase of the pulsation from *3.2rad* to *3.3rad* makes the oscillation amplitude jump from *4 inches* to *6 inches*.

From the point of view of flight dynamics, “handling qualities cliffs” may occur in the meaning that a small parameter perturbation may lead, to abrupt significant variation of the rotorcraft behaviour. They correspond to the mathematical phenomenon of resonance jumps.

III. Anti-wind up design with stability domain optimisation

This part dealing with anti-windup is organised as follows. First the main general elements of the anti-windup approach are presented. Then the practical application to the present rotorcraft case is addressed. As far as we know, that is a first attempt of application to helicopter system.

A. Elements about the anti-windup approach by LMI synthesis

Saturations in position and velocity of elements within a command channel may destabilise it and lead to state value deviation. For example, the saturation of an integrator induces often large overshoots because the integrator does not succeed in unloading itself. Several techniques exist to avoid this bad situation. The so-called anti-windup approach aims at improving the stability and performance of saturated systems. In this research contribution, the gains of the anti-windup scheme will be calculated thanks to a mathematical description of the problem under LMI form and its optimisation.

Following some recent developments as the ones of J-M. Biannic and S. Tarbouriech [9,10], the system is in fact (re-)written with dead-zones instead of saturations. It aims at relaxing the constraints of the optimisation problem by modifying the sector conditions. The problem has a less conservative formulation, therefore the solution is more exact and provides a bigger area of admissible solutions.

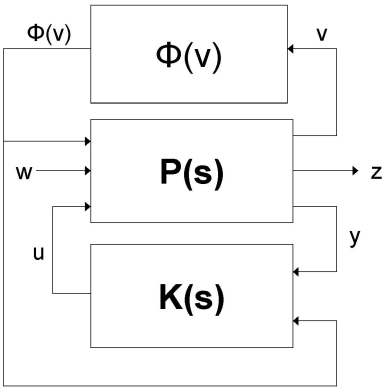


Fig. 9: Standard configuration for a system with an anti-windup compensator.

As summed up by the diagram on Fig. 9, the problem of anti-windup includes the plant *P(s)*, the controller *K(s)* (which contains also the anti-windup gain) and the set of dead-zones (or saturations) *Φ(v)*. The variables are the inputs *w*, the outputs *z*, the controls *u*, the inner states y and the entries of the nonlinear elements *v*.

B. Standard interconnection plant

Before initiating the analysis, the “standard interconnection plant” representing the components of the system and their relationships must be described.

The linear plant model corresponds to the bare rotorcraft (i.e. the rigid body and the rotor dynamics) whereas the controller is built with the (ADOCS and upper boost) actuators, the feedback stabilisation system and at the end with the anti-windup device as well. The set of saturations contains only one item which corresponds to the rate limited displacement of the swashplates (upper boost actuator) and whose bound is *R=10inch/s.*

The command channel is the same one as in the section on the open-loop system. But in order to use successfully the optimisation algorithms, the transfer functions of the elements are simplified.

After presenting the theoretical background and the concrete helicopter control system, the main design steps of an anti-windup scheme are briefly exposed hereafter.

C. Gain calculation of the anti-windup scheme and time simulations

The design of the anti-windup scheme is performed thanks to the optimisation of a LMI problem. First, stability and performance are assessed. Then time simulations allow illustrating and comparing the systems with or without the anti-windup element by considering the reaction to larger inputs or the stabilisation delay at the final value.

The desired pitch angle $\theta_c=30deg=30\pi/180rad$ is selected high (maybe too high to be fully realistic) in order to be able to observe easily the effect of the anti-windup device. The responses of the systems with or without anti-windup device and of a simplified linearised model are observed.

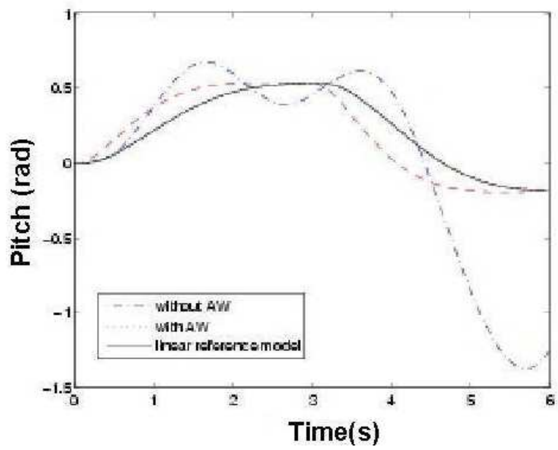


Fig. 10: Time simulations of the system without or with an anti-windup compensator.

On one hand, without any anti-windup scheme, the rate limiter saturates too much and the command channel is completely destabilised as the blue dash-dotted simulation shows. On the other hand, with an anti-windup scheme, the system succeeds in reaching the demanded value, as exposed by the red dashed simulation. The observation of the difference between this last one and the reference linearised model (continuous dark line) give some information about the efficiency and properties of the controller as the time needed for stabilisation or the robustness.

Once the stability analysis performed, the next step aims at estimating the performance of the complete command channel. Indeed it is also interesting to evaluate the discrepancy between a first reference model that has no saturation and no added device and a second system with saturations and a modified controller. Noting *z(t)* the difference of response between the real system and the linear model, the performance measures the biggest possible error $\int_0^\infty z(t)^T z(t)\,dt=\gamma$ for input signals whose maximal amplitude is θ_c.

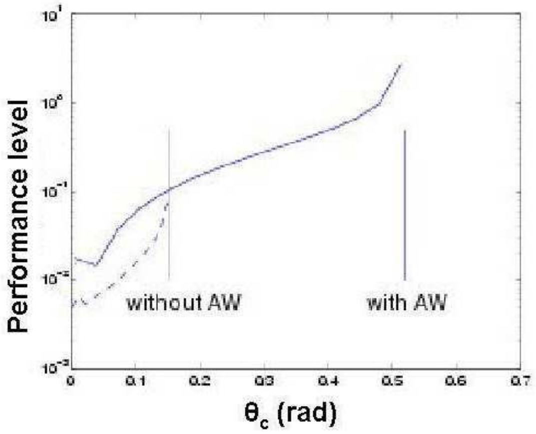


Fig. 11: Performances of both configurations.

The figure 11 shows the performance bound in function of the input magnitude θ_c. (Ignoring the numerical calculation problems for low inputs,) it is visible that the anti-windup device contributes to improve the performance at high input amplitudes where it plays actively its stabilising role.

IV. Conclusions

During this research, the impact of nonlinear elements in the command channel of a helicopter has been analysed and ways to attenuate their negative effects such as pilot induced oscillations, destabilisation or divergence have been studied. Mathematical methods and tools for nonlinear analysis and control have been used successfully in this first application to the rotorcraft case.

In a first part, the describing function method turns out to be a practical and efficient way to diagnose the existence of limit cycles and their first-order harmonic properties. Closed-loop or open-loop systems with fixed or sinusoidal inputs have been examined. As observed, the ADOCS helicopter may suffer from some PIO in the studied configurations even with little pilot gain. As the crossover model theory associates pilot nervousness to concrete pilot gain, the fact that PIO occurres for little pilot gains that is to say low nervousness is a hint (or of) about the sensitivity of the phenomenon.

In a second part, in order to be able to apply larger inputs without divergence of the system, an anti-windup scheme is designed. The level of saturation is taken into account such that a correction is added to the command value transmitted to the flight control system. The description under LMI form and the resolution of the associated optimisation problem permit to make the system as stable or as insensitive to perturbations as possible.

As a conclusion, such mathematical tools are quite efficient to analyse and improve a command channel containing nonlinear elements. They give interesting insights and concrete values which help orienting the engineer in his choices when accomplishing the next step: designing and building the effective flight control system.

References

[1] Tischler, M. B., “Digital Control of Highly Augmented Combat Rotorcraft”, NACA TM 88346, May 1987.

[2] Tischler, M. B., J. W. Fletcher, P. M. Morris, and G. T. Tucker, “Applications of Flight Control System Methods to an Advanced Rotorcraft”, NACA TM 101054, July 1989.

[3] Klyde, D. H., and Mitchell, D. G. “A PIO Case Study – Lessons learned through Analysis”, AIAA Atmospheric Flight Mechanics Conference and Exhibit, San Francisco, August 2005.

[4] NATO/AGARD, “Flight Control Design – Best Practices”, RTO-TR-029, 2000.

[5] Gelb, A., and Vander Velde, W. E., “Multiple Input Describing Functions and Nonlinear System Design”, Mc Graw Hill, 1968.

[6] Commitee on the Effects of Aircraft-Pilot Coupling on Flight Safety, “Aviation Safety and Pilot Control: Understanding and Preventing Unfavourable Pilot-Vehicle Interactions”, National Academy Press, Washington D. C., 1997.

[7] Klyde, D. H., and Mitchell, D. G., “Investigating the Role of Rate Limiting in Pilot-Induced Oscillations”, AIAA Atmospheric Flight Mechanics Conference, Austin, August 2003.

[8] Gilbreath, G. P., “Prediction of Pilot-Induced Oscillations due to Actuator Rate Limiting Using the Open-Loop Onset Point (OLOP) Criterion”, Master’s Thesis, Air Force Institute of Technology, 2001.

[9] Biannic, J-M., and Tarbouriech, S., “Stability and Performance Enhancement of a Fighter

Aircraft Flight Control System by a new Anti-Windup", 17th IFAC Symposium on Automatic Control in Aerospace, Toulouse, June 2007.

[10] Biannic, J-M., Tarbouriech, S., and Farret, D., "A Practical Approach to Performance Analysis of Saturated Systems with Application to Fighter Aircraft Flight Controllers", 5th IFAC Symposium ROCOND, Toulouse, 2006.

[11] Bedford, R. G., and Lowenberg, M. H., "Use of Bifurcation Analysis in the Design and Analysis of Helicopter Flight Control Systems", 29th European Rotorcraft Forum, Friedrichshafen, 2003.

[12] Basset, P-M., Chen, C. , Prasad, J. V. R., and Kolb, S., "Prediction of Vortex Ring State Boundary of a Helicopter in Descent Flight by Nonlinear Flight Dynamics Simulation", Journal of the American Helicopter Society, 2008.