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SOFTWARE ON PERSONAL-COMPUTERS FOR FREQUENCY-DOMAIN ANALYSIS AND SYSTEM IDENTIFICATION FOR HELICOPTERS

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Abstract

In this paper software for frequency domain analysis and system identification for helicopters, based on Matlab 4.0 (for Windows) is presented. As frequency domain analysis is graphics intensive, user friendly software is needed. In the package, graphical user interfaces are used extensively. This facilitates plotting the large number of variables and defining different structures for the parametric transfer-functions, which are fitted to the estimated non-parametric transfer-functions. Some features of the software are presented, using a flight test manoeuvre of the BO-105 helicopter as an illustrative example. The Dutch-roll damping and cross-over frequency are identified in different transfer-functions, both in individual and in simultaneous fitting sessions. The results are compared with those obtained by other institutes of NATO.

Notations

p	roll-rate	
q	pitch-rate	
r	yaw-rate	
R_{min}	Minimal required resolution	
$S_{x_iy.j!}$	conditioned cross-spectral density	
$S_{x_ix_i.j!}$	conditioned auto-spectral density	
$S_{y:x}$	multiple coherent output spectrum	$\gamma^2_{y:x}S_{yy}$
u_i	fully conditioned input x_i: $x_{i.(i-1)!}$	
x_i	input signal	
δ	control	
$\gamma^2_{xy}(\omega)$	coherence function	$\frac{\lvert S_{xy}\rvert^2}{S_{xx}S_{yy}}$
$\gamma^2_{y:x}$	multiple coherence function	$\gamma^2_{u_1y}+\gamma^2_{u_2y}+\cdots+\gamma^2_{u_xy}$
! (as in $S_{44.3!}$)	conditioned for inputs 1, 2, 3	

Abbreviations

AGARD	Advisory Group for Aerospace Research and Development
DoF	Degree of Freedom
FFT	Fast Fourier Transform
MIMO	Multi-Input/Multi-Output
NLR	Nationaal Lucht- en Ruimtevaartlaboratorium
PSD	Power Spectral Density
SISO	Single-Input/Single-Output
TUD	Technical University Delft
WG-18	Working Group 18 (AGARD), on Rotorcraft system identification

1 Introduction

At the TUD, system identification for fixed wing aircraft has been practised for many years. So far, time-domain techniques have been the main approach. The basic procedure in this technique is to define a model with initial values for the parameters and to use an iterative procedure to change all parameter-values so that the model outputs are as close as possible to the original flight-test outputs. In the two-step method, first the states and subsequently the model parameters are estimated (ref. 5). Obviously, the structure of the model is of great importance and even small changes in the model structure can result in large changes in the values of identified parameters. For helicopters, the coupling between symmetric and asymmetric movements (mainly due to the presence of the rotor) and the extra degrees of freedom of the rotor (flapping and lead-lag hinges) lead to complex models. Neglecting to model these effects will lead to poor results. However, if too many parameters are used, some of these parameters will be highly correlated and the results will become unreliable.

One of the key advantages of frequency domain methods over time domain methods is that the system can be "split" into single-input/single-output (SISO) systems. Non-parametric transfer-functions are estimated and the model (transfer-function) structure can be based on visual inspection of the non-parametric transfer-functions. Furthermore, the quality of the transfer-functions can be assessed, via the estimated coherence-functions. Also, time-delays can be estimated directly. Another important advantage is the possibility to limit the frequency-range of the transfer-functions. Thus, widely separated sub-modes can be identified separately. The simplified representation of the system (a series of transfer-functions) and the extra information (coherence functions and spectral-decompositions) enables the analyst to improve his insight.

For these reasons a project on frequency-domain analysis and identification was recently started at the TU-Delft, in cooperation with the NLR. A software-package for frequency-domain analysis and system identification for helicopters was developed, on a personal computer, using Matlab 4.0 (for Windows). As frequency domain analysis is graphics intensive, user friendly plotting routines are needed. In order be able to understand the influence of different structures for the parametric transfer-function in the fitting procedure, the analyst should be able to define different structures quickly and the influence on the fit should be visualized in a bode plot directly. Extensive graphical user interfaces with on-line help facilities were built to facilitate the interaction between the analyst and the computer, both for the plotting and for the fitting routine. In this paper, a test-flight of the BO-105 helicopter is used as an illustrative example. The roll- and yaw-rate due to lateral stick movements are used to develop parametric transfer-functions.

2 The Software Package

The package is built in a modular fashion: First the time history records are Fourier-transformed and auto- and cross PSD-functions, coherence functions and non-parametric transfer-functions are calculated. The MISO-system is conditioned to multiple SISO-systems by an iterative procedure (ref. 2, 6-8), resulting in partial PSD, coherence and non-parametric transfer-functions. The PSD and transfer-functions are smoothed to reduce their variances. Different smoothing algorithms have been implemented for this purpose. The (partial) PSD, coherence and transfer-functions can then be visualized in different plots. From these plots, the analyst is able to determine whether the conditioning was adequate or whether the number or order of the inputs should be adapted for adequate modelling. The second step in the identification process is to find parametric transfer-functions, that accurately describe the high-quality non-parametric transfer-functions. The combination of all transfer-functions, from all inputs to all outputs describes the total system.

If all parametric transfer-functions are to be combined in one state-space model, they must have one common denominator. This means ideally all transfer-functions are fitted simultaneously and all transfer-functions are of the same (maximal) order. The common denominator is built from a number of first- and second- order subsystems. For each subsystem, corresponding eigen-movements exist. Often only a few states are excited by a particular subsystem, for example, the dutch-roll (second order) mainly results in the rolling and yawing. Thus, in each transfer-function, some subsystems are more dominant than others. If in two transfer-functions the same subsystems are dominant, these transfer-functions obviously must be fitted simultaneously. If a subsystem is not apparent, it must not be used. In this way, reliable estimates for the parameters used can be found. Individual transfer-functions are fitted, to determine which sub-systems are dominant and/or to find good initial values for combined fitting procedures for transfer-functions with common sub-systems.

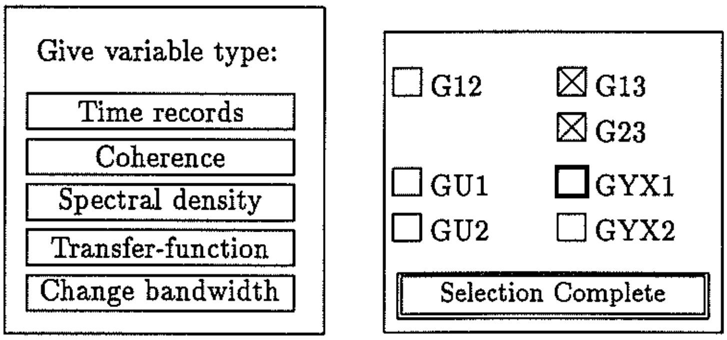
Finally, all transfer-functions are combined into one state-space model. This can only be achieved, if the fitting procedure yields one common denominator for all transfer-functions (ref. 3).

2.1 The main program

In the main program, the user must define the file with the time history records; the inputs that should be used; the outputs and the method and degree of smoothing that should be used. In order to determine the influence of the conditioning and smoothing on the estimated variables, the program must be rerun with different settings. If q denotes the number of inputs, the output is signal $q + 1$. Mouse driven menus and defaults (accepted by hitting the "return" button) were used where possible to speed-up the input of information from the user.

2.2 Plotting estimated variables

Once all variables are calculated, the plotting routine is called. The user must click one of the push-buttons in the main menu (figure 1a) with a mouse. If the button labelled "Coherence" is clicked, the sub-menu of figure 1b is shown. Depending on the number of inputs used in the conditioning process, more (or less) variables are available for plotting. The user can select a variable by clicking these "check-boxes". In figure 1b, the user has clicked the boxes for G13 and G23 ('3' stands for the output p). The selected variables will be plotted if the pushbutton "Selection completed" is clicked. For each plot, a new window is opened. Thus, all variables of one type (coherence, psd or transfer-function) can be combined in one plot and an arbitrary number of plots can be used simultaneously. At the top of each plotting window, pull-down menus are available for printing, switching to other windows, switching axis-types (linear or log-scales) etc. For every plotted variable, a different line-type and -colour is used and next to the plots, the name of each variable and the corresponding line-type is printed in the corresponding colour.



(a) Main-menu (b) Coherence selection

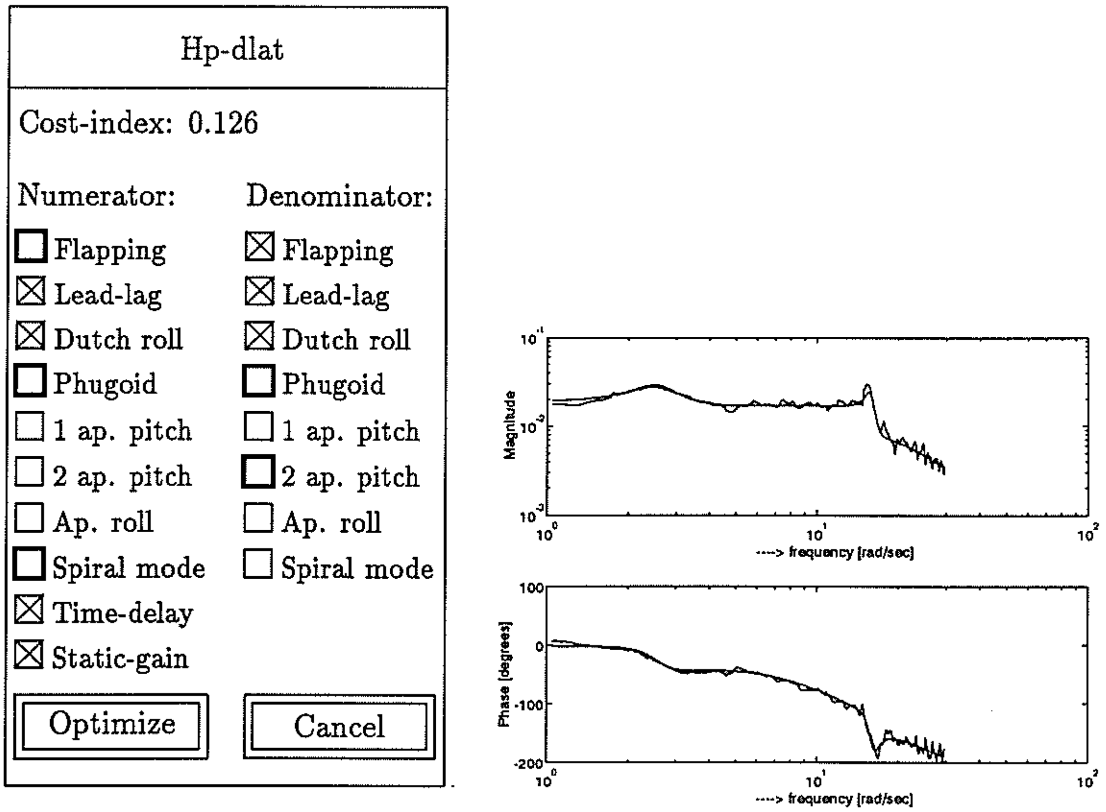
Figure 1: Main- and sub-menu of plotting routine for two inputs

2.3 Fitting individual transfer-functions

In order to determine the best structure for individual transfer-functions, an inter-active fitting routine was developed. With this routine, defining different structures for the parametric transfer-function can be done quickly. The influence on the goodness of fit is presented in the form of a bode plot and a cost-index.

By clicking the appropriate check-boxes, the analyst can switch different subsystems in the parametric transfer-function on and off, both in the denominator and the numerator. The resulting changes are instantaneously shown in the bode-plot. In this way the analyst can iteratively build the model structure to the desired degree of adequacy. Figure 2 gives an idea of the general appearance of the fitting routine. In figure 2a, the menu is shown. By viewing the checkboxes, it is clear that the fit of figure 2b is a combination of three second order systems in the denominator; two corresponding second order systems in the numerator, combined with a static-gain and a time delay.

At the top of the menu, a cost-index is given. This index is set to one if the parametric



(a) menu

(b) Bode plot

Figure 2: 6th. order fit of $H_{p\delta_{lat}}$

transfer-function is equal to zero at all frequencies. The cost-index is minimised to obtain a good fit. The index will be zero if the parametric transfer functions is exactly equal to the non-parametric estimate (a "perfect fit"). At the bottom of the menu-window, a push-button "Optimize" is found. If this button is clicked, the optimum for the current structure of the parametric transfer-function is calculated. This optimum is found with the aid of the standard Matlab function *Constr.m.* This function finds the constrained minimum of a cost function, using a combination of the Newton and the Steepest descent method. The Steepest descent assures quick convergence far from the minimum, whereas the Newton method assures quick convergence near the minimum. In the cost function, different weighting functions and constraints can be defined.

3 Smoothing Estimated PSD Functions

The Fourier transforms of the time history records, are used to estimate PSD functions. Since the measured time records are of limited lengths and discrete, the PSD estimates are also discrete and should be seen as estimates of the average value over the small frequency-band $\Delta\omega = \frac{2\pi}{T}$. Unfortunately, the large variance of these estimates is independent of the time record length. Because the coherence is a squared function, this variance leads to biased estimates for the coherence functions. This bias can (and must) be reduced by smoothing the estimated PSD functions. If the system is conditioned, squared transfer-functions are used, making the smoothing even more important.

Two smoothing approaches were implemented: In the first approach, the time-history records are divided into sub-records. A Hanning, or any other window (e.g. Bartlett, Blackmann, Hamming etc.) is used for these sub-records and the estimated PSD functions of these sub-records (the periodograms) are averaged so that smoothed estimates are found. Using m sub-records results in a reduction of variance with a factor m, but also in an identical reduction of estimate-resolution. A second advantage of this approach is, that the smaller time-records lead to less calculations for the Fourier transformation. However, the development of the Fast Fourier Transform (FFT) and the arrival of faster computers has reduced the problem of the number of calculations.

In the second approach for reducing the variance of estimated PSD functions, the raw estimates are averaged over a certain frequency-band ($m * \Delta\omega$). Again, using m points results in a reduction of variance with a factor m. Practically, a moving average is calculated. In this way, the resolution of the estimate is not changed. However, effectively this smoothing does reduce the resolution with the same factor.

Obviously, the more points are used in the smoothing procedure, the better the estimated coherence function will be. However, if the bandwidth over which was averaged becomes too large, peaks in the estimated PSD, coherence and transfer-functions will be smeared out. Thus, in a fitting routine, damping coefficients will be over-estimated. A compromise between variance and resolution must be found.

The minimal resolution depends on the cross-over frequencies and damping coefficients of the second order systems: smaller damping coefficients lead to sharper peaks in the transfer-function and that smaller cross-over frequency leads to less points where the estimate is defined. (The estimate is defined at linearly space frequencies and the influence of a systems is found in a fixed part of a decade.)

A compromise solution can be found by considering two worst-case scenarios. For instance for the case discussed in section 5, the following criteria was defined: (1) a second order system with damping coefficient 0.2 at cross-over frequency 2 rad/sec and (2) a second order system with damping coefficient 0.02 at cross-over frequency 15 rad/sec. Estimates for these transfer-functions with different resolution were calculated and fitted. This led to the conclusion, that the resolution for these systems should be at least 2 points per rad/sec. This means that the maximum number of smoothing points is $m < \frac{T}{2\pi R_{min}} = \frac{150}{2\pi 2} \approx 12$ (where T is the total length in seconds of the time signals and R_{min} the minimal required resolution).

4 Conditioning MIMO Systems

The transfer-functions describing the system, are estimated from the output and input signal. In order to obtain optimal results, the pilot tried to excite only one input. In practice, the secondary inputs are also excited. A method that deals with these unwanted secondary influences is discussed in (ref. 2). This method was applied to flight-test data (ref. 6-8).

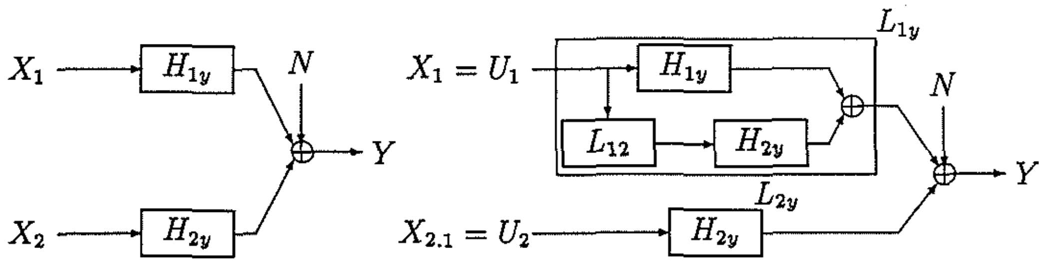
The special case of a two-input/single-output system will be considered in some detail to illustrate matters of concern in the general case of q inputs. In figure 3, X_1 and X_2 are the Fourier-transforms of the two inputs $x_1(t)$ and $x_2(t)$ that may be correlated. N is the noise, which is not correlated with the inputs ($S_{n1} = S_{n2} = 0$). From measurements of $x_1(t)$, $x_2(t)$ and $y(t)$, it is desired to determine H_{1y} and H_{2y}. The original system of figure 3a is redrawn to figure 3b. In this figure $X_{2.1}$ is the linear independent part of X_2 and L_{12} is the transfer-function describing the linear relation between the first and second input. The coherence between the conditioned input signals X_1 and $X_{2.1}$ is equal to zero and the system can be treated as two separate SISO systems.

L_{12}, L_{1y} and L_{2y} are estimated by:

$$L_{12} = \frac{S_{x_1x_2}}{S_{x_1x_1}}, \qquad L_{1y} = \frac{S_{u_1y}}{S_{u_1u_1}} \quad \text{and} \quad L_{2y} = \frac{S_{u_2y}}{S_{u_2u_2}} \tag{1}$$

The transfer-functions of the original system are found by:

$$\begin{aligned} H_{2y} &= L_{2y} \\ H_{1y} &= L_{1y} - L_{12}H_{2y} \end{aligned} \tag{2}$$



(a) Original system with correlated inputs

(b) Conditioned equivalent

Figure 3: Two-input/single-output system

Practically, the conditioned signals are not used. Only the conditioned spectra are calculated.

For MISO systems with more than two inputs, the removal of linear dependencies between inputs is done in steps. In the first step the linear effects of the first input is removed from all other inputs and the output. This results in conditioned spectra such as $S_{22.1}$, $S_{33.1}$, $S_{23.1}$, $S_{24.1}$, $S_{34.1}$, $S_{35.1}$ etc. etc. In the second step, the conditioned spectra are simply conditioned again. For example, $S_{34.12}$ is the cross-spectrum of the inputs 3 and 4, with the linear dependent parts with both input 1 and input 2 removed. Increasing the number of inputs in the conditioning process, results in a large number of variables. As stated, a user friendly plotting routine is needed in order to make full advantage of these variables. From these conditioned spectra, coherence and transfer-functions can be estimated. Returning to the 2ISO system, the coherence functions $\gamma^2_{u_1y}$ and $\gamma^2_{u_2y}$ represent the portion of the output that is a linear result of the conditioned input:

$$\begin{aligned}\gamma^2_{u_1y} &= \frac{|S_{u_1y}|^2}{S_{u_1u_1}S_{yy}} \quad \left(= \frac{|S_{1y}|^2}{S_{11}S_{yy}} = \gamma^2_{1y}\right) \\ \gamma^2_{u_2y} &= \frac{|S_{u_2y}|^2}{S_{u_2u_2}S_{yy}} \quad = \frac{|S_{2y.1}|^2}{S_{22.1}S_{yy}}\end{aligned} \tag{3}$$

The combined influence of inputs is given by the *multiple coherence functions:*

$$\begin{aligned}\gamma^2_{y:1} &= \gamma^2_{u_1y} \\ \gamma^2_{y:2} &= \gamma^2_{u_1y} + \gamma^2_{u_2y}\end{aligned} \tag{4}$$

Using these coherence functions, the output spectrum can be decomposed in spectra due to the inputs and the noise spectrum:

$$S_{yy} = \gamma^2_{u_1y}S_{yy} + \gamma^2_{u_2y}S_{yy} + S_{nn} = \gamma^2_{y:2}S_{yy} + S_{nn} \tag{5}$$

The multiple coherence functions give the *multiple coherent output spectra:*

$$\begin{aligned}S_{y:1} &= \gamma^2_{y:1}S_{yy} = \gamma^2_{u_1y}S_{yy} \\ S_{y:2} &= \gamma^2_{y:2}S_{yy} = \gamma^2_{u_1y}S_{yy} + \gamma^2_{u_2y}S_{yy}\end{aligned} \tag{6}$$

Interpretation of these results is heavily dependent upon the ordering of the two original input records x_1 and x_2. If x_1 is placed first, then any correlation between x_1 and x_2 is interpreted as being due to x_1, with $\gamma^2_{u_1y}$ yielding the maximum spectrum of y due to $u_1 = x_1$ and $\gamma^2_{u_2y}$ yielding the minimum spectrum of y due to $u_2 = x_{2.1}$. The converse occurs when x_2 is placed first: now one will obtain the maximum spectrum of y due to x_2 and the minimum spectrum of y due to $x_{1.2}$. Fortunately the ordering for helicopters is not a problem. Each output signal is meant to be controlled by one input. For example if the roll-rate is taken as the output, obviously the lateral stick must be taken as the first input and the longitudinal stick as the second input, if the roll-pitch coupling is to be modelled.

4.1 Multi-input/multi-output systems

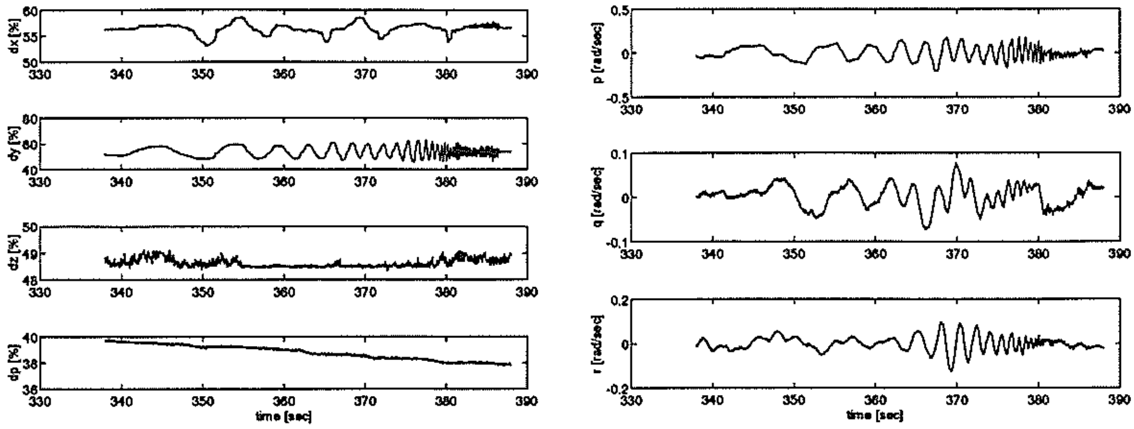
If all of the correlation between different outputs is due only to the fact that they originate from the same inputs, the MIMO system can be considered as a combination of separate and distinct MISO systems; one for each output. However, if some correlation between different output records remains, even after the effects of all input records have been removed, then the usual MISO problem must be extended to include the additional contributions due to these correlated output records.

For example, let us suppose a portion of the pitching of the helicopter during the manoeuvre is not explained with the input-signals alone. Due to the gyroscopic coupling between pitch and roll movements, the helicopter will start to roll. If the helicopter is considered as a MISO system, any rolling due to pitching which is not a linear result of the inputs, is considered as noise. By using the roll-rate (an output) as an additional input, these noise effects can be explained and taken into account (ref. 7-8).

If outputs are used as additional inputs, it is very important that the physical inputs are arranged first, prior to the outputs.

5 Analysis and Identification of a Lateral Stick Manoeuvre

A lateral stick frequency sweep will be used to illustrate the developed software. Frequency sweeps were designed especially for frequency domain identification. In figure 4, the time history records of the input signals and the resulting angular rates are given. This data is used by some of the members of WG-18 of AGARD (ref. 4) on Helicopter system identification. The pilot tried to excite only one input, but the longitudinal stick was excited as well. The conditioning discussed earlier, is used to eliminate the effects of the other inputs.



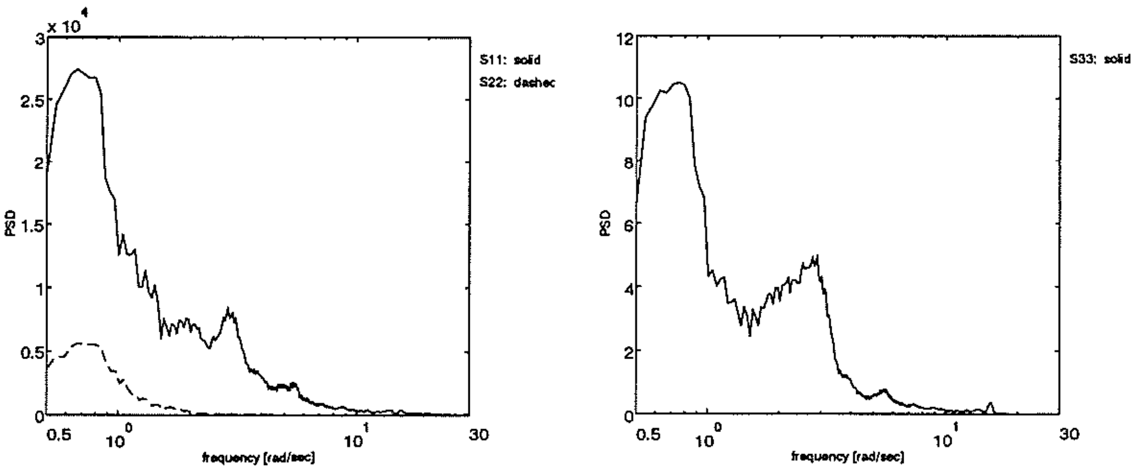
(a) Inputs $\delta_{lon}, \delta_{lat}, \delta_{ped}$ and δ_{col} (b) Outputs $p, q,$ and r

Figure 4: Time history records of a lateral sweep

5.1 Analysis of the roll-rate identification

The lateral stick was designed to make the helicopter roll, so first we will concentrate on the relation between the lateral stick and the roll-rate. In order to obtain a maximal resolution for the estimated PSD functions, three similar manoeuvres are combined. The combined length of these manoeuvres is 150 seconds. The variance of the estimated PSD functions is reduced by taking the moving average over 11 points. Thus, the effective resolution of the functions becomes $\frac{150}{11*2\pi} \approx 2.2$ per rad/sec. Using more smoothing points will improve the estimated coherence functions and the squared transfer-functions used in the conditioning process, but may lead to overestimation of identified damping-coefficients.

In figure 5 the auto spectra of these signals are given. The resolution of these estimates remains unchanged in the smoothing process because a *moving* average was used. Effectively, the resolution is 11 times smaller than the apparent resolution (11 smoothing points were used). The pilot was able to excite the lateral stick up to approximately 30 rad/sec. However, the input power for frequencies over 10 rad/sec deteriorates rapidly. The second input (the longitudinal stick) was mostly excited in the lower frequency range. The lack of input signal for higher frequencies or other unknown sources of input may contribute to unreliable estimates at these frequencies. For the conditioning, the longitudinal stick is used as the second input. This means, all of the linear dependent part between the input and output signals is assumed to be due to the lateral stick. The conditioning process produces ordinary conditioned coherence functions; multiple coherence functions; decomposed spectra (for the output and the inputs); (multiple) coherent output spectra *and* conditioned transfer-functions.



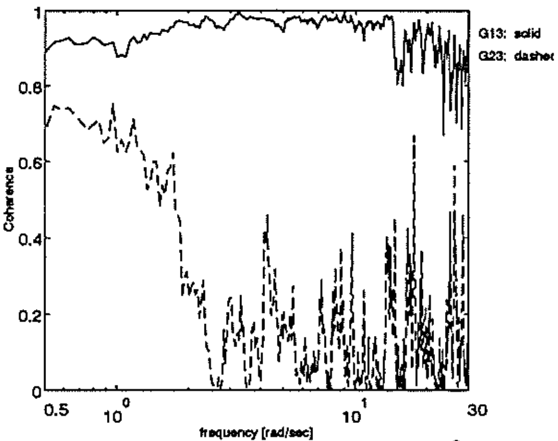
(a) Auto spectra of δ_{lat} and δ_{lon} (b) Auto spectrum of p

Figure 5: Auto spectra of the input and output signals

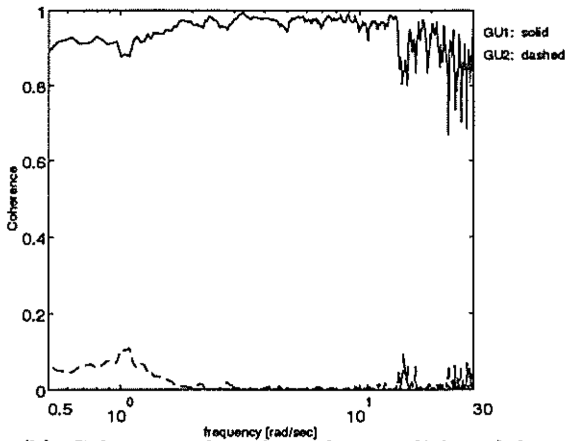
Here only a limited number of plots are presented, but the software is developed so that the analyst is able to produce different plots very easily. The figures presented in this paper, were made with this software package.

In figure 6a, the coherence functions between the (unconditioned) inputs and the output, γ_{1y}^2 (G13) and γ_{2y}^2 (G23), are given. Figure 6b shows that most of the coherence between the second input and the output is removed in the conditioning process. The same effect can be seen in figure 7. In figure 7a the roll-rate is decomposed into three parts: the part which is linearly dependent with the lateral stick; the part which is linearly dependent with the (conditioned) longitudinal stick and the remaining noise. Figure 7b shows that only a small portion of the longitudinal input remains after the conditioning process, as can be seen in the coherence plots.

Using the other inputs or even one or more of the other outputs, may not improve the multiple coherency very much in this particular case. Because of the problems of estimating the squared functions used in the conditioning process, the quality of the estimated transfer-function may even deteriorate if too many inputs are defined. Obviously, more plots are needed to visualize the influence of different degrees of smoothing and/or different inputs in the conditioning process. However, that would take us beyond the scope of this paper; the roll-rate analysis given here is only intended to demonstrate the (need for the) software.

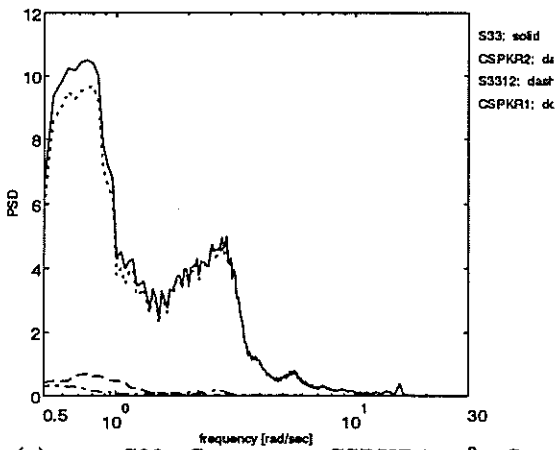


(a) Original coherence functions: $\gamma^2_{x_1y}$ and $\gamma^2_{x_2y}$

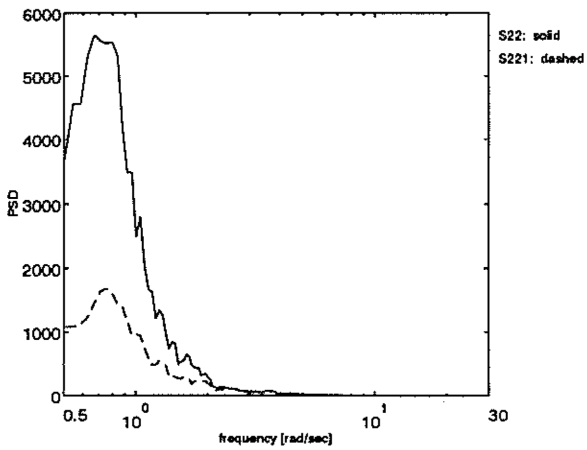


(b) Coherence functions for conditioned inputs: $\gamma^2_{u_1y}$ ($= \gamma^2_{x_1y}$) and $\gamma^2_{u_2y}$

Figure 6: Ordinary and conditioned coherence functions



(a) S33=S_{yy}, CSPKR1=$\gamma^2_{u_1y}S_{yy}$ CSPKR2=$\gamma^2_{u_2y}S_{yy}$ and S33.12=S_{nn}



(b) $S_{x_2x_2}$ and $S_{x_{2.1}x_{2.1}} = S_{u_2u_2}$

Figure 7: Decomposed roll-rate and conditioned longitudinal input

5.2 Identification of $H_{p\delta_{lat}}$

In order to demonstrate the procedure for finding a good structure for an individual transfer-function, the identification of $H_{p\delta_{lat}}(s)$ will now be discussed. As already shown in figure 2b, three second order systems are found in the estimate for $H_{p\delta_{lat}}$. The first is moderately damped with a cross-over frequency at approximately 2.5 rad/sec. This second order system, represents the Dutch-roll of the helicopter. At approximately 15 rad/sec, two second order systems are found. One moderately damped , the other very lightly damped. These systems represent the influences of the extra degrees of freedom of the rotor-blades. The first is referred to as the flapping-mode and the second as the lead-lag mode. The influence of the lead-lag mode is restricted to a small bandwidth, so neglecting this mode will not influence the dutch-roll characteristics seriously (see table 1). However, the flapping mode does influence at lower frequencies (around 5 [rad/sec]), as can be seen from the falling phase around that frequency. Neglecting this mode would affect the estimated values for the dutch-roll parameters (damping, crossover frequency) and the time delay. In table 1, the numerical values for three different ordered fits are given. Here again it becomes clear that neglecting the lead-lag results in a lower quality for the fit (a higher cost) and also affects primarily the flapping characteristics. If one desires to estimate the dutch roll characteristics using a model with only rigid body modes, it is possible to estimate them by restricting the frequency range and using a time delay to model the effects of the flapping mode.

Once the model structure is determined, small groups of linked transfer-functions (with common subsystems) can be fitted simultaneously.

Frequency band	Transfer-function	Cost-index
$0.5 < \omega < 30$	$\frac{1.74\,10^{-2}\,[0.49\ 2.97]\,e^{-0.018s}}{[0.29\ 2.61]\,[0.41\ 12.94]}$	0.23
$0.5 < \omega < 30$	$\frac{1.84\,10^{-2}\,[0.34\ 2.79]\,[0.07\ 16.56]\,e^{-0.025s}}{[0.21\ 2.60]\,[0.62\ 14.36]\,[0.04\ 15.81]}$	0.15
$0.5 < \omega < 15$	$\frac{1.81\,10^{-2}\,[0.28\ 2.64]\,e^{-0.13s}}{[0.18\ 2.54]}$	0.11

$[\zeta\ \omega_n]$ represents $\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1$
(τ) represents $\tau s + 1$
Smoothing points: 11, x_W: 1.5, squared errors
Manoeuvre: swp123, conditioning: 2iso; secondary input: δ_{lon}

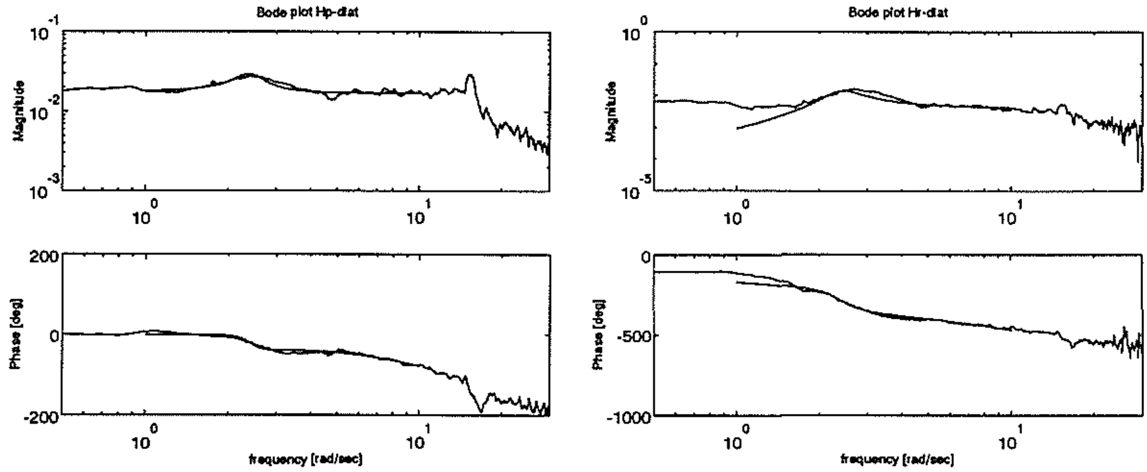
Table 1: Parametric roll-rate response models

5.3 Simultaneous fitting

For the pitch- and yaw-rate, a similar procedure can be followed, to find non-parametric estimates for $H_{q\delta_{lat}}$ and $H_{r\delta_{lat}}$. In all of these transfer-functions, the influence of a dutch roll can be identified. Fitting these transfer-functions simultaneously will result in more reliable estimates for the identified dutch-roll damping and cross-over frequency. To emphasise the transfer-functions with the highest quality, weighting functions can be used. Table 2 and figure 8 show some results of a simultaneous fitting procedure. Limiting the bandwidth makes it possible to use a small number of parameters, and leads to fits that are quite reasonable in the frequency range normally excited by the pilot. In table 3 some sample results of the dutch-roll characteristics computed from the software are compared with those presented by other institutes of NATO (ref. 1). A close comparison of the limited bandwidth identification results can be seen. In ref. 4, frequency domain and time domain methods are compared.

6 Conclusions

A user friendly software package was developed on a personal computer using Matlab 4.0 (for Windows), which makes it possible to exploit the advantages of frequency domain methods for helicopter system identification. This package includes inter-active features with extensive graphical facilities, which enables the analyst to select adequate model structures and to improve his insight into the behaviour of complex systems such as helicopters.



(a) $H_{p\delta_{lat}}$ (b) $H_{r\delta_{lat}}$

Figure 8: Non-parametric estimates and limited bandwidth fits for p and r

$p/q/r$	Transfer-function	Figure
$H_{p\delta_{lat}}$	$\frac{1.77\,10^{-2}\,[0.26\;2.44]\,e^{-0.13s}}{[0.15\;2.4]}$	8a
$H_{r\delta_{lat}}$	$\frac{4.43\,10^{-5}\,(-4.0)\,(-4.1)\,e^{-0.21s}}{[0.15\;2.4]}$	8b
Frequency range: 1 : 10 rad/sec Smoothing points: 11, Manoeuvre: swp123 Conditioning: 2iso; secondary input: δ_{lon} Relative error, weights p:r = 1:1		

Table 2: Identification of the Dutch-roll characteristics, results of a combined fit.

Institute	$[\zeta_{dr}\;\omega_{0_{dr}}]$
TUD	[0.15 2.40]
AFDD	[0.22 2.60]
CERT	[0.13 2.51]
DLR	[0.14 2.50]
Glasgow Uni	[0.16 2.27]
NAE	[0.13 2.58]
NLR	[0.17 2.17]
source: ref. 1, table 6.2.13	

Table 3: Comparison of identified dutch-roll characteristics

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