EVALUATION OF A CONSTANT FEEDBACK GAIN FOR CLOSED LOOP HIGHER HARMONIC CONTROL

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Abstract

Since Higher Harmonic Control (HHC) has been found to be very effective in suppressing helicopter vibrations, one aspect of the research work on the field of active rotor control was the developement of a controller which is suited for a closed loop application. Although several solutions have been presented, most of them are of adaptive nature and consist of a Kalman filter and a minimum variance controller. Due to the adaptibility this combination is able to deal with the time variing state of the rotor and therefore yields a good vibration reduction within the whole flight envelope.

Disadvantageous however is the relative long response time of the controller and the impossibility of demonstrating a stable behaviour analytically in advance. For that reason the evaluation of a constant feedback gain is strongly desired especially in the case of a high helicopter agility as can be achieved with a hingeless rotor.

After a short overview of the previous work concerning closed loop higher harmonic control such constant feedback gain will be determined in the paper and the resulting distribution of the eigenvalues will be demonstrated. The influence of a reduced number of feedback signals, an increased control rate limit and a reduced controller cycle time will be investigated and the resulting controller response time will be shown.

Introduction

The increasing international cooperation on economic level and the general desire of a high mobility recently lead to a strong acretion of the air traffic and therefore to a considerable load of the international airports. Due to their limited capacities, business as well as private flights often were and still are delayed, a situation which is not only unpleasant for the passengers but in addition yields higher operating costs for the airlines.

Especially with respect to the foundation of the European Market in 1992 this evolution is expected to increase in the future and therefore requires an enlargement of the starting and landing capacities in order to prevent a complete breakdown of the air traffic. Besides an extension of the large-scale airports, an expansion of the international air traffic to smaller airports actually is under discussion too.

This decentralization will reduce the amount of passengers on feeder flights, for what reason smaller aircrafts are demanded. Besides the well-known fixed wing airplanes VFW 414 etc., helicopters are well suited for this task too, because they are not restricted to the highly frequented runways but also can use realtive small, possibly aside located landing tabs.

With this respect, the helicopter vibration level which, despite of passive damping elements, still is relative high, becomes very important. It represents a considerable stress for material, passengers and crew and leads to an abridgement of the maintenance intervals associated with higher maintenance costs and, in addition, impairs the flight comfort and the flight security.

These negative effects can be eliminated by using Higher Harmonic Control which superposes a higher harmonic blade pitch angle to the conventional one. Because the optimal amplitude and phase shift of this additional control signal depends on the flight conditions, a controller is required which adjusts these parameters corresponding to the actual vibration level.

On the first view this controller has to be an adaptive one which is able to adjust itself to the variable rotor state by identifying the transfer function of the plant in every cycle. Allthough this procedure yields a high vibration reduction within the whole flight envelope it inhibibts an analytical demonstration of stability.

Nevertheless such kind of controller has been investigated intensively in the past by several institutions during flight and wind tunnel tests. As a result it turned out that a good vibration reduction could in fact be achieved at static and quasi static conditions but not during rapid maneuvers.

For that reason recent efforts at the DLR Institute for Flight Mechanics aimed upon the evolution of a controller which works with a constant feedback gain. As can be seen from the following chapters, this controller yields a good vibration reduction within a wide range of flight and, in addition, possesses an excellent transient behaviour.

Previous Work

An overview of the previous work which was conducted on international level is shown in table 1. In a chronological sequence it provides informations concerning the type of investigation, i.e. simulations, wind tunnel tests or flight tests, the characteristics of the rotor, the type of controller, the achieved vibration reduction, the controller response time and the stability range.

From this table it can be seen that the major part of the investigated controllers were adaptive ones which worked in the frequency domain [1] – [7]. Most of these controllers yielded a vibration reduction of about 80% - 90% and were stable within a wide range of the flight envelope.

Disadvantageous however was the tendency towards instability which could be observed during some flight maneuvers and the relative long controller response time.

For that reason a fixed gain, time domain approach was investigated by DuVal in 1981 [8] and 1987 [9] within the scope of simulations. This approach partially lead to a controller response time which was shorter than that of the adaptive ones and therefore seemed to be very promising. Unfortunately it turned out to be very sensitive to unmodeled phenomena and to a variation of the flight condition and therefore was not realized in practice.

Realized however was a gain scheduled and a fixed gain controller which worked in the frequency domain (see Shaw in 1980 and 1987 and Hammond in 1981 and 1983). They arised from the adaptive ones by simply suppressing the online identification of the rotor transfer function and by selecting a feedback gain which was found out to be optimal for one specific operating point.

Whereas during the first attempts stability could only be achieved for the nominal flight condition, the final tests were successful and yielded a vibration reduction of 90% and a controller response time of 4 rotor revolutions. These values represent an excellent result, especially if it is considered that they originate from a practical realization and not from simulations.

A drop of bitterness however was the increase of the dynamic rotor moments by about 20% when the dynamic rotor forces were reduced simultanoulsy. This is not a problem in the case of an articulated rotor because it's dynamic moments are nearly negelctible even if they are increased within the above mentioned scope.

For an application of Higher Harmonic Control in conjunction with a modern hingeless rotor however a simultanous reduction of all dynamic forces **and** moments is the only way to achieve the aspired flight comfort and with that the required acceptance of passengers and crew.

DLR Controller Concept and Wind Tunnel Results

Controller Architecture

Like most other investigators the DLR Institute for Flight Mechanics decided to realize a frequency domain controller too [10, 11]. It was based upon the so called "local model"

$$\Delta \underline{y} = \underline{T} \cdot \Delta \underline{\vartheta} \qquad (1)$$

which establishes a linear relationship between the vibration change $\Delta \underline{y}$ and the higher harmonic input change $\Delta \underline{\vartheta}$ via the rotor transfer matrix $\underline{T}$. This matrix was determined online by a Kalman filter (fig. 1) which usually is used for optimal state estimations of a dynamical system but, in a modified version, is well suited for the ascertainment of a transfer function too [12].

With respect to this T-matrix a minimum variance controller determined the higher harmonic control parameters leading to a minimum of the quadratic quality citerion

$$J = E\{\underline{y}^T(k) \cdot \underline{W}_y \cdot \underline{y}(k)\}$$

with

$\underline{y}(k)$	a vector including the cosine and sine components of the 4/rev part of the vibrations in the fixed system,
$\underline{\Theta}(k)$	a vector including the cosine and sine components of the higher harmonic control signals in the rotating system,
k	the sample index,

and

$\underline{W}_y$	a weighting matrix.

In order to achieve this minimum, the equation

$$\Delta\underline{\Theta}(k+1) = -(\hat{\underline{T}}(k)\cdot\underline{W}_y\cdot\hat{\underline{T}}^T(k))^{-1}\cdot\hat{\underline{T}}^T(k)\cdot\underline{W}_y\cdot\underline{y}(k)$$

with

$\hat{\underline{T}}(k)$ the optimal estimate of the rotor transfer function $\underline{T}$

had to be processed which represents an integral controller. In conjunction with the local model, it yields the block diagram shown in fig. 2 from which the closed loop system equation

$$\underline{y}(k) = (\underline{I} - \underline{T}\cdot\hat{\underline{T}}^{-1})\cdot\underline{y}(k-1) + \underline{T}\cdot\underline{w}(k-1) \quad (2)$$

can be derived. It shows that the system output, i.e. the vibration vector $\underline{y}(k)$, becomes directly proportional to the commanded vector $\underline{w}(k-1)=\underline{0}$ if the T-matrix is exactly identified by the Kalman filter, i.e. if $\hat{\underline{T}}=\underline{T}$.

Allthough this ideal case normally is not achieved within a practical application, the above mentioned integral controller yielded very good results during the wind tunnel tests.

Wind Tunnel Results

In order to allow an observation of the rotor reaction in dependance of higher harmonic control inputs and to avoid a damage of the test rig, the HHC parameters were adjusted manually during the first tests. Fig. 3 shows one result of these investigations and clearifies that a 3/rev blade pitch angle yielded nearly the same vibration reduction as was achievable with a combination of a 3-, 4- and 5/rev HHC input signal. Furthermore a simultanous reduction of all dynamic rotor forces and moments could be noticed in the case of a pure 3/rev blade pitch angle as is illustrated in fig. 4.

In respect of these results, the first closed loop tests were conducted with an adaptive 3/rev controller at stationary flight conditions. As can be seen from fig. 5 it yielded a good vibration reduction (up to 91%) within the whole range of speed, for what reason it was subjected to a continous variation of the tunnel speed in combination with a stepwise readjustement of the rotor state too. Fig. 6 shows the result and illustrates that the controller worked stable during the whole test achieving a considerable vibration reduction independant of the rotor trim.

Allthough this result was very impressing it did not provide sufficient information concerning the transient behaviour of the controller. For that reason additional tests were required which had to be evaluated especially with respect to the controller response time, the tendency of the HHC parameters and the corresponding vibrations.

As one result of these tests, fig. 7 shows the performance of the so called 3/rev **S**ingle **O**utput **C**ontroller (SOC) in comparison with that one of the (3+4)/rev **M**ultiple **O**utput **C**ontroller (MOC). It demonstrates that the MOC yielded an additional vibration reduction of about 5% but was burdened with a response time which made twice that one of the SOC.

This deterioration of the controller's transient behaviour resulted from the higher amount of free parameters which had to be determined in every cycle and which were represented by the HHC amplitudes and phase shifts as well as by the elements of the T-matrix.

For that reason a fixed gain SOC was assumed of possessing a very short response time and therefore was tested in the wind tunnel too. It originated from the adaptive one by simply suppressing the online identification of the rotor state and by selecting a T-matrix which was valid for the actual flight condition.

The result of this procedure is shown in fig. 8 which shows that the fixed gain controller worked stable too and, in addition, approached the vibration minimum directly. Disadvantageous however was it's turbulent output which did not affect the resulting vibration level but indicated a small stability distance. A continous variation of the rotor state therefore was considered to be irresponsible until a systematical investigation within the scope of a nonlinear simulation would have been performed.

Fixed Gain Controller Design

Offline Identification of the Rotor Transfer Function

Derivation of the Identification Algorithm Before the evaluation of a fixed gain controller became possible, the real rotor transfer function had to be ascertained. This ascertainment represented the basis for the achievable control quality and the resulting stability distance, for what reason a high accuracy was required.

Due to the uncertainties of the induced velocities and, with that, the aerodynamic damping and forcing terms acting on the rotor blades, this demand could not be fulfiled with a theoretical model. Therefore an idenfication algorithm became necessary which determines the rotor transfer function by relating the corresponding input and output signals of the plant.

One imaginable procedure for this determination is to assume the T-matrix to consist of several 2x2-submatrices $\underline{T}_{i,j}$ which characterize the influence of the jth control input to the ith vibration output. With this assumption, the local model (1) becomes of the form

$$\begin{bmatrix} \Delta z_{1C} \\ \Delta z_{1S} \\ \cdot \\ \cdot \\ \cdot \\ \Delta z_{6C} \\ \Delta z_{6S} \end{bmatrix} = \begin{bmatrix} \underline{T}_{1,3} & \underline{T}_{1,4} & \underline{T}_{1,5} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \underline{T}_{5,3} & \underline{T}_{5,4} & \underline{T}_{5,5} \end{bmatrix} \cdot \begin{bmatrix} \Delta\vartheta_{3C} \\ \Delta\vartheta_{3S} \\ \cdot \\ \cdot \\ \cdot \\ \Delta\vartheta_{5C} \\ \Delta\vartheta_{5S} \end{bmatrix}$$

from which with

$$\underline{T}_{1,3} = \begin{bmatrix} T(1,1) & T(1,2) \\ T(2,1) & T(2,2) \end{bmatrix}$$

and

$$\Delta\vartheta_{4C} = \Delta\vartheta_{4S} = \Delta\vartheta_{5C} = \Delta\vartheta_{5S} = 0$$

for example, the system of equations

$$\Delta z_{1C}(k) = T(1,1) \cdot \Delta\vartheta_{3C}(k) + T(1,2) \cdot \Delta\vartheta_{3S}(k)$$

$$\Delta z_{1S}(k) = T(2,1) \cdot \Delta\vartheta_{3C}(k) + T(2,2) \cdot \Delta\vartheta_{3S}(k)$$

can be derived. Since it is underconditioned, two additional measurements $\Delta z_{1c}(k+1)$ and $\Delta z_{1s}(k+1)$ with slightly varied higher harmonic control parameters $\Delta\vartheta_{3c}(k+1)$ and $\Delta\vartheta_{3s}(k+1)$ are required, in order to determine the four elements of the submatrix $\underline{T}_{1,3}$.

If, beyond it, further measurements are available, the effect of sensor noise can be reduced by applying an algorithm following the least squares method. The detailed derivation of this algorithm [13] yielded the equation

$$\hat{\underline{T}} = \Delta\underline{Y} \cdot \Delta\underline{\Theta} \cdot (\Delta\underline{\Theta}^T \cdot \Delta\underline{\Theta})^{-1}$$

with

$\Delta\underline{Y}$ — a matrix consisting of the vectors of vibration change $\Delta\underline{y}$ (k)

and

$\Delta\underline{\Theta}$ — a matrix consisting of the vectors of higher harmonic input change $\Delta\underline{\vartheta}$ (k).

From this equation it can be seen that the calculation of the optimal estimate $\hat{\underline{T}}$ requires an inversion of the matrix $\Delta\underline{\Theta}^T \cdot \Delta\underline{\Theta}$ which can become singular for certain HHC input combinations. Therefore care has to be taken that the vectors $\Delta\underline{\vartheta}(k)$ forming the matrix $\Delta\underline{\Theta}$ do not depend on each other in order to allow a successful application of the least squares identification algorithm.

Verification of the Identification Algorithm In order to verify the identification algorithm and to determine the minimum number of measurements which is required in the case of sensor noise, simulations in conjunction with the local model were performed. Within these simulations, the arbitrarily choosen T-matrix represented the transfer function to be identified which, in a first step, was assumed to be independant of the higher harmonic input vector $\Delta\underline{\vartheta}$.

With respect to this vector, the vector of vibration change $\Delta\underline{z}$ was ascertained via the choosen T-matrix, before both were subjected to the identification algorithm determining the optimal estimate of the rotor transfer function.

One exemplary result of this procedure is shown in fig. 9 from which it can be seen, that, in the case of undisturbed signals, the T-matrix was identified exactly as soon as the minimum number of two measurements were available. Therefore the identification algorithm was classified as adequately funtioning on principle and therefore was subjected to noisy signals.

Fig. 10 shows the result and verifies that, in reality, a sufficient accuracy can only be achieved if at least fourteen measurements are taken into account instead of two. For that reason twenty measurements were considered to represent a good compromise between the achievable accuracy and the required amount of computational power and therefore were used for the application of the identification algorithm to a nonlinear rotor simulation model.

Application of the Identification Algorithm After the ascertainment of the minimum number of measurements which is required in the case of noisy signals, the identification algorithm was applied to a nonlinear rotor simulation model [14]. Due to the aspired design of a fast controller which only works with one feedback signal and one control signal, a restriction to a 2x2 T-matrix became possible.

In order to determine its dependancy on the rotor condition, this T-matrix was identified for different advance ratios, and HHC operating points. Fig. 11 shows one representative result and demonstrates that the T-matrix varies with the HHC phase shift as well as with the HHC amplitude.

Since the extrema of this variation depend on the advance ratio, the overall range of the T-matrix elements can be characterized as shown in fig. 12. It demonstrates the tendency of the

T-matrix element T_{11} and it's upper and lower bound within the whole flight envelope and therefore represents the basis for the fixed gain controller design.

Evaluation of a Constant Feedback Gain

Due to the strong variation of the rotor transfer function, one precondition for the successful design of a fixed gain controller was the definition of operating points which characterize the range of the T-matrix variation. Allthough this range was covered nearly completely by the high speed case, i.e. v=85m/s (see fig. 12), the low speed case was considered for the evaluation of a constant feedback gain too.

For that reason a fixed gain controller which remains stable for the 4 operating points indicated in fig. 12 (minimum and maximum of the T-matrix elements at 20m/s and 85m/s respectively) was assumed of being able to suppress the vibrations within the whole envelope and thus was defined to represent the overall goal of the design procedure.

This procedure was based upon the method of Kreißelmeier [15] which determines the feedback gains by minimizing a quality criterion describing the controller characterisitcs via an optimization algorithm [16]. In our case the quality criterion was formulated in a way that makes it depending on the poles of the 4 operating points mentioned above and indicated in fig. 12. Corresponding to (2) these poles arised from the equation

$$\det(z\underline{I} - \underline{I} + \underline{T} \cdot \underline{\hat{T}}^{-1}) = 0$$

and therefore were a function of the rotor transfer matrix $\underline{T}$ as well as the feedback matrix $\underline{\hat{T}}$.

Since both, $\underline{T}$ and $\underline{\hat{T}}$, were choosen to be 2x2 matrices (one feedback signal and one control signal), each of the 4 operating points yielded two poles which means that 8 poles had to be taken into account in total.

In a first step the absolute values ot these 8 poles were combined within the quality criterion

$$GF_1 = \sum_{i=1}^{12} w_Z \cdot |z_i|$$

$$w_Z = \begin{cases} 0.0 & 0.0 \le |z_i| \le 1.0 \\ 1000.0 & |z_i| > 1.0 \end{cases}$$

which penalizes an unstable pole ($|z_i| > 1.0$) by the factor $w_z = 1000.0$ (see fig. 13) whereas a stable pole ($0.0 \le |z_i| \le 1.0$) is weighted with the factor $w_z = 0.0$ and therefore does not contribute to the quality criterion to be minimized by the optimization algorithm.

As can be seen from fig. 13 this formulation of the quality criterion lead to a feedback matrix which yields stable poles for all 4 operating points. Disadvantageous however was the negative real part which occured in some cases and which is known to generate alternating control sequences [17].

Therefore a second quality criterion

$$GF_2 = \sum_{i=1}^{12} (w_Z \cdot |z_i| + w_R \cdot Re(z_i))$$

$$w_Z = \begin{cases} 0.0 & 0.0 \le |z_i| \le 1.0 \\ 1000.0 & |z_i| > 1.0 \end{cases}$$

$$w_R = \begin{cases} 0.0 & Re(z_i) \ge 0.0 \\ -1000.0 & Re(z_i) < 0.0 \end{cases}$$

was formulated which, in addition to the absolut value, penalizes a negative real part of a pole by a factor of -1000.0. Fig. 14 shows the result of this attempt and makes clear that the design goal could again be achieved for the 4 operating points. All poles were located within the unit circle of the z-plane and, in addition, were associated with positive real parts.

However some of these real parts were very small and therefore run the risk to become negative again for operating points not taken into account explecitly within the design procedure. For that reason a third quality criterion

$$GF_3 = \sum_{i=1}^{12} (w_Z \cdot |z_i| + w_{RN} \cdot Re(z_i) + w_{RP}/Re(z_i))$$

$$w_{ZP} = \begin{cases} 0.0 & 0.0 \le |z_i| \le 1.0 \\ 1000.0 & |z_i| > 1.0 \end{cases}$$

$$w_{RN} = \begin{cases} 0.0 & Re(z_i) \ge 0.0 \\ -1000.0 & Re(z_i) < 0.0 \end{cases}$$

$$w_{RP} = \begin{cases} 0.0 & Re(z_i) > 0.1 \\ 1000.0 & 0.0 < Re(z_i) < 0.1 \end{cases}$$

was formulated which penalizes the real part of a pole as long as it is smaller than 0.1.

Fig. 15 shows the result of this approach and makes clear that the optimization algorithm determined a feedback matrix which leads to real poles exclusively. This case is known to generate aperiodic controller sequences and therefore was assumed to remain stable and to achieve a high vibration reduction within the whole flight envelope.

Controller Tuning and Controller Comparison

Tuning of the Adaptive Controller

In order to allow an assessment of the controller working with a constant feedback gain, the adaptive as well as the fixed gain SOC were operated in conjunction with a nonlinear rotor simulation model [14]. One result of these simulations is shown in fig. 16 which demonstrates the transient behaviour of the adaptive 3/rev SOC if working with 5 feedback signals and a control rate limit of 0.1°.

From this figure it can be seen, that the time required to achieve the vibration minimum can be diminished by more than 50% if the controller cycle time is switched from 5 to 2 rotor revolutions. A further reduction to 1 rotor revolution yields a marginale improvement of the controller's transient behaviour on the one hand but on the other hand leads to a more turbulent tendency of the vibrations.

Since this turbulent tendency is not only inconvenient for passengers and crew but, in addition, indicates a small stability distance, a controller cycle time of 2 rotor revolutions was assumed to be optimal and therefore was adjusted during the following simulations.

These simulations aimed upon an investigation of the influence of $\Delta\Theta_{max}$ on the controller response time and were conducted with the 3/rev SOC and 5 feedback signals too.

Fig. 17 shows the result and makes clear that the time required to achieve the vibration minimum can be reduced by more than 60% if the control rate limit is set to 0.7°. Disadvantageous however is the very high peak level which occures immediately after the beginning of the control process.

Like the above mentioned turbulent output of a controller working with a cycle time of 1 rotor revolution this peak level is very inconvenient for passengers and crew too and therefore has to be avoided. So, in order to find out the best compromise of the controller response time and the peak level, an **I**ntegral **E**rror **C**riterion (IEC) of the form

$$IEC = \sum_{i=1}^{100} GF^2(i)$$

was formulated and plotted versus the control rate limit.

The result is shown in fig. 18 which makes clear that the integral error criterion becomes minimal if $\Delta\Theta_{max}$ is set to 0.2°. In respect of this result a controller cycle time of 2 rotor revolutions and a control rate limit of 0.2° was regarded to represent the optimal parameter combination and therefore was adjusted during the simulations in conjunction with the fixed gain 3/rev SOC.

Comparison of the Adaptive and the Fixed Gain Controller

As already mentioned above the simulations aiming upon a comparison of the adaptive and the fixed gain 3/rev SOC were conducted with a controller cycle time and a control rate limit which had been found out to be optimal for the adaptive controller.

Fig. 19 shows the result of these simulations and demonstrates that all of the three constant feedback matrices which, as shown above, were evaluated for one feedback signal, i.e. the 4/rev part of the rotor force F_z, and one feedback signal, i.e. a 3/rev higher harmonic blade pitch angel did not succeed in improving the controller's transient behaviour but, in contrary, generated a strong undershoot of the vibrations, partially followed by a small overshoot.

In order to eliminate this disadvantage, a fixed gain controller which works with an additional feedback signal, i.e. the 4/rev part of the rotor moment M_x, was evaluated with the above mentioned design methodology. Compared with the adaptive 3/rev SOC, this type of fixed gain controller yields a further reduction of the response time accompanied with only a marginale increase of the stationary vibration level (fig. 20). Since this statement is valid within the whole range of speed it represents a result which is very promising for the application at a real helicopter.

Conclusions

After a short demonstration of wind tunnel results attained with an adaptive frequency domain controller, a constant feedback gain for closed loop higher harmonic control was evaluated. It was based upon results originating from an offline identification of the rotor transfer function performed with an algorithm following the least square estimation method.

The application of this algorithm demonstrated the strong variation of the rotor transfer function with speed as well as with the HHC parameters for what reason a special controller design methodology was applied. Being based upon the minimization of a quality criterion this design methodology lead to a fixed gain controller which remains stable within the whole flight envelope and which yields a vibration reduction similar to that one of an adaptive controller. In addition it possesses a better transient behaviour and therefore is well suited for high speed and high agility helicopters.

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