

## Method and examples for calculation of flight path and parameters while performing aerobatics maneuvers by the Ka-50 helicopter based on flight data recorder information.

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Extremely high maneuverability of the Ka-50 helicopter requires to develop software for analysis of flight dynamics.

This paper presents a brief description and functions of the NSTAR program system to provide processing, analysis and registration of the records from the on-board recording devices. This system performs analysis and error elimination in recording by test instrumentation, filtration for partial restoration of recordings by using kinematic ratios in Euler or Hamilton/Rodriges forms.

Using records made by test instrumentation, NSTAR system makes possible to restore the flight path and to calculate values of immediate inmeasurable flight parameters specifically in dead (band) of manometric/aneroid pressure transducers, vane (aerometric) sensors, i.e. at low flight speeds / large attack and slip angles. The paper describes a solution of track restoration and indirect flight parameter calculation at helicopter motion equation integration.

The paper presents examples of aircraft record processing by means of NSTAR system and tracks of Ka-50 helicopter maneuvers.

### Notation

$t_H^0, P_H$	outside air temperature / pressure, deg. C / mercury column, mm
$\Delta$	relative air density / value increment
$X_K, X_B$	pilot's cyclic stick positions (positive to left and fore), mm
$\delta X_K, \delta X_B$	blade angle amplitudes depending on cyclic stick position, deg.
$B_{lynp} / A_{lynp}$	blade tip plane incidence, lateral / longitudinal (positive to left and fore), deg.
$X_H, \delta X_H; \varphi_{OIH}$	pedal position, differential collective pitch, mm, deg; collective pitch, deg.
$\gamma, \vartheta, \psi, \psi_M; \Theta, \Psi$	roll, pitch, yaw, magnetic heading, deg.; track inclination angle, course angle, deg.
$\alpha, \beta$	angle of attack at fuselage, sideslip angle at fuselage, deg.
$\alpha_K / \alpha_\Omega$	incidence of resultant velocity to hub plane / to blade tip plane, deg.
$\omega_{x1}, \omega_{y1}, \omega_{z1}$	angular velocities in fuselage axes system, deg/sec., rad/sec.
$g; G; P$	acceleration due to gravity (m/s <sup>2</sup> ); helicopter weight, kg.; main rotor load, kg/m <sup>2</sup>
$n_{xg}, n_{yg}, n_{zg}$	g-loads in Earth axis system
$n_{x1}, n_{y1}, n_{z1}$	longitudinal, normal, lateral g-load in fuselage axis system
$\omega_R; n_b, n_{TK}$	blade tip speed, m/sec.; rotor speed, gas generator speed, %
$W_0, \delta_w$	wind speed / angle, m/sec., deg.
$V_{np}, V_i, V, V_\Omega$	Calibrated, indicated, true, equivalent airspeed, m/sec, km/h
$R; X, Y, Z$	Main rotor radius, m; helicopter c.g. coordinates in Earth axis system, m
$H_{BAP}, H_\Gamma, H_E$	pressure altitude, geometric altitude; altitude on kinetic energy, m
$Cg/\sigma_7 = \frac{16 \cdot G \cdot n_{y1} / (\Delta \cdot (\omega R)^2 \cdot \pi R^2)}{N_b \cdot c / \pi R}$	rotor bearing capacity, where $N_b$ number of blades on both main rotors
$\tilde{V}, \bar{V}, \mu; M_0, M_\Sigma$	relative flight speeds; Mach number (average, total)

Dotted value means derivative with time of the given value. Coordinates are presented in Fig.1.

## 1. Introduction

For the flight limitations monitoring while aerobatic maneuvering of the Ka-50 helicopter, a detailed analysis of track and maneuver parameters is required. For this purpose KAMOV Company developed NSTAR program system to perform processing and analysis of the parameters and visualization (animation) of the flight track.

NSTAR system is compatible both with "GAMMA" system providing record and processing of the test flight parameters and with standard "TESTER-U3" system.

Usage of universal NSTAR system enables saving in specialists, hardware / software and facilitates tracking and modification of the integrated processing algorithm.

## 2. List of recorded flight data

Measured flight test parameters may be divided into several main groups:

1. Helicopter control parameters:  $X_B, X_K, X_H, \varphi_{OH}$  etc.
2. Power plant ratings specifying parameters:  $n_{TK}, n_B$  etc.
3. Helicopter motion parameters:  $V_{TP}, H_{BAP}, H_T, \gamma, \psi_M, \vartheta, \alpha, \beta, n_{x1}, n_{y1}, n_{z1}, \omega_{x1}, \omega_{y1}, \omega_{z1}$  etc.
4. Weather conditions specifying parameters:  $t_H, t_0, P_0, W_0, \delta_W$  etc.

Additional parameters provided for the specific flight test program may be measured if required.

Parameters mentioned above are registered by on-board test instrumentation on magnetic medium in a form of digital data files with  $\Delta t$  time discrete step in a form which can provide their processing and analysis with the help of ground computer-based system.

## 3. Flight data preliminary processing

Flight parameter records, read out from on-board memory stores, can contain both casual malfunctions and systematic errors defined mostly by the test instrumentation precision (including precision of its calibration) and a preliminary processing for algorithms of the flight data on-ground processing is required.

At the preliminary processing first stage files of recorded digital flight data are checked for availability of the simple type errors and malfunctions to be easily confined and corrected algorithmically:

- single sharp measurements outliers (corrected by "three point scheme");
- time sequence ( $\Delta t$ ) discreteness malfunctions of flight data recordings;
- single repeat recordings with the same on-board time (eliminated from the data file);
- single skip of records according to on-board time (eliminated by linear interpolation on the neighboring recording portions).

More complicated types of digital record errors and malfunctions are eliminated at the second stage of the preliminary processing according to the following algorithms.

## 4. Filtration of flight parameters records

The following well known methods and algorithms solve the problem of experimental data smoothing-off: Kalman filter, Ormsby filter, Potter-Bickford-Glase filter, Brison-Frisier smoothing-off procedures [ 21, 23, 24 ].

More simple and effective procedure of the signal smoothing off is flight data filtration with the help of symmetric digital differentiating filter [3] with no phase (frequency) distortion of parameters filtered off.

Basic configuration of the symmetric filter is given in linear transformation of the following type:

$$\tilde{x}_i(t) = \sum_{s=-m}^m h_{si} x_i(t+s), \quad (t \in m+1, \dots, T-m),$$

where  $x_i, \tilde{x}_i$  - discrete signals prior to / after filtration correspondingly;  $h_{si}$  - weight factors.

At  $(2m+1)$  filtration interval points ( $m$  - preceding the  $t$  point values and  $m$  - following the  $t$  point), discrete initial data are approximated by polynomials of degrees 2 ( $q=2$ ) or 3 ( $q=3$ ):

$$x_i(t,s) = a_{i0}(t) + a_{i1}(t)s + \dots + a_{iq}(t)s^q, \quad (s \in -m, \dots, m).$$

Base of filter ( $m$ ) is selected by experiment to provide requested smoothing off.

Convenience in usage of differential filter lies in the fact that time derivatives of the flight parameters which we need, for example  $\delta\dot{X}_B, \delta\dot{X}_K, \dot{\gamma}, \dot{\psi}, \dot{\vartheta}, \dot{\omega}_{xH}, \dot{\omega}_{yH}, \dot{\omega}_{zH}$  etc., can be obtained by the polynomial smoothing factors simultaneously.

Higher requirements to the given task are made on filtration of  $\omega_{x1}, \omega_{y1}, \omega_{z1}$  angular velocity components and  $n_{x1}, n_{y1}, n_{z1}$  g-loads considered as the main parameters to be used for integration of the helicopter motion equations.

A practical usage of a differentiating filter confirmed its functioning and reliability at flight test results processing. Parameter filtration results are presented below:

- for helicopter control:  $X_B, X_K, X_H, \varphi_{OH}, n_B, (f = 8 \text{ Hertz}, m = 4)$  (ref. to Fig.6);
- for helicopter motion:  $\omega_{x1}, \omega_{y1}, \omega_{z1}, n_{x1}, n_{y1}, n_{z1}, (f = 8 \text{ Hertz}, m = 8)$  (ref. to Fig. 8, 9).

## 5. Correction of recording errors

By using of kinematic relations between measured flight parameters it is possible to upgrade substantially the quality of experimental data processing. The well-known [13] differential kinematic relations in a form of Euler / Rodrigues-Hamilton (ref. to Fig.2) are used. Convenience in usage of kinematic relations in a form of Rodrigues-Hamilton lies in the fact that (1RH) relations are symmetric and linear about parameters at  $\omega_{x1}, \omega_{y1}, \omega_{z1}$  specified angular velocities. Besides, right parts of (1RH) differential equations are always fixed unlike (1E) Euler relations which become uncertain at  $|\vartheta| \approx \pi/2$  pitch values.

It is notable that before use of Euler or Rodrigues-Hamilton kinematic relations it is necessary to put record files of the helicopter angular positions to a (4) form by the following manner:

- the  $\psi_M$  magnetic course angle recorded in the range of  $0 < \psi_M \leq 2\pi$  should be put to  $-\pi < \psi \leq \pi$  yaw angle by the following formula [ 26 ] :

$$\operatorname{tg}\psi = \sin(-\psi_M) / \cos(-\psi_M); \quad (5)$$

- if the helicopter pitch angle is registered within  $-\pi < \vartheta \leq \pi$  range and the recording encloses an information on inverted flight conditions (of Nesterow loop type), at temporal recording portions, where  $|\vartheta| > \pi/2$ , reduction to  $-\pi/2 < \vartheta \leq \pi/2$  is performed by the following formula:

$$\operatorname{tg}\vartheta = \sin \vartheta / |\cos \vartheta|; \quad (6)$$

In the same temporal record portions, pitch/roll angle records should be corrected and "jumps" inserted:

$$\gamma = \gamma - \operatorname{sign}(\gamma)\pi; \quad (7) \qquad \psi = \psi - \operatorname{sign}(\psi)\pi. \quad (8)$$

It is verified that in case of combinations (4) completed kinematic relations in forms of Euler and Rodrigues-Hamilton are fully identical except  $\varepsilon$ -vicinity of a special point:  $|\vartheta| \approx \pi/2$  where (1E) Euler relations become uncertain.

(1E), (2E), (1RH), (2RH), (3RH-E), (3E-RH) relations (ref. to Fig.2) can be used for data correction obtained as a result of tests, in different conditions, depending on measurements. Iteration process (ref. to block-diagram, Fig.3) enables to correct records of angles and angular rates. With correctly selected relaxation factor of  $0 < K \leq 1$ , iteration process may be converged stably in 5 - 6 iterations. Flight recording processing experience revealed that data correction method on the algorithm given in the block-diagram is available and a selection of kinematic relations type to be used in a form of Euler or Rodrigues-Hamilton depends on specific analyzed flight mode.

As an example (Fig.7) we refer to initial  $\gamma, \psi, \vartheta$  angle records corrected to filtered angular rate records (ref. to Fig.8).

## 6. Flight path restoration

In the practice of flight tests, different means and methods are used to provide registration of flight path [ 1, 12], as follows:

- with the help of measuring photographic camera (fixation of flight path per one still frame);
- camera-read theodolite method (this method was first used by Rynin (Russia) in 1910);
- with the help of TV and video sets;
- radar method etc.

The given methods for flight registration are carried out with the help of ground aids and depending on outside conditions they have different accuracy and demands for more labor consuming measurements.

More simple is the flight path calculation method by means of the helicopter motion equation integration (ref. to Fig.4) according to records of data concerning c. of g. / angular position / angular velocity performed by on-board recording devices.

In the practice of Russian aviation, this method has been applied originally by V.S.Vedrov in 1935 during investigation of spin in flight performed by the P-5 airplane [7, 8, 9]. This method was at a later time developed and adapted to the helicopter subjects and used by KAMOV Company successfully at processing and analyzing of aerobatic flight modes.

## 7. Motion equations of the helicopter

It is known that [10, 17] equations of the helicopter motion in fuselage coordinate system (ref. to Fig.4), in a vector form appears as a principle of momentum (9) and in fixed Earth coordinate system they have been recorded directly in a form of main dynamic equation of the material point (9g).

Two conventional differential (D.E.S.) equations of the first-order (10) / (10g) corresponded to [9,16], represented by g-loads, can be gained in the appropriate coordinate axes.

Solution to (10) / (10g) equation systems have the same result at the full compliance of flight parameters  $\gamma, \psi, \vartheta, \omega_{x1}, \omega_{y1}, \omega_{z1}$  records with the kinematic relations (ref. to Fig.2). But numeric solutions of (10g) equation system are carried out more simple algorithmically because all including equations are independent. Equation system choice ((10) or (10g)) depends upon completeness of the flight data recorded.

Initial integration conditions to solve (10) or (10g) equation systems are set at initial portion record of the maneuvers at steady-state interval. In this case the following information should be used (ref. to Fig.4):

- calibration relations [1]:  $V_i = f(V_{np}, \beta, \alpha)$  & weather information concerning wind speed ( $W_0$ ) and direction ( $\delta_w$ );
- known kinematic relations between  $\alpha, \beta, V$  flight parameters.

After D.E.S. equation solution in a form of (10) or (10g), c. of g. coordinates are defined completely in the normal Earth coordinate system by D.E.S. (14) integration according to calculated  $V_{xg}, V_{yg}, V_{zg}$  velocity projections. Flight parameters (ref. to Fig.5) are calculated additionally.

## 8. Time integration

Solution of the conventional D.E.S. of the first order (10), (10g), (14) are carried out with the help of standard Runge-Cutta method of the fourth order, in this case discrete functions of the right equation parts are interpolated between nodes by modified polynomial method [4].

It is known [9, 18] that when integrating on great time intervals, measurement errors should be added to errors of numeric equation system solutions, which, when accumulated, could change significantly the result. For example, with  $\Delta n = \pm 0.05$  steady error in recording of any g-load components in the  $\Delta t = 10$  sec time interval of integration, velocity and coordinate errors should be estimated at  $\Delta V = \Delta n \cdot g \cdot \Delta t \approx \pm 5$  m/s and  $\Delta S = \Delta n \cdot g \cdot \Delta t^2 / 2 \approx \pm 25$  m correspondingly.

Flight test processing practice demonstrated that the method mentioned above is usable for relative short time intervals up to 10...15 sec which would suffice for processing of the majority of the helicopter aerobatic flight modes.

## 9. Numerical results

Based on the mathematical model mentioned above NSTAR program system is developed to provide processing, analysis and recording of the flight information consisting of the following units:

- unit intended for analysis and error correction, filtration and partial restoration of the flight data records using kinematic relations in a form of Euler or Rodrigues-Hamilton;
- unit intended for indirect flight path and parameters calculation when helicopter motion equations integrated;
- unit intended for graphical displaying of processing results.

NSTAR system enables using 23 recorded flight parameters to calculate indirectly up to 63 flight parameters including values of immediate inmeasurable flight parameters specifically in dead band of manometric / aneroid pressure transducers, vane (aerometric) sensors, i.e. at low flight speeds / large attack and slip angles. A comparison between indirectly calculated and measured flight parameters ( $V_{TP}, H_{EAP}, \gamma, \psi, \vartheta, \omega_{x1}, \omega_{y1}, \omega_{z1}, \alpha, \beta$ ) enables to verify authenticity of records made what is the equal of installation of an additional stand-by instrumentation / recording system inside the helicopter.

The findings of the NSTAR system operation for each flight mode are presented in the "Catalogue for the coaxial helicopter maneuver elements" in a form of canonical set of graphs, tables, print-outs both in paper and magnetic records.

For example, Fig. 6-14 presents a part of graphs taken out from "Catalogue" according to record processing of the right loop maneuver performed by Maj.-Gen. B. Vorobyov.

Summarized, one can recognize quite sufficient compliance of flight parameters indirectly calculated and with the measured ones.

## 10. Conclusions

NSTAR system operation results are used for the following tasks:

- to check for flight limitations, especially while performing acrobatic maneuvers;
- to analyze pilot's actions and to assist pilots in standard and acrobatic maneuver training;
- to analyze and forecast loads and clearances [6] of the coaxial rotors blade tips;
- as initial data for numerical simulation of dynamics and loads according to ULISS-6 aerolastic mathematical model of the coaxial rotors [2, 5].

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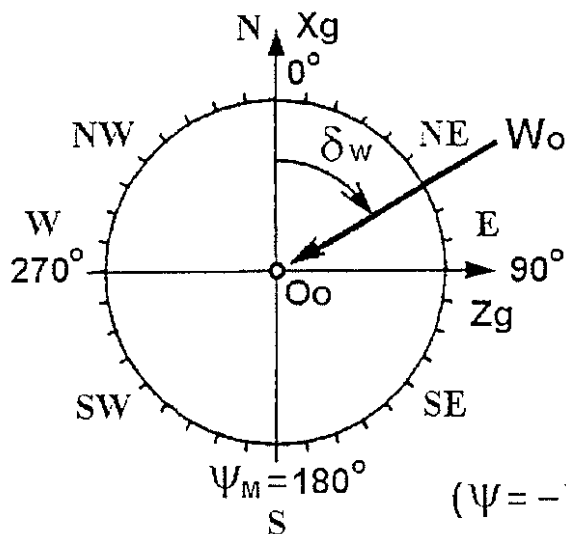
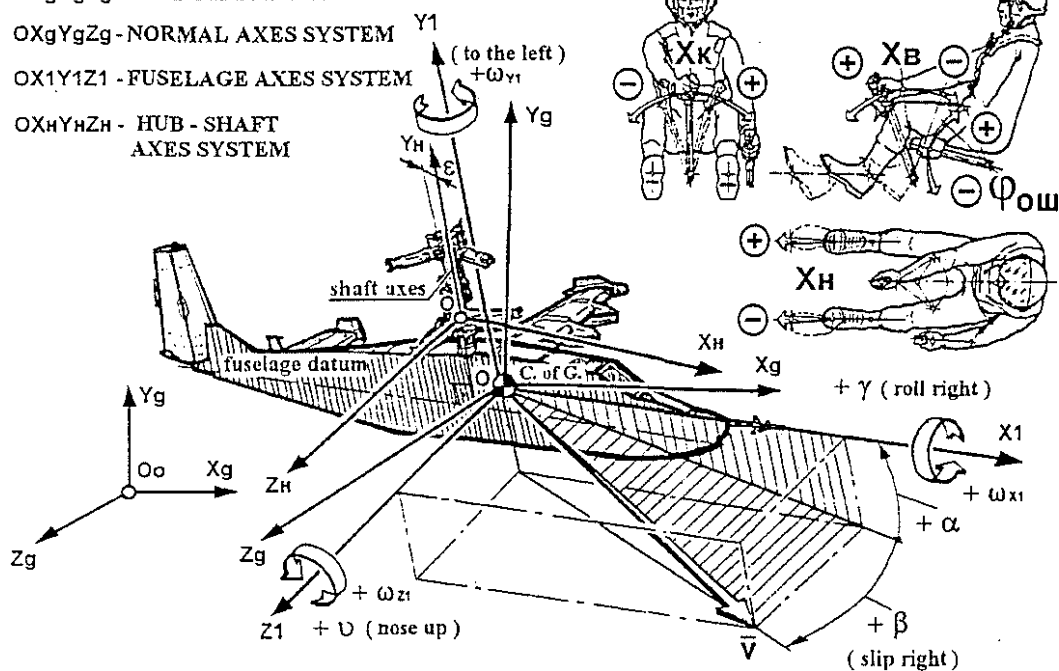
# SYSTEMS OF COORDINATES SIGN CONVENTION

OoXgYgZg - NORMAL EARTH AXES SYSTEM

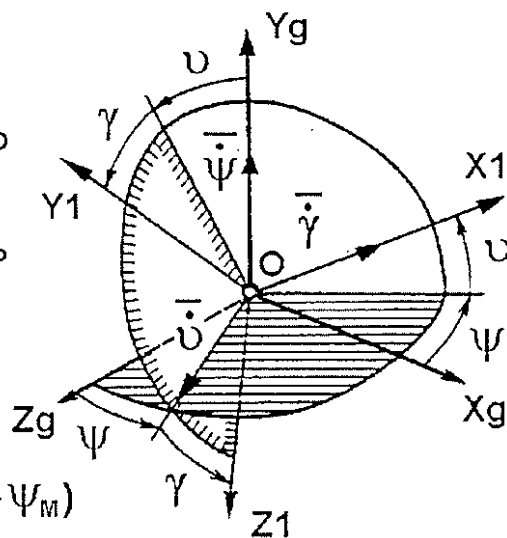
OXgYgZg - NORMAL AXES SYSTEM

OX1Y1Z1 - FUSELAGE AXES SYSTEM

OXHYHZH - HUB - SHAFT  
AXES SYSTEM



Wo - wind speed;  $\delta_w$  - wind angle  
in normal Earth axes system;  
 $\Psi_M$  - magnetic heading.



Fuselage axes system  
& normal Earth axes system.  
 $\psi$  - yaw  $\rightarrow$   $\beta$  - pitch  $\rightarrow$   $\gamma$  - roll

General rotation matrix:

$$L_{321}(\gamma, \vartheta, \psi) = L_3(\gamma)L_2(\vartheta)L_1(\psi) \quad \text{- from Earth to fuselage axes system;}$$

$$L_{123}(\psi, \vartheta, \gamma) = L_1^{-1}(\psi)L_2^{-1}(\vartheta)L_3^{-1}(\gamma) = L_{321}^{-1}(\gamma, \vartheta, \psi) \quad \text{- from fuselage to Earth.}$$

Sequential rotation matrixes:

$$L_1(\psi) = \begin{vmatrix} \cos \psi & 0 & -\sin \psi \\ 0 & 1 & 0 \\ \sin \psi & 0 & \cos \psi \end{vmatrix}; \quad L_2(\vartheta) = \begin{vmatrix} \cos \vartheta & \sin \vartheta & 0 \\ -\sin \vartheta & \cos \vartheta & 0 \\ 0 & 0 & 1 \end{vmatrix}; \quad L_3(\gamma) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{vmatrix}$$

Fig.1

## KINEMATIC RELATIONS

### Euler (E)

$$\left. \begin{aligned} \dot{\gamma} &= \omega_{x1} - \dot{\psi} \sin \vartheta; \\ \dot{\psi} &= \frac{\omega_{y1} \cos \gamma - \omega_{z1} \sin \gamma}{\cos \vartheta}; \\ \dot{\vartheta} &= \omega_{y1} \sin \gamma + \omega_{z1} \cos \gamma. \end{aligned} \right\} (1E)$$

### Rodrigues-Hamilton (RH)

$$\left. \begin{aligned} \dot{\rho} &= -(\omega_{x1} \lambda + \omega_{y1} \mu + \omega_{z1} \nu) / 2; \\ \dot{\lambda} &= (\omega_{x1} \rho - \omega_{y1} \nu + \omega_{z1} \mu) / 2; \\ \dot{\mu} &= (\omega_{x1} \nu + \omega_{y1} \rho - \omega_{z1} \lambda) / 2; \\ \dot{\nu} &= (-\omega_{x1} \mu + \omega_{y1} \lambda + \omega_{z1} \rho) / 2. \end{aligned} \right\} (1RH)$$

Inverted relations:

$$\left. \begin{aligned} \omega_{x1} &= \dot{\gamma} + \dot{\psi} \sin \vartheta; \\ \omega_{y1} &= \dot{\vartheta} \sin \gamma + \dot{\psi} \cos \gamma \cos \vartheta; \\ \omega_{z1} &= \dot{\vartheta} \cos \gamma - \dot{\psi} \sin \gamma \cos \vartheta. \end{aligned} \right\} (2E)$$

$$\left. \begin{aligned} \omega_{x1} &= 2(\rho \dot{\lambda} - \lambda \dot{\rho} + \dot{\mu} \nu - \dot{\nu} \mu); \\ \omega_{y1} &= 2(\rho \dot{\mu} - \mu \dot{\rho} + \dot{\nu} \lambda - \dot{\lambda} \nu); \\ \omega_{z1} &= 2(\rho \dot{\nu} - \nu \dot{\rho} + \dot{\lambda} \mu - \dot{\mu} \lambda). \end{aligned} \right\} (2RH)$$

Relation between Rodrigues-Hamilton parameters & Euler angles:

$$\left. \begin{aligned} \rho &= \cos \frac{\psi}{2} \cos \frac{\vartheta}{2} \cos \frac{\gamma}{2} - \sin \frac{\psi}{2} \sin \frac{\vartheta}{2} \sin \frac{\gamma}{2}; \\ \lambda &= \sin \frac{\psi}{2} \sin \frac{\vartheta}{2} \cos \frac{\gamma}{2} + \cos \frac{\psi}{2} \cos \frac{\vartheta}{2} \sin \frac{\gamma}{2}; \\ \mu &= \sin \frac{\psi}{2} \cos \frac{\vartheta}{2} \cos \frac{\gamma}{2} + \cos \frac{\psi}{2} \sin \frac{\vartheta}{2} \sin \frac{\gamma}{2}; \\ \nu &= \cos \frac{\psi}{2} \sin \frac{\vartheta}{2} \cos \frac{\gamma}{2} - \sin \frac{\psi}{2} \cos \frac{\vartheta}{2} \sin \frac{\gamma}{2}; \\ \rho^2 + \lambda^2 + \mu^2 + \nu^2 &= 1. \end{aligned} \right\} (3RH-E)$$

Euler angles through Rodrigues-Hamilton parameters:

$$\left. \begin{aligned} \operatorname{tg} \gamma &= 2(\rho \lambda - \nu \mu) / (\rho^2 + \mu^2 - \nu^2 - \lambda^2); \\ \operatorname{tg} \psi &= 2(\rho \mu - \lambda \nu) / (\rho^2 + \lambda^2 - \mu^2 - \nu^2); \\ \operatorname{tg} \vartheta &= 2(\rho \nu + \lambda \mu) / \left| (\rho^2 + \mu^2 - \nu^2 - \lambda^2) \sqrt{1 + \operatorname{tg}^2 \gamma} \right| = \\ &= 2(\rho \gamma + \lambda \mu) / \left| (\rho^2 + \lambda^2 - \mu^2 - \nu^2) \sqrt{1 + \operatorname{tg}^2 \psi} \right|. \end{aligned} \right\} (3E-RH)$$

Range  
of angles:

$$\left. \begin{aligned} -\pi < \gamma &\leq \pi; \\ -\pi < \psi &\leq \pi; \\ -\frac{\pi}{2} < \vartheta &\leq \frac{\pi}{2}. \end{aligned} \right\} (4)$$

Fig.2

**BLOCK - DIAGRAM  
OF "ANGLES - ANGULAR VELOCITIES"  
CORRECTION**

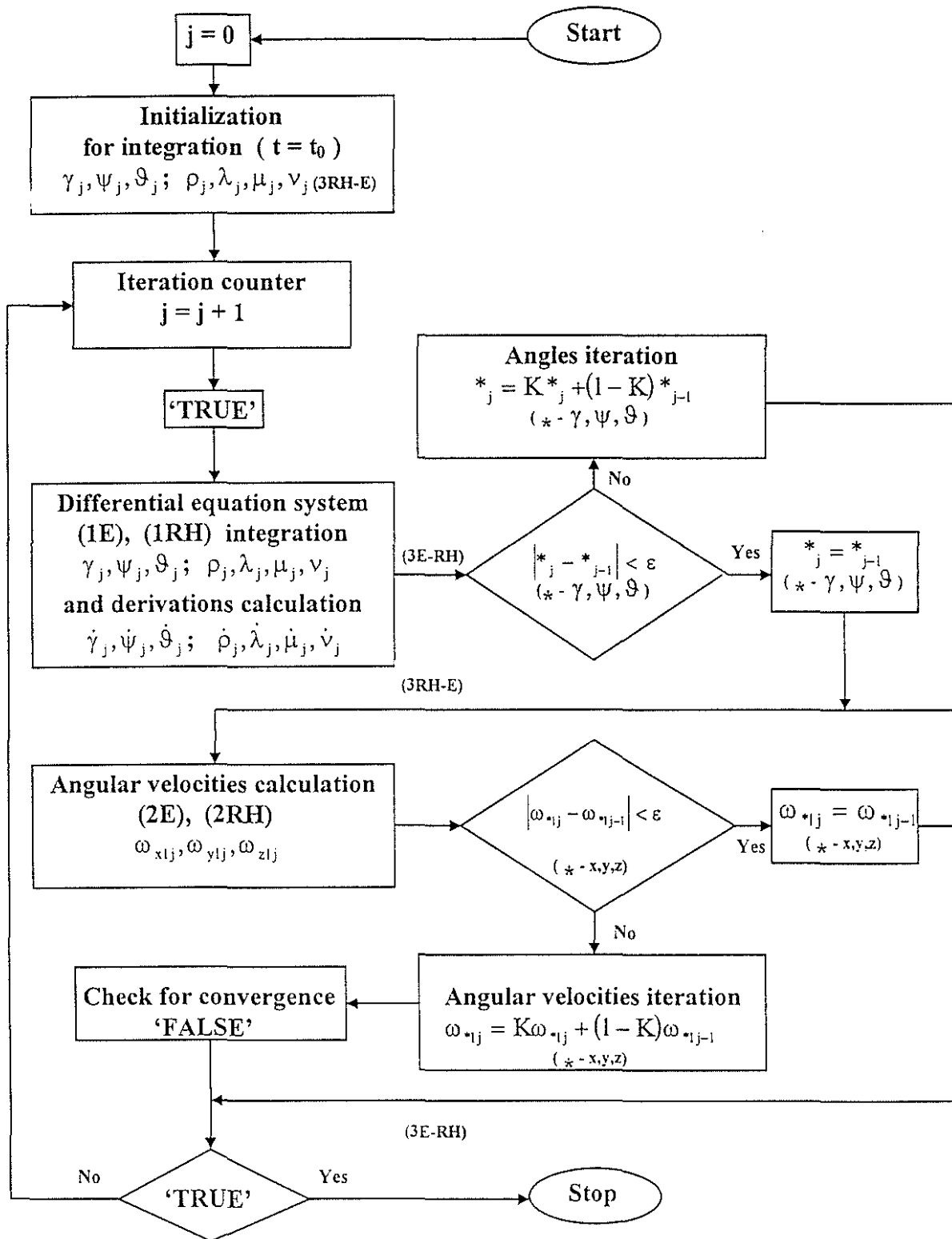


Fig.3



# MOTION EQUATION INTEGRATION ( HELICOPTER CENTER OF MASS )

**Normal Earth axes system:**

$$d\bar{P}/dt = \bar{R} \quad (9g)$$

**Fuselage axes system:**

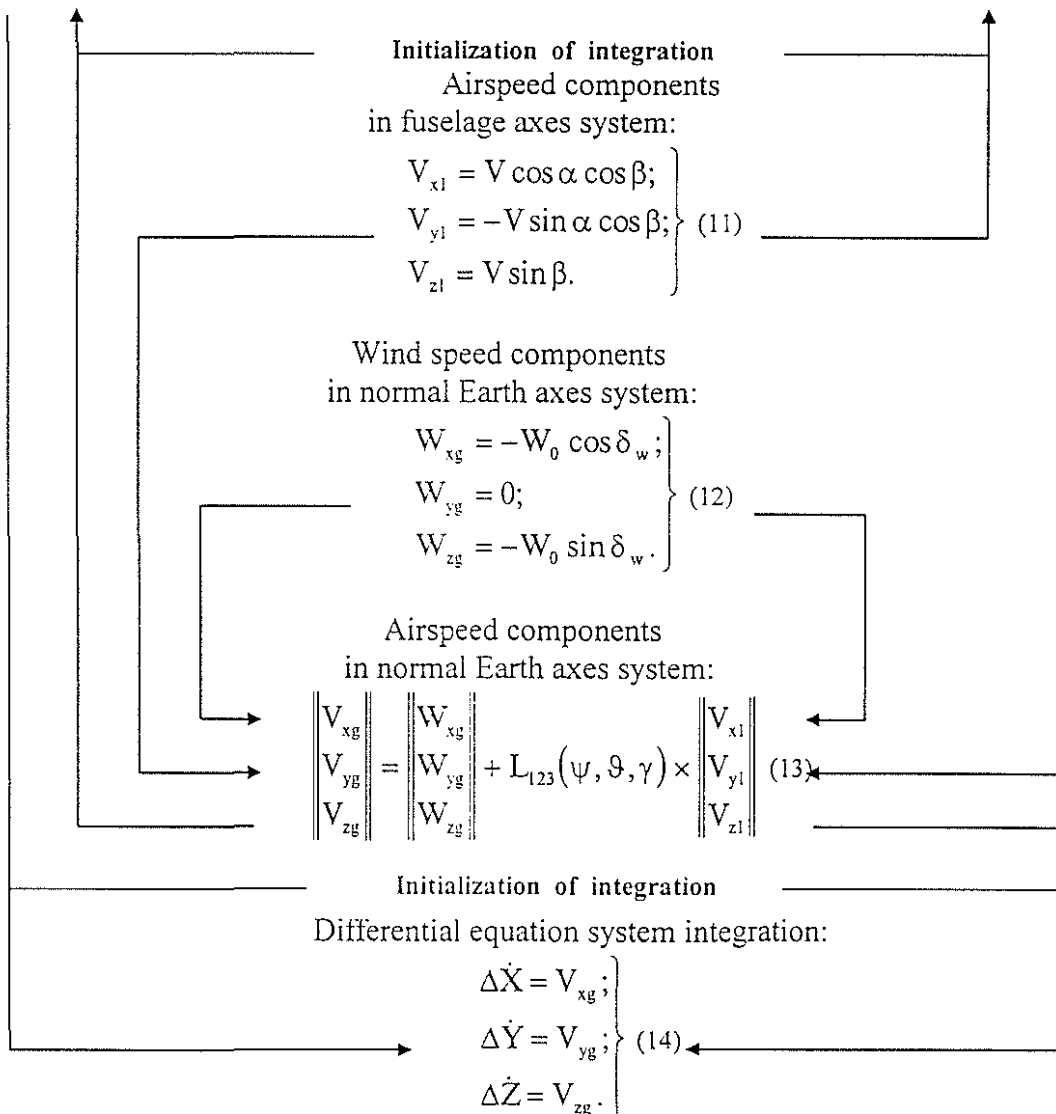
$$d\bar{P}/dt + \bar{\Omega} \times \bar{P} = \bar{R} \quad (9)$$

$\bar{P}$  - momentum vector;

$\bar{\Omega}$  - angular velocity vector of fuselage axes system;

$\bar{R}$  - total force vector.

$$\left. \begin{aligned} \dot{V}_{xg} &= gn_{xg}; \\ \dot{V}_{yg} &= g(n_{yg} - 1); \\ \dot{V}_{zg} &= gn_{zg}. \end{aligned} \right\} (10g), \quad \left\| \begin{matrix} n_{xg} \\ n_{yg} \\ n_{zg} \end{matrix} \right\| = L_{123}(\psi, \vartheta, \gamma) \times \left\| \begin{matrix} n_{xl} \\ n_{yl} \\ n_{zl} \end{matrix} \right\|; \quad \left. \begin{aligned} \dot{V}_{xl} &= \omega_{zl} V_{yl} - \omega_{yl} V_{zl} + g(n_{xl} - \sin \vartheta); \\ \dot{V}_{yl} &= \omega_{xl} V_{zl} - \omega_{zl} V_{xl} + g(n_{yl} - \cos \vartheta \cos \gamma); \\ \dot{V}_{zl} &= \omega_{yl} V_{xl} - \omega_{xl} V_{yl} + g(n_{zl} + \cos \vartheta \sin \gamma). \end{aligned} \right\} (10);$$



**Fig.4**

## FLIGHT PARAMETERS INDIRECTLY ACCOUNTED

### Helicopter speeds:

- airspeed:  $V = V_i / \sqrt{\Delta}$ ,

where:  $V_i = f(V_{np}, \beta, \alpha)$ ,  $\Delta = 0.379 P_H / (273 + t_H)$ ,  $P_H = 760(1 - H_{EAP} / 44300)^{5.256}$ ;

- in hub-shaft axes system:

$$\begin{Bmatrix} V_{xH} \\ V_{yH} \\ V_{zH} \end{Bmatrix} = L_2^{-1}(\varepsilon) \times L_{321}(\gamma, \vartheta, \psi) \times \begin{Bmatrix} V_{xg} - W_{xg} \\ V_{yg} - W_{yg} \\ V_{zg} - W_{zg} \end{Bmatrix};$$

- in tip path plane axes system:

$$\begin{Bmatrix} V_{\vartheta xH} \\ V_{\vartheta yH} \\ V_{\vartheta zH} \end{Bmatrix} = L_3^{-1}(B_{1ynp}) \times L_2^{-1}(A_{1ynp}) \times \begin{Bmatrix} V_{xH} \\ V_{yH} \\ V_{zH} \end{Bmatrix},$$

where:  $A_{1ynp} = \delta X_B \cos \delta_3$ ,  $B_{1ynp} = \delta X_K \cos \delta_3$ ;  $\delta_3$  - angle of  $\Delta$ -coupling.

### Angular velocities:

- in hub-shaft axes system:

$$\begin{Bmatrix} \omega_{xH} \\ \omega_{yH} \\ \omega_{zH} \end{Bmatrix} = L_2^{-1}(\varepsilon) \times \begin{Bmatrix} \omega_{xI} \\ \omega_{yI} \\ \omega_{zI} \end{Bmatrix};$$

- angular velocities of tip path plane tilt:

$$\left. \begin{aligned} \omega_{RxH} &= \omega_{xH} - \dot{A}_{1ynp}, \\ \omega_{RzH} &= \omega_{zH} - \dot{B}_{1ynp}. \end{aligned} \right\}, \quad \dot{A}_{1ynp} = \delta \dot{X}_B \cos \delta_3, \quad \dot{B}_{1ynp} = \delta \dot{X}_K \cos \delta_3.$$

### Angles:

- track inclination angles, course angles:

$$\operatorname{tg} \Theta = V_{yg} / V_{xgzg}, \quad \operatorname{tg} \Psi = -V_{zg} / V_{xg};$$

- attack angles, slip angles (of fuselage):

$$\operatorname{tg} \alpha = -V_{yI} / V_{xI}, \quad \operatorname{tg} \beta = V_{zI} \cos \alpha / V_{xI};$$

- design & tip path plane attack angles:

$$\operatorname{tg} \alpha_K = -V_{yH} / V_{xHzH}, \quad \operatorname{tg} \alpha_{\vartheta} = -V_{\vartheta yH} / V_{\vartheta xHzH}.$$

### X, Y, Z in normal Earth axes system:

- "energy" altitude:  $H_E = V^2 / (2g)$ .

### Similarity criteria:

- rotor bearing capacity:

$$Cg / \sigma_7 = 16P / (\Delta(\omega R)^2 \sigma_7), \quad \text{where: } P = n_{yI} G / (\pi R^2);$$

- relative flight speeds:

$$\tilde{V} = V_{\vartheta xHzH} / (2\sqrt{P/\Delta}), \quad \bar{V} = V_{\vartheta xHzH} / (\omega R), \quad \mu = \bar{V} \cos \alpha_{\vartheta};$$

- Mach number (overage, total):

$$M_0 = \omega R / (20\sqrt{273 + t_H}), \quad M_{\Sigma} = M_0(1 + \bar{V}).$$

**Fig.5**

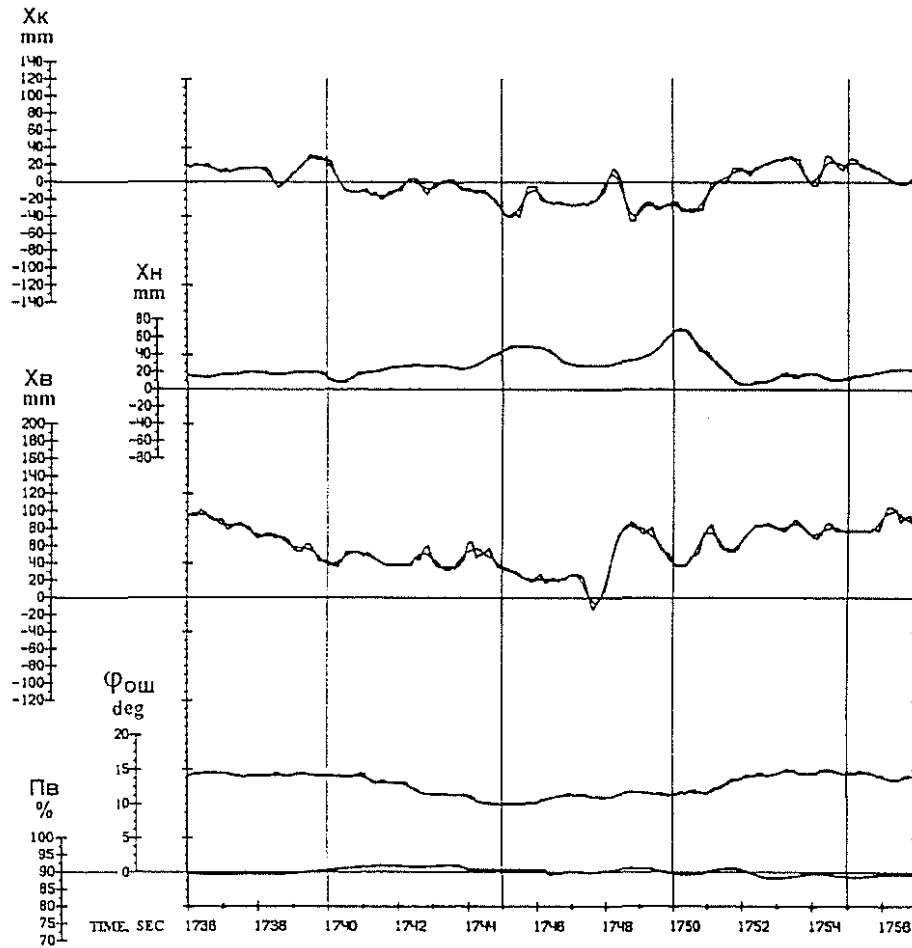


Fig.6 Position to the controls  
& rotor speed ( Пв, % )

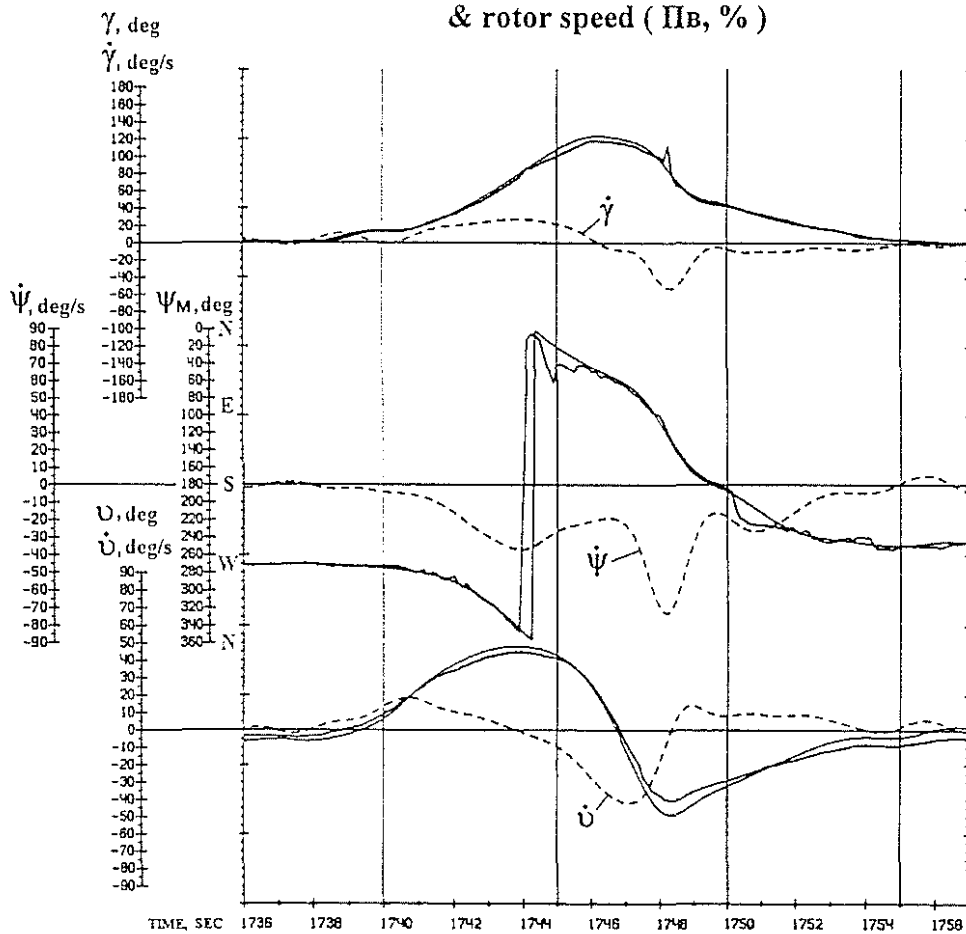


Fig.7 Helicopter roll, magnetic heading, pitch  
angles & their time derivatives

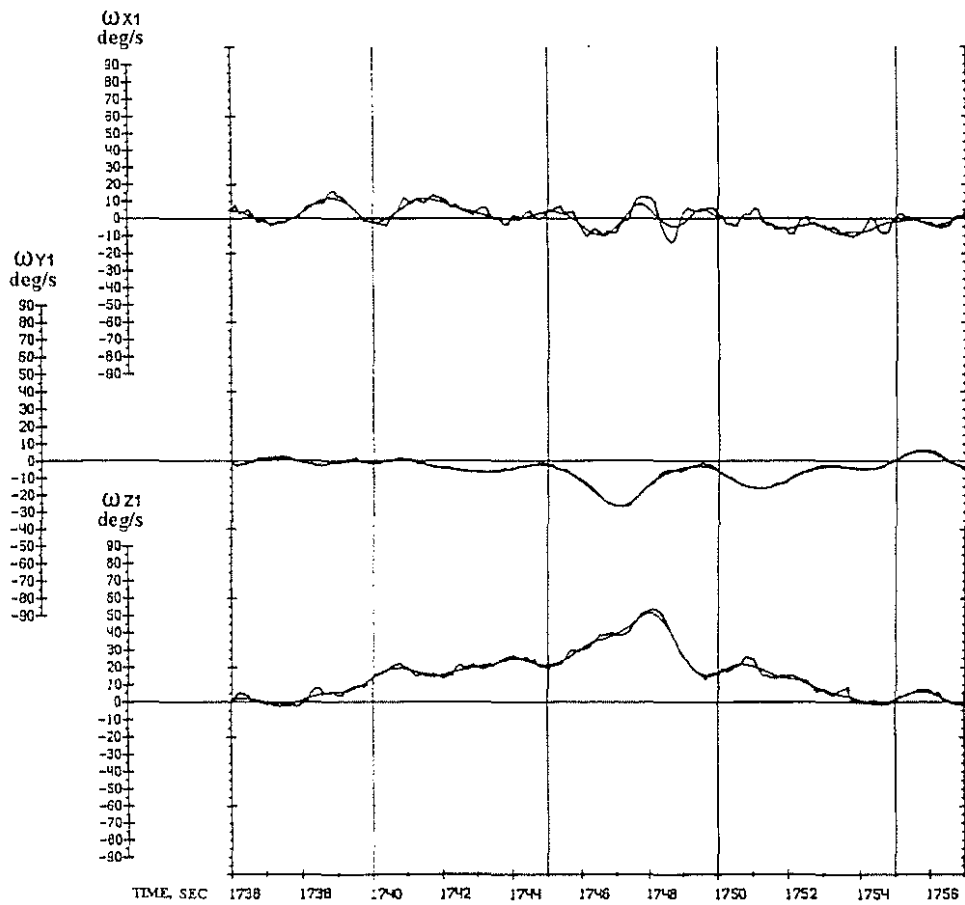


Fig.8 Angular velocities of helicopter  
( in fuselage axes system )

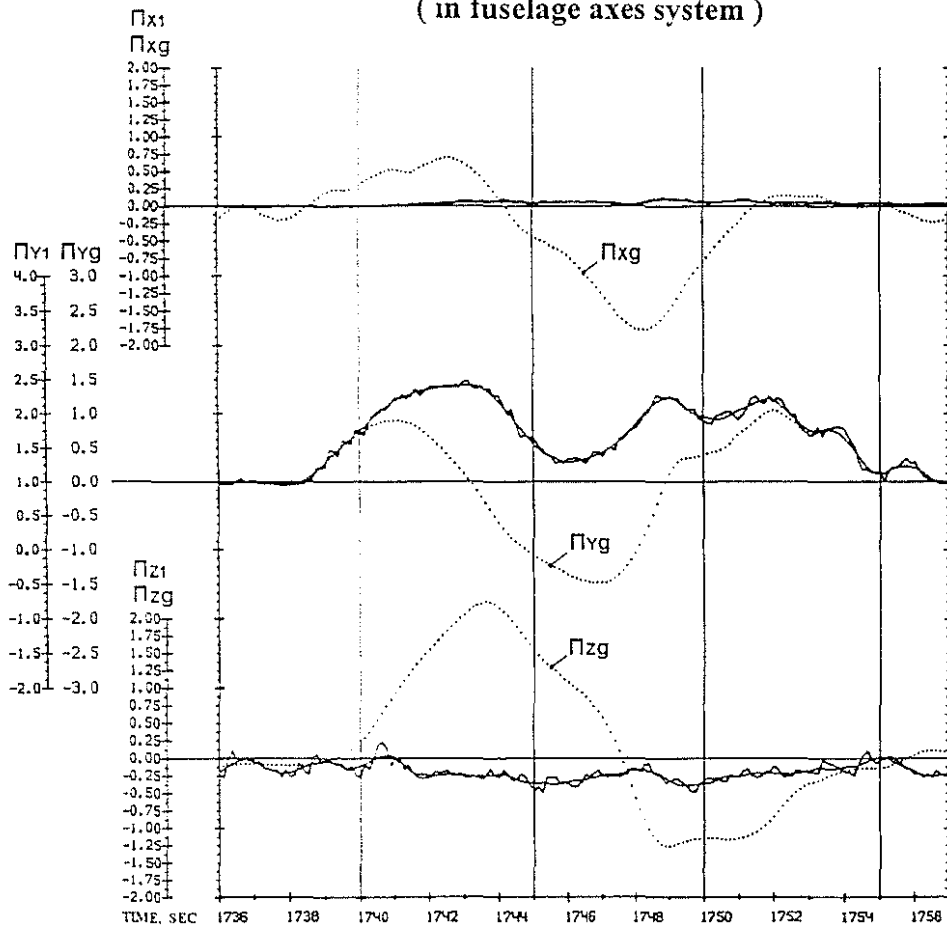


Fig.9 Longitudinal, normal, lateral g-loads of helicopter  
( in fuselage & Earth axes systems )

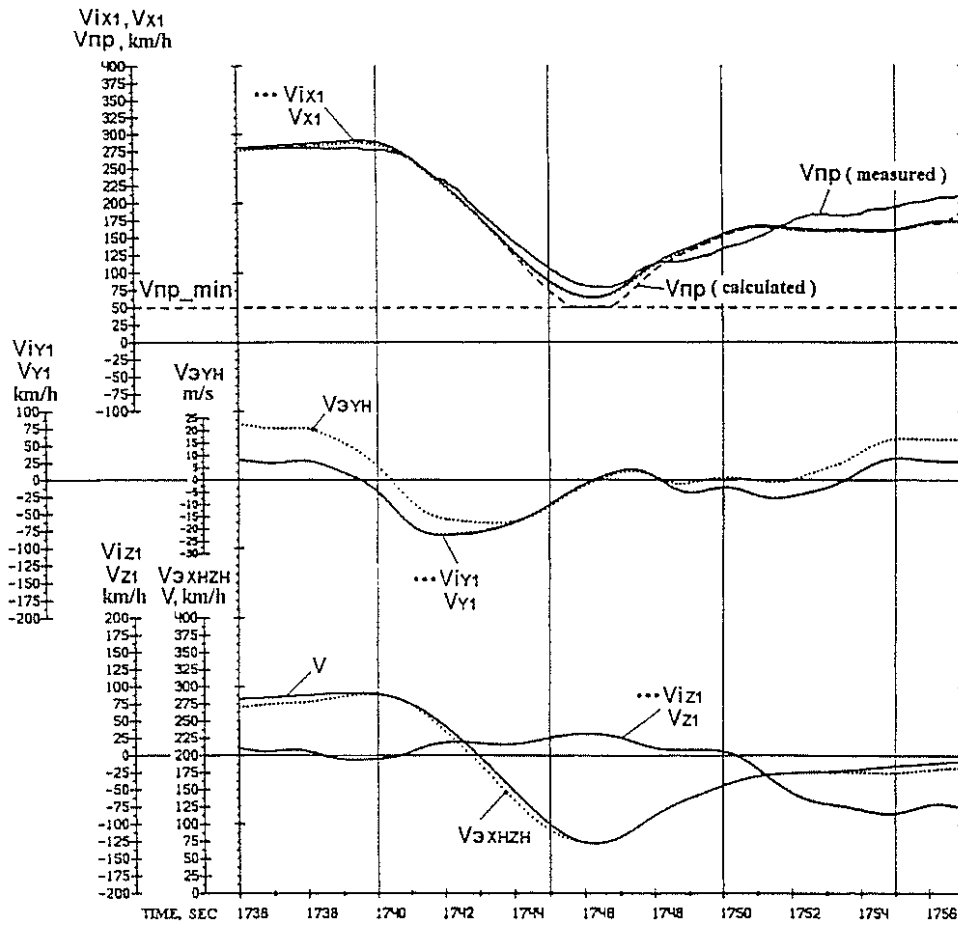


Fig.10 Calibrated, indicated, true & equivalent helicopter airspeeds projections

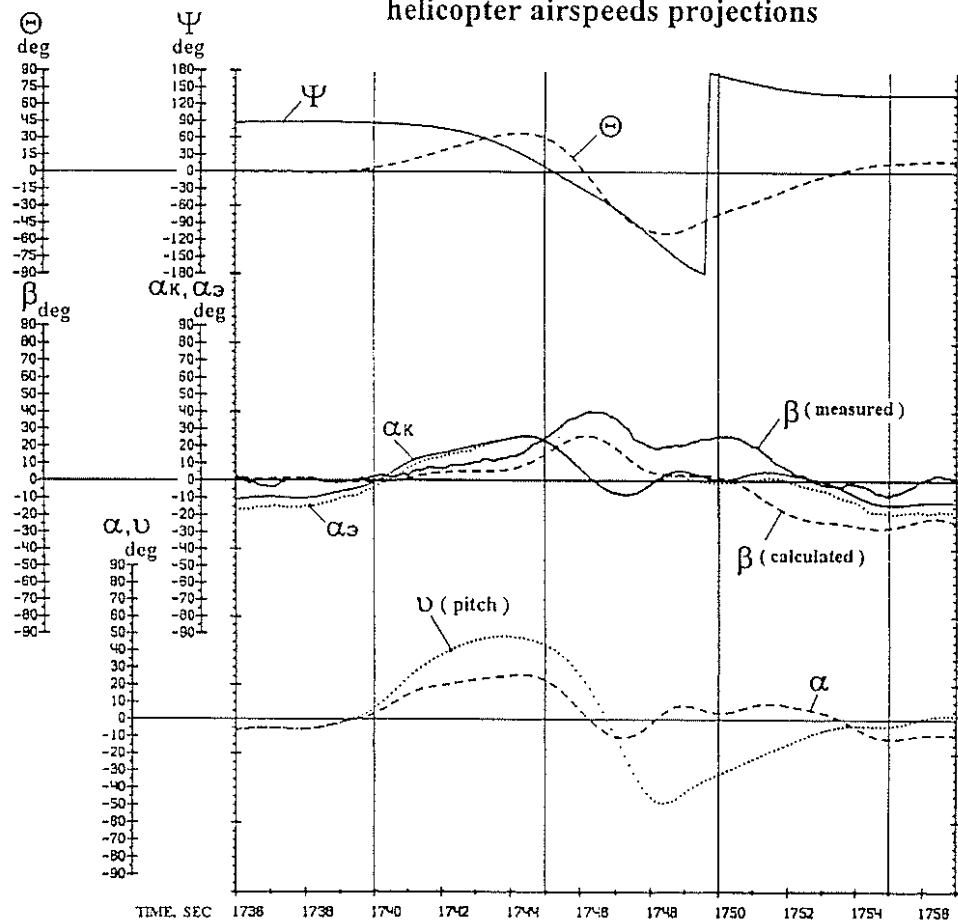


Fig.11 Inclination angles of track / course; sideslip / attack / pitch

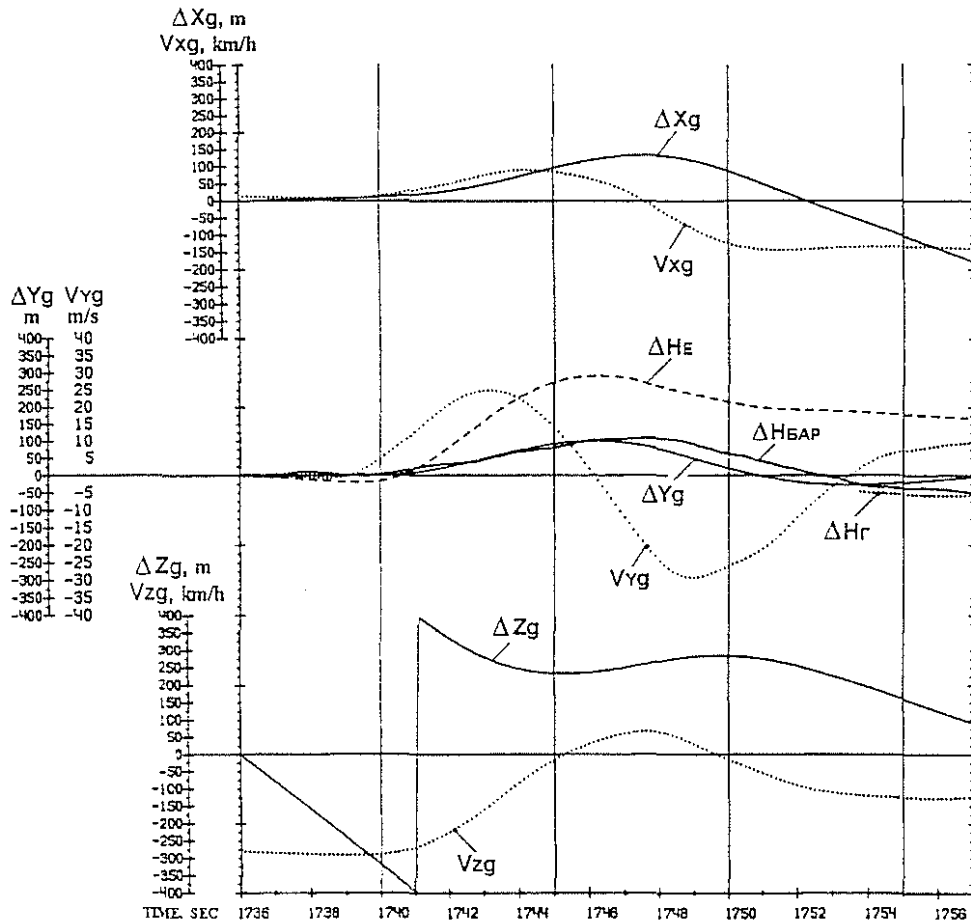


Fig.12 Differences of helicopter C.of G. coordinates & airspeeds in Earth axes system

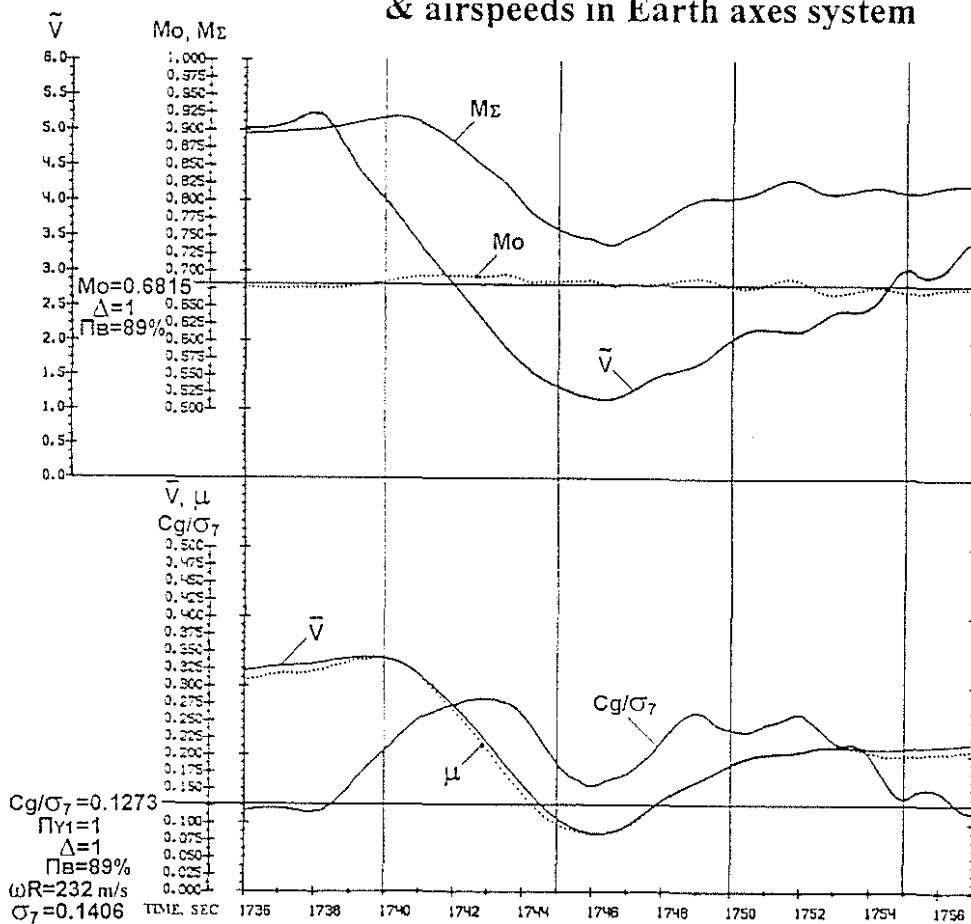
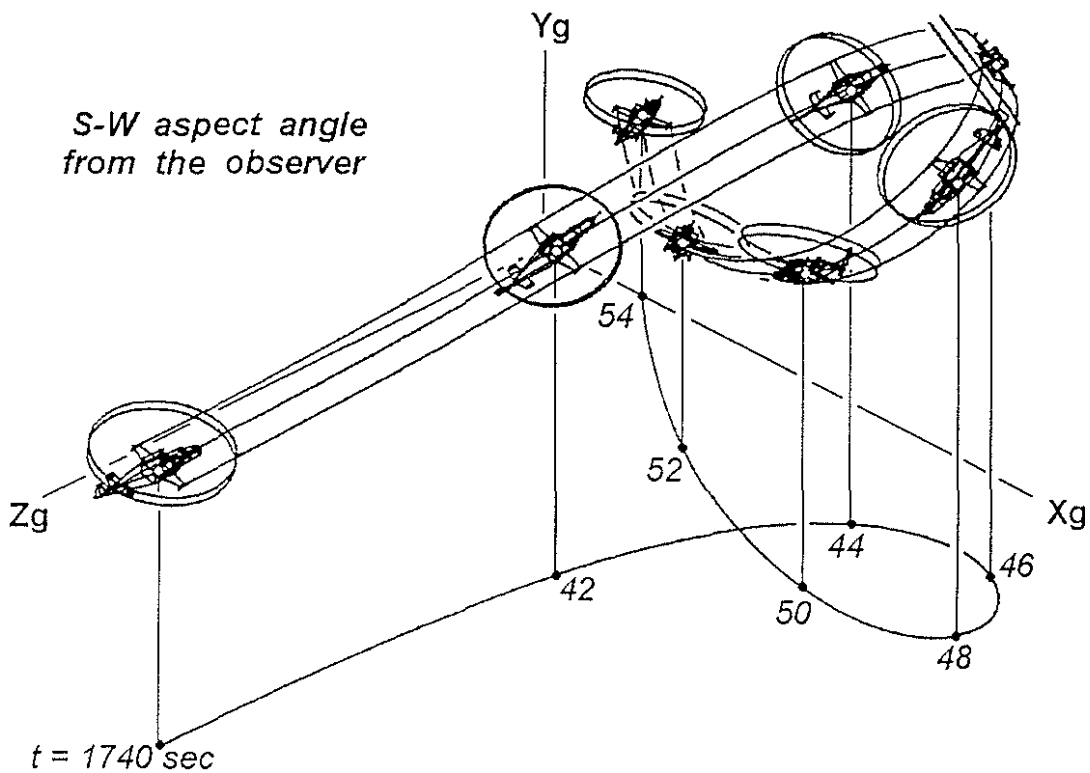
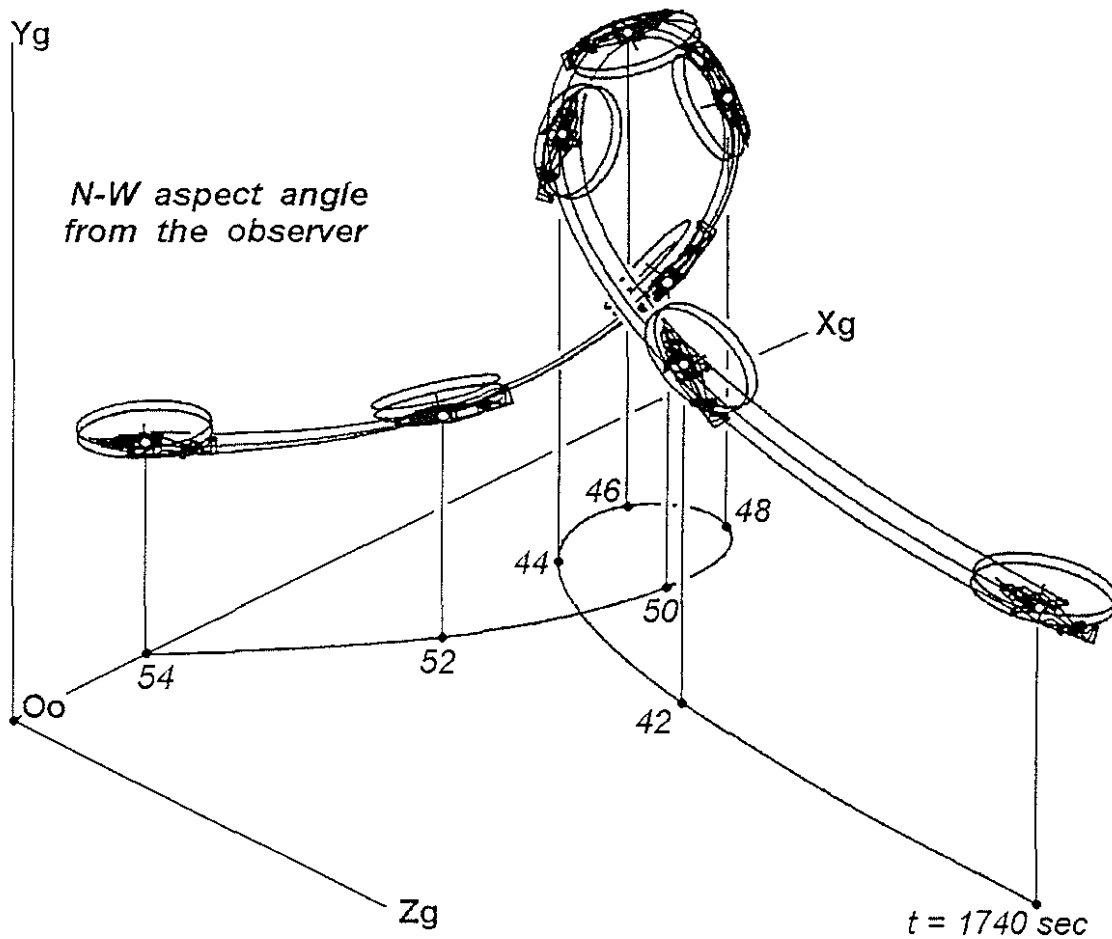


Fig.13 Similarity criteria



**Fig.14 Flight path while performing skewed loop (right)**